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Dynamic Cournot Oligopoly Models of the State Promotion of Innovative Electronic Courses in Universities

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Abstract We built and investigated a two-level dynamic game theoretic model of control of the state promotion of innovative electronic courses in universities based on Cournot oligopoly. A federal state is the Principal, and universities competing a la Cournot are agents. The agents invest in the development of new electronic educative courses that is considered as their innovative investments. The Principal gives subsidies to the agents for the promotion of innovations. The agents play a dynamic game in normal form that results in a Nash equilibrium, and the Principal solves an inverse Stackelberg game (a Germeier game of the type Γ_{2t}). We investigated different types of strategies: (1) uniform strategies for all agents; (2) type-dependent (agent efficiency-dependent) strategies; (3) action-dependent strategies. For a specific form of the model functions we found a solution in explicit form, and in the general case we used a method of qualitatively representative scenarios in simulation modeling. We analyzed the results by means of the individual and collective relative efficiency indices.

Keywords: Cournot oligopoly, inverse dynamic Stackelberg games, simulation modeling, university management.

1. Introduction

A problem of promotion of innovations is very actual and is discussed in literature. A concept of innovation funnel is considered in (Bonazzi and Zilber, 2014, Hakkarainen, 2014). A review of the mathematical models of economics with innovations is presented in (Makarov, 2009). In (Cellini and Lambertini 2002) they analyzed a dynamic oligopoly where firms invest to increase product differentiation. They compare the steady state solutions under the open-loop and the closed-loop Nash equilibrium. The authors' approach to the dynamic game theoretic modeling of the promotion of innovations in universities is proposed in (Malsagov et al., 2020, Kaluza et al., 2010). In this paper we emphasize a comparative analysis of the solutions that correspond to a selfish agents behavior, their hierarchical organization and cooperation (Ougolnitsky, 2022). For a quantitative evaluation of the different ways of organization we use individual and collective indices of the relative efficiency. As differs from (Kaluza et al., 2010), we pay the principal attention to different types of subsidies for the promotion of innovations in universities. A method of qualitatively representative scenarios in simulation modeling (Ougolnitsky and Usov, 2018) is used together with known numerical methods.

2. Problem Formulation

We consider a difference inverse Stackelberg game of the type Principal - agents. However, the equation of dynamics is a differential one. Agents are universities

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competing a la Cournot: they develop electronic educative courses for sale. Resources allocated for this development are considered as innovative investments. The Principal is the federal state or its representative bodies (for example, Ministry of Education). The Principal exerts on the agents an economic influence (impulsion) by subsidies.

The model in the case of n agents has the following form:
- the Principal's payoff functional:

$$J_0 = \sum_{t=1}^{T} \delta^t \left(\chi \overline{x}_t - \sum_{i \in N} I(x_{it}) s_{it}(x_t) \right) + \delta^T y_T \to \max \tag{1}$$

- Principal's budget constraints:

$$s_{it}(x_t) \geq 0; \quad \sum_{j \in N} s_{jt}(x_t) \leq S_t; \quad t = 1, 2, ..., T; i \in N = \{1, 2, ..., n\} \tag{2}$$

- agents' payoff functionals:

$$J_i = \sum_{t=1}^{T} \delta^t ((D - \alpha \overline{x}_t) x_{it} - \frac{x_{it}^2}{2\left(r_i + \beta \sum_{j=1; j \neq i}^{n} I(x_{jt}) r_j\right)} - c_i I(x_{it}) + \tag{3}$$

$$I(x_{it}) s_{it}(x_t)) \to \max$$

- agents' control constraints

$$0 \leq x_{it} \leq x_{max}; \quad i = 1, 2, ..., n; \quad t = 1, 2, ..., T \tag{4}$$

- an equation of dynamics

$$y_{t+1} = y_t + \sum_{i \in N} k_i x_{it} - m y_t; \quad y(0) = y_0 \tag{5}$$

Here J_0, J_i - are payoffs of the Principal and the agents respectively; $i \in N$; $s_{it}(x_t)$- a subsidy from the Principal to the i-th agent; S_t – an annual Principal's budget; x_{it} - an output volume of the innovative product by the i-th agent in the moment of time t; $\overline{x}_t = \sum_{i=1}^{n} x_{it}$; $x_t = (x_{1t}, x_{2t}, ..., x_{nt})$; r_i - an agent's type that characterizes an efficiency of his technologies; $D, \chi, x_{max} > 0$; $\alpha, \beta \geq 0$ - model parameters; $\delta \in (0, 1)$ a discount factor; c_i- constant agent's cost; $I(x_{it})$ - indicator function;

$$I(x_{it}) = 0, \quad \text{if } x_{it} = 0 \ \text{ and } I(x_{it}) = 1, \text{ if } x_{it} \neq 0;$$

y_t — a general innovative level of the education system (a number of the used innovative products); m — a coefficient of decreasing of this level in the case when new innovative products are not developed; k_i - an impact coefficient for the i-th product; y_0- an initial value of the innovative level; T - a length of the game.

Thus, the model (1)–(5) is a difference inverse Stackelberg game (a Germeier game Γ_{2t}) that is similar to a continuous version described in (Malsagov et al., 2020).

The Principal chooses her open-loop strategies with a feedback on control s_{it} and reports them to the agents. Given the Principal's control mechanism, the agents choose their actions x_{it} so that to attain a Nash equilibrium in their game in normal form (3)–(4). The Nash equilibrium is treated as the agents' best response to the Principal's strategy. As the Principal anticipates the agents' best response, she chooses her strategies so that to solve the problem (1)–(2), (5) on the set of Nash equilibria in the game (3)–(4). If there are several Nash equilibria then the Principal uses the guaranteed result principle. The Principal's ϵ -optimal strategy together with any best response of the agents form a solution in the inverse Stackelberg game (Germeier game Γ_{2t}).

Notice that the agents in the model (1)–(5) are myopic, i.e. their payoff functionals may be rewritten as

$$J_i = \sum_{t=1}^{T} \delta^t J_{it};$$

$$J_{it} = (D - \alpha \overline{x}_t) x_{it} - \frac{x_{it}^2}{2\left(r_i + \beta \sum_{j=1;j\neq i}^{n} I(x_{jt}) r_j\right)} - c_i I(x_{it}) + I(x_{it}) s_{it}(x_t)$$

and their optimal values do not depend on the state value, or on the solution of a differential equation (5). Therefore we can pass from an optimization problem for the functional (3) for the i-th agent to the optimization problem for T functions in the form

$$J_{it} \to \max; t = 1, 2, ..., T \tag{6}$$

Each function (6) is maximized by the variable x_t at a fixed moment of time t subject to the constraints (4). Thus, each agent solves T optimization problems (6),(4).

3. Nash Equilibrium

Consider a case of the indifferent Principal without her own objectives. Assume that the Principal's strategies are linear functions of the agent's actions: $s_{it} = s_{it}(x_{it}) = \gamma_{it} x_{it};\ i = 1, 2, ..., n$. Then we receive a game of n agents (4)–(6) where a Nash equilibrium is built. For the i-th agent a maximal payoff is attained when $x_{it} = 0$, and then it is equal to zero, or when $x_{it} > 0$. Let us consider the latter case. Using a necessary first order condition

$$\frac{\partial J_{it}}{\partial x_{it}} = 0; i = 1, 2, ..., n,$$

in the case of symmetrical agents

$$c_i = c;\ x_{it} = x_t;\ r_{it} = r_t; \gamma_{it} = \gamma_t;\ J_i = J;\ J_{it} = J_t; i = 1, 2, ..., n$$

we receive an equation for determination of their stationary controls

$$\frac{\partial J_t}{\partial x_t} = D - 2\alpha n x_t - \frac{x_t}{(r + \beta(n-1)r)} + \gamma_t = 0 \tag{7}$$

Notice that

$$\frac{\partial^2 J_t}{\partial x_t^2} = -2\alpha n - \frac{1}{(r + \beta(n-1)r)} < 0.$$

Therefore, a solution of the equation (7) determines a maximum point, and if $x_t > 0$ then an optimal control of the agent is given by the formula

$$x_t^0 = \frac{(D + \gamma_t) r (1 + \beta(n-1))}{1 + 2\alpha n r (1 + \beta(n-1))}$$

In this case the agent's payoff is equal to

$$J = \sum_{t=1}^{T} \delta^t \frac{(D+\gamma_t)(r + r\beta(n-1))^2}{(1+2\alpha nr(1+\beta(n-1)))^2} \left((D+\gamma_t - c)\alpha n + \frac{D+\gamma_t}{2r(1+\beta(n-1))} \right) =$$

$$\sum_{t=1}^{T} \delta^t A_t$$

Thus, equilibrium strategies and payoffs of the agents are determined by the formulas

$$x_t^* = 0, \quad if \quad x_t^0 \leq 0 \quad or \quad A_t < 0 \quad \text{and} \quad x_t^* = x_t^0, \text{ otherwise} \tag{8}$$

$$J^* = \sum_{t=1}^{T} J_t; \quad J_t = \max\left(0, \delta^t A_t\right)$$

Therefore, the following proposition is proved.
Proposition 1. Formulas (8) determine a maximum point of the payoff functions (6) and payoffs of n symmetrical agents in a Nash equilibrium in the case of an indifferent Principal.

4. Cooperation of the Principal with Agents

In the case of cooperation of the Principal with n agents they form a grand coalition and solve together an optimal control problem with a payoff functional in the form

$$J^C = \sum_{t=1}^{T} \delta^t \left(\chi \overline{x}_t + \sum_{i\in N} \left((D - \alpha \overline{x}_t)x_{it} - \frac{x_{it}^2}{2\left(r_i + \beta \sum_{j=1;j\neq i}^{n} I(x_{jt})r_j\right)} - c_i I(x_{it}) \right) \right)$$

$$+\delta^T y_T \to \max \tag{9}$$

The maximum is searched by n functions $(x_{it})_{i=1}^n$ subject to the constraints (4) and equation of dynamics (5). The game is reduced to an optimal control problem.

If controls of all agents are equal to zero: $x_{it} = 0; i = 1, 2, ..., n$ then the coalitional payoff is equal to $\delta^T y_0(1-m)^T$.

Otherwise, for determination of the maximum in (9) a discrete Pontryagin maximum principle is used (Boltyanskii, 1978). An integrand in (9) is convex, the equation of dynamics is linear by the control variables that belong to a convex closed set. Therefore, for the solution of the problem (4),(5),(9) we can use a discrete Pontryagin maximum principle (Boltyanskii, 1978). A Hamilton function of the grand coalition has the form:

$$H_t\left(y_t, \lambda_{t+1}, x_t\right) = \delta^t \left(\chi \overline{x}_t + (D - \alpha \overline{x}_t)\overline{x}_t - \sum_{i\in N} \left(\frac{x_{it}^2}{2\left(r_i + \beta \sum_{j=1;j\neq i}^{n} r_j\right)} - c_i \right) \right)$$

$$+\lambda_{t+1} \left(\sum_{i\in N} k_i x_{it} + (1-m)y_t \right),$$

where λ_{t+1} is a conjugate variable. From the necessary condition of extremum we receive the system of n equations $i = 1, 2, ..., n$

$$\frac{\partial H_t}{\partial x_{it}} = \delta^t \left(\chi + D - 2\alpha \overline{x}_t - \sum_{i \in N} \left(\frac{x_{it}}{r_i + \beta \sum_{j=1;j\neq i}^{n} r_j} \right) \right) + \lambda_{t+1} k_i = 0 \qquad (10)$$

and for determination of the conjugate variable - a simple initial value problem

$$\lambda_t = (1-m)\lambda_{t+1}; \quad \lambda_T = \delta^T,$$

therefore,

$$\lambda_t = (1-m)^{T-t}\delta^T.$$

In general case the system of equations (10) is solved numerically. For n=2 the system takes the form

$$\frac{\partial H_t}{\partial x_{1t}} = \delta^t \left(\chi + D - 2\alpha(x_{1t} + x_{2t}) - \frac{x_{1t}}{r_1 + \beta r_2} \right) + \lambda_{t+1} k_1 = 0;$$

$$\frac{\partial H_t}{\partial x_{2t}} = \delta^t \left(\chi + D - 2\alpha(x_{1t} + x_{2t}) - \frac{x_{2t}}{r_2 + \beta r_1} \right) + \lambda_{t+1} k_2 = 0.$$

Its solution gives

$$x_{1t}^0 = \frac{A_{1t} - B_{1t}}{2\alpha + 1 + 1/(2\alpha(r_1 + \beta r_2))};$$

$$x_{2t}^0 = \frac{A_{2t} - B_{2t}}{2\alpha + 1 + 1/(2\alpha(r_2 + \beta r_1))}$$

where

$$A_{1t} = \chi + D + k_2(1-m)^{T-t-1}\delta^{T-t}; A_{2t} = \chi + D + k_1(1-m)^{T-t-1}\delta^{T-t};$$

$$B_{1t} = \left(2\alpha + \frac{1}{r_2 + \beta r_1}\right)\left(\frac{\chi + D}{2\alpha} + \frac{k_1(1-m)^{T-t-1}\delta^{T-t}}{2\alpha}\right);$$

$$B_{2t} = \left(2\alpha + \frac{1}{r_1 + \beta r_2}\right)\left(\frac{\chi + D}{2\alpha} + \frac{k_2(1-m)^{T-t-1}\delta^{T-t}}{2\alpha}\right).$$

The found pair of points (x_{1t}^0, x_{2t}^0) is a maximum point of the Hamilton function for positive controls x_{1t}, x_{2t}. Really,

$$\frac{\partial^2 H_t}{\partial x_{1t}^2} = -\delta^t \left(2\alpha + \frac{1}{r_1 + \beta r_2}\right) = E < 0; \quad \frac{\partial^2 H_t}{\partial x_{2t}^2} = -\delta^t \left(2\alpha + \frac{1}{r_2 + \beta r_1}\right) = F < 0;$$

$$\frac{\partial^2 H_t}{\partial x_{1t} \partial x_{2t}} = -2\alpha\delta^t = G < 0; \quad \Delta = E\ F - G^2 > 0; \quad E < 0.$$

Therefore, a maximum of the Hamilton function subject to the control constraints (4) is attained in one of the vector points

$$(x_{1t}^0, x_{2t}^0); \ (x_{1t}^0, 0); \ (0, x_{2t}^0); \ (0), 0); \qquad (11)$$

$$(x_{1t}^0, x_{max}); \ (x_{max}, x_{2t}^0; \ (x_{max}, x_{max})$$

and the following proposition is proved.

Proposition 2. Formulas (11) determine a point of maximum of the Hamilton function in the case of cooperation of the Principal with two agents.

5. Solution of the Inverse Stackelberg Game (Germeier Game Γ_{2t})

From the point of view of the Principal her interaction with agents is described by an inverse Stackelberg game (Germeier game Γ_{2t}). An algorithm of solution of this game is based on the approach proposed in (Ugolnitskii and Usov, 2014, 2016).

1. A strategy of punishment by the Principal of the agents who refuse to cooperate with her is calculated:

$$x_{it}^P(\{s_{it}^P\}_{t=1}^T) = arg \max_{0 \le x_{it} \le x_{max}} J_i(\{s_{it}\}_{t=1}^T, \{x_{it}\}_{t=1}^T);$$

$$\{s_{it}^P\}_{t=1}^T = arg \min_{s_{it} \ge 0; \sum_{i \in N} s_{it} \le S_t} J_i(\{s_{it}\}_{t=1}^T, \{x_{it}\}_{t=1}^T).$$

If an agent refuses to cooperate then his guaranteed payoff is equal to ($i = 1, 2, ..., n$)

$$L_i = J_i(\{s_{it}^P\}_{t=1}^T, \{x_{it}^P\}_{t=1}^T) =$$

$$\max_{0 \le x_{it} \le x_{max}} \min_{s_{it} \ge 0;\ \sum_{i \in N} s_{it} \le S_t} J_i(\{s_{it}\}_{t=1}^T, \{x_{it}\}_{t=1}^T)$$

and is determined by the formula similar to (8) when $s_{it} = 0; i = 1, 2, \ldots, n; t = 1, 2, \ldots, T$.

2. An optimal control problem (1), (2), (4), (5) is solved with conditions

$$L_i < J_i(\{s_{it}\}_{t=1}^T, \{x_{it}\}_{t=1}^T); \quad i = 1, 2, \ldots, n. \tag{12}$$

A maximum is searched by two grid functions $\{s_{it}\}_{i,t=1}^{n(T)}, \{x_{it}\}_{i,t=1}^{n(T)}$. Denote a solution of this optimal control problem by $\{s_{it}^R\}_{t=1}^T, \{x_{it}^R\}_{t=1}^T$, where $\{s_{it}^R\}_{t=1}^T$ is a strategy of reward of the i-th agent when he chooses $\{x_{it}^R\}_{t=1}^T$.

3. The Principal reports to each agent a strategy with a feedback on his action:

$$s_{it} = s_{it}^R, \quad if \quad x_{it} = x_{it}^R \ \text{ and } \ s_{it} = s_{it}^P, \text{ otherwise}$$

The condition (12) provides that for the agents a reward strategy is more profitable than a punishment strategy. Thus, the solution has the form $(\{s_{it}^R\}_{t=1}^T, \{x_{it}^R\}_{t=1}^T)$.

A solution of the inverse Stackelberg game (Germeier game Γ_{2t}) is built numerically by means of the method of qualitatively representative scenarios in simulation modeling (QRS SM method) (Ougolnitsky and Usov, 2018).

The QRS SM method is based on the idea that for evaluation of the consequences of control impacts on a dynamic system it is sufficient to consider a small number of control scenarios that reflect qualitatively different variants of the impact.

Assume that

$$\Omega = S_1 \times \ldots \times S_n \times X_1 \times \ldots \times X_n.$$

Here

$$S_i = (s_i \ge 0; \sum_{i=1}^n s_i \le S); X_i = (x_i \ge 0), i = 1, 2, \ldots, n$$

are the sets of feasible controls of the Principal and agents.

Definition (Ougolnitsky and Usov, 2018). A set

$$QRS = S^{QRS} \times X^{QRS} = S_1^{QRS} \times S_2^{QRS} \times \ldots \times S_n^{QRS} \times X_1^{QRS} \times X_2^{QRS} \times \ldots \times X_n^{QRS} =$$

$$\{(s,x)=(s_1,\ldots,s_n;x_1,\ldots,x_n);s_i\in S_i^{QRS}\in S_i;x_i\in X_i^{QRS}\in X_i\}$$

is a QRS set in a Stackelberg game with precision Δ If:
(a) for any two elements $(s,x)^{(i)},(s,x)^{(j)}\in QRS \quad |J_0^{(i)}-J_0^{(j)}|>\Delta$;
(b) for any element $(s,x)^{(l)}\notin QRS$ there is an element $(s,x)^{(j)}\in QRS$ such that $|J_0^{(l)}-J_0^{(j)}|\le\Delta$.

An algorithm of solution of the inverse Stackelberg game (Germeier game Γ_{2t}) by means of the QRS SM method has the following form.

1. An initial set $QRS^{(k)}$ has the form $(k=0)$

$$QRS^{(k)}=(S^{QRS})^{(k)}\times(X^{QRS})^{(k)};$$

$$(S^{QRS})^{(k)}=(S_1^{QRS})^{(k)}\times(S_2^{QRS})^{(k)}\times...(S_n^{QRS})^{(k)};$$

$$(X^{QRS})^{(k)}=(X_1^{QRS})^{(k)}\times(X_2^{QRS})^{(k)}\times...(X_n^{QRS})^{(k)};$$

$$(S_i^{QRS})^{(k)}=\{s_1^{(k)};s_2^{(k)};s_3^{(k)}\};(X_i^{QRS})^{(k)}=\{x_1^{(k)};x_2^{(k)};x_3^{(k)}\};$$

$$s_1^{(k)}=0;s_2^{(k)}=s_{max}/2;s_3^{(k)}=s_{max};\;\;x_1^{(k)}=0;x_2^{(k)}=x_{max}/2;x_3^{(k)}=x_{max},$$

where values s_{max},x_{max} are big enough and are chosen specifically for each control system.
2. The set $QRS^{(k)}$ contains 3^{2N} elements. All of them are checked for satisfaction of both conditions in the mentioned definition of a QRS set. If it is necessary then an initial set $QRS^{(k)}$ is reduced or extended by new elements.
3. A strategy of punishment of the agent who refuses to cooperate with the Principal is found. First, by enumeration of the strategies from the set $(X^{QRS})^{(k)}$ Nash equilibria for a given Principal's control $NE^{QRS}((S^{QRS})^{(k)})$ are found. Then a guaranteed payoff of the i-th agent who refuses to cooperate with the Principal is calculated:

$$L_i^P=\max_{x_i\in NE^{QRS}((S^{QRS})^{(k)})}\;\min_{s_i\in(S^{QRS})^{(k)}}J_i(s_i,x_i).$$

4. By the complete enumeration of the qualitatively representative strategies of the Principal from $(S^{QRS})^{(k)}$ and the agents from $(X^{QRS})^{(k)}$ a maximum in the problem (1), (2), (4) with conditions $J_i>L_i^p(i=1,2,...,n)$ is found.

The values that provide the maximum form a k-th approximation to the solution of the game. Denote them by $(s^R)^{(k)},(x^R)^{(k)}$.
5. The QRS sets of the Principal and the agents $(k:=k+1)$ are refined in the vicinity of the built equilibrium as follows.

if $(s_i^*)^{(k-1)}=s_1^{(k-1)}$, then $s_1^{(k)}=s_1^{(k-1)};s_2^{(k)}=(s_1^{(k-1)}+s_2^{(k-1)})/2;s_3^{(k)}=s_2^{(k-1)}$.

If $(s_i^*)^{(k-1)}=s_2^{(k-1)}$, then $s_1^{(k)}=(s_1^{(k-1)}+s_2^{(k-1)})/2;s_2^{(k)}=s_2^{(k-1)};s_3^{(k)}=(s_2^{(k-1)}+s_3^{(k-1)})/2$.

If $(s_i^*)^{(k-1)}=s_3^{(k-1)}$, then $s_1^{(k)}=s_2^{(k-1)};s_2^{(k)}=(s_2^{(k-1)}+s_3^{(k-1)})/2;s_3^{(k)}=s_3^{(k-1)}$.

New sets $QRS^{(k)}$ for the agents are built similarly.

If at an iteration we receive that $(s_i^R)^{(k)}=(s_i^R)^{(k-1)};\;(x_i^R)^{(k)}=(x_i^R)^{(k-1)};\;i=1,2,\ldots,n$ then a solution of the game by means of the QRS SM method is built. Otherwise, go to step 2 of the algorithm.

6. Numerical Calculations

We considered the following types of the Principal's strategies (her subsidies to the agents):

(a) uniform subsidies to all agents in a fixed moment of time. If an output volume of the innovative product for all agents is positive then $\forall i \;\; s_{it} = s > 0$, otherwise $\forall i \;\; s_{it} = 0$;

(b) type-dependent (agent efficiency-dependent) strategies $s_{it} = s_i(r_{it})$; a linear dependency is used $s_i(r_{it}) = \alpha_i r_{it}$; constants α_i are to be determined;

(c) action-dependent strategies $s_{it} = s_i(x_{it})$; a linear dependency is also used $s_i(x_{it}) = \theta_i x_{it}$; constants θ_i are to be determined.

All computer simulations were conducted on a personal computer with a processor AMD Ryzen 5 3550H with operative memory 8 Gb by means of an object-oriented programming language C++. An average time of one computer simulation for determination a QRS set is less than one second.

An analysis of the received results was based on the following indicators:
(1) a total discounted payoff of the Principal;
(2) values of the individual and collective relative efficiency indices (Ougolnitsky, 2022).

The collective relative efficiency indices demonstrate a need in a hierarchical control in a dynamic system. The closer are their values to one, the better the system is coordinated, and a hierarchical control by the Principal is less actual.

In the computer simulations we varied the following parameter values:
1. χ from 0.01 to 3;
2. D from 5 to 100;
3. A from 0.001 to 0.1 year/mln.rub.;
4. $r_{1,2}$ from 0.5 to 50 thousand rub./year;
5. $c_{1,2}$ from 50 to 1000 mln.rub./year;
6. β from 0.01 to 0.6;
7. m from 0.0001 to 0.11/year;
8. $k_{1,2}$ from 0.001 to 0.051/year;
9. y_0 from 30 to 500 mln.rub./year;
10. S_t from 100 to 500 mln.rub./year.

Input data for numerical calculations are presented in Table 1. The results of calculations for these data and $T = 6; n = 2$ for different control scenarios are given in Table 2. The upper index in the values $J_0^{(k)}, J_1^{(k)}, J_2^{(k)}$ stands for a type of scenario, namely: (a) - uniform subsidies; (b) - type-dependent strategies; (c) - action-dependent strategies; $J^{(C)}$ denotes a payoff of the coalition of the Principal with agents in the case of their cooperation.

For a comparative analysis of the different scenarios of the Principal's control we used a system of the individual and collective relative efficiency indices (Ougolnitsky, 2022). The collective relative efficiency indices correlate the values of social welfare (a total payoff of all players) for different scenarios with the maximal value of social welfare that is attained in the case of cooperation of all players: $SCI = \sum_{i=0}^{n} J_i / J^C$. Here J_i is a payoff of the respective agent in a specific scenario (a),(b),(c) of the Principal's control, and J^C is a cooperative payoff of the grand coalition in the case of cooperation.

The individual relative efficiency indices correlate payoffs of the agents in a specific scenario (a),(b),(c) of the Principal's control with their symmetrical coop-

Table 1. Input data for numerical calculations

N	D	r_1	r_2	c_1	c_2	y_0	k_1	k_2	χ	α	β	m
1	200	20	30	500	700	200	0.02	0.4	1	0.2	1	0.03
2	200	60	50	300	400	200	0.03	0.3	1	0.1	0.8	0.03
3	200	20	70	350	200	100	0.04	0.06	0.5	0.3	0.1	0.03
4	200	60	20	200	400	100	0.04	0.03	0.5	0.25	0.5	0.03
5	200	30	10	600	500	200	0.01	0.05	1	0.05	0.3	0.05
6	200	20	50	400	300	200	0.05	0.02	0.5	0.1	1.4	0.05
7	300	40	60	200	500	200	0.12	0.2	1	0.12	0.8	0.05
8	300	20	10	400	300	200	0.1	0.1	1.5	0.15	0.6	0.1
9	300	40	20	500	600	50	0.01	0.03	0.4	0.05	0.7	0.02
10	300	20	10	200	500	50	0.07	0.01	0.1	0.1	1	0.02
11	300	50	30	400	300	50	0.05	0.01	0.3	0.1	0.8	0.01
12	500	20	10	400	300	200	0.03	0.25	0.4	0.2	1	0.03
13	500	10	50	400	200	200	0.07	0.03	0.3	0.2	0.8	0.03
14	500	40	25	200	500	200	0.01	0.03	0.2	0.1	1.2	0.03
15	500	30	50	400	300	200	0.05	0.3	0.1	0.5	1.5	0.03
16	500	20	40	100	500	200	0.2	0.1	0.3	0.3	1	0.03
17	500	15	10	150	100	200	0.1	0.5	0.1	0.4	0.5	0.02
18	200	20	30	450	350	100	0.06	0.04	0.1	0.2	2	0.01
19	200	60	25	500	450	100	0.01	0.03	0.4	0.15	1	0.01
20	200	10	40	400	300	100	0.03	0.02	0.3	0.2	1	0.01
21	200	30	40	500	700	100	0.01	0.05	0.5	0.05	2	0.01
22	200	40	50	600	300	100	0.02	0.04	0.3	0.1	2	0.01
23	200	15	20	400	600	200	0.01	0.08	0.7	0.2	1	0.02
24	100	50	40	300	400	200	0.05	0.01	0.8	0.2	2	0.02
25	100	50	15	500	600	200	0.01	0.06	0.5	0.1	2	0.01
26	100	30	5	500	300	200	0.05	0.08	0.5	0.1	1	0.02
27	100	5	20	400	500	200	0.03	0.06	0.5	0.2	1	0.02
28	100	10	5	300	400	200	0.02	0.05	0.5	0.1	2	0.01
29	100	20	10	400	700	100	0.07	0.05	1	0.3	1	0.01
30	100	30	10	600	500	100	0.05	0.02	0.5	0.1	1.5	0.02
31	100	10	20	500	700	100	0.05	0.01	1.5	0.05	1.2	0.01
32	100	20	30	500	400	100	0.01	0.02	1	0.2	1	0.01
33	50	20	25	500	400	50	0.01	0.02	0.9	0.23	1.4	0.01
34	50	30	5	500	600	50	0.03	0.05	1.4	0.25	1.7	0.01
35	50	5	10	700	400	100	0.02	0.04	1.7	0.15	1.5	0.01
36	50	5	20	400	600	50	0.05	0.03	1.2	0.2	1.2	0.02
37	50	10	5	500	400	100	0.01	0.02	1.5	0.15	1	0.02
38	50	20	15	500	600	100	0.05	0.01	1.5	0.15	0.5	0.01
39	50	15	10	500	400	50	0.03	0.05	1	0.1	1	0.02
40	50	10	15	700	500	50	0.01	0.03	1.2	0.15	0.7	0.02

Table 2. Results of the numerical calculations

N	J^c	$J_0^{(a)}$	$J_1^{(a)}$	$J_2^{(a)}$	$J_0^{(b)}$	$J_1^{(b)}$	$J_2^{(b)}$	$J_0^{(c)}$	$J_1^{(c)}$	$J_2^{(c)}$
1	58756	1018	25434	24834	1260	25445	25065	1450	25475	25225
2	60027	868	27572	27371	1380	28042	27311	1300	27612	27662
3	53185	335	24320	24857	897	24331	25408	767	24361	25248
4	54733	285	25605	24984	767	26076	24995	717	25646	23375
5	62544	378	27345	27561	610	27576	27562	810	27386	27952
6	59429	238	27267	27540	640	27278	27961	670	27307	27961
7	89206	638	42566	41662	1140	45525	42139	1070	42557	42059
8	88442	568	41477	41748	880	41788	41748	1000	41518	42139
9	92185	938	42689	42381	560	43000	42391	670	42730	42771
10	89611	248	42784	41884	400	42935	41885	680	42825	42275
11	89601	408	42258	42255	620	42649	42576	840	42300	42946
12	146967	605	70684	70984	667	70735	71735	1036	70735	71735
13	146306	275	70734	71344	667	70721	71708	707	70702	71631
14	149337	145	72855	71957	472	73166	71972	577	72896	72348
15	137793	631	66273	66569	1043	66294	66960	1063	66314	66560
16	143530	608	70146	68946	930	70157	69257	1040	70188	69337
17	140595	1547	68371	68494	1660	68482	68496	1980	68412	6888
18	57034	548	25612	25906	790	25623	26136	980	25653	26296
19	56674	338	26214	26364	825	26685	26381	770	26255	26755
20	56674	358	25734	26034	670	25735	26345	790	25775	26425
21	62622	468	27724	27121	800	27746	27432	900	27765	27512
22	60146	408	26682	27580	830	26713	27971	840	26723	27971
23	57653	453	25701	25101	610	25707	25252	885	25742	25492
24	26546	408	11080	10782	830	11471	10813	840	11121	11173
25	30908	518	11962	11676	915	12353	11682	950	12003	12067
26	30312	493	11902	12502	720	12133	12498	925	11943	12893
27	27544	393	10659	10359	540	10655	10510	825	10700	10750
28	29975	518	12421	12159	585	12493	12155	950	12462	12550
29	25481	918	9184	8284	1070	9335	8285	1350	9225	8675
30	31258	643	11601	11859	875	11832	11860	1074	11643	12250
31	32984	678	12665	12051	830	11267	11202	1109	12704	12442
32	27306	468	10434	10734	710	10445	10965	900	10475	11125
33	8360	438	2491	2788	640	2501	2979	870	2532	3179
34	7944	838	2162	1886	1067	2163	2091	1270	2203	2283
35	10060	828	2972	3845	895	2968	3916	1260	3013	4236
36	9986	578	3180	2564	725	3176	2716	1009	3221	2956
37	10288	543	3509	3809	610	3580	3805	975	3550	4200
38	11842	768	3622	3309	760	3066	4011	1200	3663	3700
39	11512	518	4359	4659	630	4470	4660	950	4400	5050
40	9694	328	3025	3620	440	3026	3611	760	3066	4011

erative payoffs: $K_i = (n+1)J_i/J^C; i = 0,1,2$. It is supposed that all payoffs are non-negative. The received values of relative efficiency indices are presented in Table 3.

The last row of the Table 3 contains average values of the indices. Thus, we receive the following preference systems:

society: $C \succ (c) \succ (b) \succ (a)$;

Principal: $C \succ (c) \succ (b) \succ (a)$;

agents: $(b) \sim (c) \succ (a) \succ C$;

Thus, the whole society and the Principal prefer cooperation, and the agents (followers) prefer type-dependent or action-dependent subsidies.

Besides, the following conclusions are made.

1. A parameter χ characterizes a dependency of the Principal's payoff on a total output volume of the innovative products. If its value increases then the Principal's payoff increases linearly for all types of subsidies. The agents' payoffs do not change.

2. If demand parameters D and α increase then the agents' payoffs increase exponentially. The Principal's payoff does not change.

3. If an agent's type changes (an efficiency of his technologies increases or decreases) then his payoff changes slightly. For example, if the efficiency increases twice then the payoff increases on 10% approximately. The Principal's payoff does not change.

4. If the agents' costs increase then their payoffs expectably fall.

5. Remind the parameters of the equation of state dynamics: m - a coefficient of decreasing of the innovative level; k_i - an impact coefficient for the i-th product. If these parameters change then the agents' payoffs do not change. However, the Principal's payoff decreases when m increases, and increases abruptly when k_i increase. Also, it increases together with an initial value of the innovative level.

7. Conclusion

We built and investigated a two-level control system aimed at promotion of innovations in the universities competing a la Cournot. The system is formalized as a difference inverse Stackelberg game (Germeier game Γ_{2t}) of the type Principal-agents with a differential equation of dynamics. Based on a discrete Pontryagin maximum principle, for a specific class of model functions in the case of an indifferent Principal we found analytically a Nash equilibrium in the game of agents in normal form. An algorithm of solution of the inverse Stackelberg game (Germeier game Γ_{2t}) is proposed and implemented on the base of the method of qualitatively representative scenarios in simulation modeling. The received results allowed for some conclusions given above. The main conclusion is that the whole society and the Principal prefer cooperation, and the agents (followers) prefer type-dependent or action-dependent subsidies.

Universities are often myopic that we considered in the model. That's why a promotion of innovations advocates for the interested Principal who provides innovations by means of subsidies to the agents. The Principal's direct payoff may be quite small.

In the future we suppose to investigate the considered model in a cooperative dynamic game theoretic setup with different characteristic functions and to conduct a comparative analysis.

Table 3. The values of relative efficiency indices for different scenarios: (a) uniform strategies; (b) type-dependent strategies; (c) action-dependent strategies

N	$SCI^{(a)}$	$K_0^{(a)}, K_1^{(a)}, K_2^{(a)}$	$SCI^{(b)}$	$K_0^{(b)}, K_1^{(b)}, K_2^{(b)}$	$SCI^{(c)}$	$K_0^{(c)}, K_1^{(c)}, K_2^{(c)}$
1	0.87	0.05/1.3/1.27	0.88	0.06/1.29/1.28	0.88	0.07/1.3/1.29
2	0.93	0.04/1.38/1.87	0.95	0.07/1.39/1.36	0.94	0.06/1.38/1.39
3	0.99	0.02/1.31/1.4	0.95	0.05/1.37/1.43	0.95	0.04/1.37/1.42
4	0.93	0.02/1.4/1.4	0.95	0.04/1.43/1.37	0.91	0.04/1.41/1.28
5	0.88	0.02/1.31/1.32	0.89	0.03/1.32/1.32	0.9	0.04/1.31/1.34
6	0.93	0.01/1.38/1.39	0.94	0.03/1.38/1.41	0.94	0.03/1.38/1.41
7	0.95	0.02/1.43/1.40	0.99	0.04/1.53/1.42	0.96	0.04/1.43/1.41
8	0.95	0.02/1.41/1.42	0.95	0.03/1.42/1.41	0.96	0.04/1.43/1.41
9	0.97	0.03/1.39/1.38	0.93	0.02/1.4/1.38	0.93	0.02/1.39/1.39
10	0.95	0.01/1.43/1.4	0.95	0.01/1.44/1.4	0.96	0.02/1.43/1.42
11	0.95	0.01/1.42/1.41	0.96	0.02/1.43/1.43	0.96	0.03/1.42/1.44
12	0.97	0.01/1.44/1.45	0.97	0.01/1.44/1.46	0.98	0.02/1.44/1.46
13	0.97	0.01/1.45/1.44	0.97	0.01/1.44/1.46	0.97	0.01/1.44/1.46
14	0.97	0.01/1.46/1.45	0.92	0.01/1.47/1.45	0.98	0.01/1.46/1.45
15	0.97	0.01/1.44/1.45	0.97	0.02/1.48/1.46	0.97	0.02/1.44/1.45
16	0.97	0.01/1.47/1.44	0.98	0.02/1.47/1.45	0.98	0.02/1.47/1.45
17	0.98	0.03/1.46/1.47	0.98	0.04/1.46/1.47	0.99	0.04/1.46/1.48
18	0.91	0.03/1.35/1.36	0.92	0.04/1.35/1.37	0.93	0.02/1.35/1.38
19	0.93	0.02/1.38/1.39	0.95	0.04/1.41/1.4	0.95	0.04/1.39/1.42
20	0.92	0.02/1.36/1.38	0.93	0.04/1.36/1.39	0.95	0.04/1.39/1.42
21	0.88	0.02/1.33/1.31	0.89	0.04/1.33/1.38	0.9	0.04/1.33/1.32
22	0.91	0.02/1.33/1.35	0.92	0.04/1.33/1.4	0.92	0.04/1.33/1.4
23	0.89	0.02/1.34/1.31	0.89	0.03/1.34/1.31	0.89	0.05/1.34/1.32
24	0.84	0.05/1.25/1.22	0.87	0.09/1.3/1.22	0.87	0.09/1.26/1.26
25	0.78	0.05/1.16/1.13	0.82	0.09/1.2/1.13	0.81	0.09/1.17/1.17
26	0.82	0.05/1.18/1.21	0.83	0.07/1.2/1.24	0.85	0.09/1.18/1.28
27	0.78	0.04/1.16/1.13	0.79	0.06/1.16/1.14	0.81	0.09/1.16/1.17
28	0.84	0.05/1.24/1.21	0.84	0.06/1.25/1.22	0.87	1.1/1.25/1.26
29	0.72	0.11/1.08/0.99	0.74	0.13/1.1/0.98	0.76	0.16/1.09/1.02
30	0.77	0.06/1.11/1.09	0.79	0.08/1.14/1.14	0.8	0.1/1.1/1.18
31	0.77	0.06/1.15/1.11	0.71	0.08/1.02/1.02	0.8	0.1/1.16/1.13
32	0.79	0.05/1.15/1.19	0.81	0.08/1.15/1.2	0.82	0.1/1.15/1.22
33	0.68	0.16/0.89/1	0.73	0.23/0.9/1.07	0.79	0.31/0.92/1.14
34	0.63	0.32/0.82/0.71	0.67	0.4/0.82/0.79	0.72	0.48/0.83/0.86
35	0.75	0.25/0.89/1.15	0.77	0.27/0.89/1.17	0.85	0.38/0.9/1.2
36	0.63	0.17/0.96/0.77	0.66	0.22/0.95/0.82	0.72	0.3/0.97/0.9
37	0.76	0.16/1.02/1.11	0.74	0.18/1.04/1.11	0.85	0.2/1.04/1.22
38	0.65	0.19/0.92/0.84	0.66	0.19/0.78/1.02	0.72	0.3/0.93/0.94
39	0.83	0.13/1.14/1.21	0.85	0.16/1.16/1.21	0.9	0.25/1.15/1.32
40	0.72	0.1/0.94/1.12	0.73	0.14/0.94/1.12	0.81	0.24/0.95/1.24
Average	0.86	0.06/1.25/1.24	0.87	0.07/1.27/1.26	0.89	0.1/1.26/1.3

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