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E-Games: A Very Short Introduction

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Abstract The Dirichlet's Unit Theorem describes the structure of the group of units as follows: let K be an algebraic number field with r_1 real and $2r_2$ complex embeddings and ring of integers O_K . Then the group of units of O_K is equal to the direct product of the finite cyclic group E(K) of roots of unity contained in K and a free abelian group of rank $r := r_1 + r_2 - 1$. Dirichlet's E-symbol ("container" of the numbers of a special type) became the object of Nikolai Bugaev's mathematical dissertation.Bugaev and his followers of Moscow's Philosophical Mathematical School have attempted to develop a sort of E-games which we define today as noncooperative signaling games in post-Quantum perspective. Our very short introduction in E-games describes historical circumstances and the reasons for introduction of E-games in post-quantum game theory. Fundamental Riemann problem and ABC conjecture in Number theory are considered also as an examples of E-game, hence, game-theoretical approach in Number theory is firstly justified.

Keywords: acausality, Schopenhauer, Moscow's Philosophico-Mathematical School, Bugaev, penny-flip game, 2+1 players game, quantum leadership, E-game, signaling game, Riemann problem, ABC conjecture.

1. Introduction

The Dirichlet's Unit Theorem describes the structure of the group of units as follows. Let K be an algebraic number field with r_1 real and $2r_2$ complex non-real embeddings and ring of integers O_K. Then the group of units of O_K is equal to the direct product of the finite cyclic group E(K) of roots of unity contained in K and a free abelian group of rank $r := r_1 + r_2 - 1$. In his publication of 1856 Dirichlet showed that E - symbol ("container") can have a mathematical theory applied in general mathematics. Nikolai Bugaev (1837–1912) a legendary founder of Moscow's Idealistic Philosophico-Mathematical School (P.A. Nekrasov 1853–1924, V.Ia. Zinger 1836–1907, L.M. Lopatin 1855–1920, V.G. Alekseev 1866–1943, D.F. Egorov 1869–1931, P.A. Florensky 1882–1937, independent St Petersburg's N.M. Gunter 1871–1941) and distinguished number - theorist made the next step towards post-Dirichlet's E-symbol theory. He created a new kind of E(x) Calculus and E-game theory, based on a very original idealistic philosophy of Leibniz's monads (Bugaev 1866, 1905, 1879). Similar with Arthur Schopenhauer - who had a decisive influence in the middle of the 19th century on German mathematics, Nikolai Bugaev (being PhD student of Dirichlet) proclaimed that physical causality was only one of the rulers of the world; the other was a metaphysical acausality investigated by mathematicians.

As earlier Schopenhauer wrote "coincidence is the simultaneous occurrence of causally unconnected events. If we visualize each causal chain progressing in time as

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a meridian on the globe, then we may represent simultaneous events by the parallel circles of latitude... All the events in a man's life would accordingly stand in two fundamentally different kinds of connection: in the objective, causal connection of the natural process; secondly, in the subjective connection which exists only in relation to the individual who experiences it, and and which is thus as subjective as his own dreams, whose unfolding content is necessarily determined, but in manner in which the scenes in a play are determined by the poet's plot. That both connection exist simultaneously, and the self-same event, although a link in two different chains, nevertheless falls into place in both, so that the fate of one individual invariably fits the fate of the other, and each is the hero of his own drama while simultaneously figuring in a drama foreign to him — this is something that surprises our powers of comprehension, and can only be conceived as possible by virtue of the most wonderful pre-established harmony... (Kostler 1972, 107–108).

In the given passage Schopenhauer is describing main ideas of Bugaev-Necrasov's acausal or E-games conjecture (Nekrasov 1902, 1904, 1916), which was a subject of my presentation for GTM 2024 International Conference at St. Petersburg State University.

2. General Remark

I'd like to begin our discussion of E-game theory by reviewing my old result of this field which initiated anthropological studies in quantum games. In 2009 I proposed that one can construct 2-players games with quantum-like systems in which if one (collective) player is restricted to classical strategies, while the other player has access to quantum (non-classical) strategy, the quantum-like player will win (Popov, 2009). What is meant for E-game theory is described below.

3. The Classical Penny-Flip Game

If for Arthur Schopenhauer and Moscow's idealists-mathematicians coincidence is the simultaneous occurrence of causally unconnected events (or, there is an acausality), a famous Nash equilibrium occurs when all the players are simultaneously making a best reply to the strategy choices of the others. In other words, a Nash equilibrium is just a pair of strategies whose use results in a cell which both payoffs are causally circled. Let a 2-players classical penny-flip gam consisting of players A and B, and a coin. The coin starts out in a state Heads. Player A is asked to select between one of two strategies — Flip (F)) or non-Flip (N).In other words A can choose to either Flip the coin or to leave it. Correspondingly, player B is asked to choose between F or N without revealing the coin to him. Next phase of the game: player A is asked again to choose between one of the 2 moves. The coin is revealed to both players at the end of 3 moves. Following game rules, each player can only perform operations on the coin but cannot know a real state during the game. In the case when at the end of the 3 moves the state of the coin is Heads, we define that player A wins, alternatively, if it is Tails, player B wins. The payoff function π_B of player B in a classical penny - flip game can be presented in the following form (Table 1):

For any play, ρ, playoff of player A is $\pi_A(\rho) = -\pi_B(\rho)$. In other words, this is a zero sum game. It is easy to see hence, that none of the players have a dominant strategy and, correspondingly, there are no Nash - Equilibria in Pure strategies.

Table 1. Playoff of player B

	NN	NF	FN	FF
N	-1	+1	+1	-1
F	+1	-1	-1	+1

However, there is a Nash equilibrium in Mixed strategies ρ^{NF} (ρ_B, ρ_A) where $\rho_B = [\frac{1}{2}\frac{1}{2}]$, $\rho_A = [\frac{1}{4}\frac{1}{4}\frac{1}{4}\frac{1}{4}]$.

4. The Quantum Penny-Flip Game

4.1. This is a nonclassical game consisting of players A and B, and the coin (in "quantum box" assuming an existence of Verifier). Quantum games are an area of experimental quantum mechanics and in order to describe the quantum penny-flip game we need some sort of "cross-disciplinary translator" (because any quantum game is usually associated with a quantum experimental setup in a specialised quantum lab - this is introducing the quantum probabilistic definition of game strategy). Please, see Appendix . In particular, physically speaking, in the classical penny-flip game player A chooses the side of the coin (Tails, Heads), locks it in a "box" and sends it to player B. Player B has proof that the coin was specifically prepared, but he cannot predict the actual result. If it is Tails, player B wins. Quantum cryptographers may say in this context that "there is no classical protocol which allows unrestricted security against cheating for the "coin tossing" protocols". However, using quantum mechanics, it is possible, at least, to limit probability in the situations when player B, for instance, is always losing.

4.2. In quantum games we replace the "box" by a quantum state (in quantum laboratory). Player A chooses one among a series of non-orthogonal states and sends it to player B. Each of the states encodes the result of the transformation of the coin. Thus without previous knowledge, player B cannot know with certainty which of the states he possesses. At this point player B makes his bet or he chooses between two possible moves. To "unlock" the state player A can have dialogue (question-answer) with player B on which state A sent. Player B can measure it in an orthogonal basis to check player A's answers. Thus, it is easy to see, hence, that in quantum interpretation classical penny-flip game became 2 + 1 - players cooperative game consisting of player A, player B and the Verifier. In the terms of generalised algebra of quantum cooperative games, we have actually a non - local game as a tuple $G = (I_A, I_B, O_A, O_B)$: where I_A, I_B are finite sets that represent questions for A and B; O_A, O_B are finite sets that represent answers for players A and B. and, when Rule function is defined as $\lambda : I_A * I_B * O_A * O_B \to \{0, 1\}$. Verifier randomly sends a question x (which belongs to I_A) to player A and y (belonging to I_B) to player B separately. The players respond with individual answer a (belonging O_A) and b (belonging O_B) respectively. Correspondingly, players win the round $<-> \lambda(x, y, a, b) = 1$.

Experimental physicists usually say in this context about a necessity to use three dimensional quantum states or so-called "qutrits" (Molina-Terriza, 2004): the protocols using qutrits are better suited for this particular game. Founder of quantum game theory physicist Meyer (1999) optimistically suggested that a player, equipped with a quantum computer, may always win no matter what classical player does in any type of classical games actually.

5. Post-Quantum E-games

5.1. Always winning players in the history of science and management ("the Challenge of Perfect Play") represent an open (may be elusive) problem of classical game theory and psychology of management today. Some historical observations, nevertheless, may suggest some analogy for that elusive problem in quantum game theory, where a quantum player (hypothetical "quantum leader") equipped with a quantum device can certainly win some types of classical games no matter what other classical players do indeed (Meyer, 1999). Similar reflections (Khan et al., 2018; Leung, 2011, Popov, 2009) on the problem of "quantum leadership", however, can produce ideas of the End of Classical game theory and some sort of "Superdeterminism" in post-quantum game theory. But it is not clear here what kind of post-quantum games could be useful in the future leadership paradigm. Current experiments in the area of Artificial Intelligence and Quantum Artificial Intelligence, unfortunately, are not able to provide essential clarification of such question, also (Dilmegani 2022, Dey et al., 2023)

5.2. It is not surprisingly, that working systematically under philosophy of global leadership and genius psychology, Schopenhauer and Moscow's idealists - mathematicians made some attempts to develop intuitive E-games or "post-quantum" noncooperative non-local (players are separated and cannot communicate physically with each other during game) and without real Verifier (in the modern terms) games which could be associated with a problem of mathematical winners beyond materialism and "fundamental fatalism".

5.3. In his "Математика и научно-философское миросозерцание" (1905), in particular, Nikolai Bugaev attempted to define quite puzzling "free values function" (функцию произвольных величин), which in comparison with analytical continuous functions of Analysis and discrete functions of Number theory (Bugaev's "Аритмологии") pretended to have manifold of the meanings for the same meaning of the independent variable ("Между тем в аритмологии встречаются особые функции, обратные прерывным. Их можно назвать функциями произвольных величин. Оне обладают свойством иметь бесчисленное множество значений для одного и того же значения независимой переменной")(Bugaev, 1905, p. 364). Bugaev's free values function is a nonanalytical discontinuous function, which could be used as a marker of an existence of fundamental error in theory or conjecture. Semi-analytical E(x) function, correspondingly, may be used as "an antidote" against speculative free values functions and it contains constantly changing random independent variable x, applicable in E-games in science making. In agreement with Pavel Nekrasov, hypothetical E-games can continue the tradition of idealistic probability mathematics (Nekrasov 1902, 1916). He predicted that E-games could be noncooperative-like games, where an essential role is played by mathematical language of probability theory (Bernoulli-Chebyshev theorems), when analytical and nonanalytical applications are useless and there is no "hidden" scientific law for scientific explanation in principle.

5.4. Closest example of such sort of E-Game is Riemann problem in Number theory. This is 2-players noncooperative signaling game consisting of two players-mathematician **M** and Nature ("Creator of Mathematics"), **N**. It is assumed that a formal role in that game is played by nonnatural language (Number theoretical mathematical language), which is commonly accepted by both players **M** and **N**. Some non exact analogies for this new kind of E-game could be found in contemporary literature on communication games (Crawford & Sobel 1982), signaling games (Kohlberg & Mertens 1986, Crossman & Perry 1986, Cho & Kreps 1987) and language games (Reny 2024). In the terms of signaling games, in our Riemannian E-game there are two players: a Sender (**M**) and a Receiver (**N**). We allow **M** to make mathematical statements (proofs) about the strategy which he is using. To be effective **M** - mathematical language must include some conventions. Following Nikolai Bugaev, in order to provide the effective language for communication with **N**, the false free values function must be replaced by the true semi analytical E-function. In positive probability communications mathematical proof by **M** can take an effective meaning. In some cases (please, see below ABC conjecture example) mathematical formulation of the problem can be defined as an ill-posed problem and false convention is rejected by both players **M** and **N** (in order to achieve some kind of Nash's "mathematical-language-equilibrium"). Hence, **M** wins when the wrong convention is rejected by **M** and **N** simultaneously. Similar with quantum games, E-game is played as a series of rounds involving questions and answers between the imaginary Vetifier and player **M**. Players are separated and cannot physically communicate with each other during the E-game. A Vetifier is merely a speculative entity, but in player **M**'s imagination two players try to convince an imaginary Verifier by giving correct pair answers to pairs of questions posed by imaginary Verifier… Because nobody knows in the Riemannian task why and what for distribution of the prime numbers (**N** created "proto-integers" having only two divisors) has no analytical mathematical law, mathematicians try to find unknown probability theory in order to prove the Riemann hypothesis. It is quite possible that Riemann complex function could be associated with some unknown free values function and **M** is needed for some unknown discontinuous semi analytical function, indeed.

5.5. In the 1950s legendary game theorist John Nash also attempted unsuccessfully to find an unknown type of noncooperative game in order to prove the Riemann hypothesis (he introduced a Nash Embedding Theorem). However, as we know today, nobody knows what kind of randomness theory is used by **N**'s prime numbers in their chaotic distribution, indeed.

5.6. Another example is connected with ABC conjecture in Number theory where Nikolai Bugaev made his first historical contribution. Contemporary interpretation of ABC conjecture by Masser-Osterle (1985) suggests that for every positive real integer there can exist only finitely many triples (a, b, c) of coprime positive integers with $a+b=c$ such that $c > Rad(abc)^{1+\varepsilon}$. In about 15.000.000.000 cases ($c < 10000$) it became clear that as rule $Rad(abc)^{1+\varepsilon} > c$, and, only in 120 cases of them (for instance, $1 + 8 = 9$) $Rad(abc)^{1+\varepsilon} < c$.

Hence, because nobody knows what kind of probability theory is used by **N**'s triples of coprime positive integers indeed, every new player **M** is faced with unknown false free values function and with a necessity to find a true semi analytical E(x) function. Following the earlier described heuristics of E-games, we may assume that the Masser-Oesterle's formulation of ABC conjecture contains some sort of a convention between mathematicians invented to reduce a level of indeterminacy of the task (to make effective language) (see, also Mochizuki, 2024). Let **N** be a positive integer. Thus, in agreement with the fundamental theorem of arithmetic **N** must have unique prime factorisation. However, Masser and Osterle have invented a special class of deformation of that fundamental theorem, when **N** is transforming into a quasi - round number called Rad (n) or " radical of the integer **N**". Thus, for example , in a normal case 60 is a product of 2.2.3.5 prime divisors. But in Masser-Oesterle arithmetic, 60 became a product of 2.3.5. Correspondingly, $Rad(60) = 2.3.5 = 30$. Similarly, in the normal case $100 = 2.2.5.5$. After unlawful free values deformation by Masser-Oesterle $Rad(100) = 2.5 = 10$. Free values function Rad (x) by Masser-Oesterle is introducing also a new sort of "pathology " of fundamental theorem of arithmetic: every prime number for some unknown reasons cannot have Masser-Oesterle deformation, hence $Rad(5) = 5$ and $Rad(11) = 11$. Lack of proof of an existence of free values function (Rad (x)) is producing another kind of difficulty, connected with the introduction of so-called "indexes of compositionality"(ε).

1260, 1680, 5040, 7560, 10080, 15120, 20160, 25200, 45360 and 50400 are different integers, however, their free value function $Rad(x)$ for $x = 1260$, $x = 1680$,… must have the same value, because $Rad(1260) = Rad(1680)= Rad(5040) = Rad(7560) = Rad(10080) = Rad(15120) = Rad(20160) = Rad(25200) = Rad(45360) = Rad(50400) = 2.3.5.7 = 210\ldots$! In order to distinguish them Masser and Oesterle introduced artificial log - criterion $LogN/LogRad$ (abc). Hence, say, (ε)1260 and (ε)25200 must have different meanings: 1.335089… and 1.89534.

In his "Introduction in Number theory" Nikolai Bugaev (1905, see, also 1891) formulated an earlier version of ABC conjecture without free values function $Rad(x)$. He introduced the semi analytical function $\rho(n)$, where $\rho(2) = 2$, $\rho(3) = 2$, $\rho(4) = 3$, $\rho(5) = 2$, $\rho(6) = 4$, $\rho(7) = 2$, etc. Hence, hypothetical Bugaev's ABC conjecture could be formulated in the form:

ABC conjecture: *Suppose a, b, c are coprime positive integers such that $a+b = c$. Then define $d = Rad(abc)$ using $\rho(n)$ function.*

Formal comparisons of the two different interpretations of ABC conjecture may suggest at least one conclusion: Masser-Oesterle interpretation of ABC conjecture is an ill-posed problem, based on pure speculative free value function.

Masser-Oesterle's ABC conjecture

$9 + 4 = 13$, then $d = Rad(9.4.13) = 2.3.13 = 78$, hence, $78 > 13$ ($d > c$)

$25 + 7 = 32$, then $d = Rad(25.7.32) = 2.5.7 = 70$, hence, $70 > 32$ ($d > c$)

$8 + 1 = 9$, then $d = Rad(8.1.9) = 2.1.3 = 6$, hence, $6 < 9$ ($d < c$)

Bugaev's ABC conjecture

$9 + 4 = 13$, then $d = Rad(9.4.13) = 3.3.2 = 18$ (because $\rho(9) = 3$, $\rho(4) = 3$, $\rho(13) = 2$), hence, $18 > 13$ ($d > c$)

$25 + 7 = 32$, then $d = Rad(25.7.32) = 3.2.6 = 36$ (because $\rho(25) = 3$, $\rho(7) = 2$, $\rho(32) = 6$), hence, $36 > 32$ ($d > c$)

$8 + 1 = 9$, then $d = Rad(8.1.9) = 4.1.3 = 12$ (because $\rho(8) = 4$, $\rho(1) = 1$, $\rho(9) = 3$), hence, $12 > 9$ ($d > c$)

6. Conclusion

Moscow's idealists-mathematicians want to see more radical changes in the future of probability theory and game theory because things haven't gone nearly far enough. I think the re-examination of well forgotten Bugaev's E-games in today's post-quantum game theory could be quite suitable and productive indeed — a new kind of game — theoretical approach in Number theory (applied to Riemann problem and ABC conjecture) can become firstly justifiable. We showed that following a pioneering attempt by John Nash (1950s) and combining a signaling E-game theory with current Number theory (for instance in the case of successful analysis of ABC conjecture) we may open new ways for game theory development assisted by AI and Quantum AI achievements. It's about time that it happened.

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Appendix

Quantum game theory is a relatively new field that extends the concepts of classical game theory to the quantum realm. Here are comparative characteristics of key notions of quantum game theory.

QUANTUM PLAYER

Quantum player is defined as a maker of quantum measurements.

QUANTUM STRATEGY

Strategy is an effective program of quantum measurements involving superposition and entanglement (entanglement is a counterintuitive nonlocal behaviour of quantum particles which was mistakenly rejected by Albert Einstein (EPR article 1935) as "a game with Solipsism in scientific physics" (Nobel Prize in physics 2022)).

QUANTUM PAYOFF FUNCTIONS

Quantum mechanical expectations values.

QUANTUM EQUILIBRIUM CONCEPT

Quantum Nash-equilibrium. Quantum entanglement-assisted equilibrium.

QUANTUM GAME DYNAMIC

Quantum dynamic governed by the Schrödinger equation.

QUANTUM INFORMATION FLOW

Quantum information can be entangled and non-local in quantum games.

QUANTUM INFORMATION PROCESSING

The basic idea of quantum information processing is that information is stored in quantum bits and processed by quantum logic gates. Just as classical logic gates take classical bits of information from one site to another (in classical computers), quantum logic gates take so-called "qubits" (or quantum bits) from one state to another.

QUANTUM HAMILTONIAN

Quantum logic gates take qubits by modifying the system's Hamiltonian, by applying additional control fields to the background Hamiltonian, which underlies the system.

QUANTUM EVOLUTION UNDER UNITARY TRANSFORMATION

Applying Hamiltonians will cause qubits to evolve under unitary transformations (which are reversible).

"QUANTUM BOX"

"Quantum box" with the coin in quantum penny-flip game can be considered as the Bloch sphere or as quantum logic gate and it can be implemented by applying an appropriate Hamiltonian for an appropriate time.

HADAMARD GATE

Hadamard gate, usually indicated by the letter H, is self - inverse quantum logic gate $|+ > -> | 0 >$ and. $| - > -> | 1>$, so that applying it twice is equivalent to doing nothing. This means that the Hadamard gate must correspond to a 180° rotation, and it is in fact equivalent to a 180° rotation around an axis tilted at 45° degrees from the x axis toward the x axis.

CHOICE OF QUANTUM GATES IN GAME PLANNING

It is possible to describe quantum gates in many different ways. The choice is often a matter of context and the background of the quantum game - theorist. Researchers with a background in computer science would tend to use the most abstract notation X, while theoretical physicists might instead choose the Pauli matrix form σ_x. By contrast, experimental physicists would usually use the H.

Key differences and similarities

QUANTUM STRATEGIES

Quantum game theory allows players to employ quantum strategies leveraging the principles of superposition and entanglement to gain strategic advantages.

QUANTUM ENTANGLEMENT

Entanglement enables players to correlate their strategies in a way that it is not possible in classical game theory.

QUANTUM MEASUREMENT

The act of measurement in quantum mechanics can introduce randomness and uncertainty, affecting the outcome of the game.

NASH EQUILIBRIUM

Both theories utilized the concept of Nash equilibrium , a state where no player can improve their payoff by unilaterally changing their strategy. However, the definition of Nash equilibrium is extended in quantum game theory to account for quantum strategies.