Contributions to Game Theory and Management, XVII, 74–85

Game Theoretic Model of Venture Capital Investment*

Suriya Sh. Kumacheva

Faculty of Economy, St. Petersburg State University,
7/9, Universitetskaya nab., St. Petersburg, 199034, Russia
E-mail: s.kumacheva@spbu.ru

Abstract The study focuses on one case of modelling investment decision making when the decision maker has a minimum of information about the state of the environment. Venture capital investment is an example of such a case. The decision maker in this setting is an investor who makes a choice whether or not to invest in a venture capital project based on beliefs (opinions) about whether the project will be profitable in the future. Formalising the problem and proceeding to the mathematical model, we obtain a game with incomplete information, or Bayesian game. The players in this formulation are the Investor and the Project. In this case, we consider a set of states of nature (market) and a set of types that can be possessed by the player Project. Based on this, the profit functions of both players and the set of their strategies are formed. The Bayesian Nash Equilibrium is sought.

Keywords: investments, venture capital projects, game theoretic model, incomplete information, Bayesian game, Bayesian Nash Equilibrium (BNE).

1. Introduction

Nowadays, the importance of information cannot be overemphasized. Depending on the completeness of the information one can possess, she makes certain decisions. Whether they turn out to be effective or not is directly related to the extent to which the knowledge we relied on to make them corresponds to the reality. From this the models of description and forecasting of various processes, opinion dynamics, dissemination of information and many problems of decision-making theory, appear in applied sciences. In turn, game-theoretic models also take into account the completeness or incompleteness of information about the properties and moves of the opponent.

One of the most well-known classes of problems related to decision making based on available information is investment problems. The choice of an effective investment strategy is related to how much detailed and qualitative information an investor has about the market, its trends and dynamics, and the behavior of other market participants. Quantitative risk management, whose decisions are based on previous statistics, as well as qualitative analysis based, in particular, on the expert opinion of experienced analysts, helps to assess financial risks and build forecasting models competently.

However, in a situation when we are talking about a completely new project involving an area that has not been sufficiently researched, both statistical data

*The authors acknowledge Saint-Petersburg State University for research project 116814048.

https://doi.org/10.21638/11701/spbu31.2024.07

and qualitative information may be lacking. Such projects include various start-ups, IT-projects, innovative activities in various spheres, and investment in them is called venture capital. Venture capital is a form of private equity financing provided by firms or funds to startup, early-stage, and emerging companies, that have been deemed to have high growth potential. According to (Innovation overview, 2023), the most demanded sectors for this type of investment are information technologies (IT), including cyber security and artificial intelligence (AI), biotechnology, energy, transport, space technologies, completely new and emerging industries and segments of the economy. If rational economic agents make decisions to invest in such a project, it is unconditionally an investment in the future prospect, in the opportunity of receiving high income in the future, rather than in real guarantees. By investing in such companies, investors provide financing them. At the same time, taking into account the peculiarities of such companies, venture investments have a high degree of risk and, in fact, are long-term investments in high-risk securities.

One of the most important concepts of modern risk management is the concept of 'risk as a resource'. The emergence of the latter was a major step in the development of risk theory and, in particular, in the resolution of the classic for this theory 'Risk-return paradox', studied both in classical works (Markowitz, 1952) and in modern studies related to financial risk management (Damodaran, 2012). This concept, first formulated in 1998 by M.A. Greenfield (Greenfield, 1998), is based on the idea that risk management should be in many respects similar to resource management. If costs are understood as the costs of risk management, and benefits are understood as the losses prevented, then the task is reduced to cost optimisation by comparing marginal costs and benefits.

It is accepted to distinguish the following main features of resource-like risk. Firstly, the risk must have positive factors. This means that an increase in the level of risk has the potential to lead to additional benefits. Otherwise, it would not be a resource for the decision maker. Secondly, it is certainly positive that the resource-like risk can be avoided. However, along with the first and the second, it is also worth taking into account the third feature of resource-like risk: increasing the level of risk can be effective only up to a certain limit. Based on the analysis of the listed features we can conclude that the management of resource-like risk should consist in finding a certain level of saturation and maintaining its optimal values.

Thus, when talking about venture capital investment, we are faced with an example of resource-like risk. Venture capital investments are distinguished from classical ones by a higher risk, determined by a much lower probability of return, but often by a much higher return (Russian Venture Capital Investment Association, 2024).

When taking such a high risk, investors need to from an understanding of the type of company/project they are facing. This type is determined not so much by the characteristics of the project (or company) itself, but by external market conditions.

Venture investors can be individuals (retail investors, personally or through representatives) or organizations of various levels – from state corporations or state investment funds to private investment funds or companies. Each of them determines the degree of their involvement in the project, the amount of capital allocated to finance it, the stage at which this financing is being carried out, and the point at which it should be stopped or suspended.

Thus, we have a model in which there are two players – the Investor and the Project. Additionally, we should take into account the influence of external market which plays the role of environment. Formalising this model, we obtain a decision-making problem with incomplete information, which is mathematically closest to Bayesian games, which were described and studied first in (Harsanyi, 1967/68).

In this study it is assumed to consider the investor's game with one project, but in future it can be extended to the case of n projects. Player Investor decides whether or not to invest a certain amount of money in a particular venture project, based on beliefs (opinions) about the type of player Project. This also considers a set of states of nature (in our case the market). Based on this, the profit functions of both players and the set of their strategies are formed. In the following, the problem is solved in the classical formulation of the Bayesian game (Aumann and Hart, 1992, Zamir, 2008): considering a set of states of nature, possible scenarios are analyzed and their posterior probabilities are estimated, based on which players make decisions and choose their strategies. Each player seeks to maximize their payoff based on their beliefs about the type of opponent.

The paper has the following structure. Section 1 justifies the motivation for modelling the venture capital investment problem using a game-theoretic approach. Section 2 describes the problem statement by formalising the model under study. Section 3 analyses this model, formulates the main results and ways to further improve the model. Section 4 discusses the conclusions of the study and its future prospects.

2. Model Formulation

To study the concept of resource-like risk (Greenfield, 1998) using venture capital investments as an example, we will consider a model with two counterparties: an investor and a project. Without detailing their nature and properties, it should be taken into account that their decisions and interactions are influenced by external market conditions (Sharpe and Alexander and Bailey). Thus, we can say that in our model the external market plays the role of the environment. Thus, we face the convergence of two approaches: game theory and decision theory. This convergence leads to the study of the game with incomplete information, or Bayesian game (Harsanyi, 1967/68, Aumann and Hart, 1992, Aumann and Hart, 1994, Zamir, 2008).

To formulate the model, we first construct a simple basic game of investment in normal form (Pechersky and Belyaeva, 2001, Vasin and Morozov, 2005). Then we extend this basic model to the game with incomplete information (Harsanyi, 1967/68, Zamir, 2008) and obtain it in extensive form (Gibbons, 1992).

2.1. Example of Investment Game

First, we consider non-antagonistic game (Petrosyan and Zenkevich and Shevkoplyas, 2012), in which players are the Investor and the Project. Formally, the Investor's behavior can be represented by the strategies A (to invest) and $\overline{A}$ (not to invest). The Project's strategies are B (to be profitable) and $\overline{B}$ (to be unprofitable).

How does an investor's outcomes form? It consists of two components, the value I of investment (the amount the investor puts into the project) and the return R from the project realisation. In the example under consideration, we will assume that the amount of investment is a conditional 1. The Project's earned benefit can take the value 0 or 2. It is worth considering that the project can only be launched when it has received initial investor approval and initial capital.

Suppose the Investor reasonably decided to invest in the project and the Project is implemented. This situation corresponds to the strategy profile (A, B), which gives the players payoffs $(3, 1)$: the Project receives funding 1 and generates profit 2, the Investor spends $1(-1)$ and earns 2. In the case of investing in an unrecoverable project (profile $(A, \overline{B})$), the Project (since it is not implemented) wins -1 and the Investor loses 3 (wins -1). In the opposite case (profile $(\overline{A}, B)$) the Project wins 0 as it failed to raise funds and was not implemented ($I = 0$, $R = 0$). At the same time the Investor saves her money (does not invest 1), but does not earn the profit that the Project could generate. Her payoff is similar to the regret $I - R$ in the case of the profitable Project. Thus, in this case, for given values of I and R, the players have payoffs $(0; -1)$. When both players do nothing (profile $(\overline{A}, \overline{B})$), both players get 0.

Thus, the extensive form of this game is represented in the figure 1.

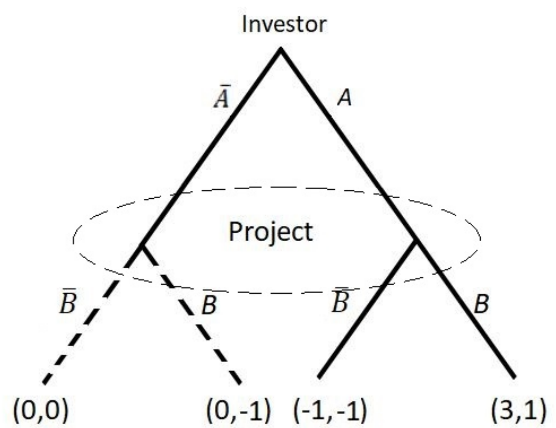

Fig. 1. Example of investment games in extensive form

and has the following profit matrix:

	A	$\overline{A}$
B	$(3, 1)$	$(0, -1)$
$\overline{B}$	$(-1, -1)$	$(0, 0)$

Analysing strategy profiles, it is obtained that in this game there exist two Nash equilibriums: profiles (A, B) and $(\overline{A}, \overline{B})$.

Considering this example we should notice that the main issue that defines, whether the game starts or not, is does Investor choose A or $\overline{A}$. It depends on various factors we need to study. In order to bring this game model closer to the reality we are trying to describe with it, we must realise that this situation actually involves decision making under conditions of uncertainty. Let us now build a model with this fact in mind.

First of all, let's restrict ourselves by considering the Investor's game with a single venture Project (further this assumption can be extended to the case of n projects).

2.2. Basic Game

As it was considered in subsection 2.1, devoted to the example of investment game, players are Investor and Project and their strategies are generally unchanged from the given example. To generalize it, we need to analyse players' payoff functions, taking into account that both's aims are to maximise their payoffs. To do this let's make the following denotations: I is initial investment amount; R is the Project's return (as it was denoted earlier); β is Investor's share in the project, $\beta \in (0, 1)$.

We continue to generalize the game with complete information which particular case was considered earlier in 2.1. Its extensive form is represented in the figure **??**. In practice, if the Investor chooses $\overline{A}$, the game is finished. To say it more correctly, it is not started: Project is not supported by financing, thus, it is not loaded. So, we should consider only the branch when the Investor chooses the strategy A and the Project can response by B or $\overline{B}$. But hypothetically the game could have continued

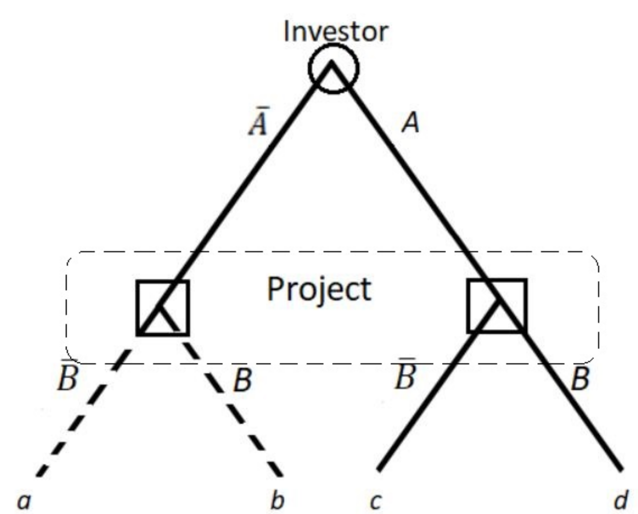

Fig. 2. Investment game in extensive form

if the Investor had even chosen a strategy $\overline{A}$ (the figure shows these branches in dotted lines). If we develop this idea, it is also necessary to take into account the Investor's lost profit in case the project would have been successfully realised with its financing.

When we considered the previous example, we assumed that the strategies of the player Project is the set $\{$*be profitable*, *be unprofitable*$\}$ and assumed that the choice of a strategy is a rational action of the player. At the same time, in economic reality, the profitability of a project depends on a number of external factors, not only on the internal choices of the player herself. At this stage it is necessary to differentiate the strategy of the player Project (it can be interpreted as reasonable actions of the project management) and whether the Project can be profitable or not under given external conditions. We will refer the former to the strategies of the player Project – *to be implemented* (B) or *not to be implemented* ($\overline{B}$).

If the Investor decided to invest (chooses the strategy A), then, receiving the funding, the Project is implemented (strategy B) and generates the profit. If, oppositely, it is not implemented, the Investor only looses her money I. But if the Investor chooses $\overline{A}$, she looses unearned income (but saves the investment I), if the Project is implemented (similar to the concept of regret in the sense of the Savage criterion (Friedman and Savage, 1948)), and wins 0 when the second player chooses $\overline{B}$. Such representation of the game corresponds to the following profit matrix:

	A	$\overline{A}$
B	$(I+(1-\beta)R, -I+\beta R)$	$(0, I-\beta R)$
$\overline{B}$	$(I, -I)$	$(0, 0)$

In this case we have only considered that the project is implemented (generates profit R) or not (generates profit 0) and have not considered the degree of payback. Returning to the conversation about the profitability and unprofitability of the project, it is worth noting that the Investor does not know exactly how much his investment will pay off in the future. Moreover, since we consider the venture capital investment game, we assume that the absence of forward-looking statistics exacerbates this uncertainty. Hence, the uncertainty factor arises, which leads us to the need to correct the problem statement.

In reality, profitability of the Project depends on the **state of nature**. In the framework of the considered model (Hu, Loo, 2014) the set of states of nature (scenarios) can be defined by the following factors: market environment, competitiveness, external economic conditions and many others. According to the classical form of the decision-making problem, the Chance (Nature) defines a set of types, which the player Project can possess. Taking into account such conditions, we face a **game with incomplete information** (Zamir, 2008), or Bayesian game (Harsanyi, 1967/68) in which we consider a set of states of nature and two rational opponents.

2.3. Game with Incomplete Information

In classical literature devoted to Bayesian games (Harsanyi, 1967/68, Aumann and Hart, 1992, Aumann and Hart, 1994 and Zamir, 2008), a game with incomplete information in normal form is usually defined as the set

$$\langle N, S, T, p, u \rangle \tag{1}$$

which consists of the following elements:

1. N is a set of players;

2. S is a set of strategies: $S = s_1 \times \ldots \times s_N$, where s_i is the set of possible strategies for player i;
3. T is a type set: $T = t_1 \times \ldots \times t_N$, where t_i is the (finite) set of possible types for player i;
4. Prior p: a joint probability distribution $p(t_1, \ldots, t_N)$ over all possible type profiles: the probability that Player i has type t_i, where $i = \overline{1, N}$;
5. Payoff functions $u_i : S \times T \to R$ assign a payoff to a player given her type and the strategy.

We continue to study the basic game considered in subsection 2.2, in which players are the Investor and the Project and the sets of their strategies are the same. But, according to the classical formulation of Bayesian game (Harsanyi, 1967/68, Gibbons, 1992), the players can possess various types.

For example, the behavior of the Investor depends on the stage of project: early stage (pre-seed and seed rounds), middle stage (start-up) and late stage (ramp-up). Various stages define the availability of information about the Project, which influences on the Investor's decision. Moreover, Investor can be an economic agent with different risk propensities: risk averse, risk neutral or risk loving. In the current study we will restrict and simplify the model by considering only risk neutral (rational) type of the Investor with strategies A (*to invest*) and $\overline{A}$ (*not to invest*).

The type of the layer Project is determined by whether a player is *profitable* (t_1) and *unprofitable* (t_2). The type of the player Project influences on the value of return R.

Remark 1. Generally speaking, the return of the Project can be represented by its distribution on the segment $[\underline{R}, \overline{R}]$ with probability density function $f_R(r)$. But for further study of the Bayesian game, a number of simplifying assumptions will be made.

First, we will abandon the idea of a continuous distribution of profits in favor of a discrete one, i.e., we will assume that R takes only values from the set $\{\underline{R}, \overline{R}\}$.

Second, by saying that the Project has only two types t_1 and t_2, we will mean that in the first case, the return can be $\overline{R} = R$ if the Project is implemented and 0 if it is not implemented; in the second case, when the Project has type t_2, it can generate only losses $\underline{R} = -R$ for the Project itself if it is implemented and generate nothing (0) if it is not implemented. In both cases, the Investor does not get profit from the Project.

Third, a player Investor makes a decision based on her beliefs (opinions) about the type of player Project (Zamir, 2008), rather than on precise knowledge, which she does not have a priori: will the Project be profitable or not talking about future prospect? It is assumed that p is a belief of the Investor that the Project is profitable and, correspondingly, $1-p$ relates to a belief that it is unprofitable (general a priori probability distribution $(p; 1-p)$).

2.4. Extensive Form of the Game

If we take into account the incompleteness of information, we obtain a new form of the game. Together with the strategies of the players Investor and Project, we also need to consider the move by nature that determines the prior distribution of the players' types. In our model, this move will determine the prior distribution of the Project's type. We will assume that this distribution corresponds to the Investor's beliefs about the type of the second player. The game tree (Petrosyan and Zenkevich and Shevkoplyas, 2012, Vasin and Morozov, 2005), that represents the game in extensive form considering the move of nature (Gibbons, 1992), is shown in figure 3.

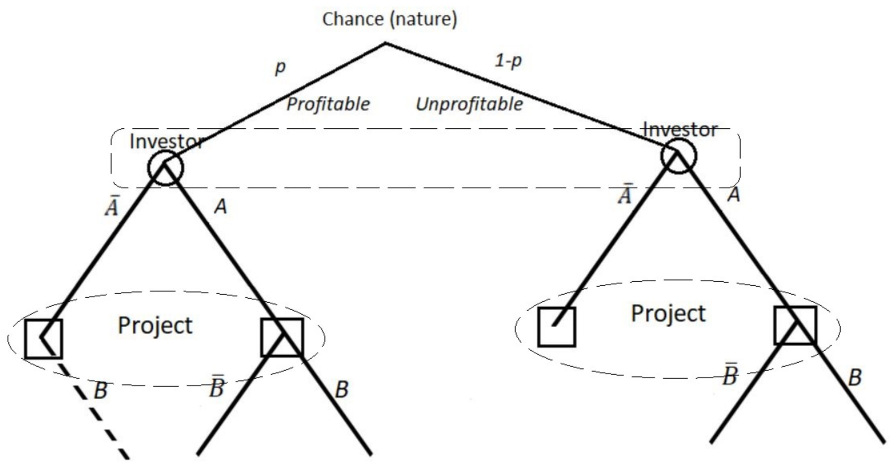

Fig. 3. The tree of the game with incomplete information

To analyze this bimatrix game, we need to study two types of Project, and, correspondingly, the Investor's beliefs.

First, let's consider the case when the Project is profitable (has type t_1). It corresponds to the positive scenario which realises with probability p, represented by the left branch of the game tree in figure 3, and the following profit matrix (similar to the matrix of the game with complete information from subsection 2.2):

	A	$\overline{A}$
B	$(I+(1-\beta)R, -I+\beta R)$	$(0, I-\beta R)$
$\overline{B}$	$(I, -I)$	$(0,0)$

If we analyze this game where according to the structure of Bayesian game (Pechersky and Belyaeva, 2001, Gibbons, 1992), we obtain that a dominant Project's strategy is B.

The second case (right branch of the decision tree) is the negative scenario when nature makes the move with probability $1-p$, defining the Project as unprofitable (has the type t_2). In this case the correct decision for the Investor is to stop the game, choosing $\overline{A}$. If the Investor mistakenly chooses strategy A, the implementation of any strategy by the player Project only causes losses for the Investor.

	A	$\overline{A}$
B	$(I-(1-\beta)R, -I)$	$(0, I)$
$\overline{B}$	$(I, -I)$	$(0,0)$

The analysis of the game shows that a dominant Project's strategy is $(\overline{B})$.

The final representation of the game in extensive form can be seen in figure 4.

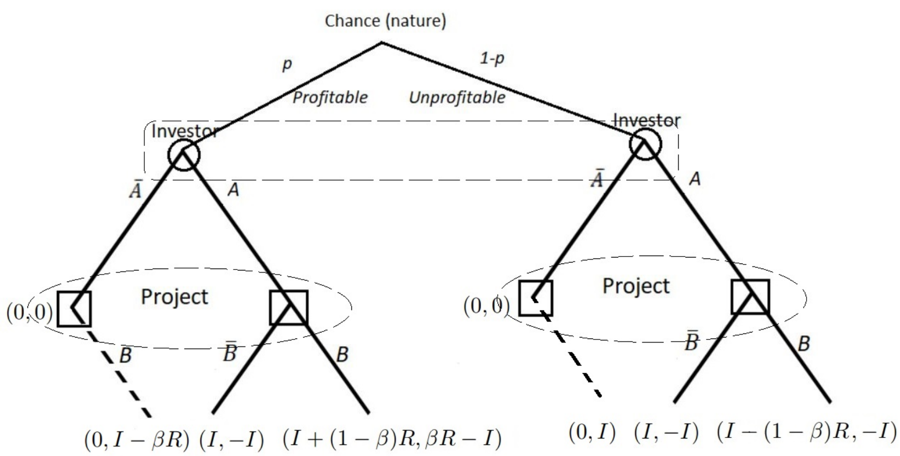

Fig. 4. The game with incomplete information in extensive form

3. The Results

To formulate the results obtained for the studied game we need to search for the Bayesian Nash equilibrium (BNE) (Aumann and Hart, 1992, Aumann and Hart, 1994, Zamir, 2008). First of all, we need to analyse how the Investor makes her decision when the Project chooses dominant strategies with various types.

3.1. The Analysis of Venture Capital Investments Game

Previously in subsection 2.4 the dominant strategies of the player Project were found for both types of the player; alternative strictly dominated strategy can thus be removed. Given this, if the Investor chooses A, her profit is

$$u_I(A) = p(\beta R - I) + (1-p)(-I) = p\beta R - I.$$

If she chooses $\overline{A}$, then

$$u_I(\overline{A}) = p(I - \beta R) + (1-p)(0) = pI - p\beta R.$$

Since in the current study we restricted ourselves to considering only the risk-neutral (rational) Investor, she will prefer a strategy *to invest* when $u_I(A) \geq u_I(\overline{A})$. Thus, the condition of decision to invest:

$$p \geq \frac{I}{2\beta R - I}. \tag{2}$$

Taking into account the mathematical meaning of probability (2), we obtain that the following double inequality must be fulfilled:

$$0 \leq \frac{I}{2\beta R - I} \leq 1.$$

Therefore, we get that the investment amount and the project return should satisfy: $I \leq \beta R$. The last inequality corresponds to the requirement of profitability of investments.

But what is the relation between R and I? To answer this question we can assume that a rational economic agent invests in funds expecting to receive a certain amount of profit from the project in the future. In other words, the initial amount of investment I can be found as an Investor's discounted profit in the current investment period:

$$I = \beta R e^{-r\tau}, \tag{3}$$

where τ is an investment period, r is an interest rate. Thus, according to the (3), we obtain the following discount factor restrictions:

$$e^{-r\tau} \leq 1.$$

Last inequality is equivalent to condition $r\tau \geq 0$, which can be achieved at any non-negative interest rate r.

Summarising all the preceding considerations and the analysis of the game, we can formulate the proposition which consists of the following statements.

Proposition 1. *In the considered game (1) of Venture Capital Investments with incomplete information*

1. *Bayesian Nash Equilibrium (BNE) is achieved for the profiles (A, B) for Project type t_1 with probability p and $(\overline{A}, \overline{B})$ for Project type t_2 with probability $1 - p$;*
2. *The investments start when prior probability p satisfies (2);*
3. *No additional restrictions on the investment period τ and interest rate r need to be imposed, other than the requirement of profitability of investments ($r \geq 0$).*

3.2. Further Elaboration of the Model

Taking into account the specifics of venture capital investment, it should be understood that the considered game is only a stage of modelling a large complex dynamic process. Moreover, within the framework of the Bayesian approach, the played game should be perceived not only as a completed stage of interaction between the Investor and the Project, but also as a way of obtaining new information about the opponent and, consequently, updating one's beliefs (Zamir, 2008) and estimates of the model parameters.

What parameters and values can be re-evaluated after the described game is played?

First of all, it becomes clear that real payback period T can be longer or shorter than initially assumed τ. Here, the life cycles and payback period of a particular project must be taken into account (Damodaran, 2012). At the same time, at the end of the first stage, the real value of the real value of the Project return $\widehat{R}$ instead of expected R is clarified. The Investor also receives a new information about profitability of the given project and makes correction of her beliefs, thus, the value of a posteriori probability $\widehat{p}$ should also change. Taking into account this new knowledge she also can correct the interest rate r and, therefore, discount factor β.

4. Conclusions and Prospects

The current study is focused on the investigation the concept of resource-like risk. Venture capital investment is considered as an example of the implementation of such concept.

The modeling of the problem related to the venture capital investments was conducted in the form of a Bayesian game (or the game with incomplete information). The game was studied and the following results were obtained. First of all, the Bayesian Nash equilibrium is found given the prior probability distribution of the state of nature. Second, the constraints on the model parameters (investor's share in the project and interest rate) were found. These results are formulated as a Proposition.

Speaking about the future prospects of this model, it is worth considering a multistage game based on the studied static game (investing in the project at different stages). Speaking of such development of the model, it is necessary to take into account the ways of refining the key parameters for the next funding periods in the multi stage game, which were identified earlier in subsection 3.2. Among such methods, the first to be mentioned is the derivation of posterior probabilities of various states of nature, to reassess the Investor's beliefs. Also, using the information gathered during the first stage of the game, it becomes possible to estimate the payback period of the project. In addition, it is possible to use the information collected about the real profit of the project to adjust the discount factor (interest rate).

Also, the restrictive assumption of risk-neutrality of the player Investor should be dropped, and different types of investors depending on their risk appetite and stage of the project should be considered. For example, only risk loving agents are inclined to invest at an early stage of the project. Investments at a middle stage are typical for risk neutral players. Risk averse investors will wait for the late stage.

References

Aumann, R., Hart, S. (1992). *Handbook of Game Theory with Economic Applications*, **vol.1**, pp. 1–733.

Aumann, R., Hart, S. (1994). *Handbook of Game Theory with Economic Applications*, **vol.2**, pp. 735–1520.

Bank, European Investment. Innovation overview, 2023. *https://www.eib.org/en/publications/innovation-overview-2023.*

Damodaran, A. (2012). *Investment Valuation: Tools and Techniques for Determining the Value of Any Asset.* 3rd Edition, Wiley, Hoboken.

Friedman, M., Savage, L. J. (1948). *Utility Analysis of Choices Involving Risk.* Journal of Political Economy, **56(4)**, 279–304. https://doi.org/10.1086/256692

Gibbons, R. (1992). *Game Theory for Applied Economists.* Princeton University Press, 144–152.

Greenfield, M. F. (1998). *Risk management 'Risk as a resourse'. URL: http://www.hq.nasa.gov/office/codeq/risk/risk.pdf.*

Harsanyi, J. C. (1967/68). *Games with Incomplete Information Played by Bayesian Players, I-III.* Management Science, **14(3)**, 159–183 (Part I), **14(5)**, 320–334 (Part II), **14(7)**, 486–502 (Part III).

Hu, Yu., Loo, C. K. (2014). *A Generalized Quantum-Inspired Decision Making Model for Intelligent Agent.* The Scientific World Journal. Article ID 240983, 8 p. https://doi.org/10.1155/2014/240983

Markowitz, H. (1952). *Portfolio Selection*. The Journal of Finance, **7**, 77–91.

Official website of the Russian Venture Capital Investment Association (RVIA). *http://www.rvca.ru/rus/*.

Pechersky, S. L., Belyaeva, A. A. (2001). *Game Theory for Economists*. St.Petersburg: Europian University (in Russian).

Petrosyan, L., Zenkevich, N., Shevkoplyas, E. (2012). *Game theory*. St. Petersburg: BHV-Peterburg, 432 p. (in Russian).

Sharpe, W. F., Alexander, G. J., Bailey, J. V. (1999). *Investments*. Prentice Hall, 962 p.

Vasin, A., Morozov, V. (2005). *The Game Theory and Models of Mathematical Economics*. Moscow: MAKSpress (in Russian).

Zamir, S. (2008) *Bayesian Games: Games with Incomplete Information*. Encyclopedia of Complexity and Systems Science. p. 426–454. Encyclopedia of Complexity and Systems Science. https://doi.org/10.1007/978-0-387-30440-329