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Cooperative Solutions for Working Capital Cost Management

Vladislav Novikov and Andrey Zyatchin

St.Petersburg State University,
7/9, Universitetskaya nab., St. Petersburg, 199034, Russia
E-mail: nov.vlad.serg@gmail.com*,* zyatchin@gsom.spbu.ru

Abstract An advancement of the framework for reduction and redistribution of joint working capital costs in financial supply chain networks with combined topology is represented: factoring, reverse factoring, and inventory financing were chosen as the financial supply chain instruments to reduce the cost of joint working capital, a characteristic function is set and Core and Shepley values are chosen as optimality principles, the developed framework is compared with similar methods constructed by other authors, and also tested on the case study of the automotive industry.

Keywords: working capital management, financial supply chain, financial supply chain solutions, cooperative games.

1. Introduction

Working capital management (WCM) is one of vital tools that helps companies leverage their liquidity, CAPEX and general profits. Thus, business aims to implement different tools and methods to increase the possibility of WCM application. Moreover, in recent global recession, with immense inflation, political and economic instability, companies are struggling with both access and efficiency of an outside financing of day-to-day operations. Thus, it is beneficial to find ways to increase liquidity and free the cash locked up in the operation.

Moreover, in recent decades, the nature of competition has shifted from competition between individual companies to a competition between supply chains. Thus, it is reasonable to explore means of enhancing cooperation in supply chains, as well as improving its efficiency, rather than focus on companies themselves. The majority of the contemporary research on cooperative working capital management solutions still revolves around physical flows between companies, i.e. inventory management, transportation and procurement optimization, and others. Whereas research on financial flows is undermined and not being studied as much. The presented paper aims at addressing this gap, studying the cooperation in network structures from the perspective of financial flows to enhance the working capital of both each company in the chain and the chain in total.

The subject of the presented paper is joint working capital costs of members of financial supply chains with combined topology (many-one-many structure).

The object of the paper is financial supply chains themselves.

The main purpose of the study is advancing the framework for reduction and redistribution of joint working capital costs in financial supply chain networks with combined topology.

To achieve the stated purpose the following objectives are set:

- Based on the analysis of academic and practical literature, analyze possible financial supply chain solutions to decrease working capital costs.

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– Improve the framework for reduction and redistribution of joint working capital costs of networks with combined topology affected by usage of chosen financial supply chain solutions.
– Evaluate the effectiveness of improved framework and figure out its limitations based on case studies of supply chains.

The paper is divided into four parts. Chapter 1 is introduction. Chapter 2 covers the tools of financial supply chain management and settles the methodology of joint working capital costs calculation. Chapter 3 establishes the framework in which the cooperative game is set, i.e. the characteristic function is set out, imputation principles are chosen and implemented. Chapter 4 implements the developed framework on a case study with data.

2. Working Capital in Financial Supply Chains

2.1. Regular and Financial Supply chain cooperation

Supply chain management. The term 'Supply Chain Management' originates from (Oliver and Webber, 1982), who were discussing the potential benefits from linking internal business processes like manufacturing, procurement, distribution, and sales – operations. However, over the time the term evolved, leading to management of the flow of goods from point of their origin to point of their final consumption. This leads to the need of cooperation with all the performers of business operations during the flow, making it a network. (Harland, 1996) gives one of the definitions as 'management of a network of interconnected businesses involved in the ultimate provision of product and service packages required by end customers.

Basically, Supply Chain Management deals with interconnections between partners along the flows of goods, finance and information, revolving around a product. The flows are both upstream and downstream: goods are directed downwards, finance – upwards, information – both. Supply Chain Management is seen as an alternative to the previously predominant concept of vertical integration (Christopher, 1992) (Coase, 1937) .

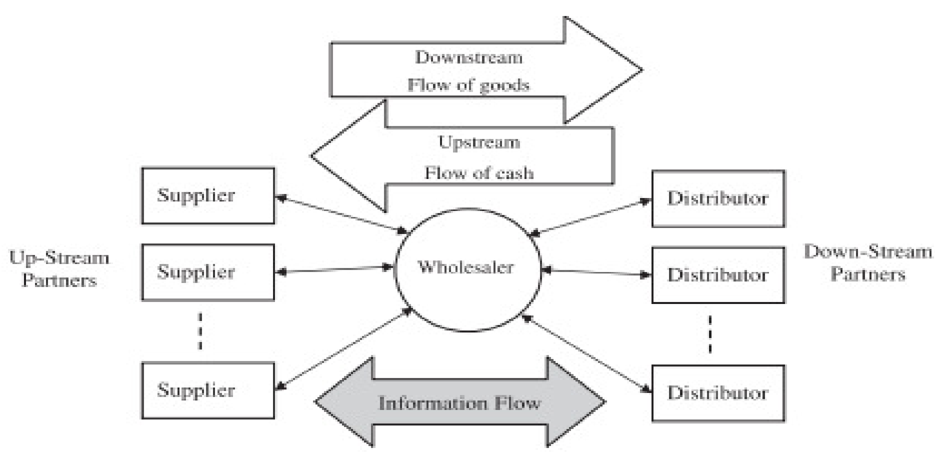

Fig. 1. Scheme of a supply chain. Source: (Gupta and Dutta, 2011, p. 48)

Vertical integration might be considered as the starting point of the development of the concept, as together with SCM they form a trade-off spectrum of business integration. As seen in figure 2, vertical integration is bringing the business to level 1 – internal chain. It has its benefits, such as serving as a barrier to entry, source of economy of scale and, generally, being a part of a competitive advantage for a firm (Comanor, 1967). However, internalization and centralization lead to losses in flexibility for a business (Prater et al., 2001). SCM, on the other hand, revolves around vertical *disintegration* (Harland, 1996). In its extreme form, SCM deals on level 4 – network.

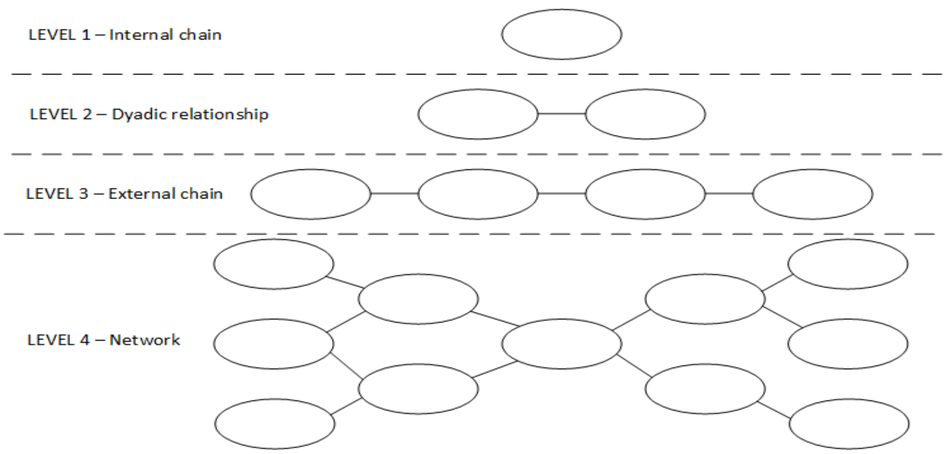

Fig. 2. Levels of supply chains. Source: (Harland, 1996, p. S72)

The development of SCM has been greatly boosted by the development of Japanese manufacturing practices as Total Quality Management (TQM), Just-in-Time (JIT) and others. (Harrison, 1995) mentions that these approaches are driving the inventories away from the assemblers, more complex logistical schemes. Moreover, (Hayashi, 2004) states that these Japanese manufacturers, who invented the practices, are shifting to global operations models. Such a trend is obviously not aiding the simplification of supply chains, thus further development of SCM was and is required.

Considering that SCM is an integral part of any business nowadays, the continuous development is still relevant. The recent years have shown that the main driver for it would be technological development. Usage of WMS/TMS integration, barcodes and previously mentioned Japanese-originated practices is already a standard for most of businesses, with new innovations disrupting the industries. Thus, it is possible to state that overall, the market is in stage of Supply Chain Management 4.0. According to (Frazzon et al., 2019), SCM 4.0 can be defined as "the integration and synchronization of the product's entire value chain across different companies, using smart technologies (IoT, IoS and others) to build an interconnected and transparent system with real-time communication that can manage flows and optimize itself, leading to an autonomous, adaptive, intelligent, agile, and dynamic network that focuses on customers' requirements". It is vital to highlight the phrase 'product's entire value chain across different companies. With context of technology implementation, collaboration between partners becomes more important than ever before.

Financial supply chain management. As seen from before, Supply Chain Management as a discipline is primarily focused on physical flows of goods. Despite the technological advancements and pull of partners closer to each other, financial side of the flow was undermined and did not receive proper attention. However, over time the importance of FSC has become more and more obvious. For example, (Blackman et al., 2013) conducted thorough research of Motorola supply chain and concluded that physical and financial supply chains are immensely interconnected, especially in case of globally operating companies. Moreover, they figured out that standardization of FSCM practices throughout such supply chains is beneficial for companies.

Despite the urge to split SCM and FSCM into different categories, they are closely interconnected and not to be studied separately. It is worth to mention that the term "Supply Chain Finance" (or SCF) is predominant in academic discussion. However, in general view the term SCF is misinterpreted with the *factoring* instrument (for example, by (PWC, 2018), which will be explored later. Thus, due to practical orientation of the paper, to avoid confusion, FSCM is a more appropriate term and will substitute SCF when possible. Otherwise, they should be considered interchangeable.

The interconnection between SCM and FSCM is clearly seen when studied from the point of definition. Several relevant terms are represented in the table 1.

Table 1. Definitions of FSCM. Source: [created by author V. Novikov]

Paper	Definition of FSCM
"Focusing the financial flow of supply chains: An empirical investigation of financial supply chain management". (Wuttke et al. 2013).	Optimized planning, managing, and controlling of supply chain cash flows to facilitate efficient supply chain material flows
"Supply Chain Finance: some conceptual insights" (Hofmann, 2005)	SCF is an approach for two or more organizations in a supply chain, including external service providers, to jointly create value through means of planning, steering, and controlling the flow of financial resources on an inter-organizational level
"Supply chain finance" (Camerinelli, 2009)	SCF is the set of products and services that a financial institution offers to facilitate the management of the physical and information flows of a supply chain
"Supply chain finance: optimizing financial flows in supply chains" (Phofl and Gomm, 2009)	SCF is the inter-company optimization of financing as well as the integration of financing processes with customers, suppliers, and service providers in order to increase the value of all participating companies

Definitions from all four papers share the idea of exploiting financial management practices in order to enhance supply chain performance. Moreover, each implicitly emphasize the importance of collaboration between partners, i.e. optimized planning or controlling of supply chain flows requires a timely and complex communication between several companies. Thus, it is possible to state that role of FSCM is broader than improvement of financial performance exclusively.

However, authors take different approaches: Wuttke is focusing on internal effort in optimizing the flows, Hofmann and Phofl are including external service providers into FSCM, while Camerinelli explicitly states that SCF is impossible without an external financial institution. It is unreasonable to argue any of the opinions, as there are many ways to manage the financial supply chain, and usage of both internal and external services is possible.

Financial supply chain solutions. As seen from before, FSCM may rely on external and internal sources of collaboration. There is a wide variety of instrument used in both ways. To determine solutions, relevant for this research, a review of academic literature was concluded. Table 2 represents the ones widely studied.

However, to determine which solutions are most appropriate in context of this research, a more thorough understanding of listed ones is required.

Factoring

Factoring is a type of financing that allows companies to improve their cash flow by selling their receivables to a third-party financial institution, such as a bank. This financial interaction between the two partners is facilitated by the third-party financial institution, which takes on the responsibility of collecting payments for the receivables.

In general, factoring works as follows: the supplier sells its receivables to the third-party financial intermediary at a discount (which can be interpreted as commission), and the latter then collects payment for the receivables from the buyer on the due date.

The seller benefits from this arrangement by receiving immediate cash for the receivables, which can be used for other expenses instead of waiting for the transaction to happen. Consequently, exploiting factoring increases liquidity for the supplier, freeing the 'locked up' cash to fund further operations or invest into growth opportunities. Moreover, factoring could be used as a tool for 'bad debt' management, transferring the risk onto the third party.

However, the latter benefit is limited, as factoring agreements are typically one of two types: recourse or non-recourse. The first means that in case of buyer not adhering to payment terms, the finance intermediary has the right to demand repayment from supplier. Non-recourse, in turn, does not provide such a right (GSCFF, 2016).

In case of factoring, a third-party involvement is unavoidable, usually being a bank or a fintech company. However, it does not require a high level of cooperation between the participants of the agreement. As the initiative and communication mostly goes between supplier and buyer, the commission charged is usually based off supplier's credit rating.

Factoring can be an effective tool for managing cash flows, reducing financial risks and improving supply chain efficiency. However, effective cooperation and collaboration between all stakeholders is necessary for success, and the introduction of factoring itself requires careful planning and coordination.

Reverse factoring

Another type of factoring is *reverse* factoring. The result of using the instrument is the same – the supplier converts the receivables into cash faster. However, in case of reverse factoring, the initiative is on buyer's side. Generally, the invoices

Table 2. Financial supply chain solutions. Source: [created by author V. Novikov]

Solution	Definition	Main benefit	Key researchers
Factoring	Factoring is a form of Receivables Purchase, in which sellers of goods and services sell their receivables (represented by outstanding invoices) at a discount to a finance provider. (GSCFF, 2016)	Factoring helps with risk management, working capital optimization, liquidity management, etc.	(Klapper, 2006), (Mian and Smith Jr., 1992)
Reverse factoring	The technique provides a seller of goods or services with the option of receiving the value of receivables (represented by outstanding invoices) prior to their actual due date and typically at a financing cost aligned with the credit risk of the buyer (GSCFF, 2016)	Just as usual factoring helps with risk management, working capital optimization, liquidity management, etc. Moreover, as the initiative for instrument usage comes from the buyer's side – it enhances the relationships between buyers and suppliers	(Klapper, 2006), (Wuttke et al., 2016)
Inventory financing	Solution in which a logistics intermediary becomes a synthetic 'merchant', buying and then selling the goods at a determined price, while getting a commission. (Hofmann, 2009)	Reduction of inventory levels for both supplier and retailer, faster cash flows and reduced risks.	(Hofmann, 2009) , (Chen and Cai, 2011)
Dynamic or invoice discounting	An instrument, or, rather, agreement between buyer and seller, in which earlier payment of an invoice leads to a reduced price (discount on nominal value of invoice). (Gelsomino et al., 2016)	Dynamic/invoice discounting provides a possibility to reduce the level of accounts payables and receivables for both parties, thus reducing the working capital. Moreover, it enhances the relationship between parties	(Gelsomino et al., 2016), (Templar et al., 2016)
Revenue-sharing contracts	Agreement, in which a company pays its supplier a wholesale price for each unit purchased, plus a percentage of the revenue the retailer generates. (Cachon and Lariviere, 2005)	Revenue-sharing contracts help to incentivize suppliers to improve the quality of their goods and services and can help to improve supply chain relationships.	(Cachon and Lariviere, 2005)
Vendor-managed inventory (VMI)	Agreement in which the supplier is planning production and replenishment schedule as long on his own as the agreed customer service levels are met. (Claasen et al., 2008)	VMI helps to improve inventory management and can reduce supply chain costs and risks.	(Claasen et al., 2008), (Waller et al., 1999)
Consignment stock	Consignment stock is an inventory management arrangement where the supplier retains ownership of the inventory until the customer uses or sells it. The customer only pays for the inventory when it is used or sold. (Gumus et al., 2008)	Consignment stock helps to improve inventory management and can reduce supply chain costs and risks	(Valentini and Zavanella, 2003), (Gumus et al., 2008)

are again transferred to the factor from the supplier with an unconditional and irrevocable commitment to pay from the buyer. However, it is the customer who chooses which invoices it is possible to pay out earlier. On the stage, the supplier narrows down the range, by choosing the particular invoices it needs to be paid to earlier (GSCFF, 2016).

As seen, the model of reverse factoring is more complex than usual factoring – the process includes several steps and involves a three-way communication. Thus, it requires a higher level of collaboration between all three parties. However, it pays off via lower commissions and higher demand meet. Moreover, as on the first steps reverse factoring is a commitment between buyer and financial provider, there are grounds for managing the payment terms, thus influencing its own working capital. One other difference from usual factoring is that commission charged is based off buyer's credit rating, not supplier's.

Reverse factoring improves cash flow, fosters innovation, and improves supply chain efficiency. Reverse factoring is beneficial for the supplier when the spread in deadweight financing costs is high, nominal payment periods are long, and demand volatility is high (Tanrisever et al., 2015). However, initial payment terms and purchase volume strongly influence the optimal time to implement reverse factoring, as well as the optimal extension of payment terms (Wuttke et.al, 2019).

Inventory financing

Traditionally, inventory financing is understood as a type of financing that allows businesses to use their inventory as collateral to obtain a loan or line of credit (Selviaridis and Spring, 2007). However, (Hofmann, 2009) proposed a different approach to using the instrument. Hofmann's idea brings a 'network perspective' to inventory management is supply chain. In his view, involvement of a third-party logistics provider as a virtual or synthetic merchant in the chain let ease the friction, created by different goals of suppliers and buyers, e.g. the schedule of shipments.

The solution works as a three-way agreement: the supplier sells the inventory to the LSP and pays the commission for holding it. In return, he frees up his own inventory and does not bear the costs of holding it. The buyer renegotiates the shipment procedures, and buys the inventory at the same price, also reducing the holding costs.

The scheme resembles the *consignment stock* solution with extra logistics party involvement, but there is a core difference. Costs for CS solution with 3PL involvement are based on the physical features of the inventory transferred (dimensions, weight, etc.), while IF – directly with value.

Dynamic/invoice discounting

Another form of factoring, which is recognized as factoring only partially (for example, US GAAP doesn't recognize it as factoring), is invoice discounting. This method comprises of the supplier taking loans with invoices serving as collateral. Such a process serves the same purpose – improving liquidity, freeing cash, but due to a different nature – it is up for discussion, whether invoice discounting is factoring as well.

Dynamic/invoice is a type of financing that allows businesses to receive immediate payment for unpaid invoices from a financial institution. Which, in turn, provides a percentage of the invoice value up front and then collects the full payment from the customer when the invoice is due. After receiving payment, the bank returns the remaining amount to the business minus a discount fee.

The benefits of dynamic/invoice discounting include improved cash flow for businesses because they can get paid for unpaid invoices much faster than with traditional payment methods. Dynamic/actual discounting also helps reduce the risk of bad debts, as the bank assumes responsibility for obtaining payment from the customer. Thus, by obtaining capital quickly, businesses can invest in expansion and growth.

The disadvantage of this method is that it can be more expensive than traditional bank loans or other types of financing. Because the discounting fee charged by the financial institution may be higher than the interest rate charged by the bank. Also, dynamic/invoice discounting does not eliminate risk because the business is responsible for making sure its customers pay their bills, and if a customer defaults, the business may be required to pay a discounting fee to the financial institution. Dynamic/invoice discounting may not be available to businesses with a limited number of customers or, conversely, with a high concentration of customers, which is considered already a high credit risk.

Revenue-sharing contracts

Revenue-sharing contracts are agreements where costs for goods are wholesale, while the value added comes from the percentage of final revenue. In simpler terms – the retailer pays a wholesale price to the supplier of the good, and then pays a percentage of revenue to the same supplier. One of the first authors on the topic of such contracts are (Cachon and Lariviere, 2005). They have compared the performance of revenue-sharing contracts in different contexts: supply chains of different sizes, other types of contracts, different industries. The authors find out that these contracts perform differently in different conditions, however, the overall impact is positive.

The advantage of using revenue sharing agreements is the ability to reduce risk for all parties, because the income generated is shared by all parties. This instrument aligns incentives between the parties, as each party has an interest in the success of the product or service. And because of this interest, revenue sharing agreements are an effective way to stimulate innovation and creativity.

A study by (Dana and Spier, 2001) examines revenue-sharing contracts in a demand uncertainty model, where the results show that a linear transfer payment can provide supply chain coordination. This was confirmed in a later paper by (Giannoccaro and Pontrandolfo, 2004). It follows that through this process the revenue sharing process can contribute additional income for the supplier compared to a situation without exchange rate risk.

Such an instrument, though, has its own disadvantages. Revenue-sharing contracts can be complex and require careful planning and coordination between the sides concerned. They can also be risky for all stakeholders because the income received can vary depending on market conditions, customer demand, and other factors. And in this case, ambiguity can make it challenging to predict future revenues and plan accordingly. Moreover, such contracts require an overburdening amount of paperwork, which, in the long term might be critical for the entire supply chain. Thus, a very advanced level of cooperation is required to maintain such contractual relationship.

Vendor-managed inventory (VMI)

Vendor Managed Inventory (VMI) is a type of inventory management where the supplier is responsible for managing inventory levels at the customer location.

That is, the supplier himself performs inventory level control and replenishment as needed, based on agreed inventory levels and delivery schedules.

The main advantage of VMI is that it increases inventory accuracy and reduces inventory costs for customers. As a result, supply chain performance is increased, and supplier demand predictions become possible. VMI also shrinks inventory, enhancing customer satisfaction. After all, in this case, suppliers can better accommodate customer demand by maintaining optimal inventory levels.

However, VMI involves the supplier managing inventory levels, and this contributes to limiting the customer's control over his own inventory. Also, for the same reason, if the supplier fails to effectively manage inventory levels, it will lead to shortages or oversupply of goods. Another disadvantage of this tool is the limitation of flexibility in the customer's decision-making, which will lead to difficulty responding to changes in demand or even finding a better offer from another supplier.

Consignment stock

A consignment warehouse is an inventory management where the supplier owns and manages the transportation and shipping of inventory that is held by the retailer or distributor until it is sold to the end customer. The retailer or distributor does not pay for the goods until after they are sold, and any unsold goods can be returned to the supplier.

Consignment stocking reduces inventory costs for retailers and distributors, and in this case they do not need to purchase goods in advance. Likewise, this method reduces the risk for suppliers who retain ownership of goods until they sell them. Consignment inventory can help improve cash flow for retailers and distributors because they do not pay for goods until after they are sold.

However, consignment inventory can limit a customer's control over their inventory, and it can also be risky because the supplier retains ownership of the inventory until it is sold. And since the customer is responsible for selling the supplier's inventory, subject to unsold goods, he will have to deal with the remaining inventory, being responsible for any associated costs. Not unimportant is the fact that the consignment warehouse requires storage space for the supplier's stock. After all, if there is a large amount of supplier inventory, the customer may have difficulty storing his own inventory.

Choice of solutions

Out of seven listed solutions, the most appropriate for the research are factoring, reverse factoring and inventory financing. The basis for such conclusion is the simplicity of the methods and the best fit for the model developed further. For example, out of both factoring types, dynamic/invoice discounting and revenue-sharing contracts the first ones prevail, because they target the working capital components directly (which revenue-sharing contracts do not address explicitly) and require less data for quantifying the model (the D/I discounting also requires the information about splits between payments to the supplier). At the same time, out of inventory financing, VMI and consignment stock, the first one is the most beneficial, as it has a direct financial effect, while the others' effect is secondary. Moreover, IF is less focused on day-to-day management than the other two solutions.

Therefore, there are three financial supply chain solutions, which will later be used: factoring, reverse factoring, and inventory financing.

2.2. Working Capital

Single company's perspective. Working capital is a vital part of managing corporate financial well-being on a short-term level. Before the financial crisis of 2008, both theoretical and practical focus was concentrated on another aspect of corporate performance – areas of long-term investments and financial decision-making (Singh and Kumar, 2014). However, due to a significant shift in risk-profit balance due to the crisis, interest of researchers and business shifted to working capital management as a means to boost operational performance (Wang, 2002).

(Jones, 2006) defined the working capital as the company's ability to cover its short-term debt with current assets, calculating it as:

$$Working\ capital = Current\ assets - Current\ liabilities$$

Currents assets include (Pass and Pike, 2007):

1. Inventory (raw materials, work-in-progress, finished goods awaiting sale or delivery)
2. Debtors/accounts receivables (unpaid bills for which the profit has been realized in the accounts)
3. Trade credit
4. Cash in hand
5. Short-term securities

Current liabilities include (Pass and Pike, 2007):

1. Monies owned to trade creditors/ accounts payables (mainly for raw materials and other suppliers)
2. Bank overdrafts
3. Other short-term loan
4. Outstanding tax, dividend, and interest obligations

Jones's approach is widely known and used, however, there are others. (Pirtilla et al., 2014), for example, approaches the working capital from a cyclical view, focusing on the production to the sale of goods. This leads to a more operational evaluation of company's performance.

The formula in this case becomes as below:

$$Working\ capital = Inventories + Accounts\ receivable - Accounts\ payable.$$

Management of Working Capital (WCM) is essential for companies of any industry, as it has its impact on the profitability of the company. (Deloof, 2003) studies the influence of WCM on profitability of the Belgian firms, and reaches the conclusion, that reduction of accounts payables leads to growth in profitability. (Yazdanfar and Ohman, 2014) are performing a similar study across Swedish companies and come to the same conclusion – WCM does affect profitability of businesses. Thus, its management is essential.

There are several ways of reaching optimal levels of working capital. The first and major is inventory management. (Harrison, 1995), for example, mentions that companies are driving away the inventories to 3rd party vendors, up to the point of almost free inventory holding. There are lots of practices, invented and used throughout the world. Just-In-Time approach, created by Toyota, is aimed at synchronizing the material flows with production. Vendor-Managed Inventory, created by P&G and Walmart, transfers the responsibility of inventory management decisions upstream.

Another way of optimizing WC is changing the time periods, i.e. payment terms with suppliers and clients, as well as inventory turnover with the aforementioned practices. Cash conversion cycle is a widespread and recognized way of measurement of those time periods.

Introduced by (Richards and Laughlin, 1980), the concept of Cash Conversion Cycle is aimed at representing the dynamics in liquidity of the firm. The main idea of CCC is stating, that 'static balance sheet' ratios are misinterpreting the actual state of firm's financial position due to focusing only on periodic results.

In general terms, CCC represents, how much time it takes for the cash invested into raw materials to return to the company via consumer's purchase. It consists of three components:
DIO – Days Inventory Outstanding
DRO – Days Receivables Outstanding
DPO – Days Payables Outstanding

As a result, CCC could be found as follows:

$$CCC = DIO + DRO - DPO,$$

where:

$$DIO = \frac{Average\ Inventory}{Cost\ of\ Goods\ Sold} \times 365,$$

$$DRO = \frac{Average\ Accounts\ Receivables}{Revenue} \times 365,$$

$$DPO = \frac{Average\ Accounts\ Payables}{Cost\ of\ Goods\ Sold} \times 365.$$

Collaborative working capital. However, the Cash Conversion Cycle, introduced by Richards and Laughlin, is primary focused on the operations of only one company. Considering the goal of the paper and supply chains instead of separate companies being observed, the authors' approach requires 'an upgrade'. (Hofmann and Kotzab, 2010) have introduced such advanced concept, the Collaborative Cash Conversion Cycle (CCCC). Despite the authors focusing on the shareholders' value being influenced by collaboration across the supply chain, i.e. the stock price, the model they develop is useful for this research.

Via some calculations (especially, mutual exclusions of DPOs and DROs) it is possible to conclude, that the formula for CCCC is as follows:

$$CCCC = \sum_{i=1}^{n} DIO_i + DRO_n - DPO_1.$$

As seen from the formula, the internal payment processing times do not affect the overall liquidity of the investigated supply chain. However, changes in them may affect the WCs of separate companies in a supply chain. These changes become possible and mutual beneficial only with increased level of collaboration amongst members of a supply chain. Moreover, with such collaboration, it becomes possible to manage inventories in a more precise manner via JTI, VMI or other practices, cutting down individual and cumulative CCCs.

3. Joint Working Capital Cost Reduction and Redistribution Methodology

3.1. Working capital costs

Single company costs. Once the concept of CCCC is established, it is required to decide, how to calculate the costs burdening the company regarding its working capital.

One of the possible ways to calculate working capital costs of a single company was proposed by (Viskari and Karri, 2013):

$$FC = Inv \times \left[(1+c)^{\frac{DIO}{365}} - 1\right] + AR \times \left[(1+c)^{\frac{DRO}{365}} - 1\right] - AP \times \left[(1+c)^{\frac{DPO}{365}} - 1\right],$$

where:
Inv – value of inventory in the end of the year,
AR – value of accounts receivables in the end of the year,
AP – value of accounts payables in the end of the year,
DIO, DRO, DPO – components of CCC,
c – cost of capital.

As seen, the formula takes into consideration both the values of working capital components, their intensity over the time period, and the company-unique cost of capital. This means, that the resulting value will be corresponding with the specifics of each business, making the formula quite representative.

What's also worth mentioning is the resulting value of costs for the entire chain. If we consider supply chain participants as separate entities, then the total costs of working capital for the supply chain will be as follows:

$$FC_{SC} = \sum FC_{each\ participant}.$$

Considering the chosen solutions, it is also required to determine the way of calculation of commissions for their usage.

Factoring costs. As was previously mentioned, the commission which the factor gets for providing the service is a discount from the receivables factored. Therefore, it is logical to calculate it as the share of accounts receivables. As this research is aimed at a period overall instead of gradual influence, average of AR is used. Furthermore, the commission is based on the payment period of the 'buyer', just as a usual loan. If only factoring was examined in this research, then the mention of payment periods would be appropriate, as, most probably, the payment terms do not change for the buyer. However, as this is not the case and manipulation of terms is a vital part in the study, it seems proper to include it in the formula.

Thus, the cost of using factoring solution between 2 partners is calculated as follows:

$$FC_F = x \times AAR_1^0 \times DPO_2^0 \times r(f)_{daily},$$

where:
x – % of total financial flow between 2 companies, which is being factored

AAR_1^0 – value of average accounts receivables of the 1st company (supplier) before implementation of factoring
DPO_2^0 – Days Payables Outstanding of the 2nd company (buyer) before implementation of factoring
$r(f)_{daily}$ – factor's daily rate for services.

Factoring leads to changes in the values of accounts receivables, as well as the days receivables outstanding. The reason is in essence of the solution: supplier converts his receivables into cash instantly, thus both change by the amount factored. Thus, the values change:

$$AAR_1^1 = AAR_1^0 \times (1 - x),$$

$$DRO_1^1 = DRO_1^0 \times (1 - x),$$

where:
AAR_1^1 – value of average accounts receivables of the 1st company(supplier) after implementation of factoring
DRO_1^1 – days receivables outstanding of the 1st company (supplier) after implementation of factoring.

Reverse factoring costs. Calculation of commission for using reverse factoring is very much similar to usual factoring. However, due to the specifics of the research, some changes should be implemented. It would be quite logical to calculate the cost based on share of accounts receivables of the supplier. But firstly, the initiative is on the buyer's side, and secondly, the network studied is isolated, i.e. the there are no other companies. To illustrate, the second remark means that the sum of payables of the companies from left side of the network is equal to the receivables of the middle company. These terms are yet to be established later in the Chapter, however, considering this remark, it is logical to state that using *share of payables* is more appropriate in this case. Furthermore, the influence of payment terms is the same as for the factoring solution.

Thus, the cost of using reverse factoring solution between 2 partners is calculated as follows:

$$FC_{RF} = y \times AAP_2^0 \times DPO_2^0 \times r(rf)_{daily},$$

where:
y – % of total financial flow between 2 companies, being reverse factored by buyer,
AAP_2^0 – value of average accounts payables of the 2nd company (buyer) before implementation of reverse factoring,
DPO_2^0 – days payables outstanding of the 2nd company (buyer) before implementation of Factoring/Reverse Factoring,
$r(rf)_{daily}$ – factor's daily rate for services.

Unlike usual factoring, the reverse one leads to more changes, as both companies are influenced. Accounts receivables and days receivables outstanding of the supplier are changed as with usual factoring, but accounts payables and days payables outstanding of the buyer are changing as well. As was mentioned before, reverse factoring provides grounds for management of payment terms for the buyer. Therefore, for the purpose of the research it is appropriate to assume that the buyer pays out the factored sum immediately, thus lowering its accounts payables and days payables outstanding for the factored share.

This way, when reverse factoring is used, the following values change:
$AAP_2^1 = AAP_2^0 \times (1-y)$,
$DPO_2^1 = DPO_2^0 \times (1-y)$,
$AAR_1^1 = AAR_1^0 \times (1-y)$,
$DRO_1^1 = DRO_1^0 \times (1-y)$,
where:
AAP_2^1 – value of accounts payables of the 2nd company (buyer) after using reverse factoring,
DPO_2^1 – days payables outstanding of the 2nd company (buyer) after using reverse factoring,
AAR_1^1 – value of account receivables of the 1st company (supplier) after using reverse factoring,
DRO_1^1 – days receivables outstanding of the 1st company (supplier) after using reverse factoring.

Inventory financing costs. As was described earlier, inventory financing is about using a third-party logistics provider (LP) as a synthetic merchant, which buys and sells the goods between supplier and buyer at a predetermined price, receiving a commission. For sake of simplicity, the study assumes that payments and inventory (physically) are transferred immediately. Therefore, the commission received by the third party is based on the value of transferred inventory (*share of average*) and averaged turnover of the inventory. However, research does not revolve around how efficient inventory utilization is, but around how long the inventory stays obsolete. Therefore, it is more appropriate to use days inventory outstanding to determine how long it stays in the logistics provider warehouse. What is worth mentioning is that the 2nd company's value of inventory and days inventory outstanding does not influence the outcome, as physically, only disposition of inventory is changed, while the processes stay the same.

Thus, the cost of using inventory financing solution is calculated as follows:

$$FC_{IF} = z \times AI_1^0 \times DIO_1^0 \times r(IF)_{daily},$$

where:
z – % of average value of inventory of the 1st company (supplier) being financed,
AI_1^0 – value of average inventory of the 1st company (supplier) before implementation of inventory financing,
DIO_1^0 – days inventory outstanding of the 1st company (supplier) before implementation of Inventory Financing,
$r(IF)_{daily}$ – LP's daily rate for services.

As was mentioned, commission amount is not influenced by the buyer. However, this is applicable vice versa. The scheme of receiving inventory for the buyer stays the same, unless he decides to change it himself. However, this would require additional agreements with the logistics provider. Thus, by using inventory financing, no changes are applied to values of inventory or days inventory outstanding of the buyer. However, as the supplier is transferring the inventory to a third party from its own warehouse, the values of average inventory and days inventory outstanding alternate.

Thus, when inventory financing is used, the following variables change:
$AI_1^1 = AI_1^0(1-z)$,
$DIO_1^1 = DIO_1^0(1-z)$,

where:
AI_1^1 – average inventory of the 1st company (supplier) after implementation of inventory financing.
DIO_1^1 – days inventory outstanding of the 1st company (supplier) after implementation of inventory financing.

3.2. Cooperative game for reduction of joint working capital costs

Characteristic function.

Definition 1. The *characteristic function* of a game with a multitude of players N is the real function defined on all possible coalitions $S \subseteq N$, and for any pair of non-overlapping coalitions T, S ($T \subset N$, $S \subset N$) the superadditivity condition is satisfied:

$$v(T) + v(S) \leq v(T \cup S),\ v(\emptyset) = 0.$$

Superadditivity principle is a core concept in cooperative game theory, stating that two or more coalitions working together achieve a higher total payoff than they could achieve individually. Therefore, there is a motive for players to cooperate.

Considering the goal to both reduce and redistribute the working capital costs, the characteristic cost function is to be defined as follows:

$$v(S) = FC^0_{Total} - FC^1_{Total},$$

where:
FC^0_{Total} – total costs of the network before the game,
FC^1_{Total} – total costs of the network after the game.

As sometimes not all members are part of the coalition – it can be derived as: $FC^1_{Total} = FC_{Total}\,(S) + FC_{Total}(-S)$, where $(-S)$ is an additional coalition. This way the payoff for SC members is the cash saved by using the instrument. Further, the cash savings are divided among members of the coalition.

Characteristic function of the cooperative game therefore takes the following form:

$$v(S) = \max(FC^0_{Total} - FC^1_{Total}).$$

As seen, there is no conflict between the formed and additional coalitions. As the payoff is achievable only with using the instruments, the additional coalition simply doesn't have any leverage to impact the outcome.

Considering the stability of costs for the additional coalition $FC_{Total}(-S)$, FC^1_{Total} is to be decomposed further.

As was previously mentioned, $FC_{SC} = \sum FC_{each\ participant}$. Thus:

$$FC^1_{Total}(S) = \sum_{i=1}^{n} FC^1_i + FC_{F_i} + FC_{RF_i} + FC_{IF_i},$$

where: i – number of a member of a coalition S;
n - amount of member of coalition S;
FC^1_i – working capital costs of member i;
FC_{F_i} – costs for using factoring solution for member i;
FC_{RF_i} – costs for using reverse factoring solution for member i;
FC_{IF_i} – costs for using inventory financing solution for member i.

Game with combined topology. Pieces of a supply chain might be organized differently based on the distribution of members along the chain. Basically, there are three basic types of such distribution (Beamon and Chen 2001): distribution (divergent) network, fig. 3; assembly (convergent) network, fig. 4; sequential (Harland, 1996) – in the recent years often related to as *Stackelberg game* in papers related to integration of IT-systems (i.g. (Li and Qu, 2023)), fig.5. By *combining* these networks, a network with *combined* topology is created, fig.6.

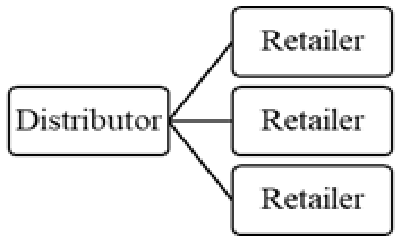

Fig. 3. Divergent network. Source: [created by author V. Novikov]

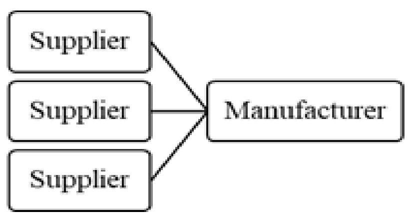

Fig. 4. Convergent network. Source: [created by author V. Novikov]

Fig. 5. Sequential network. Source: [created by author V. Novikov]

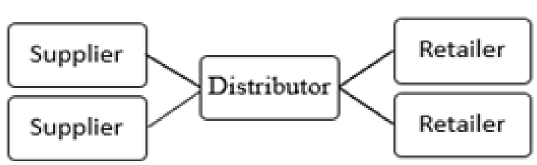

Fig. 6. Combined topology network. Source: [created by author V. Novikov]

These schemes are the topologies in focus of this research. There are 5 members of the chain: 2 suppliers, 1 distributor and 2 retailers. They are numbered by their respective position (column) and position within a column. For example, the higher supplier is numbered 1.1, the lower retailer – 3.2. The exact numbering is shown on the figure 7.

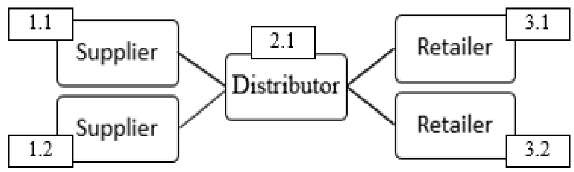

Fig. 7. The numbering of network members. Source: [created by author V. Novikov]

The network is assumed to be isolated, meaning there are no other companies connected to either of the listed. Without the creation of coalition a player may only use inventory financing. Factoring and revese factoring a not available. This leads to several limitations:

Limitation 1. $AAR_{Distributor} = \sum AAP_{Retailers}$;

Limitation 2. $AAP_{Distributor} = \sum AAR_{Suppliers}$;

Limitation 3.1. As there are no companies *before* the suppliers, they cannot use reverse factoring;

Limitation 3.2. As there are no companies *after* the retailers, they cannot use regular factoring.

Limitations 3.1 and 3.2 reveal the necessity to draw another limitation on usage of factoring and reverse factoring.

Limitation 4. If member 2.1 is not in coalition, usage of factoring or reverse factoring is impossible.

Management of working capital costs is a wide term, which can be perceived differently. However, in context of this research, the leverage used is the shares of accounts receivables, payables and inventory, which is being attributed to the instruments. Thus, making x (factoring), y (reverse factoring) and z (inventory financing) the variables of the model. This leads to another limitation:

Limitation 5. $0 \leq x \leq 1;\ 0 \leq y \leq 1;\ 0 \leq z \leq 1$, where 0 means not using the instrument at all and 1 – attributing the entire amount of AR, AP or inventory.

Once the topology is drawn and limitations established, it is possible to decompose the characteristic function fully. Characteristic function represents total savings of working capital for corresponding coalitions.

$$v(S) = \max(FC^0_{Total} - FC^1_{Total}) = \max(\textstyle\sum_{i=1}^{a}\sum_{j=1}^{b} FC^0_{ij} -$$
$$-(\textstyle\sum_{m=1}^{k}\sum_{n=1}^{l} FC^1_{mn} + FC_{F_{mn}} + FC_{RF_{mn}} + FC_{IF_{mn}} +$$
$$\textstyle\sum_{p=1}^{w}\sum_{q=1}^{r} FC^0_{pq}) = max\left(\sum_{i=1}^{a}\sum_{j=1}^{b} EI^0_{ij} \times \left[(1+c_{ij})^{\frac{DIO^0_{ij}}{365}} - 1\right] + \right.$$
$$+EAR^0_{ij} \times \left[(1+c_{ij})^{\frac{DRO^0_{ij}}{365}} - 1\right] - EAP^0_{ij} \times \left[(1+c_{ij})^{\frac{DPO^0_{ij}}{365}} - 1\right] -$$

$$- \Big(\sum_{m=1}^{k} \sum_{n=1}^{l} EI_{mn}^{1} \times \left[(1 + c_{mn})^{\frac{DIO_{mn}^{1}}{365}} - 1 \right] +$$
$$EAR_{mn}^{1} \times \left[(1 + c_{mn})^{\frac{DRO_{mn}^{1}}{365}} - 1 \right] - EAP_{mn}^{1} \times \left[(1 + c_{mn})^{\frac{DPO_{mn}^{1}}{365}} - 1 \right] +$$
$$+ x_{mn} \times AAR_{mn}^{0} \times DPO_{m+1,n}^{0} \times r\,(f)_{daily} +$$
$$+ y_{mn} \times AAP_{m+1,n}^{0} \times DPO_{m+1,n}^{0} \times r\,(rf)_{daily} +$$
$$+ z_{mn} \times AI_{mn}^{0} \times DIO_{mn}^{0} \times r\,(IF)_{daily} + \sum_{p=1}^{w} \sum_{q=1}^{r} EI_{pq}^{0} \times \left[(1 + c_{pq})^{\frac{DIO_{pq}^{0}}{365}} - 1 \right] +$$
$$+\ EAR_{pq}^{0} \times \left[(1 + c_{ij})^{\frac{DRO_{pq}^{0}}{365}} - 1 \right] - EAP_{pq}^{0} \times \left[(1 + c_{pq})^{\frac{DPO_{pq}^{0}}{365}} - 1 \right] \Big) \Big),$$

where

a, i, k, m, p, w – refers to relative position (column)

j, b, n, l, q, r – refers to position within the column

$a, b \in N; k, l \in S; p, q \in (-S)$

EI – value of inventory at the end of the year

EAR – value of accounts receivables at the end of the year

EAP – value of accounts payables at the end of the year

c – cost of capital of the company $i.j$ or $m.n$ or $p.q$.

Obviously, it is hard to perceive the calculation in general form as presented above. However, actual calculations are performed in Excel, making it way simpler.

As was mentioned before, x, y and z are the variables, unique to every member of the network. This way, after creation of a certain coalition, its members determine the values of the variables with the intent to minimize the costs of the coalition, thus maximizing the payoff. The additional coalition's variables values are always equal to 0, as without cooperation – there are no changes. Therefore, after choosing the values, the players achieve the characteristic function's value.

Technically, the values are determined via Excel solver, which maximizes the characteristic function above. An example of variables values is presented in the table 3.

Table 3. Example of variables values. Source: [created by author V. Novikov]

	1.1 (supplier 1)	1.2 (supplier 2)	2.1 (distributor)	3.1 (retailer 1)	3.2 (retailer 2)
x	0.00	0.29	0.25	0.00	0.00
z	0.54	0.52	0.51	0.49	0.50
y	0.00	0.00	0.23	0.46	0.20

The calculations of new values of working capital and CCC components are all made automatically via MS Excel. The final figures are presented in the table 4.

Table 4. Example of resulting value of characteristic function. Source: [created by author V. Novikov]

FC_1	122.1865
FC_0	3100.07
$v(s)$	2977.884

As it has already been stated, there are 5 players in the network. Via combinations, it is possible to determine, that there will be 31 possible coalitions in the network, each having its own *payoff.*

Examples of characteristic functions. The subparagraph is devoted to listing several characteristic functions not over-burdened by operators. To better realize the variety, which the general form of characteristic function provides, functions for some coalitions will be listed below.

Coalition of 1 supplier

This coalition consists of only 1 participant – the first supplier, fig. 8. Overall, in 2-1-2 structure there are 5 possible such one-element coalitions, and they all have one similarity – as this is a single-sided action, it is impossible for members to use solutions like factoring or reverse factoring, limiting the reduction only to inventory financing. Thus, x and y are equal to 0.

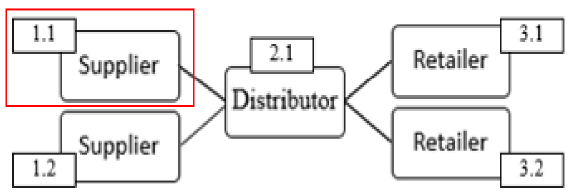

Fig. 8. The coalition of 1 supplier. Source: [created by author V. Novikov]

$$v\left(s\right) = \max\left(FC^0_{Total} - FC^1_{Total}\right) = \max(FC^0_{1.1} - (FC^1_{1.1} + FC_{IF_{1.1}})).$$

The reason for *totals* to be replaced by 1.1 is simple - FC^1_{Total} in this case consist of the changed costs for the 1st supplier and original costs for every other member of the network. Thus, they eliminate each other.

$$v\left(s\right) = \max(FC^0_{1.1} - (FC^1_{1.1} + FC_{IF_{1.1}})) =$$
$$= \max\left(EI^0_{1.1} \times \left[(1 + c_{1.1})^{\frac{DIO^0_{1.1}}{365}} - 1\right] - EI^1_{1.1} \times \left[(1 + c_{1.1})^{\frac{DIO^1_{1.1}}{365}} - 1\right] - \right.$$
$$\left. - z_{1.1} \times AI^0_{1.1} \times DIO^0_{1.1} \times r(IF)_{daily}\right).$$

As was mentioned before, x and y are equal to 0. Therefore, there are no changes to accounts receivables, accounts payables or their days outstanding, leading to their elimination from the function. It is possible to expand the function by substituting the changed values, but this will overburden the visual representation. However (as it will be explained in the paragraph 4.2):

$$AI = \frac{BI + EI}{2},$$

and as it was shown in the paragraph 3.1.4.:

$$AI^1_1 = AI^0_1(1 - z),$$

the functions $EI^1_{1.1}$ and $DIO^1_{1.1}$ are presented below:

$$EI^1_{1.1} = 2 \times AI^0_{1.1} \times (1-z) - BI^0_{1.1},$$

where $BI^0_{1.1}$ – value of inventory of company 1.1 in the beginning of the observed period.

$$DIO^1_{1.1} = DIO^0_{1.1} \times (1-z).$$

Coalition of 1 supplier and distributor (1.1, 2.1)

This coalition consists of 2 members – 1st supplier and the distributor, fig. 9. In this case, it is possible for supplier to use factoring (however, not possible for distributor, as none of the retailers are in coalition), and for distributor – reverse factoring. As was mentioned before, reverse factoring is locked for suppliers in any case. Thus, $y_{1.1}$ is equal to 0, $x_{2.1}$ is equal to 0.

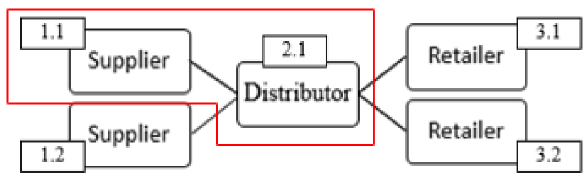

Fig. 9. The coalition of 1 supplier and the distributor. Source: [created by author V. Novikov]

$$\begin{aligned}
&v(s) = \max(FC^0_{1.1,2.1} - (FC^1_{1.1,2.1} + FC_{IF_{1.1}} + FC_{IF_{2.1}} + FC_{F_{1.1}} + FC_{RF_{2.1}})) = \\
&= \max\Bigg(EI^0_{1.1} \times \left[(1+c_{1.1})^{\frac{DIO^0_{1.1}}{365}} - 1\right] + EAR^0_{1.1} \times \left[(1+c_{1.1})^{\frac{DRO^0_{1.1}}{365}} - 1\right] - \\
&- EAP^0_{1.1} \times \left[(1+c_{1.1})^{\frac{DPO^0_{1.1}}{365}} - 1\right] + EI^0_{2.1} \times \left[(1+c_{2.1})^{\frac{DIO^0_{2.1}}{365}} - 1\right] + \\
&+ EAR^0_{2.1} \times \left[(1+c_{2.1})^{\frac{DRO^0_{2.1}}{365}} - 1\right] - EAP^0_{2.1} \times \left[(1+c_{2.1})^{\frac{DPO^0_{2.1}}{365}} - 1\right] - \\
&- \Bigg(EI^1_{1.1} \times \left[(1+c_{1.1})^{\frac{DIO^1_{1.1}}{365}} - 1\right] + EAR^1_{1.1} \times \left[(1+c_{1.1})^{\frac{DRO^1_{1.1}}{365}} - 1\right] - \\
&- EAP^1_{1.1} \times \left[(1+c_{1.1})^{\frac{DPO^1_{1.1}}{365}} - 1\right] + EI^1_{2.1} \times \left[(1+c_{2.1})^{\frac{DIO^1_{2.1}}{365}} - 1\right] + \\
&+ EAR^1_{2.1} \times \left[(1+c_{2.1})^{\frac{DRO^1_{2.1}}{365}} - 1\right] - EAP^1_{2.1} \times \left[(1+c_{2.1})^{\frac{DPO^1_{2.1}}{365}} - 1\right] + \\
&+ z_{1.1} \times AI^0_{1.1} \times DIO^0_{1.1} \times r(IF)_{daily} + z_{2.1} \times AI^0_{2.1} \times DIO^0_{2.1} \times r(IF)_{daily} + \\
&+ x_{1.1} \times AAR^0_{1.1} \times DPO^0_{2.1} \times r(f)_{daily} + y_{2.1} \times AAP^0_{2.1} \times DPO^0_{2.1} \times r(rf)_{daily}).
\end{aligned}$$

The *totals* are replaced in this function for the same reason, as in the previous example. As seen, even for two-player coalition the characteristic function becomes complex and long.

However, the algorithm is quite obvious:

1. Define the coalition;

2. Determine, which solutions are applicable;
3. Determine the function based the information from steps 1 and 2;
4. Minimize the function under the limitations stated before;
5. Determine the imputation best in the business context.

To determine the function, there are several checkpoints to focus on:

- If the player is in the coalition, there will always be the *original value* of costs and the value, determined by share of usage of given instruments.
- If the instrument is impossible to use due to coalition specifics or the limitations stated before – the corresponding value of the variable is equal to 0
- If the instrument is usable for a particular member of the coalition – the corresponding addendum should be in the function

3.3. Methods of fair allocation of joint working capital costs

After minimizing the costs of the network via maximizing the payoff of the coalition, the costs of each participant may be redistributed between the members to achieve the best individual payoffs for each participant. Such allocation among the coalition participants in real economic conditions could be realized through a decision of the governing board of representatives of companies in the supply chain.

Before determining the methods of imputations construction, it is required to highlight the importance of superadditivity principle, stated before. Adherence of the function to the principle is vital for any imputations to be present. However, it is related not exclusively to the function itself, but to the data used as well. As there no quantifiable requirements to the data, it is required to check for the adherence with every new set of data.

Core.

Definition 2. A set of non-dominating imputations of game (N, v) is Core.

For imputation α to be in the Core, it is required and enough for it to adhere to the following inequality for any possible coalition S (Petrosyan, Zenkevich, 2016):

$$v\,(S) \leq \alpha(S) = \sum_{i \in S} \alpha_i .$$

To check non-emptiness of the Core it is possible by using Excel solver once more. The variables in this case are the individual payoffs of coalition members. There are 31 conditions: 30 inequalities (for each coalition apart from grand coalition) and 1 equality (for grand coalition). For example:

$v\,(1.2, 2.1) \leq \alpha\,(1.2, 2.1)$ *(and 29 more, for each coalition)*

$v\,(1.1, 1.2, 2.1, 3.1, 3.2) = \alpha\,(1.1, 1.2, 2.1, 3.1, 3.2)\,.$

Shapley value. It is possible for the Shapley value to be included in the Core. If this is the case – it can be perceived as the most stable and just allocation of payoffs, as it is both reflective of the inputs and not worse than other imputations in the Core (Petrosyan, Zenkevich, 2016).

4. Evaluation of Effectiveness of Improved Framework.

4.1. Comparison of the improved framework to the existing.

Two papers can be drawn as examples for comparisons and evaluation.

The first paper is "The Three-Level Supply Chain Finance Collaboration under Blockchain: Income Sharing with Shapley Value Cooperative Game" by S. Li and S. Qu (Li and Qu, 2023). The authors are studying the impact of blockchain technology on Supply Chains in terms of its possibility of enhancement of FSC performance. Li and Qu propose a cooperative game, as well as this paper. However, this is the only similarity between the papers. Focus of the authors revolve around the flow of information and money within the supply chain in the context of a single order. The network used in the study comprises of three participants, not including the financing institutions. The characteristic function is determined based on the revenue increase, thus the payoff is the increase itself. The only imputation principle implemented is the Shapley Value, which can be a subject to be argued.

The second paper is "Shapley Value in Cooperative Working Capital Cost Game for Distributive Supply Network" by A. Ivakina, E. Lapin and N. Zenkevich (Ivakina et al., 2019). The authors aim at developing a methodology for reduction and redistribution of working capital costs. The papers are quite similar in terms of object, subject and approach, however, there are differences. First is the method of establishment of the cooperative game. The authors assume that non-cooperating members of the chain will form a coalition against the existing one. Therefore, the characteristic function becomes more complex, turning into a minimax function, i.e. the original coalition is acting to minimize the impact of additional coalition's actions (which are harmful to the original). This is far from reality, when decision to cooperate is voluntary, and the case of non-cooperation is only preventing the changes. Moreover, Ivakina, Lapin and Zenkevich do not take FSCM solutions into the function. One of the core differences between the papers is the choice of variables. This research uses 'shares of usage of FSC solutions', i.e. the percentages of financial items (average inventory, accounts receivables and payables) that are attributed to financial intermediary for factoring, reverse factoring and inventory financing. Thus, there is a direct connection 'exploiting instruments – changes in working capital components'. The research by Ivakina, Lapin and Zenkevich uses the component of Cash Conversion Cycle as variables, not considering *how* they are changed. Therefore, authors take a less in-depth point of view. In terms of the structure of the game – authors are able to construct a game only for a three–member chain, due to complexity of calculations, while this research manages to include five with potential for more. Regarding imputation principles in use – authors focus on Shapley value and its inclusion in Core, but don't calculate the Core itself. The comparison of papers is represented in the table 5.

As seen, there is a set of matters to compare, but in both cases the proposed framework is either equal or improved. Thus, it may be concluded that the framework is indeed improved.

4.2. Combined topology network case study.

The case represents a network of 5 players: 2 suppliers, 1 distributor and 2 retailers, on the period of a year. Members of the network are operating in the automotive industry. Suppliers are producers, which sell the cars to the distributor, which, in turn, passes the cars further to retailers for realization, fig. 10.

Table 5. The comparison of papers. Source: [created by author V. Novikov].

	This paper	S.Li & S. Qu	A. Ivakina, E. Lapin & N. Zenkevich
Payoff principle	Decrease in costs	Increase in revenue	The costs themselves (with no limitation on non-negativity)
Network	5 members & potential for more (2-1-2)	3 members (1-1-1)	3 members (1-1-1)
Variables	Rate of usage of FSC solutions	Prices within the chain	Components of CCC
Optimality principle	Core, Shapley Value (with Core belonging check)	Shapley Value	Shapley Value (with Core belonging check)

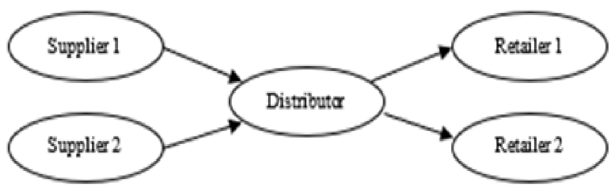

Fig. 10. Case study network. Source:[created by author V. Novikov]

There is factual data on all 5 participants for the beginning of the period and expected values for the end of the period for cost of capital, working capital components, cost of goods sold and revenue. The data are represented in the table 6.

Table 6. The original data. Source: [created by author V. Novikov]

	Supplier 1	Supplier 2	Distributor	Retailer 1	Retailer 2
WACC	8.4%	9.1%	8.9%	11.7%	10.8%
BI	2345	3856	5435.6	3004	6132
EI	2782	4202	5616.2	2876	6011
BAR	834	3245	4395	1543	2845
EAR	793	3403	4261	1403	2554
BAP	455	1452	4079	894	3501
EAP	632	1328	4196	765	3496
COGS	1623.25	3862.5	3738	2129	4105.75
Revenue	2179	4781	3564	2505.75	4829.5

where:
WACC – weighted average cost of capital (%);
BI – Inventory in the beginning of the period (year);
EI – Inventory in the end of the period (year);
BAR – Accounts Receivables in the beginning of the period (year);
EAR – Accounts Receivables in the end of the period (year);
BAP – Accounts Payables in the beginning of the period (year);

EAP – Accounts Payables in the end of the period (year);
COGS – Cost of Good Sold.

The exact unit of measurement of financial data, except WACC, is not exactly relevant. The reason is the research focuses on dynamics of the measures, rather than absolute levels. Thus, whether it be rubles, dollars, euros, etc., the study is not affected. To be usable in the model, it is required to calculate the averages and CCC components, which will serve as the *original* values, table 6. In this case, as data for only two time points is available, the averages are calculated as usual. For example, average inventory is calculated as:

$$AI = \frac{BI + EI}{2}.$$

The average values and CCC components are represented in the table 7.

Table 7. The average values and CCC components. Source: [created by author V. Novikov]

	Supplier 1	Supplier 2	Distributor	Retailer 1	Retailer 2
AI	2563	4029	5525.6	2940	6071.5
AAR	813.5	3324	4328	1473	2699.5
AAP	543.5	1390	4137.5	829.5	3498.5
DIO	550.84	391.4	539.57	510.64	538.03
DRO	131.01	247.5	443.25	209.66	206.06
DPO	116.79	135.03	404.03	144.07	310.02

The daily rates for commission for using the chosen instruments are represented in the table 8.

Table 8. Values of daily rates of commissions for using FSC instruments.

	Factoring	Reverse factoring	Inventory financing
Daily rate	0.0040%	0.0020%	0.0241%

By quick calculation, it is seen that the total working capital costs for the network is: $FC^0_{Total} = 3100,07$.

Maximum coalition payoff. Considering that there are 31 possible coalitions in such network, only values for the maximum coalition will be displayed. The characteristic function takes the following form:

$$v(N) = \max[\sum_{i=1}^{m=3}\sum_{j=1}^{l=2} FC^0_{i.j} - \sum_{i=1}^{m=3}\sum_{j=1}^{l=2}\left(FC^1_{i.j} + FC_{IF_{i.j}} + FC_{F_{i.j}} + FC_{RF_{i.j}}\right)].$$

Unfortunately, substitution of general form with the already known figures will not simplify the function for perception. The reason for this is that variables are present in two parts of each addendum. For example:

$$EI^1_{1.1} \times \left[(1 + c_{1.1})^{\frac{DIO^1_{1.1}}{365}} - 1\right] =$$

$$= (2 \times AI^0_{1.1} \times (1 - z) - \ BI^0_{1.1}) \times \left[(1 + c_{1.1})^{\frac{DIO^0_{1.1}\times(1-z)}{365}} - 1\right].$$

Thus, in the table 9 there are presented the values of the variables after maximization of the characteristic function of coalition N (maximum, all members of the network).

Table 9. Variables values for the maximum coalition. Source: [created by author V. Novikov]

	Supplier 1	Supplier 2	Distributor	Retailer 1	Retailer 2
x (Factoring)	0.00	0.29	0.25	0.00	0.00
z (Inventory Financing)	0.54	0.52	0.51	0.49	0.50
y (Reverse Factoring)	0.00	0.00	0.23	0.46	0.20

The numbers above represent the shares of corresponding components of working capital, attributed to the instruments. Accounts receivables – to x (Factoring), account payables – to y (Reverse factoring), inventory – to z (Inventory Financing). For instance, the value of $z_{1.1}$=0,54 means, that 54% of inventory of supplier 1 is attributed to using inventory financing solution.

The expected results for the period are represented in the table 10.

Table 10. The resulting values after minimization. Source: [created by author V. Novikov]

	Supplier 1	Supplier 2	Distributor	Retailer 1	Retailer 2
AI	1172.50	1928.00	2717.81	1502.00	3066.00
EI	0.00	0.00	0.00	0.00	0.00
AAR	813.50	2360.05	3250.37	1473.00	2699.50
EAR	793.00	1475.10	2105.73	1403.00	2554.00
AAP	543.50	1390.00	3173.55	447.00	2803.37
EAP	632.00	1328.00	2268.10	0.00	2105.73
DIO	251.95	187.30	265.39	260.88	271.70
DRO	131.01	175.73	332.89	209.66	206.06
DPO	116.79	135.03	309.90	77.64	248.42
FC	7.27	20.24	1.04	92.55	1.08

It is also possible to evaluate the percentage changes in the values, table 11.

Table 11. The percentage changes in values. Source: [created by author V. Novikov]

	Supplier 1	Supplier 2	Distributor	Retailer 1	Retailer 2
AI	-54%	-52%	-51%	-49%	-50%
EI	-100%	-100%	-100%	-100%	-100%
AAR	-0%	-29%	-25%	-0%	-0%
EAR	-0%	-56%	-50%	-0%	-0%
AAP	-0%	-0%	-23%	-46%	-20%
EAP	-0%	-0%	-46%	-100%	-0,39%
DIO	-54%	-52%	-51%	-49%	-50%
DRO	-0%	-29%	-25%	-0%	-0%
DPO	-0%	-0%	-23%	-46%	-20%
FC	-98%	-96%	-100%	-83%	-100%

It is noticeable, that percentage changes in AI, AAR and AAP correspond to the values of the variables. The reason for that is simple – as there are only two time points in the model, the change is discrete, instead of cumulative. For example, if the model consisted of 12 monthly periods with the game taking place each period, the cumulative change would not be equal to the variables' values (except for the first period).

Furthermore, it is possible to calculate the total costs and total payoff of the coalition, table 12.

Table 12. Total costs and payoff of the maximum coalition. Source: [created by author V. Novikov]

FC^1_{Total}	122.1865
% change of FC	-96%
$v(S)$	2977.884

Payoffs for all coalitions. As was mentioned, there are 31 possible coalitions. The payoffs for all the coalitions are calculated in order to proceed to checking for superadditivity and calculating imputations. The payoffs are presented in the table 13.

Table 13. Payoffs of each possible coalition. Source: [created by author V. Novikov]

	$v(S)$		$v(S)$
1	359.6084	1,2,4	1251.427
2	410.8085	1,2,5	1584.68
3	753.309	1,3,4	1625.911
4	481.0096	1,3,5	1983.449
5	814.2628	1,4,5	1654.881
1,2	770.4169	2,3,4	1702.548
1,3	1112.917	2,3,5	2120.208
1,4	840.618	2,4,5	1706.081
1,5	1173.871	3,4,5	2098.127
2,3	1164.118	1,2,3,4	2062.157
2,4	891.8181	1,2,3,5	2479.817
2,5	1225.071	1,2,4,5	2065.689
3,4	1266.303	1,3,4,5	2464.458
3,5	1617.118	2,3,4,5	2618.275
4,5	1295.272	1,2,3,4,5	2977.884
1,2,3	1523.726		

According to these data it is possible to check, that superadditivity is held.

Core. Next, Core is checked on non-emptiness. As it is a five-dimensional multitude, there is no benefit in trying to visualize it. Thus, proof that non-dominating payoffs exist can be displayed only by presenting at least one such payoff. In the table 14 there is an imputation form the Core. Consequently, the Core is not empty.

Table 14. An imputation, belonging to Core. Source: [created by author V. Novikov]

α_1	α_2	α_3	α_4	α_5
359.6	513.4	809.6	481.0	814.3

Shapley value. The Shapley value is represented in the table 15.

Table 15. Shapley value. Source: [created by author V. Novikov]

	1	2	3	4	5
$\phi_i[v]$	360.2	446.3	821.1	492.7	857.6

As was noted in Chapter 3, Shapley value might belong to the Core. The checking for compliance with conditions shows, that payoff of coalition {1.2,2.1,3.1,3.2} (2nd supplier, distributor, both retailers) is higher than the one proposed by Shapley vector. Therefore, it is not included in the Core. Such a result means, that if all five players constitutes the grand coalition and consider Shapley value as an imputation, players 1.2, 2.1, 3.1, 3.2 would disagree with such a proposition and leave the grand coalition, because for them it is more profitable to play in the coalition {1.2,2.1,3.1,3.2}.

5. Conclusion

The research is aimed at advancing the framework for reduction and redistribution of joint working capital costs in financial supply chains. It is achieved through creating a cooperative game to reduce the costs and using different approaches for finding imputations suitable to all players to redistribute the benefits acquired.

First, to determine the suitable FSC solutions, the review of main SCM concepts and solutions was conducted. The nature and scope of supply chains, financial supply chains and supply chain finance were determined. Consequently, solutions applicable to the research were chosen: factoring, reverse factoring and inventory financing.

Second, a game was developed, Core and Shapley value were chosen as methods for imputation construction.

Third, the game and imputations of costs were compared to the existing approaches. Furthermore, the game and imputation principles were implemented on a real-world example from automotive industry. The network consists of 2 suppliers, a distributor and 2 retailers. 31 possible coalitions' payoffs were calculated, Core and Shapley value for maximum coalition were calculated as well.

Theoretical implications. The main theoretical contribution of the research is a unique approach to constructing the game itself. For example, a very similar goal was established in the paper by (Ivakina et al., 2019). The authors build a cooperative game, but the additional coalition acted against the original coalition. This led to a more complex characteristic function, i.e. the min-max function, which, in turn, made the further calculations more complex. Consequently, the game was limited to a certain number of members of the chain, as further additions made the calculations more and more complicated. The approach taken in this research, lets the game be scalable to more players. This was demonstrated by the real example – (Ivakina et al., 2019) managed to build it only for 3 players, while this research advanced to 5 with possibility to expand further.

Managerial implications. Overall, the approach gives an instrument for members of the supply chain to manage financial flows between them. After minimization of the working capital costs via the characteristic function of the game, the payoff (which, in turn, is the cost reduction amount) can be distributed among participants in different ways. For instance, Shapley value gives the 'fair' imputation, as allocation is determined according to each members' input. Such approach may be used by managers when company's partners are already cooperative and aim just at being fair with each other. Core, in turn, is a more elastic way of payoff distribution. In case some partners require a higher share of payoff, Core may provide this flexibility. Thus, managers can balance out the requests to enhance the cooperation along the chain.

Limitations and grounds for future research. One of the base assumptions of the paper is that financial transactions are instant, which is not the case for real world. Thus, adding extra details is required to overcome this certain degree of simplification. Also, the paper wasn't taking into consideration the general industrial optimums for working capital components values, as this was not the aim of the research. However, it gives grounds to another research with an additional goal of reaching optimums.

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