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Edge Ranking in a Transport Graph of Petrozavodsk Basing on Equilibrium Flows*

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Abstract Game-theoretic methods are frequently used to model the dynamic processes in urban road networks and evaluate network efficiency. One of the key concepts in this field is a Wardrop equilibrium (user equilibrium), the situation where no driver can reduce their journey time by unilaterally choosing another route. The concept is widely used in the literature to model the distribution of regular trips in a transport network. In general case, the equilibrium is unstable, and the construction of a new equilibrium trip distribution may require significant time. At the same time, when analyzing transport networks, one is often interested in the impact of short-term changes, when a road is closed for a short time, and the drivers do not seek for a new equilibrium but rather select acceptable routes in the current situation. Here, Wardrop equilibrium can be seen as the basic flow distribution, and the temporal changes in agents' strategies have some impact on local or global characteristics of the network. One may predict the scale of this impact by estimating the importance of the edge being temporarily unavailable. In this work, we analyze edge centralities within a Wardrop equilibrium in the transport graph of Petrozavodsk. We propose a modification of edge betweenness centrality that incorporates precalculated equilibrium flows passing through the road segment. We illustrate how the resulting edge ranking can be used to enhance the classical betweenness centrality to consider not only the topological graph properties, but also the actual flow distribution. One can use the modified centrality measure to estimate the properties of Wardrop equilibrium and to increase the efficiency of its recalculation upon graph modifications. The results can be used in the traffic analysis and planning the development of the transport network.

Keywords: transport graph, betweenness centrality, Wardrop equilibrium.

1. Introduction

Game-theoretic methods are frequently used to model the dynamic processes in urban road networks and evaluate network efficiency (see, for example, a recent review (Ahmad and Al-Fagih, 2023)). One of the key concepts in this field is a Wardrop equilibrium (or user equilibrium) (Wardrop, 1952), the situation where no driver can reduce their journey time by unilaterally choosing another route. Wardrop's first principle reads: the journey times on all the routes actually used are equal, and less than those which would be experienced by a single vehicle on any unused route (Wardrop, 1952). The equilibrium is reached when every driver

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independently selects a route with the minimal travel costs, and the costs depend on congestion. The concept of Wardrop equilibrium is widely used in the literature to model the distribution of regular trips in a transport network.

It is known (Englert et al., 2010) that in general case, the Wardrop equilibrium distribution of agents over the routes is unstable, and even for linear delays, there are network instances where for any $\epsilon > 0$, the removal of an edge with ϵ-fraction of traffic will lead to the full recomposition of the agents' strategy profile, as every agent will change their route in order to recover equilibrium. In transport networks applications, such a situation may arise when a road is permanently closed, and the drivers have to reach an equilibrium in the new setting. The construction of a new equilibrium may require significant time.

From the other side, when analyzing transport networks, one is often interested in the impact of short-term changes, when a road is closed for a short time (for example, due to a road accident), and the drivers do not seek for a new equilibrium but rather select the routes that seem acceptable in the current situation. Here, the Wardrop equilibrium can be seen as the basic flow distribution, and the temporal changes in agents' strategies have some impact on local or global characteristics of the network. One may predict the scale of this impact by estimating the importance of the edge being temporarily unavailable.

Edge ranking plays a crucial role in applications addressing the topology and efficiency of transport graphs. A few examples of widely studied problems include determining which road segments, when blocked or partially restricted, have a significant impact on network efficiency; other examples include identification of bottlenecks, over- or underloaded roads, optimal points for public service allocation, etc. Methods of edge ranking in transport networks have been recently reviewed, for example, in (Mattsson and Jenelius, 2015).

One of the most popular concepts used for edge ranking is centrality. Betweenness centrality (Bavelas, 1948), (Freeman, 1977) of a vertex (edge) is the fraction of all shortest paths in a graph that pass through this vertex (edge). In the simplest case, the shortest path is defined via edge count or length. Employment of various structural or dynamical graph properties as edge weights to select shortest paths is a natural extension of the classic betweenness centrality. Such modifications have been used in multiple works to address the specifics of problems being analyzed. In particular, in transportation networks, travel time may depend on link capacities, speed restrictions and traffic rather than mere path length.

The authors of (Akbarzadeh et al., 2019) analyzed variants of betweenness centrality considering road congestion, lengths, capacities, simulated travel time and equilibrium flows. In (Zadeh and Rajabi, 2013), a "targeted constrained betweenness" edge centrality was calculated iteratively basing on road capacities, travel demands and the equilibrium flows.

When analyzing possible consequences of temporal road closures, authors widely use another metric called edge criticality: a characteristic estimating how partial or full edge unavailability will influence the network performance. A recent review (Mylonas et al., 2023) suggests hybrid measures of criticality considering topological properties of transport networks together with the data on Wardrop equilibrium in the unmodified network. The authors of (Kurmankhojayev et al., 2024) propose a modified criticality measure to work with the Wardrop equilibrium.

In general, the consideration of equilibrium data is known to increase the efficiency of centrality in estimation of edge importance. In this work, we analyze edge centralities within a Wardrop equilibrium. We propose a modification of edge betweenness centrality that considers equilibrium flows passing through road segments and illustrate the results on an example of Petrozavodsk (Chirkova, 2024). The results can be used in the traffic analysis and planning the development of the transport network of Petrozavodsk.

2. Edge betweenness centrality and the user equilibrium

The road network graph of Petrozavodsk was constructed in (Ermolin et al., 2022); the vertices correspond to road intersections, and the edges correspond to highway sections connecting them. In the present work, we use a version of the graph with 1520 vertices and 3739 directed edges. The analysis was performed using R Statistical Software (https://www.R-project.org/).

The Wardrop equilibrium has been calculated in (Chirkova, 2024) with BPR-type delay functions (U.S. Bureau of Public Roads, 1964) on road segments corresponding to graph edges ($e \in E(G)$),

$$f_e(x) = t_e \left(1 + \alpha \left(\frac{\delta_e(x)}{c_e}\right)^{\beta}\right). \tag{1}$$

Here, t_e is free-flow travel time on edge e, δ_e is the traffic volume on edge e, c_e is edge capacity, α and β are the parameters that determine the sensitivity of travel time to changes in traffic volume; the used values are $\alpha = 0.15$, $\beta = 4$.

In general case, calculation of a Wardrop equilibrium is a computationally complex problem related to the optimization of a global potential function on the graph. A suitable flow distribution might reduce the algorithm runtime by providing an initial solution. Let us consider a variant of edge betweenness centrality that takes into account delay functions when counting shortest paths:

$$BPR(e) = \sum_{\substack{s,t \in V \\ s \neq t, \sigma_{st} > 0}} \frac{\sigma_{st}(e)}{\sigma_{st}}. \tag{2}$$

Here, σ_{st} is the number of shortest paths between s and t; $\sigma_{st}(e)$ is the number of shortest paths between s and t passing through edge e, and the shortest paths are counted, considering BPR delay functions (1) with fixed traffic x. For example, in Fig. 1, we present edge ranking by the centralities (2) calculated with equally distributed traffic between all edges of the graph. The centralities were normalized and broken into 100 categories. The edges are colored according to the categories.

In addition, Fig. 2 presents equilibrium edge loads, normalized and broken into categories in the same way as centralities. The edges depicted in gray are not used in the Wardrop equilibrium due to the structure of the underlying correspondence matrix; centrality measure (2) does not take into account the correspondences. Despite this difference, we observe that the edge ranking provided by BPR-based centralities has similarities with the equilibrium load distribution. For 76% of the edges used in the equilibrium, the calculated characteristics differ by at most 0.1. Only for 8% of the edges, they differ by more than 0.3.

This result illustrates that the edge ranking obtained by a simple centrality variant (2) allows setting an initial flow distribution (with traffic volumes proportional to normalized edge centralities) when computing the Wardrop equilibrium, in order to speed up its computation.

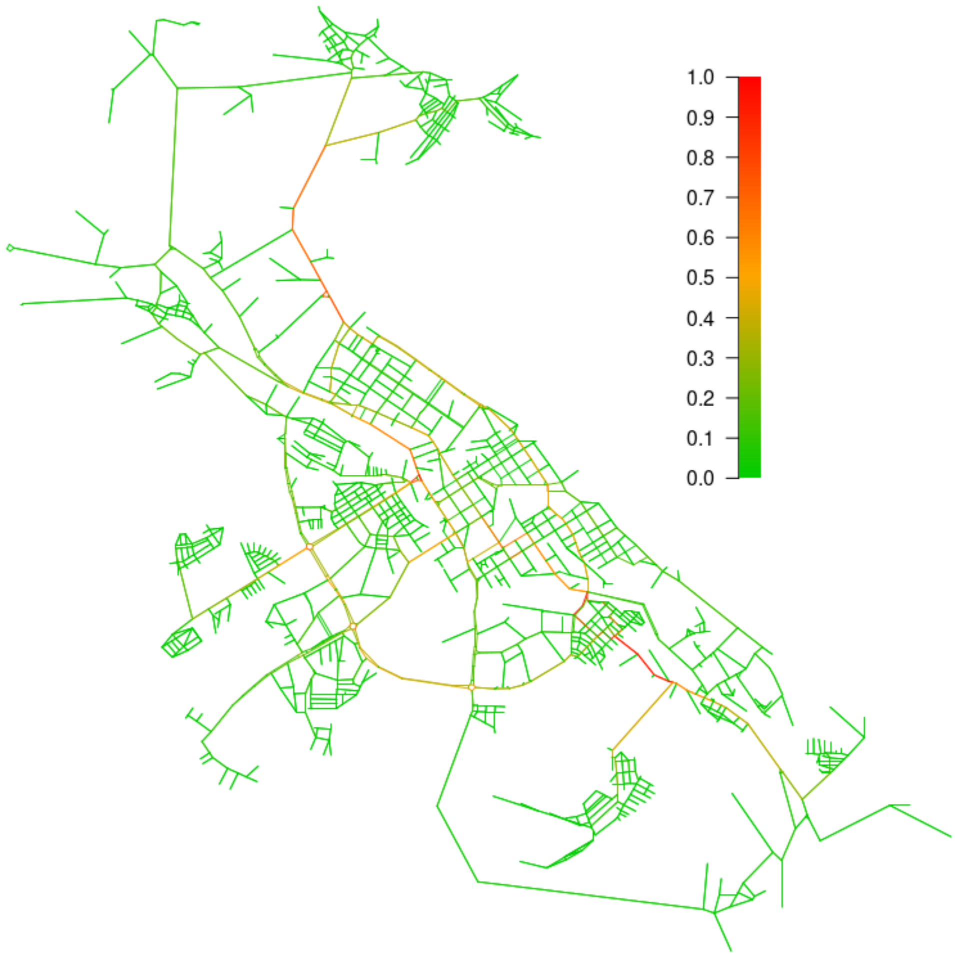

Fig. 1. Edge betweenness centralities calculated with BPR delays

3. Edge Ranking in a User Equilibrium

Betweenness centrality of an edge is the fraction of all shortest paths that pass through this edge. In order to model the real road situation and take into account not only the topological structure of the graph but also the dynamic processes occurring in it, one may modify the betweenness centrality to consider trip distribution in the form of a correspondence matrix.

Another approach is to use equilibrium flows in the graph instead of the correspondence matrix. For any pair of vertices s and t, we have W_{st} amount of equilibrium flow passing $s \to t$, and $W_{st}(e)$ of the flow passes through the edge e. We

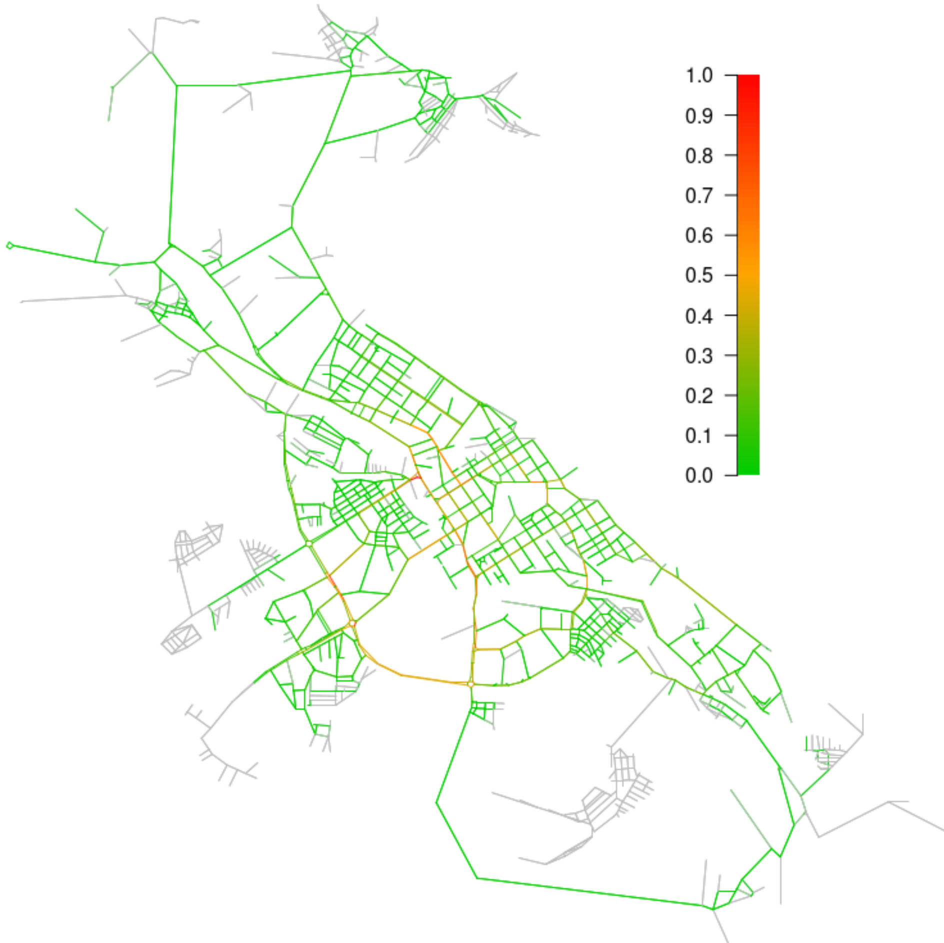

Fig. 2. Edge loads in a Wardrop equilibrium calculated with BPR delays

define edge betweenness centrality by equilibrium flows:

$$BE(e) = \sum_{\substack{s,t \in V \\ s \neq t, W_{st} > 0}} \frac{W_{st}(e)}{W_{st}}. \tag{3}$$

Figure 3 illustrates the Wardrop equilibrium for Petrozavodsk. Here, we restrict ourselves to a fragment of the city center for the sake of visibility. We depict equilibrium flows (a) and the corresponding betweenness centralities (b). In order to compare the results of different edge rankings, the calculated characteristics of the edges were normalized and broken into 10 categories. The edges are colored according to the categories. Edges with zero equilibrium flows are shown in gray.

We observe that the overall picture of edge betweenness centralities with equilibrium flows (3) is very similar to the picture of edge loads in the equilibrium. More precisely, 68% of the edges used in the equilibrium have the same rank calculated by the equilibrium flows and betweenness centralities, 87% differ by at most 0.1, and only 0.4% of the edges differ by more than 0.4. In such a way, when the equilibrium has been calculated, one can easily obtain betweenness centralities (3) and recalculate them upon any graph modifications.

At the same time, the proposed variant of betweenness centrality allows to understand the relative importance of separate edges. For example, if an edge has high betweenness centrality, it might play an important role in connecting different parts of the city, even despite the low load. Such an edge may become overloaded if it has low capacity and some of the other edges of the graph become temporarily unavailable (for example, due to a road accident).

4. Conclusion

The concept of Wardrop equilibrium is widely used to model the distribution of regular trips in a transport network. In this paper, we analyze edge centralities within a Wardrop equilibrium in the transport graph of Petrozavodsk. We propose two simple modifications of edge betweenness centrality: the first one uses BPR delay functions to define the shortest paths in the graph, and the second one incorporates precalculated equilibrium flows passing through the road segment.

We illustrate how BPR-based edge ranking may help to obtain initial flows distribution to facilitate the computation of the Wardrop equilibrium. Application of a BPR delay function, which considers congestion effects on travel time, provides a more realistic estimate of edge importance.

We also show how equilibrium flows-based edge ranking extends the knowledge about importance of city roads and can be used to enhance the classical betweenness centrality to consider not only the topological graph properties, but also the actual flow distribution and congestion levels reached at the equilibrium. Such a measure can be convenient when modeling short-term modifications of the road network.

One can use the modified centrality measures to elaborate on complex pictures of flow distribution, estimate the properties of Wardrop equilibrium and to increase the efficiency of its recalculation upon graph modifications. The results can be used in the traffic analysis and planning the development of the transport network.

(a) Edge loads

(b) Edge betweenness centralities

Fig. 3. Wardrop equilibrium in the transport graph of Petrozavodsk (fragment of the city center)

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