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About the Instability of Cooperative Communication Structures in Differential Network Games*

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Abstract In the paper, a cooperative differential network game is considered. We suppose that players simultaneously and independently choose the neighbor with whom they intend to interact during the game. Each player can choose neighbors from a fixed subset of players. Such subsets can be different for different players, and for each player, the number of its possible neighbors is limited. The players create the network to miximize the joint payoff. But network which is optimal at the initial time instant may cease to be so afterwords.
As solution the Shapley value is proposed. The results are illustrated on an example.

Keywords: cooperative communication structures, dynamic network game, the Shapley value.

1. Introduction

In the paper, a cooperative differential n-person network game is considered. In (Cao et al., 1963, Gao and Pankratova, 2017, Meza and Lopez-Barrientos, 2016, Petrosyan, 2010, Yeung and Petrosyan, 2018, Petrosyan and Yeung, 2020) different approaches for the definition of the characteristic function in differential network games are presented. In some of them, a special type of characteristic function is introduced, which not only simplifies the finding of a cooperative solution for differential network games, but also provides solutions with such an important property as time consistency (Petrosyan and Zaccour, 2003, Yeung, 2010). This paper is a continuation of (Tur and Petrosyan, 2020, Petrosyan et al., 2021, Petrosyan et al., 2024).

We assume that starting from t_0 at specified time instants players adjust (have the possibility to change) the network by making simultaneous neighbour selections at time instants t_k, $k = 1, \ldots, n$ (just as it happened at the beginning of the game), and then act cooperatively in accordance with the trajectory maximizing the total payoff in the network selected at the begining of the game (time instant t_0).

2. Differential Network Games

Consider a class of n-person differential games on the network with game horizon $[t_0, T]$.

– $N = \{1, 2, \cdots, n\}$ is the set of players (nodes) in the network.

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– Denote the set of all arcs in network N by P. $P = \{arc\ (i,j) : \ i,j \in N, \ i \neq j\}$.
– The set of players connected to player i is $K(i) = \{j : \ arc(i,j) \in P\}$, for $i \in N$.

Let $x^i(\tau) \in R^m$ be the state variable of player $i \in N$ at time τ, and $u^i(\tau) \in U^i \subset R^k$ control variable of player $i \in N$, U_i is a compact set.

At t_0 players simultaneously and independently choose neighbors with whom they intend to interact during the game. Player i can choose neighbors from a fixed subset of players $N_i \subset N \backslash \{i\}$, where N is the set of all players. The sets N_i can be different for different players, and for each player i, the number of its possible neighbors is limited by the number n_i. Communication is established (that is, the link is created in the network) between players i and j if $i \in N_j$, $j \in N_i$.

The state dynamics of the game is

$$\dot{x}^i(\tau) = f^i(x^i(\tau), u^i(\tau)), \ x^i(t_0) = x^i_0, \quad \text{for } \tau \in [t_0, T] \ \text{ and } i \in N. \tag{1}$$

Functions $f^i(x^i, u^i)$ are continuously differentiable in x^i and u^i and satisfy the conditions of existence, uniqueness and continuability of the solution on the interval (t_0, T) for all admissible piecewise continuous controls with a finite number of discontinuity points. For notational convenience, we use $x(t)$ to denote the vector $(x^1(t), x^2(t), \cdots, x^n(t))$.

We consider a special case, when the payoff of player i depends upon his state variable and the state variables of players from the set $K(i)$. Thus, if the connections remain valued, the payoff of player i is given as

$$H_i\left(x_1^0, \ldots, x_n^0, u^1, \ldots, u^n\right) = \sum_{j \in K(i)} \int_{t_0}^{T} h_i^j\left(x^i(\tau), x^j(\tau)\right) d\tau, \ i \in N \tag{2}$$

provided that the players do not interrupt communication. In case, the player i interrupts the communication with player j at some time instant t functions $h^j_i(x^i(\tau), x^j(\tau))$ and $h^i_j(x^j(\tau), x^i(\tau))$ will be set 0 for all $\tau \geq t$, $t \in [t_0, T]$. Function $h_i^j\left(x^i(\tau), x^j(\tau)\right)$ is the instantaneous gain that player i can obtain through network links with player $j \in K(i)$ (note that $(i,i) \notin P$), and non-negative for $j \in K(i)$.

3. Cooperative Differential Game

In what follows we consider the cooperative version of the game. In [1-3], it is assumed, that at any time instant, players can break connection between themselves and other players. Taking into account the non-negativity of players' payoffs, this assumption greatly simplified the construction of the characteristic function of the game and, as a result, the calculation of optimality principles from cooperative game theory based on it.

Define the value of the characteristic function for coalition N in the network G.

$$V_G\left(x_0, T - t_0; N\right) = \max_{u_i, i \in N} \sum_{i \in N} \sum_{j \in K(i)} \int_{t_0}^{T} h_i^j\left(x^i(\tau), x^j(\tau)\right) d\tau$$

$$= \sum_{i \in N} \sum_{j \in K(i)} \int_{t_0}^{T} h_i^j\left(\bar{x}^i(\tau), \bar{x}^j(\tau)\right) d\tau$$

where maximum is taken over the set of all admissible controls.

Denote by $V(x_0, T-t_0; N) = \max_G V_G(x_0, T-t_0; N) = V_{\bar{G}}(x_0, T-t_0; N)$.

Define values of the characteristic function for coalitions $S \subset N$ as

$$V(x_0, T-t_0; S) = \sum_{i \in S} \sum_{j \in S \cap K(i)} \int_{t_0}^{T} h_i^j \left(\bar{x}^i(\tau), \bar{x}^j(\tau)\right) d\tau, \ S \subset N.$$

Note that the value of $V_G(x_0, T-t_0; N)$ depends on the network that was formed at the initial time instant t_0 as a result of the simultaneous selection of neighbors by players. We assumed that players choose such a network $\bar{G}$, which gives the maximum total payoff of players from the set N, i.e. $V_{\bar{G}}(x_0, T-t_0; N)$. We will call such network $\bar{G}$ *a cooperative network or a cooperative interaction network*.

Note that during evolution of the game along the cooperative trajectory $(\bar{x}^i(\tau))$ there is the following non-trivial property, which, it seems to us, has not been noticed by anyone before, that at some intermediate time instant on the cooperative trajectory, the cooperative network may cease to be such, since the maximum total payoff of players in the subgame from the initial states on the cooperative trajectory can be achieved on another network. We shall show that on the example.

For convenience, introduce the following notation

$$\gamma_{ij}(t) = \gamma_{ij}(\bar{x}(t), T-t) = \int_t^T h_i^j(\bar{x}^i(\tau), \bar{x}^j(\tau)) d\tau, \tag{3}$$

Example 1. Consider a network of six players (A, B, C, D, E, F). Assume that all players can create no more than three connections.

Suppose that

$$\gamma(x_0, T-t_0) = \{\gamma_{ij}\}_{i,j=1,\dots 6} = \begin{pmatrix} 0 & 7 & 4 & 8 & 4 & 2 \\ 6 & 0 & 7 & 10 & 3 & 1 \\ 3 & 6 & 0 & 6 & 8 & 4 \\ 7 & 9 & 7 & 0 & 5 & 6 \\ 5 & 3 & 7 & 4 & 0 & 3 \\ 3 & 5 & 2 & 4 & 2 & 0 \end{pmatrix}$$

where γ is a matrix of payoffs that players can get during the time $T-t_0$ by cooperation.

Define the maximazing network (Fig.1) at time instant t_0, or in other words, define a network that maximizes the sum of players' payoffs at the initial time t_0, provided that players cannot create more than three connections. The maximum total payoff is 105.

Suppose that, in this case, along the cooperative trajectory, players' payoffs change as follows ($\gamma(x_0, T-t_0)$ is determined by formula (3)).

$$\gamma(x_0, T-t_0) = \gamma(x_0, t'-t_0) + \gamma(\bar{x}(t'), T-t') =$$

$$\begin{pmatrix} 0 & 7 & 4 & 8 & 4 & 2 \\ 6 & 0 & 7 & 10 & 3 & 1 \\ 3 & 6 & 0 & 6 & 8 & 4 \\ 7 & 9 & 7 & 0 & 5 & 6 \\ 5 & 3 & 7 & 4 & 0 & 3 \\ 3 & 5 & 2 & 4 & 2 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 3 & 1 & 8 & 2 & 0 \\ 2 & 0 & 5 & 0 & 0 & 1 \\ 0 & 4 & 0 & 6 & 7 & 2 \\ 7 & 8 & 0 & 0 & 5 & 6 \\ 3 & 0 & 6 & 4 & 0 & 1 \\ 3 & 2 & 1 & 4 & 1 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 4 & 3 & 0 & 2 & 2 \\ 4 & 0 & 2 & 10 & 3 & 0 \\ 3 & 2 & 0 & 0 & 1 & 2 \\ 0 & 1 & 7 & 0 & 0 & 0 \\ 2 & 3 & 1 & 0 & 0 & 2 \\ 1 & 3 & 1 & 0 & 1 & 0 \end{pmatrix}$$

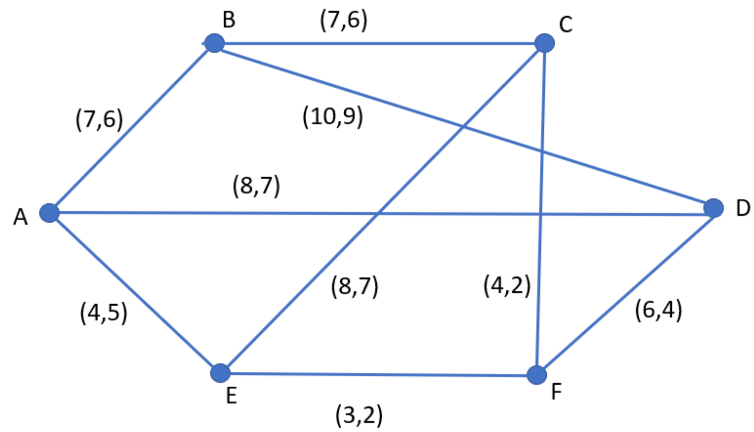

Fig. 1. The maximum network at time instant t_0 under condition that players cannot create more than three connections

Define a network at time instant t', or in other words, define a network that maximizes the sum of total payoff of players at time instant t', provided that players cannot create more than three connections. We see that at time instant t' on time interval $[t', T]$ the initial network has ceased to be optimal, i.e. there is another network that gives the maximum total payoff of players on remaining time interval $[t', T]$.

The maximazing network in the subgame, starting at t', has the following structure (Fig. 2).

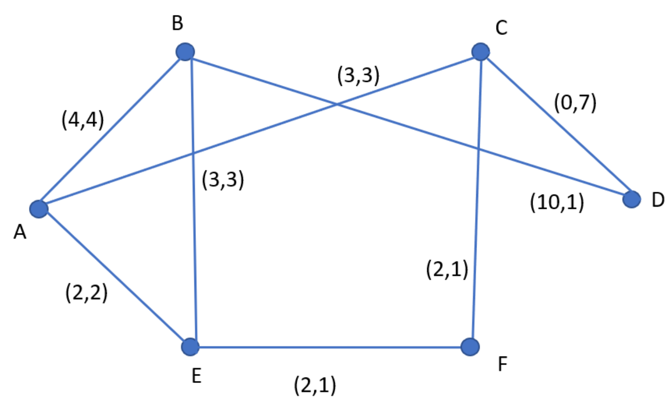

Fig. 2. The network at time instant t' provided that players cannot create more than three connections

The maximum total payoff of players in subgame defined in time interval $[t', T]$ is 48.

As we have already seen, even in simple cases, a permanent network (which does not change throughout the game) does not provide maximum total payoff to players, i.e. it cannot be considered as the basis of cooperation on the entire time interval $[t_0, T]$ of the game.

In this paper, we assume that at specified time instants $t_1, t_2, \ldots, t_r$, players adjust (have the possibility to change) the network by making simultaneous neighbour selections (just as it happened at the beginning of the game), and then on the subsequent time interval act cooperatively in the corresponding subgame in accordance with the trajectory maximizing the total payoff in the network $\bar{G}$ selected in time instant t_0 on the time interval $[t_k, T]$ until the moment of the next correction t_{k+1}.

Denote the trajectory adjusted at time instants $t_1, t_2, \ldots, t_r$ by $\bar{\bar{x}}(t)$. In this setting, we propose as cooperative solution the so-called *locally cooperative trajectory*. Namely, the time interval $[t_0, T]$ is divided into sub-intervals $[t_0, t_1)$, $[t_1, t_2)$, $\ldots$, $(t_{r-1}, T]$, and the following local cooperative behavior is proposed:

- at time instant t_0, the network that maximizes the total payoff on the interval $[t_0, T]$ is selected, and the game evaluates on the time interval $[t_0, t_1)$ along the corresponding maximizing (cooperative) trajectory without changing the network.
- at time instant t_1, the choice is made to find a network that maximizes the total payoff of players on the interval $(t_1, T]$ (this network may differ or coincide with the previous network) and move on the time interval $[t_1, t_2)$ along the previous maximizing (cooperative) trajectory or if the new network coincides with the previous one moves without changing the network.
- next, at the time instant t_k, the network is selected that maximizes the total payoff for the remaining period of time $(t_k, T]$, and so continues to act similarly until the time instant t_{r-1}.

As a result, we get the sequence of networks

$$G^{t_0}, G^{t_2}, \ldots, G^{t_{r-1}}$$

and the trajectory

$$\bar{\bar{x}}(t) = \bar{x}(t),\ t \in [t_k, t_{k+1}),\ k \in 0, 1, \ldots, r-1,$$

consisting of pieces of cooperative trajectory defined in the game on the time interval $[t_0, T]$. We shall denote this trajectory the *conditional cooperative trajectory.*

Introduce an analogue of the characteristic function on the time interval $[t_0, T]$ corresponding to the conditional cooperative trajectory $\bar{\bar{x}}(t)$.

$$\bar{V}\left(x_0, T - t_0; S\right) = \sum_{k=0}^{r-1} \sum_{i \in S} \sum_{j \in S \cap K_{G^{t_k}}(i)} \int_{t_k}^{t_{k+1}} h_i^j\left(\bar{\bar{x}}^i\left(\tau_k\right), \bar{\bar{x}}^j\left(\tau_k\right)\right) d\tau_k, \qquad (4)$$

for all $S \subset N$, and $K_{G^{t_k}}(i)$ is set of players connected to player i in the network G^{t_k}

Proposition 1. *The function $\bar{V}\left(x_0, T - t_0; S\right)$ defined by (4) is superadditive.*

4. The Shapley Value

Calculate the Shapley value for the introduced above analogue of the characteristic function (Shapley, 1953, Petrosyan and Yeung, 2020, Petrosyan and Zaccour, 2003)

$$\bar{Sh}_i(x_0, t_0) = \sum_{\substack{S \subset N, \\ S \ni i}} \frac{(|S|-1)!(n-|S|)!}{n!} \times \tag{5}$$

$$[\bar{V}(x_0, T-t_0, S) - \bar{V}(x_0, T-t_0, S \backslash \{i\})]. \; i \in N,$$

and we shall call vector $\bar{Sh}$ — *modified Shapley value.*

It can be shown that the modified Shapley value is time-consistent (dynamically stable) (Yeung, 2010).

Proposition 2. *Modified Shapley value determined by the formula (5) with the analogue of the characteristic function (4) is time consistent.*

Example 2. Consider an example of a network game $|N| = 6$ with players A, B, C, D, E. On the Fig. 1. we have the network $\bar{G}^{t_0}$ which maximizes the players' joint payoff on time interval $[t_0, T]$.

Find the Shapley value for the initial network which maximizes the joint payoff of players. Denote by $\gamma_{ij}(x_0, T-t_0) = \gamma_{ij}$, $i, j = A, B, C, D, E, F$.

$$Sh_A(x_0, t_0) = \frac{\gamma_{AB} + \gamma_{BA} + \gamma_{AE} + \gamma_{EA} + \gamma_{AD} + \gamma_{DA}}{2} = \frac{37}{2},$$

$$Sh_B(x_0, t_0) = \frac{\gamma_{AB} + \gamma_{BA} + \gamma_{CB} + \gamma_{BC} + \gamma_{BD} + \gamma_{DB}}{2} = \frac{45}{2},$$

$$Sh_C(x_0, t_0) = \frac{\gamma_{CB} + \gamma_{BC} + \gamma_{CE} + \gamma_{EC} + \gamma_{CF} + \gamma_{FC}}{2} = \frac{34}{2},$$

$$Sh_D(x_0, t_0) = \frac{\gamma_{DA} + \gamma_{AD} + \gamma_{BD} + \gamma_{DB} + \gamma_{DF} + \gamma_{FD}}{2} = 22,$$

$$Sh_E(x_0, t_0) = \frac{\gamma_{CE} + \gamma_{EC} + \gamma_{EA} + \gamma_{AE} + \gamma_{EF} + \gamma_{FE}}{2} = \frac{29}{2}$$

$$Sh_F(x_0, t_0) = \frac{\gamma_{FE} + \gamma_{EF} + \gamma_{FD} + \gamma_{DF} + \gamma_{FC} + \gamma_{CF}}{2} = \frac{21}{2}.$$

Compute the vector of payments to players on the interval $[t_0, t')$, if these payments are made in accordance with the Shapley value calculated for the initial network which maximizes the joint payoff on time interval $[t_0, T]$ of players without the correction of the network at the time instant t'. Denote by $\gamma_{ij}(x_0, t'-t_0) = \gamma_{ij}^0$,

$i, j = A, B, C, D, E, F$

$$Hs_A(x_0,t_0) = \frac{\gamma^0_{AB}+\gamma^0_{BA}+\gamma^0_{AE}+\gamma^0_{EA}+\gamma^0_{AD}+\gamma^0_{DA}}{2} = 9,$$

$$Hs_B(x_0,t_0) = \frac{\gamma^0_{AB}+\gamma^0_{BA}+\gamma^0_{CB}+\gamma^0_{BC}+\gamma^0_{BD}+\gamma^0_{DB}}{2} = 12.5,$$

$$Hs_C(x_0,t_0) = \frac{\gamma^0_{CB}+\gamma^0_{BC}+\gamma^0_{CE}+\gamma^0_{EC}+\gamma^0_{CF}+\gamma^0_{FC}}{2} = 8,$$

$$Hs_D(x_0,t_0) = \frac{\gamma^0_{DA}+\gamma^0_{AD}+\gamma^0_{BD}+\gamma^0_{DB}+\gamma^0_{DF}+\gamma^0_{FD}}{2} = 9,$$

$$Hs_E(x_0,t_0) = \frac{\gamma^0_{CE}+\gamma^0_{EC}+\gamma^0_{EA}+\gamma^0_{AE}+\gamma^0_{EF}+\gamma^0_{FE}}{2} = 6.5,$$

$$Hs_F(x_0,t_0) = \frac{\gamma^0_{FE}+\gamma^0_{EF}+\gamma^0_{FD}+\gamma^0_{DF}+\gamma^0_{FC}+\gamma^0_{CF}}{2} = 3.$$

On the figure (Fig.2) we present $\bar{G}^1$ which maximizes the players' joint payoff on time interval $[t', T]$.

Find the Shapley value for the network which maximizes the joint payoff of players on time interval $[t', T]$ (the network $\bar{G}^1$). Denote by $\gamma_{ij}(\bar{\bar{x}}(t'), T - t') = \gamma'_{ij}$, $i, j = A, B, C, D, E, F$

$$Sh_A(\bar{\bar{x}}(t'),t') = \frac{\gamma'_{AB}+\gamma'_{BA}+\gamma'_{AE}+\gamma'_{EA}+\gamma_{AD}+\gamma'_{DA}}{2} = 12.5,$$

$$Sh_B(\bar{\bar{x}}(t'),t') = \frac{\gamma'_{AB}+\gamma'_{BA}+\gamma'_{CB}+\gamma'_{BC}+\gamma'_{BD}+\gamma'_{DB}}{2} = 11,$$

$$Sh_C(\bar{\bar{x}}(t'),t') = \frac{\gamma'_{CB}+\gamma'_{BC}+\gamma'_{CE}+\gamma'_{EC}+\gamma'_{CF}+\gamma'_{FC}}{2} = 12.5,$$

$$Sh_D(\bar{\bar{x}}(t'),t') = \frac{\gamma'_{DA}+\gamma'_{AD}+\gamma'_{BD}+\gamma'_{DB}+\gamma'_{DF}+\gamma'_{FD}}{2} = 16.5,$$

$$Sh_E(\bar{\bar{x}}(t'),t') = \frac{\gamma'_{CE}+\gamma'_{EC}+\gamma'_{EA}+\gamma'_{AE}+\gamma'_{EF}+\gamma'_{FE}}{2} = 10,$$

$$Sh_F(\bar{\bar{x}}(t'),t') = \frac{\gamma'_{FE}+\gamma'_{EF}+\gamma'_{FD}+\gamma'_{DF}+\gamma'_{FC}+\gamma'_{CF}}{2} = 7.5$$

We see (Table 1) that after the correction of network at time instant t' the joint payoff of players increases.

– This justifies the correction of the network that we have introduced. As a result, the components of the Shapley value for four out of 6 players increase as well, i.e.

$$\bar{Sh}_i(x_0,t_0) = HS_i(x_0,t_0) + Sh_i(\bar{\bar{x}}(t'),t') > Sh_i(t_0,x_0), \;\; i = A, B, C, D, E$$

$$\bar{Sh}_F(x_0,t_0) = HS_F(x_0,t_0) + Sh_F(\bar{\bar{x}}(t'),t') < Sh_F(t_0,x_0).$$

Table 1. Comparison of the solutions

Players	$Sh_i(x_0, t_0)$	$\bar{Sh}_i(x_0, t_0)$
A	18.5	$9 + 12.5 = 21.5$
B	22.5	$12.5 + 11 = 23.5$
C	17	$8 + 12,5 = 20.5$
D	22	$9 + 16.5 = 25.5$
E	14.5	$6.5 + 10 = 16.5$
F	**18.5**	**3+7.5=10.5**
Joint Payoff	105	117.5

The example shows only a lower estimate for the possibility of increasing the cooperative joint payoff, since the increase occurs only due to the correction of the network structure, without changing the optimal cooperative control, using the same as calculated for a time interval $[t_0, T]$. Additional optimization of controls gives an even greater increase in the maximum joint payoff (cooperative payoff).

5. Conclusion

We considered a cooperative differential network game where players simultaneously and independently choose neighbors with whom they intend to interact during the game. The players create the network to miximize the joint payoff. We proved that network which is optimal at the initial time instant may cease to be so at some intermediate time instant.

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