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Coordinating Buyback Contract in Supply Chains with Limited Funding

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Abstract The paper considers the problem of supply chain profit maximization with the buyback contract. The solution is given for a two-echelon supply chain where a retailer is faced with limited funding and chooses between bank financing and trade credit provided by a supplier. It is shown that the buyback contract does not coordinate the considered supply chain as the supplier's profit in this case does not achieve its maximum. Therefore conditional coordination of the supply chain with limited funding is considered. It identifies the situation where the retailer's and the supply chain's profits are maximized while the supplier's profit is greater than that earned with the wholesale-price contract. It is proved that the examined supply chain achieves conditional coordination both with bank loan and trade credit, and trade credit is preferred to bank financing as the supply chain's individual and total profits are higher than those earned with bank loan. Based on the model solution an algorithm for selecting the parameters of conditionally coordinating buyback contract with limited funding is proposed.

Keywords: supply chain, coordination, buyback, contract, limited funding, supplier financing, trade credit.

1. Introduction

Optimal performance of supply chains commonly implies an optimal set of actions taken by supply chain members so as to maximize supply chain's total profit while maximizing their own profits. To ensure that the supply chain members act in accordance with the optimal set of actions, their relationships and interdependencies should be managed through cooperation, with coordination being its main component and the subject of this study.

To coordinate the actions of supply chain parties several mechanisms can be applied, including contracts. The ubiquity of the contracts' application makes the problem of setting optimal contract parameters that ensure the maximum of supply chain's profit highly relevant today. Nonetheless, practice shows that the process of managing contractual relationships often fails to provide the contracting parties with the best possible results. Despite the fact that companies can save up to 80% on transaction costs with generally accepted and clearly regulated rules and procedures related to contracting, only 48% of business entities actually run an organized process of contract management (Berezinets, Meshkova, Nikolchenko, 2019).

Besides addressing general management issues, the study touches upon such particular aspect of financial resources management as credit used by the retailer to pay the supplier in exchange for the goods that retailer wants to purchase for

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further retailing. This issue is indicated by the term "limited funding", which refers to a situation where the retailer's holdings of cash are constrained and might be insufficient to pay the supplier. The paper considers two types of credit – bank loan and trade credit.

The issue of limited funding has grown since the start of the COVID pandemic. A research on the state of US supply chains carried out in 2022 by SAP, one of the world's largest enterprise software companies, found that more than half of businesses suffered a decrease in revenue and faced the need for taking new financing measures, such as business loans. Researchers also have been showing interest in studying supply chain coordination with contracts with limited funding considered. However, the majority of those studies focus on the contract types other than the buyback contract (Kouvelis, Zhao, 2012; Chen, 2015; Zhan, Chen, Hu, 2018). Besides, the papers dedicated to the buyback contract do not exhaustively consider the problem of limited funding – they analyze coordinating properties of the contract with only one form of borrowing applied and focus on different aspects of limited funding, such as bankruptcy risk tolerance or working capital management (Xiao et al., 2017; Fu, Liu, 2019). This body of research differs in the way that it studies the buyback contract with two forms of credit commonly applied in business practice.

Therefore, the goal of this study is to build a model of the buyback contract with limited funding that allows for two forms of borrowing – bank loan and trade credit, and develop an algorithm for selecting parameters of the buyback contract with limited funding that provide conditional coordination of a supplier-retailer supply chain. In this paper the term "supply chain coordination" refers to a situation where the contract parameters agreed by the supply chain members allow both the supplier and retailer to maximize their profits, thus achieving the maximum of supply chain profit. Supply chain coordination by this definition fails virtually for all contract types studied in the field of supply chain contracting, which is why the term "conditional coordination" is used. The term "conditional coordination" of the supplier-retailer supply chain indicates a situation where the set of the contract parameters ensures maximal retailer's profit, improves the supplier's profit relative to the wholesale-price contract instead of maximizing it, and thus maximizes supply chain's profit (Berezinets et al., 2020).

The paper is organized as follows. The first part gives an overview of research on supply chain coordination with contracts, especially with buyback contract, and discusses limited funding and the ways of dealing with it studied by the researchers in supply chain contracting. The second part focuses on modeling the buyback contract with limited funding and presents the solution of the problem of supply chain coordination with this contract. The third part is dedicated to conditionally coordinating buyback contract with limited funding, especially to the case of uniform distribution of demand and presents an algorithm for selecting its parameters based on the constructed model. The model and the algorithm are then applied to a numeric example. Conclusion summarizes the results obtained in the study.

2. Literature Overview

2.1. Contracts as a Mechanism of Supply Chain Coordination

Supply chain can be considered as a network of interconnected firms involved in the process of producing value in the form of products and services for the ul-

timate consumer, and its management can adopt different levels of centralization, depending on concentration of the decision-making power exercised by one supply chain member. Thus, centralization can be defined as a degree to which a single firm makes decisions concerning the supply chain operations that all the firms belonging to this supply chain are to comply with. High centralization implies that only one supply chain member fully exercises the decision-making authority for all the partners in the supply chain, while low centralization is characterized by each firm making decisions independently from the rest of the supply chain members (Giannocaro, 2018).

The choice of the level of centralization greatly affects the supply chain's performance and its financial result, such as profit. Researchers in the field of supply chain management and supply chain contracting agree that high level of centralization is preferable to its low level. Centralization brings such benefits as improved efficiency and reduced distortions of demand information moving up the supply chain from the retail end to the manufacturing end (Lee, Padmanabhan, Whang, 1997). It also allows the partners along the supply chain to resolve their conflicting goals (Giannocaro, 2018). Decentralization, on the other hand, can increase flexibility and communication between the supply chain members, and improve innovation. However, it implies that the decision-making authority is distributed among the members, and this creates an incentive conflict in the supply chain as each firm is primarily concerned with her own performance and does not align individual objectives with those of the supply chain, which results in poor performance of the supply chain as a whole.

Consequently, coordination mechanisms are required to motivate the supply chain parties to align their goals and ensure optimal performance and financial result of the supply chain. Supply chain coordination counteracts negative effects of decentralization (e.g., inaccurate forecasts and excessive inventory) and suggests the way to allocate its total profit so that each partner is better off. The most common mechanisms are information technology, information sharing, joint decision-making, and contracts (Arshinder, Deshmukh, 2008). Applied in decentralized supply chains, contracts allow to manage actions of the supply chain partners so that total profit of the supply chain increases and is arbitrarily divided among its members.

From economic point of view, contract is an agreement between the buyer and the supplier of a product or service in which the terms of exchange are determined by price, asset specificity and guarantees (Williamson, 1985 cited in Berezinets, Meshkova, Nikolchenko, 2019). Conclusion of the contract results from a negotiation, which serves the purpose of determining the terms of exchange, or the contract parameters, acceptable to each party involved. The contract parameters are "acceptable" when the party's expected profit from participation in the contract is not lower than without it (Bernstein, Marx, 2006 cited in Berezinets, Meshkova, Nikolchenko, 2019). If the parties fail to reach an agreement on these conditions, the contract is not concluded.

With supply chain contracting, firms manage their incentive conflicts and align their objectives with the supply chain's objective, through which optimal supply chain performance is achieved. Among positive effects of supply chain contracting there can be distinguished improvement of supply chain performance, sharing risks between contracting parties, and facilitation of long-term partnerships (Tsay, Nahmias, Agrawal, 1999).

Researchers in supply chain contracting deal with several contract types. These contract types all have such parameters as wholesale price and order quantity, but differ in the incentives created in order for the supply chain parties to implement optimal actions required for supply chain coordination. Based on those incentives, the following contract types are distinguished: revenue-sharing contract, quantity-flexibility contract, sales-rebate contract, quantity-discount contract, and buyback contract, which is of particular interest in this study and discussed in detail further.

Based on different approaches to interpreting coordination, coordinating contract can be defined in different ways. One condition commonly included in definitions of coordinating contract is maximization of expected total profit earned by the supply chain, although this condition alone does not take into account the profit of each participant and leaves room for avoiding compliance with the contract terms if individual profit does not seem acceptable.

In (Cachon, 2003) coordinating contract is defined as a contract that has such parameters that provide a unique Nash equilibrium, so that the partners have no incentive to deviate from the contract terms, and allows each partner to maximize their individual expected profit so as to maximize the expected supply chain profit.

Another approach suggests that coordinating contract must maximize supply chain profit and provide supplier and retailer with a "win-win" situation, which implies that their individual profits with a coordinating contract should be higher than without it (Taylor, 2002; Saha, 2013).

Definition proposed in (Heydari, Choi, Radkhah, 2017) enhances the definition introduced in (Cachon, 2003) and states that coordinating contract should maximize the supply chain profit and provide a Pareto-optimal solution.

Another approach also based on the definition from (Cachon, 2003) suggests that under the assumption of risk neutrality of the supply chain parties coordinating contract motivates each party to make decisions that provide maximum of the expected supply chain profit, i.e., each party intends to maximize individual profit and arrives at the contract parameters that maximize the supply chain profit (Berezinets, Meshkova, Nikolchenko, 2019; Berezinets et al., 2020). The authors also emphasize the equivalence of the problem of supply chain profit maximization and supply chain coordination as the latter is achieved with maximal supply chain profit.

Applying this approach to studying such contract types as revenue sharing (Zenkevich, Gladkova, 2018), sales rebate (Berezinets, Meshkova, Nikolchenko, 2019) and buyback contract (Berezinets et al., 2020), the authors come to a conclusion that these contracts do not allow to achieve supply chain coordination for their application maximizes only the retailer's and the supply chain profit, and not the supplier's. They introduce the definition of conditionally coordinating contract that allows to achieve maximal retailer's profit, improve the supplier's profit compared to that earned with the wholesale-price contract, and, thus, maximizes the supply chain profit, which is the sum of the supplier's and retailer's profits. All the three types of supply contracts are proven to be conditionally coordinating.

2.2. Supply Chain Coordination with Buyback Contract

Parameters of the buyback contract include – wholesale price ω, quantity of goods q ordered by the retailer, and buyback price b which the supplier will pay to the retailer for all the items left unsold at the end of the selling season. The supplier's promise of buying unsold goods back from the retailer gives the latter an

incentive to order more as typically the retailer is reluctant to order large amounts of the product and run the risk of overstocking caused by uncertain demand.

Buyback contracts are commonly applied when the product demand is difficult to predict and the risk of obsolescence of the product may be high (Tsay, 2001). Examples of such industries are publishing and book selling business, fashion and beauty retail, retailing seasonal products and those with short life cycle, such as FMCG and grocery retail (Pasternack, 1985; Tsay, 2001; Bose, Anand, 2007; Berezinets et al., 2020).

One of the classic works dedicated to the buyback contract is (Cachon, 2003). The author analyzes the newsvendor model transformed into a game, which is relatively simple, but sufficiently rich to study coordinating properties of supply contracts, and then scrutinizes six contract types, including the buyback and wholesale-price contracts, which are explored in this body of research.

The newsvendor model considers a supplier-retailer supply chain during one selling season. The supplier manufactures a single type of product and sells it to the retailer who then sells it to the end consumers. The relationship between the supplier and retailer is regulated by their contract. According to this contract, the supplier offers certain terms that vary depending on the contract type, and the retailer reacts with an amount of goods he decides to order from the supplier. The retailer can place his order only in advance of the selling season and thus is faced with the newsvendor problem – during the selling season he is supposed to satisfy stochastic market demand. As the supplier receives the retailer's order quantity, she produces and fulfills the order before the start of the selling season. At the end of the selling season, payment for the goods is transferred in accordance with agreed contract terms. The model assumes that both parties are risk-neutral, have full access to the information related to the contract and aim to maximize their profits.

Regarding the buyback contract, the model studied in (Cachon, 2003) is an extension of the model introduced in (Pasternack, 1985), one of the pioneering studies of buyback contracts that proves its coordinating properties provided that total supply chain profit is maximized. The author comes to the same conclusion that supply chain coordination can be achieved under this contract type with any profit allocation, including extreme situations where the supply chain profit is completely earned by the retailer or the supplier. Nonetheless, the author highlights ambiguity of these cases since the optimal set of the contract parameters providing coordination is not a unique Nash equilibrium, which means that there also exists a suboptimal set of the contract parameters that may also show coordinating properties.

In (Cachon, 2003) there is evidence that profit allocation between the supplier and retailer would depend on simultaneous adjustment of the wholesale price and the buyback rate. This is shown by presenting the wholesale price as a function of the buyback rate. In practice it means that these parameters should be negotiated simultaneously and not one after another, which is another reason to apply the game-theoretic approach, as it reflects the real process of negotiating the contract terms in the best way.

Another revision of the buyback contract is conducted in (Cachon, Lariviere, 2005). The authors primarily study coordinating properties of the revenue-sharing contract and then compare it to other supply contracts, such as the buyback, price-discount, quantity-flexibility, franchise and sales-rebate contracts. Using the game-theoretic approach, the authors introduce a general

model which is applicable for both stochastic and deterministic demand and does not depend on the revenue function. Their findings show that the revenue-sharing contract is equivalent to the buyback contract in the newsvendor model and coordinates the supply chain with a fixed retail price. It is shown that for any buyback contract there exists a revenue-sharing contract that allows to generate the same cash flows regardless of the product demand. But this conclusion cannot be generalized as the authors provide evidence that revenue-sharing contract in some cases can coordinate the supply chain while buyback contract cannot.

These three papers (Pasternack, 1985; Cachon, 2003; Cachon, Lariviere, 2005) provide a solid framework for further research on the topic of supply chain coordination with contracts, and there is an extensive body of research that further develops their approach in various directions, including:

- expansion of the supply chain by increasing the number of its partners, as in (Wu, 2013);
- extension of the model timespan, as in (Zou, Pokharel, Piplani, 2008);
- coordination with a price-dependent demand, as in (Granot, Yin, 2005) and (Zhao et al., 2014);
- information asymmetry, e.g., (Babich, Ritchken, Wang, 2012);
- relaxing the risk-neutrality assumption, e.g., (Tsay, 2002); and
- capital constraints faced by the supply chain partners, which is discussed in detail further.

2.3. Supply Chain Contracting with Limited Funding

In this paper the term "limited funding" indicates a situation where the retailer's holdings of cash are constrained and might be insufficient to pay the supplier for the amount of goods that retailer decides to purchase from her. Similar restrictions are widely explored in existing literature on supply chain contracting.

For example, in (Chen, 2015) a supplier-retailer supply chain with capital constraints is examined, and two options of funding the retailer's business are analyzed – taking a bank loan and borrowing from the supplier (a trade credit). The paper states that there exists a competition between the bank and the supplier as the retailer can choose how to finance short-term operations, and the contract conditions differ depending on that decision. The cost of the trade credit is supposed to be lower than the cost of the bank loan, which confirms the practice of trade credit usage among firms.

The author compares the two financing options under the wholesale-price and revenue-sharing contracts, and comes to a conclusion that under the wholesale-price contract trade credit is more beneficial to both partners and provides a unique financing equilibrium. This result is explained by the fact that under bank financing of the retailer's activity the supply chain acts as if it is not capital-constrained. The retailer's marginal cost is equal to the wholesale price; his default risks are shared with the bank. However, with trade credit these risks are shared between the supplier and retailer, thus decreasing the retailer's marginal cost and increasing the supplier's profit and, hence, the supply chain profit.

Examination of the bank and trade credit under the revenue-sharing contract shows different results, though. Bank credit results in the problem setting similar to that of the non-constrained supply chain, and, thus, coordination is achieved. Using the trade credit policy, however, does not allow to directly implement such contract

in its traditional setting used for the non-constrained supply chain to reach its coordination. A feasible region exists in the model of revenue-sharing contract in both cases, but the authors show that it is larger under the bank credit policy than under the trade credit policy. The profits expected by the supplier and retailer are also higher under bank financing than under trade credit. However, the author suggests that under the game-theoretic framework coordinating revenue-sharing contract allows the supplier to negotiate such terms that provide her with the same profit under both types of financing.

A supply chain wherein the supplier and retailer both are capital-constrained is also studied in (Kouvelis, Zhao, 2012). The authors study the wholesale-price contract and include working capital and collateral into their model to allow for the bankruptcy risk as they consider two alternatives for borrowing — bank loan and trade credit. They state that when the retailer is offered an optimally structured trade credit, it is always preferable as the supply chain profit improves, implying that the supplier's and retailer's profits also should improve. The optimally structured trade credit has the interest rate between 0 and risk-free rate, which confirms the authors' prediction based on the empirical data that the trade credit rate should be lower than the bank's rate to improve the supply chain performance in the presence of financial constraints. Risk-free rate is preferable as it motivates the retailer to use all his working capital to pay up front for the order quantity. The authors show that despite the profit improvement, supply chain coordination is not fully achieved.

Further enhancement of this approach is presented in (Kouvelis, Zhao, 2015), with main focus on the bank loans, namely secured loans with current assets as a collateral. The paper aims to answer how the structure of default costs can affect the design and performance of the contract, and whether it is possible to allocate expected default loss among the supplier and retailer and meet the coordination requirement of the supply chain's maximal expected profit.

Three types of the supplier-retailer supply chain are considered: two firms without coordinated working capital management, two firms with coordinated working capital management, and the case where one firm has control of all decisions related to the supply chain's activity, including working capital management. The term "coordinated working capital management" can be explained as an alignment of funds borrowed by the supplier and retailer with their revenue shares. It is assumed that the modeled supply chain operates under a general contract which subsumes the quantity-discount, revenue-sharing and buyback contracts, and can be reduced to any of these by setting proper values of the contract parameters.

Analyzing the cash flow constraints and bankruptcy risks, the authors highlight that default risk does not affect the properties of coordinating contracts when bank credit is used, which coincides with the conclusion made in (Chen, 2015). The authors conclude that only the revenue-sharing contract preserves coordinating properties in the presence of both variable and fixed bankruptcy costs. They show evidence that buyback contract coordinates the supply chain in the presence of only variable costs, but is Pareto-dominated by revenue-sharing contract when there are fixed default costs because profits under the buyback contract are lower than with the revenue-sharing contract. Quantity-discount contract does not coordinate the supply chain in any setting of default costs. As a result, new revenue-sharing contract with adjusted wholesale price is proposed as an instrument of working capital coordination in the supply chain with certain limitations.

In (Moon, Feng, Ryu, 2015) the effect of budget constraints on coordinating properties of the revenue-sharing contract is examined and basic supplier-retailer model is extended to a multistage one. The multistage model takes into account that each partner can deal with several upstream partners, so the supply chain has a tree structure. Discussing the issue of budget constraints, the authors distinguish absolute budget constraints, costs connected with the default risk taken into account in the terms of a bank loan, and administrative costs. An absolute budget constraint means a limited order quantity that the retailer can really afford to meet the budget when there is no opportunity to raise a loan. Since the order quantity cannot be increased, profit maximization cannot be achieved if optimal order quantity is larger than the quantity affordable to the retailer. Default risk being one of the factors affecting the bank's interest rate requires that all partners agree with the loan terms, which can add costs related to the negotiation process. Administrative costs increase as the borrower's and, hence, the supply chain's flows are monitored in case the loan is raised in the financial market. The paper focuses on the case of absolute budget constraints when the supply chain partners have no access to the financial market.

The authors prove that with traditional revenue-sharing contract a budget-constrained supply chain does not achieve coordination when the partners exceed the budget that matches the contract terms. A modified revenue-sharing contract is proposed, with new wholesale price settings and a penalty factor to improve the revenue sharing mechanism under the budget constraints. These contract terms allow the partners with insufficient budget to negotiate a lower wholesale price and transfer a share of their revenues, so the supply chain profit is still higher than without a coordinating contract. However, limitations to this approach include a position of the budget-constrained partners. If a partner at the end of the supply chain is faced with insufficient budget, the proposed contract cannot be implemented and the profit maximum is not achieved.

In (Xiao et al., 2017) the focus is on the problem similar to that studied in (Kouvelis, Zhao, 2012). The authors consider a supplier-retailer supply chain where the retailer receives a trade credit from the supplier due to lack of access to bank financing. This restriction implies a variable default cost that is included in the model. Comparison of the revenue-sharing, quantity-discount and buyback contracts is carried out and leads to the following conclusions. Firstly, quantity-discount contract fails to coordinate a capital-constrained supply chain. Secondly, revenue-sharing and buyback contracts equally coordinate the supply chain under financing restrictions, however, only as long as total working capital is sufficient and above a specific threshold.

To overcome the limitation related to the level of total working capital, the authors propose a generalized revenue-sharing contract that provides flexible profit allocation between the supplier and retailer and outperforms traditional revenue-sharing and buyback contracts. Under the generalized contract, the wholesale price depends on the order quantity, and the ratio of revenue sharing depends on the retailer's sales as his revenue is used to repay the bank loan before sharing.

Discussing possible ways of further research, the authors mention that in real supply chains the retailer may be more creditworthy whereas the supplier faces capital constraints and has no access to bank financing. This problem setting has not been widely examined in existing literature and gives a novel perspective to the research on coordination of capital-constrained supply chains.

A case similar to (Xiao et al., 2017) is examined in (Zhan, Chen, Hu, 2018). The authors consider the sales-rebate contract and compare its conditions with traditional trade credit financing. They come to a conclusion that even though unique equilibrium exists in both cases, trade credit under the sales-rebate contract brings better opportunities for the supplier's pricing policy and improves financial results of the supply chain partners relative to traditional trade credit policy.

The study by (Fu, Liu, 2019) focuses on the buyback contract. The authors consider the same setting as in (Xiao et al., 2017) and (Zhan, Chen, Hu, 2018) and relax the assumption of risk-neutrality by considering a risk-averse retailer. In this setting it is assumed that in the presence of capital constraints the supplier provides a trade credit to the retailer and raises a bank loan. The authors prove that with these assumptions held traditional revenue-sharing and buyback contracts do not coordinate the supply chain since the risk-averse retailer is willing to choose an order quantity that is lower than optimal. To achieve coordination in this setting, the supplier, which is considered to be risk-neutral, should share default risks with the retailer. The authors propose a new risk-sharing contract that addresses this issue, motivates the retailer to increase the order quantity and, thus, allows to achieve coordination of the supply chain.

The terms of new contract suggest that when the order quantity is lower than optimal, the contract is reduced to traditional buyback contract, whereas for a larger order quantity additional risk-sharing terms arise. The authors demonstrate that under these terms both the supplier and retailer reach the profit levels satisfying their capital constraints, properly manage their risk constraints and achieve coordination, meaning that the supply chain profit is maximized.

(Berezinets et al., 2022) study coordinating properties of the revenue-sharing contract when the retailer has short-term financing necessity and needs to borrow either from the supplier or the bank. Financial constraint faced by the retailer is modeled as limited holdings of cash that he can use to pay the supplier for purchasing goods. The solution proposed by the authors draws upon the definitions of coordinating and conditionally coordinating contracts discussed in Section 2.2., and proves that revenue-sharing contract does not allow to coordinate the supply chain with the retailer's financial constraint neither with a trade credit, nor with a bank loan. However, the contract can be called conditionally coordinating in both cases. The authors also prove that the trade credit allows the supplier to achieve higher profit than with the bank loan.

It is straightforward to confirm from this overview that even though there exists substantial literature on buyback contracts, it yields controversial results and does not give a clear answer whether the buyback contract coordinates the supply chain. Additionally, scholars are not in agreement about the effect of financing constraints on coordinating properties of the contracts. The overview also shows that buyback contract has not been exhaustively extended with respect to the limited funding.

3. Coordinating Buyback Contract with Limited Funding

3.1. Model Description

In this study a decentralized supply chain consisting of one retailer (R, he) and one supplier (S, she) is considered within a single period. A single type of product is manufactured by the supplier and sold by the retailer to end consumers. Retailer

is to order the product from the supplier once prior to the start of the selling season and no additional orders or amendments to the order are allowed.

Market demand for the product is denoted ξ and assumed to be stochastic. ξ is a continuous random variable with probability density function $f_\xi(x)$ and strictly increasing and differentiable distribution function $F_\xi(x)$ with $x \geq 0$. Let τ denote sales volume, where

$$\tau = g(\xi) = \begin{cases} \xi, 0 \leq \xi < q, \\ q, \xi \geq q. \end{cases}$$

Then expected sales volume is $E[\tau] = q - \int_0^q F_\xi(x)\mathrm{d}x$, and its first derivative is $\frac{\mathrm{d}}{\mathrm{d}q}E[\tau] = 1 - F_\xi(q)$.

Under a buyback contract the supplier offers the following terms: a wholesale price ω per unit and a buyback price b per unit. In response to the supplier's offer the retailer chooses some order volume q that he will sell at the retail price p during the selling season. At the end of the selling season the supplier will pay up the buyback price for unsold units of the product; then the retailer will be able to sell unsold goods at the salvage value ν. Notations used in the model are listed in Table 1.

Table 1. List of notations.

ω	Wholesale price per unit (c.u.)
b	Buyback price per unit (c.u.)
q	Order quantity (u.)
τ	Sales volume (u.)
p	Retail price per unit (c.u.)
ν	Salvage value per unit (c.u.)
c_S	Supplier's cost per unit (c.u.)
c_R	Retailer's cost per unit (c.u.)
$c = c_S + c_R$	Supply chain's total costs (c.u.)
K_R	Retailer's cash available for the order payment (c.u.)
r_B	Interest rate of the bank loan
r_S	Interest rate of the trade credit
$Prof_S$	Supplier's profit per transaction (c.u.)
$Prof_R$	Retailer's profit per transaction (c.u.)
$Prof_{SC} = Prof_S + Prof_R$	Supply chain profit per transaction (c.u.)
$\pi_S = E[Prof_S]$	Supplier's expected profit per transaction (c.u.)
$\pi_R = E[Prof_R]$	Retailer's expected profit per transaction (c.u.)
$\pi_{SC} = E[Prof_{SC}$	Expected supply chain's profit per transaction (c.u.)

Approach to modeling the retailer's limited funding is based on the framework proposed in (Berezinets et al., 2022). The retailer is faced with limited funding, i.e., has insufficient volume of cash available for paying to the supplier. He considers two options for the short-term credit to pay for the order quantity he chooses. The first option is bank loan, the second is trade credit (supplier financing). Regardless of the chosen source of financing, the retailer is supposed to repay the borrowed money with interest at the end of the selling season. The size of interest payment depends on the interest rate, which is r_B if the retailer takes a bank loan or r_S if he decides to use supplier financing.

The set of the model assumptions includes the following (Berezinets et al., 2022):

1. Both the supplier and retailer are risk neutral;
2. Both the supplier and retailer are rational and aim to maximize their profits;
3. There is no information asymmetry between the supplier and retailer, they both know their costs, cash available, distribution of the product demand, interest rate on the bank loan, interest rate on the trade credit and retail price;
4. Retailer may choose between the bank and supplier financing; bankruptcy risk is not considered as it is assumed that the retailer can cover his loan obligation in full at the end of the selling season;
5. No moral hazard issues are considered as both players have no ex ante (i.e., before the event) intention to breach the contract;
6. Capital market is considered without taxes and transaction costs;
7. Wholesale price is lower than the retail price and higher than the supplier's costs. Salvage value is assumed to be lower than the supplier's costs: $0 < \nu < c_S < \omega < p$;
8. Buyback price should not exceed the wholesale price, but should be higher than the salvage value: $\nu < b < \omega$;
9. Interest rates on both sources of borrowing are given parameters that should belong to the interval between 0 and 1: $0 < r_B < 1$, $0 < r_S < 1$.

The process of negotiating the parameters of buyback contract is modeled as a two-step game with two players. At the first step the supplier chooses two contract parameters ω and b from the available set of the supplier's strategies X_S. At the second step, in response to the supplier's offer the retailer chooses the order volume q from the available set of the retailer's strategies X_R:

$$\begin{gathered} X_S = \{(\omega, b) | \nu < b < \omega, 0 < c_S < \omega\}, \\ X_R = \{q(\omega, b) | q \geq 0\}. \end{gathered} \tag{1}$$

The model assumes that the supplier is the leader in decision making and chooses her strategy first, while the retailer is the follower and chooses his strategy in accordance with the one chosen by the supplier. In this setting the supplier has the advantage of optimizing her profit function based on the retailer's response and knowing his profit function. Consequently, the buyback contract with limited funding can be defined as a combination of three parameters (b, ω, q), which is the same way as usual buyback contract with no additional conditions is defined (Berezinets et al., 2020).

Definition 1. A buyback contract with limited funding (b^*, ω^*, q^*) will coordinate the supply chain if the following conditions are met:

1. $\max\limits_{q} \pi_R = \pi_R\Big(b, \omega(b), q_R^*\big(b, \omega(b)\big)\Big),\ \forall b \in X_S$;
2. $\exists\, \omega^*(b) : q_R^*\big(b, \omega^*(b)\big) = q_{SC}^* = q^*,\ \forall b \in X_S$;
3. $\max\limits_{b} \pi_S\big(b, \omega^*(b), q^*\big) = \pi_S(b^*, \omega^*, q^*),\ \omega^* = \omega(b^*)$.

According to Definition 1, the following steps need to be taken in order to determine parameters of coordinating buyback contract for a supply chain with limited funding:

1. Determine the retailer's optimal order volume q_R^* that allows the retailer to maximize his individual profit π_R;

2. Determine the supply chain's optimal order volume q^*_{SC} that allows to maximize the supply chain profit π_{SC};
3. Determine the wholesale price ω^* for which the retailer's optimal order volume q^*_R will coincide with the supply chain's optimal order volume q^*_{SC} : $q^*_R = q^*_{SC} = q^*$;
4. Determine the buyback price b^* for which the supplier's expected profit $\pi_S(b^*, \omega^*, q^*)$ is going to be maximized when the retailer orders the volume q^* that allows to maximize both the retailer's and supply chain profits.

Depending on the lender (the bank or the supplier), expressions for the profit and expected profit of each player and of the supply chain will be different. Nevertheless, the size of the loan in both cases will be determined by the amount of cash at the retailer's disposal K_R and the order volume that the retailer chooses in response to the buyback and wholesale prices suggested by the supplier. The size of the loan is expressed as

$$\max\left\{(\omega + c_R)q - K_R, 0\right\}, \tag{2}$$

which is the amount of money he lacks to make the order q.

In case retailer's amount of cash is sufficient to make the order, the loan size will be equal to zero. Otherwise, it will be equal to $((\omega + c_R)q - K_R)$, which is the difference between the amount of retailer's cash and his total costs associated with the order he is going to make, namely wholesale price and retailer's costs.

3.2. Coordinating Buyback Contract with Bank Loan

As the retailer can have insufficient funds to finance his procurement decision, he has an opportunity to take a bank loan with the rate r_B. The principal and interest charged by the bank will be repaid at the end of the selling season. In this case profit functions depend on the order volume q that is to be chosen by the retailer, volume of the retailer's sales τ, and the parameters related to his limited funding – the amount of cash at disposal K_R and the rate charged for the bank loan r_B. Expressions for the supplier's, retailer's and supply chain profits with the buyback (BB) contract look as follows:

$$\begin{aligned}
Prof_S^{BB} &= b\tau + (\omega - c_S - b)q, \\
Prof_R^{BB} &= (p - b - \nu)\tau + (b + \nu - \omega - c_R)q - r_B \times \max\left\{(\omega + c_R)q - K_R, 0\right\}, \\
Prof_{SC}^{BB} &= (p - \nu)\tau + (\nu - c)q - r_B \times \max\left\{(\omega + c_R)q - K_R, 0\right\}.
\end{aligned}$$

Expected profits earned by the supplier, retailer and supply chain are expressed in the following way:

$$\begin{aligned}
\pi_S^{BB} &= (\omega - c_S)q - b\int_0^q F_\xi(x)\mathrm{d}x, \\
\pi_R^{BB} &= (p - \omega - c_R)q + (b + \nu - p)\int_0^q F_\xi(x)\mathrm{d}x - r_B \times \max\left\{(\omega + c_R)q - K_R, 0\right\}, \\
\pi_{SC}^{BB} &= (p - c)q + (\nu - p)\int_0^q F_\xi(x)\mathrm{d}x - r_B \times \max\left\{(\omega + c_R)q - K_R, 0\right\}.
\end{aligned}$$

Since the retailer's funds can be either sufficient or insufficient, the problem of determining the parameters of coordinating contract by Definition 1 is solved for these two cases – the case where the retailer has sufficient funds and does not need any credit and the case where the retailer takes the bank loan.

Retailer has sufficient funds. In this case the retailer's cash K_R is enough to pay up for the order he decides to make, so he does not need to take a bank loan, and expressions for the supplier's, retailer's and supply chain expected profits look as follows:

$$\begin{aligned}
\pi_S^{BB} &= (\omega - c_S)q - b\int_0^q F_\xi(x)\mathrm{d}x,\\
\pi_R^{BB} &= (p - \omega - c_R)q + (b + \nu - p)\int_0^q F_\xi(x)\mathrm{d}x,\\
\pi_{SC}^{BB} &= (p - c)q + (\nu - p)\int_0^q F_\xi(x)\mathrm{d}x.
\end{aligned} \tag{3}$$

Then the case becomes identical to that examined in (Berezinets et al., 2020) – coordination cannot be achieved as the third condition stated in Definition 1 is not met.

Retailer takes the bank loan. In this case the retailer's funds are insufficient to finance his decision on the order quantity, so he borrows from the bank. This condition is expressed as

$$(\omega + c_R)q > K_R, \tag{4}$$

where the retailer's amount of cash is not enough to pay the supplier for ordering at the wholesale price ω and bearing costs c_R of carrying and retailing the ordered goods during the selling season.

The model does not include the bank as a player as its only function is to provide the retailer with the interest rate on the loan that the retailer intends to take. This information is supposed to be shared between the supplier and retailer in accordance with the third assumption of the model as the interest rate r_B charged by the bank affects the values of the retailer's and supply chain profits, as well as the amount of the retailer's cash K_R.

Compared to the formulas (2.), expression for the supplier's expected profit remains unchanged while expressions for the retailer's and supply chain expected profits have modifications – they include the retailer's cash available K_R and interest rate on the loan r_B – reflecting the bank loan taken by the retailer. The formulas for these profits look as follows:

$$\begin{aligned}
\pi_S^{BB} &= (\omega - c_S)q - b\int_0^q F_\xi(x)\mathrm{d}x,\\
\pi_R^{BB} &= \big(p - (\omega + c_R)(1 + r_B)\big)q + (b + \nu - p)\int_0^q F_\xi(x)\mathrm{d}x + r_B K_R,\\
\pi_{SC}^{BB} &= \big(p - c - r_B(\omega + c_R)\big)q + (\nu - p)\int_0^q F_\xi(x)\mathrm{d}x + r_B K_R.
\end{aligned} \tag{5}$$

As shown in the expressions (2), when the retailer takes a bank loan, expected supply chain profit π_{SC} is lower compared to the situation where the retailer's funds are sufficient (see the formulas (2.)); this is reflected by the loan principal and interest subtracted from the retailer's and supply chain profits. The contract parameters that meet the conditions of Definition 1 are determined with the use of the 4-step approach given in Section 3.1.

The first step of the approach, which is to determine an optimal order volume that maximizes the retailer's expected profit $\pi_R^{BB}(q)$, requires finding the stationary

point of this function. The stationary point of the function $\pi_R^{BB}(q)$ exists where its first derivative at q is zero, i.e., $\frac{\partial \pi_R^{BB}}{\partial q} = 0$. Solving this equation allows to find the stationary point q_R^0. Then the second-derivative test needs to be done in order to determine whether the found stationary point q_R^0 is a local maximum of the profit function $\pi_R^{BB}(q)$. To use this test, the second derivative $\frac{\partial^2 \pi_R^{BB}}{\partial q^2}$ must be derived. If $\frac{\partial^2 \pi_R^{BB}}{\partial q^2} < 0$, then retailer's profit function has a local maximum at q_R^0, i.e. $q_R^0 = q_R^*$, where q_R^* denotes the found local maximum of retailer's profit function. By choosing the order volume q_R^* retailer will be able to achieve the highest possible profit.

At the second step the same solution is found for the supply chain profit $\pi_{SC}^{BB}(q)$ in order to determine the supply chain's optimal order volume q_{SC}^* that is a local maximum of the function π_{SC}^{BB}. With the order volume q_{SC}^* the supply chain profit achieves its maximum, which is to be divided between the supplier and retailer. As the optimal order volume q_{SC}^* is to be ordered and then sold to end consumers by the retailer, at the third step the following equation is solved:

$$q_R^* = q_{SC}^*,$$

and its solution brings the formula for the wholesale price ω^* that depends on the buyback price b. This relationship between ω^* and b allows to calculate such buyback price that would ensure maximal supply chain profit for any value of the wholesale price that the supplier and retailer agree upon during their negotiation.

At the final step an optimal buyback price b^* needs to be determined as the supplier's profit is maximized with regard to this parameter of the buyback contract. The function of supplier's expected profit is expressed through the wholesale price ω^* determined at the previous step:

$$\pi_S^{BB} = b\left(\frac{p - c - cr_B}{p - \nu + br_B} q^* - \int_0^{q^*} F_\xi(x)\mathrm{d}x\right).$$

Determining the buyback price b^* requires finding the stationary point of the supplier's profit function π_S^{BB} and performing the second-derivative test in order to prove that the stationary point is a local maximum of the function π_S^{BB}. Solution shows that the function $\pi_S^{BB}(b)$ does have a stationary point that is a local maximum b^*. The results of the solution are summarized in Table 2.

Table 2. Parameters of coordinating buyback contract with bank loan.

Retailer's optimal order quantity	$q_R^* = F_\xi^{-1}\left(\frac{p-(\omega+c_R)(1+r_B)}{p-b-\nu}\right)$
Supply chain's optimal order quantity	$q_{SC}^* = F_\xi^{-1}\left(\frac{p-c-cr_B}{p-\nu}\right)$
Wholesale price	$\omega^* = b\frac{p-c-cr_B}{p-\nu+br_B} + c_S$
Buyback price	$b^* = \frac{\sqrt{\frac{q^*(p-c-cr_B)(p-\nu)}{\int_0^{q^*} F_\xi(x)\mathrm{d}x}} + \nu - p}{r_B}$
Conditions	$b < p - \nu$, $p > c(1 + r_B)$

Table 2 shows that coordination can be achieved by setting the optimal buyback price b^*. However, the determined parameter b^* does not satisfy the conditions set in the model assumptions and therefore does not belong to the set of coordinating buyback contracts that is defined by the sets of the players' strategies X_S and X_R given in (1). This leads to a conclusion that the found optimal buyback price b^* does not allow to build a coordinating buyback contract with limited funding by Definition 1 for the case with the bank loan.

3.3. Coordinating Buyback Contract with Trade Credit

Trade credit is a business-to-business agreement in which a customer can buy goods without paying up front and pay the supplier at a later scheduled date. This type of financing is usually encouraged globally by regulators as it can potentially free up cash flow and finance short-term growth. However, trade credit can put suppliers at a disadvantage as they receive deferred payment (Investopedia, 2022). Trade credit is usually cheaper than bank financing (Kouvelis, Zhao, 2012), and this condition is met in the contract model:

$$0 < r_S < r_B < 1.$$

The model includes the rate r_S as a given parameter.

Based on the information about the retailer's volume of funds, the supplier can offer him a trade credit to cover the difference between the retailer's volume of cash and costs associated with the order he is going to make. In case the retailer accepts the supplier's offer and decides to use trade credit, the supplier agrees to postpone the payment for the retailer's order ωq until the end of the selling season and to bear the retailer's costs $c_R q$ of carrying and retailing the ordered goods. In return the supplier charges r_S, which denotes the interest rate on the trade credit and the retailer's cost of borrowing from the supplier.

At the end of the selling season the retailer will have to repay the principal (2), which is equal to the difference between the retailer's cash and costs, plus interest charged by the supplier. In this way trade credit is modeled exactly like the bank loan, although with a different interest rate. As a result, a coordinating buyback contract with trade credit can be determined based on Definition 1.

Profit expressions for the buyback contract with trade credit look as follows:

$$\begin{aligned}
Prof_S^{BB} &= b\tau + (\omega - c_S - b)q + r_S \times \max\{(\omega + c_R)q - K_R, 0\}, \\
Prof_R^{BB} &= (p - b - \nu)\tau + (b + \nu - \omega - c_R)q - r_S \times \max\{(\omega + c_R)q - K_R, 0\}, \\
Prof_{SC}^{BB} &= (p - \nu)\tau + (\nu - c)q.
\end{aligned}$$

Expressions of the expected profits earned by the supplier, retailer and supply chain are:

$$\begin{aligned}
\pi_S^{BB} &= (\omega - c_S)q - b\int_0^q F_\xi(x)\mathrm{d}x + r_S \times \max\{(\omega + c_R)q - K_R, 0\}, \\
\pi_R^{BB} &= (p - \omega - c_R)q + (b + \nu - p)\int_0^q F_\xi(x)\mathrm{d}x - r_S \times \max\{(\omega + c_R)q - K_R, 0\}, \\
\pi_{SC}^{BB} &= (p - c)q + (\nu - p)\int_0^q F_\xi(x)\mathrm{d}x.
\end{aligned}$$

Like with the bank loan, in negotiations over the buyback contract with trade credit the retailer's funds can be either sufficient or insufficient, which is why the

problem of determining parameters of coordinating buyback contract with trade credit by Definition 1 is solved for two cases – when the retailer has sufficient funds and does not borrow and when he uses trade credit.

Retailer has sufficient funds. In case the retailer has enough cash and can pay for the order without trade credit, $\max\{(\omega + c_R)q - K_R, 0\}$ will be equal to zero, and the problem will be identical to that described in Section 3.2., where the retailer has sufficient funds and does not need any bank loan.

Retailer uses trade credit. In case the retailer's funds are insufficient to pay for the order quantity, he takes a trade credit, which allows him to postpone the order payment ωq and have the supplier run his costs $c_R q$. The retailer's limited funding is expressed the same way as in (4). Expressions for the supplier's, retailer's and supply chain's expected profits look as follows:

$$\begin{aligned}
\pi_S^{BB} &= \big(\omega - c_S + r_S(\omega + c_R)\big)q - b\int_0^q F_\xi(x)\mathrm{d}x - r_S K_R,\\
\pi_R^{BB} &= \big(p - (\omega + c_R)(1 + r_S)\big)q + (b + \nu - p)\int_0^q F_\xi(x)\mathrm{d}x + r_S K_R,\\
\pi_{SC}^{BB} &= (p - c)q + (\nu - p)\int_0^q F_\xi(x)\mathrm{d}x.
\end{aligned} \tag{6}$$

As shown in the expressions (6), with trade credit the supplier's expected profit increases by the interest paid by the retailer compared to (2). The interest added to the supplier's profit is subtracted from the retailer's profit as he must repay the borrowed funds in full; hence, the supply chain profit, which is the sum of individual profits earned by the two players, remains the same as in (2.) and is higher than the supply chain profit from (2).

The process of determining the parameters of coordinating buyback contract with trade credit is the same as the solution described in Section 3.2.

At the first step of the solution an optimal order volume that maximizes the function of the retailer's expected profit $\pi_R^{BB}(q)$ is determined through finding the stationary point of this function. The stationary point of the function $\pi_R^{BB}(q)$ is proved to be a local maximum of the profit function $\pi_R^{BB}(q)$, and thus the first condition of Definition 1 is met.

At the second step a local maximum of the function π_{SC}^{BB} is found, and the second condition of Definition 1 is met as well.

At the third step the formula for the wholesale price ω^* is derived and then, at the fourth step an optimal buyback price b^* can be determined through finding the stationary point of the supplier's profit function π_S^{BB} and performing the second-derivative test in order to prove that the stationary point is a local maximum of the function π_S^{BB}. This function expressed through the optimal wholesale price ω^* looks as follows:

$$\pi_S^{BB} = b\left(\frac{p - c}{p - \nu}q^* - \int_0^{q^*} F_\xi(x)\mathrm{d}x\right) - r_S K_R.$$

Solution shows that the first derivative of the function π_S^{BB} at b is always positive, i.e., $\frac{\partial \pi_S^{BB}}{\partial b} > 0$, and its second derivative is equal to zero, i.e., $\frac{\partial^2 \pi_S^{BB}}{\partial b^2} = 0$. This implies that the supplier's profit function is increasing at b, and thus does not have

any local maximum. Therefore, the third condition of Definition 1 is not met, hence, the buyback contract with trade credit does not coordinate the supply chain with the retailer's limited funding. The results of the solution are summarized in Table 3.

Table 3. Parameters of coordinating buyback contract with trade credit.

Retailer's optimal order quantity	$q_R^* = F_\xi^{-1}\left(\frac{p-(\omega+c_R)(1+r_S)}{p-b-\nu}\right)$
Supply chain's optimal order quantity	$q_{SC}^* = F_\xi^{-1}\left(\frac{p-c}{p-\nu}\right)$
Wholesale price	$\omega^* = b\frac{p-c}{(p-\nu)(1+r_S)} + \frac{c}{1+r_S} - c_R$
Conditions	$b < p-\nu$, $p > c(1+r_S)$, $\frac{\nu-c_R}{p-\nu+c_R<r_S<\frac{c_S}{c_R}}$

4. Conditionally Coordinating Buyback Contract with Limited Funding and Demand Distributed as Uniformly

Results obtained in Sections 3.2. and 3.3. coincide with those presented in existing body of research, according to which coordination with the buyback contract as defined in Definition 1 cannot be achieved. Nonetheless, if the found parameters (see Tables 2 and 3) can provide the supplier with expected profit higher than the profit she could get by entering into a wholesale-price contract with the retailer, the supply chain can reach conditional coordination as defined by (Berezinets et al., 2020).

Definition 2. A buyback contract with limited funding (b^*, ω^*, q^*) that complies with the the following conditions:

1. $\max\limits_q \pi_R = \pi_R\Big(b, \omega(b), q_R^*\big(b, \omega(b)\big)\Big),\ \forall b \in X_S$;
2. $\exists\, \omega^*(b) : q_R^*\big(b, \omega^*(b)\big) = q_{SC}^* = q^*,\ \forall b \in X_S$;
3. $\pi_S^{BB}(b, \omega^*, q^*) > \pi_S^{WP}(\omega^0, q^0),\ q^* = q_{SC}^* : \max\limits_q \pi_{SC}^{BB} = \pi_{SC}^{BB}\big(\omega^*, q_{SC}^*(\omega^*)\big)$, $q^0 = q_R^* : \max\limits_q \pi_R^{WP} = \pi_R^{WP}\big(\omega, q_R^*(\omega)\big)$

can be called conditionally coordinating.

The first two conditions are those from Definition 1, the third condition shows that conditionally coordinating buyback contract improves the supplier's profit compared to the profit she can earn with the wholesale-price contract.

Wholesale-price contract is the simplest contract type, defined as a set of two parameters (ω, q). Researchers in supply chain contracting agree that the wholesale-price contract fails to coordinate the supply chain and is always dominated by other contract types, including the buyback contract (Cachon, 2003).

To determine the parameters of a conditionally coordinating buyback contract, parameters of a wholesale-price contract that maximize the retailer's profit need to be found, then the supplier's expected profit with the buyback contract can be compared with her profit with the wholesale-price contract. Solution for the

wholesale-price contract that maximizes the retailer's profit accords with the 4-step approach used to determine coordinating properties of the buyback contract in Section 3.1..

Solution proves that neither source of borrowing allows to achieve coordination with the wholesale-price contract. In the case of the bank loan raised by the retailer solution is the same as in the case where the retailer has sufficient funds. In both cases the wholesale-price contract coordinates the supply chain only if the wholesale price is equal to the supplier's cost:

$$\omega^* = c_S.$$

If so, the retailer succeeds in maximizing his profit, but the supplier's profit is equal to 0. Zero profit obviously makes the supplier prefer a higher wholesale price and non-coordinating wholesale-price contract.

With trade credit, the wholesale-price contract coordinates the supply chain if the wholesale price is expressed as

$$\omega^* = \frac{c}{1+r_S} - c_R = c_S - c\frac{r_S}{1+r_S}.$$

It is straightforward to confirm that in this case wholesale price is lower than the supplier's cost, i.e., $\omega^* < c_S$. Consequently, the function of the supplier's expected profit that depends on the interest rate r_S charged by the supplier and is below zero, which implies that with coordinating wholesale-price contract with trade credit the supplier suffers losses when the retailer chooses the order quantity that maximizes his profit. So the supplier also prefers to charge a higher wholesale price, and the contract does not coordinate the supply chain.

Determining the set of conditionally coordinating buyback contracts requires the values of the supplier's expected profit to be compared as in Definition 2; precisely the following values of the supplier's expected profit are compared:

- profit that is achieved when the retailer orders a quantity of goods that maximizes the supply chain profit with buyback contract, and
- profit that is achieved when the retailer orders a quantity that maximizes his own profit with wholesale-price contract.

The contract models analyzed further consider the supply chain producing and selling the product that is assumed to follow a uniform distribution of demand. Uniform distribution of the product demand has the following properties:

- demanded quantity lies within the closed interval with the minimal value equal to 0 and maximal value equal to β, i.e., $\xi \in [0, \beta]$;
- distribution function of the demand is $F_\xi(x) = \begin{cases} 0, x < 0 \\ \frac{x}{\beta}, 0 \le x \le \beta \\ 1, x > \beta \end{cases}$;
- expected sales volume is $E[\tau] = q - \int_0^q \frac{x}{\beta} \mathrm{d}x = q - \frac{q^2}{2\beta}$.

To construct a conditionally coordinating buyback contract with limited funding comparison of individual and total profits earned with the buyback and wholesale-price contracts is carried out through considering the difference between the profit with the buyback contract and that with the wholesale-price contract with respect

to the same wholesale price ω^* that maximizes the retailer's expected profit with the buyback contract. Three cases are considered: when retailer has sufficient funds, when retailer raises a bank loan, and when retailer uses supplier financing.

4.1. Conditionally Coordinating Buyback Contract with Sufficient Funds

As coordination allows to arbitrarily allocate the supply chain profit between the supplier and retailer, a parameter that allows for the profit allocation has to be included into the model. This step is done in (Berezinets et al., 2020), the paper that studies the setting equivalent to the case of retailer's sufficient funds in this body of research.

The authors find that the buyback contract is conditionally coordinating when $b > \frac{p-\nu}{2}$, and express the buyback price through a parameter λ that belongs to an open interval between 0 and 1, i.e.,

$$\lambda \in (0,1): \; b = \lambda(p-\nu). \tag{7}$$

The authors show that with λ the supply chain profit can be arbitrarily split between the supplier and retailer in proportion $\lambda : (1-\lambda)$ and, thus, λ determines the supplier's share of total profit. The authors state that the parameter λ can be considered as the supplier's leading power in negotiation, i.e., "negotiation leverage".

In the case where the retailer's funds are sufficient, the buyback price should satisfy the condition $b > \frac{p-\nu}{2}$, so it is clear that in this case λ should be greater than 0.5, i.e., $0.5 < \lambda < 1$.

With the use of the formula (7) the wholesale price and order quantity that maximize the supply chain profit expressed through the supplier's share of profit λ look as follows:

$$\omega^*(\lambda) = c_S + \lambda(p-c), \tag{8}$$

$$q^*_{SC} = \frac{p-c}{p-\nu}\beta, \tag{9}$$

and expressions for the expected profits are:

$$\begin{aligned}
\pi_S^{BB}\big(\omega^*(\lambda), q^*\big) &= \lambda\frac{\beta}{2}\frac{(p-c)^2}{p-\nu}, \\
\pi_R^{BB}\big(\omega^*(\lambda), q^*\big) &= (1-\lambda)\frac{\beta}{2}\frac{(p-c)^2}{p-\nu}, \\
\pi_{SC}^{BB}\big(\omega^*(\lambda), q^*\big) &= \frac{\beta}{2}\frac{(p-c)^2}{p-\nu}.
\end{aligned} \tag{10}$$

(Berezinets et al., 2020) also state that dependence of the buyback price on negotiation leverage given in (7) should always hold and arbitrarily divide the expected profit of the two-echelon supply chain between its members. Based on this property of the buyback price, the models of conditionally coordinating buyback contract with demand distributed as uniformly can also be constructed for the cases where the retailer takes the bank loan and uses trade credit.

4.2. Conditionally Coordinating Buyback Contract with Bank Loan

Solution shows that with the bank loan taken by the retailer the buyback contract is conditionally coordinating if the buyback price meets the condition $b > \frac{p-\nu}{2}$.

With the formula (7) inserted into expressions of the optimal wholesale price and order quantity, they look as follows:

$$\omega^*(\lambda) = \frac{c + \lambda(p-c)}{1+\lambda r_B} - c_R, \tag{11}$$

$$q^*_{SC} = \frac{p - c - cr_B}{(p-\nu)(1+\lambda r_B)}\beta. \tag{12}$$

Expected profits expressed through (7) look as:

$$\begin{aligned}
\pi_S^{BB}\big(\omega^*(\lambda), q^*\big) &= \lambda\frac{\beta}{2(p-\nu)}\left(\frac{p-c-cr_B}{1+\lambda r_B}\right)^2,\\
\pi_R^{BB}\big(\omega^*(\lambda), q^*\big) &= (1-\lambda)\frac{\beta}{2(p-\nu)}\left(\frac{p-c-cr_B}{1+\lambda r_B}\right)^2 + r_B K_R,\\
\pi_{SC}^{BB}\big(\omega^*(\lambda), q^*\big) &= \frac{\beta}{2(p-\nu)}\left(\frac{p-c-cr_B}{1+\lambda r_B}\right)^2 + r_B K_R,
\end{aligned} \tag{13}$$

and it is straightforward to confirm that the parameter λ splits not the entire supply chain profit, but the part of it that depends on the order quantity.

Parts of the profits that are dependent on the order quantity are divided between the players in proportion $\lambda : (1-\lambda)$. The second term of the retailer's expected profit is the product of the bank interest rate and retailer's holdings of cash. As the retailer raises the loan in the amount that he lacks to pay for the order quantity and adds this sum to the cash at his disposal K_R, this term in the formula of the retailer's profit can be considered as interest that retailer gets at the end of the selling season for the cash available. The condition $b > \frac{p-\nu}{2}$ shows that to achieve conditional coordination the supplier's negotiation leverage should be greater than 0.5, then her share of the supply chain profit will be between 0.5 and 1, i.e., $0.5 < \lambda < 1$.

4.3. Conditionally Coordinating Buyback Contract with Trade Credit

Analysis of the model of the buyback contract with trade credit shows that this contract conditionally coordinates the supply chain when the same constraint as with the bank loan is met: $b > \frac{p-\nu}{2}$. With this condition met the supplier's share of total profit will be greater than 0.5, i.e. $0.5 < \lambda < 1$. Solution shows that with trade credit the parameter λ splits total supply chain profit the same way it does with the bank loan – only parts of the profits that are dependent on the order quantity are arbitrarily divided between the players in proportion $\lambda : (1-\lambda)$. Optimal wholesale price, order quantity and expected profits expressed through the buyback price from (7) look as follows:

$$\omega^*(\lambda) = \frac{c + \lambda(p-c)}{1+r_S} - c_R, \tag{14}$$

$$q^*_{SC} = \frac{p-c}{p-\nu}\beta, \tag{15}$$

$$\begin{aligned}
\pi_S^{BB}\big(\omega^*(\lambda), q^*\big) &= \lambda\frac{\beta}{2}\frac{(p-c)^2}{p-\nu} - r_S K_R,\\
\pi_R^{BB}\big(\omega^*(\lambda), q^*\big) &= (1-\lambda)\frac{\beta}{2}\frac{(p-c)^2}{p-\nu} + r_S K_R,\\
\pi_{SC}^{BB}\big(\omega^*(\lambda), q^*\big) &= \frac{\beta}{2}\frac{(p-c)^2}{p-\nu}.
\end{aligned} \tag{16}$$

Formulas (9) and (15) show that the order volume q^*_{SC} that maximizes the supply chain profit in the case with trade credit is the same as the optimal order volume in the case with the retailer's sufficient funds. It is also clear that maximal supply chain profit with trade credit is equal to its profit with the retailer's sufficient funds (see formulas (10) and (16)).

Explanation for this observation is that in the model with trade credit the credit payment is transferred between the supplier and retailer and not to the third party as in the model with the bank loan (the bank). Therefore, the costs of borrowing are shared between the supply chain members and no additional costs associated with external sources of borrowing incur. As a result, the supply chain's profit with trade credit reaches its maximum equal to the case where no credit is used.

Therefore, conditionally coordinating buyback contract with limited funding allows to allocate the supply chain's profit so as to provide the supplier with more than its half. This is mathematically justified by the condition $0.5 < \lambda < 1$ imposed on the supplier's negotiation leverage (i.e., her share of total profit). This condition holds in all three cases considered in the model – with the retailer having sufficient funds and not using any credit, with the retailer taking the bank loan, and with the retailer using the trade credit offered by the supplier.

To justify the supplier's motivation to offer the trade credit instead of letting him raise the bank loan and possible retailer's preference for one type of credit over another comparative analysis of the contracts with the two types of credit is carried out further.

5. Selecting the Parameters of Conditionally Coordinating Buyback Contract with Limited Funding

5.1. Comparative Analysis of the Contracts

As the decision to offer a trade credit to the retailer is made by the supplier, it is important to justify the supplier's motivation that drives this decision. The supplier aims to improve her own profit, taking into account the goal of maximizing the supply chain profit. So the supplier will offer a trade credit to the retailer only if her own profit increases with this credit type relative to that with the bank loan.

Comparative analysis of the models with trade credit and bank loan has been conducted both for the conditionally coordinating buyback and wholesale-price contracts. Analysis of the buyback contract showed that the supplier's profit with trade credit is higher than with bank loan if the trade credit rate meets the following constraint:

$$r_S < \frac{1}{\frac{2(p-c)(1+\lambda r_B)^2}{\lambda r_B\left((p-c)(2+\lambda r_B)-cr_B\right)} - 1},$$

so it is in the supplier's interest to offer the trade credit to the retailer in order to increase individual and supply chain's profit.

Besides, from the formulas of the wholesale price in (11) and (14) it can be confirmed that for the same value of the wholesale price the trade credit rate can be expressed as $r_S = \lambda r_B$, hence the constraint imposed on the rate of trade credit

offered by the supplier with buyback contract looks as follows:

$$r_S \leq \min\left\{\lambda r_B; \frac{1}{\frac{2(p-c)(1+\lambda r_B)^2}{\lambda r_B\left((p-c)(2+\lambda r_B)-cr_B\right)}-1}\right\}. \tag{17}$$

To justify the supplierвЂ™s motivation to offer trade credit with the wholesale-price contract, the models with trade credit and bank loan are also compared. The supplier's profit with trade credit is compared with that with the bank loan using the formulas for the wholesale price that maximize the supply chain profit with buyback contract (see the formulas (8), (11) and (14)). Those formulas are inserted into the expressions for the order volume that maximizes the retailer's profit with wholesale-price contract and those for the supplier's profit. The formulas used in the comparative analysis of the wholesale-price contract are:

$$q_R^* = (1-\lambda)\frac{p-c}{p-\nu}\beta, \tag{18}$$

$$\pi_S^{WP}\left(\omega^*(\lambda), q^*\right) = \lambda(1-\lambda)\frac{(p-c)^2}{p-\nu}\beta, \tag{19}$$

for the wholesale-price contract with the retailer having sufficient funds,

$$q_R^* = \frac{1-\lambda}{1+\lambda r_B}\frac{p-c-cr_B}{(p-\nu)}\beta, \tag{20}$$

$$\pi_S^{WP}\left(\omega^*(\lambda), q^*\right) = \frac{\lambda(1-\lambda)}{(1+\lambda r_B)^2} \times \frac{(p-c-cr_B)^2}{p-\nu}\beta, \tag{21}$$

for the wholesale-price contract with the retailer taking the bank loan, and

$$q_R^* = (1-\lambda)\frac{p-c}{p-\nu}\beta, \tag{22}$$

$$\pi_S^{WP}\left(\omega^*(\lambda), q^*\right) = \lambda(1-\lambda)\frac{(p-c)^2}{p-\nu}\beta - r_S K_R, \tag{23}$$

for the wholesale-price contract with the retailer using trade credit.

Comparative analysis of the wholesale-price contract shows the same result as conditionally coordinating buyback contract. It shows that, in this case, the supplier also obtains a greater profit when offering the trade credit to the retailer instead of letting him borrow from the bank. The supplier's profit with trade credit is higher than with the bank loan if

$$r_S < \frac{1}{\frac{(p-c)(1+\lambda r_B)^2}{\lambda r_B\left((p-c)(2+\lambda r_B)-cr_B\right)}-1}.$$

Considering the equation $r_S = \lambda r_B$ derived from the expressions for the wholesale price (8), (11) and (14), the condition imposed on the trade credit rate for the wholesale-price contract looks as follows:

$$r_S \leq \min\left\{\lambda r_B; \frac{1}{\frac{(p-c)(1+\lambda r_B)^2}{\lambda r_B\left((p-c)(2+\lambda r_B)-cr_B\right)}-1}\right\}. \tag{24}$$

5.2. Selection Algorithm

As a result, a framework for negotiating the terms of a contract with limited funding can be proposed. The framework is presented as the flowchart of the negotiating process in Figure 1. It allows:

- to choose the contract type out of two options – buyback contract when the supplier's potential share of profit for a given wholesale price is higher than 0.5, and wholesale-price contract when her share of total profit is lower than 0.5;
- to decide which form of borrowing – bank loan or trade credit – to use when the retailer's funds are not enough to pay for the order quantity;
- to determine the contract terms so that each supply chain party is better off considering the type of credit chosen by the retailer.

Application of the framework starts from the point where the supplier and retailer come to an agreement about some value of the wholesale price ω. Using this value the supplier can calculate her share of the supply chain profit λ with buyback contract from the formula (8) as at this stage of the negotiation supplier is not aware of the retailer's possible limited funding; for a given value of the wholesale price this formula allows to find the value of λ. If the value of λ is between 0.5 and 1, buyback contract is chosen and further negotiation draws upon the left part of the flowchart; otherwise, when $0 < \lambda < 0.5$, it is more beneficial for the supplier to suggest the wholesale-price contract (right side of the flowchart).

Supplier's profit share $0.5 < \lambda < 1$. If for a given value of the wholesale price the supplier's share of total profit is greater than 0.5, she will be able to offer the following contracts:

- conditionally coordinating buyback contract $\big(\omega(\lambda), b(\lambda), q(\lambda)|0\big)$,
- conditionally coordinating buyback contract with trade credit $\big(\omega(\lambda), b(\lambda), q(\lambda)|r_S\big)$,
- conditionally coordinating buyback contract with bank loan $\big(\omega(\lambda), b(\lambda), q(\lambda)|r_B\big)$,
- wholesale-price contract with trade credit $\big(\omega(\lambda), q(\lambda)|r_S\big)$,or
- wholesale-price contract with bank loan $\big(\omega(\lambda), q(\lambda)|r_B\big)$.

The first three contract types relate to the conditionally coordinating buyback with limited funding and take into account the case where the retailer's funds are sufficient and no credit is used. If the retailer does not accept the contract terms offered by the supplier in those three cases, they switch to the wholesale-price contract either with the bank loan or trade credit. If the agreement about the contract terms is not reached again, the contract is completely rejected.

The algorithm suggests that having calculated the value of λ the supplier further calculates the buyback price b for the determined λ and q^*_{SC} from the formulas (7) and (9), and offers this buyback price to the retailer.

Retailer responds with some order quantity q that is either equal or less than the optimal order quantity from (9), as the retailer is always reluctant to order large amounts and the design of coordinating contracts aims to incentivize him to increase the order quantity. If the retailer's order quantity is equal to the optimal one from (9), supplier and retailer sign a conditionally coordinating buyback contract $\big(\omega(\lambda), b(\lambda), q(\lambda)|0\big)$, where zero stands for the interest rate in the case with sufficient funds as no borrowing.

If the order quantity q chosen by the retailer is lower than the one from (9), he is supposed to share information about the interest rate on the bank loan r_B

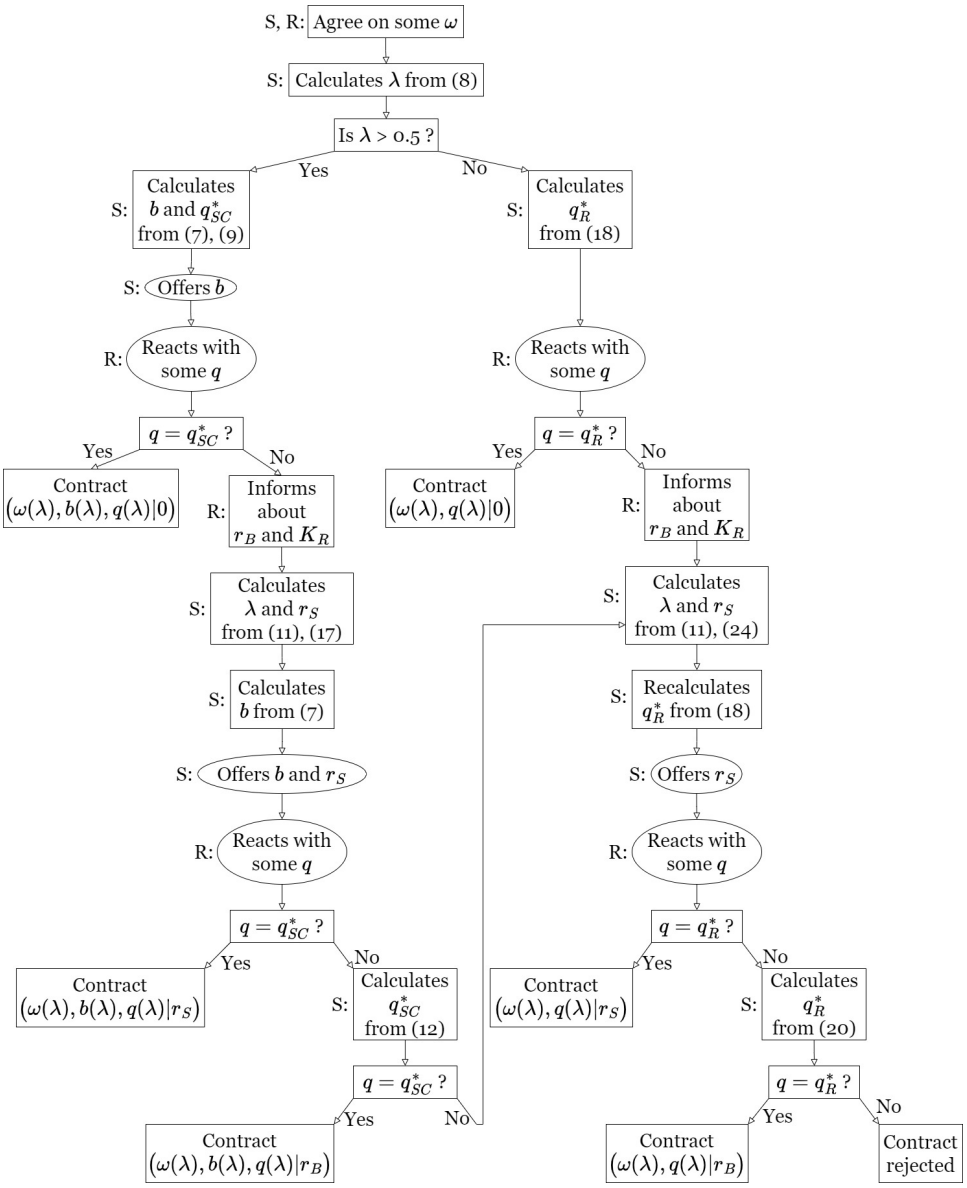

Fig. 1. Algorithm for selecting conditionally coordinating buyback contract with limited funding

and the amount of cash K_R at his disposal. With newly obtained information on the retailer's limited funding the supplier recalculates her share of the supply chain profit that can be earned with the retailer's bank loan. To obtain a new value of λ that takes the bank loan into account, the formula of the wholesale price (11) is used.

As justified by the mathematical solution in Section 5.1., under the buyback contract that conditionally coordinates the supply chain, its total profit is always

higher with trade credit than with bank loan, provided the condition (17) is satisfied. So the supplier is better off when offering trade credit to the retailer.

Based on the updated value of λ, the supplier can recalculate the buyback price (7) and determine the interest rate for the trade credit that satisfies the condition (17). Optimal order volume for a conditionally coordinating buyback contract with trade credit remains the same as in the case of the retailer's sufficient funds (see (9) and (15)).

After the supplier offers a new buyback price and trade credit rate, the retailer responds with a new order volume q. If the retailer's reaction q increases to the optimal one, they sign a conditionally coordinating buyback contract with trade credit $\big(\omega(\lambda), b(\lambda), q(\lambda)|r_S\big)$. Otherwise, the supplier turns down her offer of the trade credit and allows the retailer to take the bank loan in case the retailer's latest response is equal to (12). This order quantity is lower than (15), but provides conditional coordination under the buyback contract with bank loan $\big(\omega(\lambda), b(\lambda), q(\lambda)|r_B\big)$ that is signed at this step.

However, if the order quantity chosen by the retailer is lower than (12), the buyback contract is not considered anymore and the supplier continues with negotiating the wholesale-price contract, namely the wholesale-price contract with trade credit. For the share of the supply chain profit λ previously determined from (11) supplier determines the order quantity q_R^* that provides her with this profit share and the trade credit rate that meets the condition (24) for this contract type. To calculate the order quantity at this step, the formula (22) is used, and the supplier offers the trade credit rate determined at the previous step to the retailer.

Retailer responds with some order quantity, which he decides to purchase for the given wholesale price. If the retailer's choice is equal to (22), supplier and retailer enter into a wholesale-price contract with trade credit $\big(\omega(\lambda), q(\lambda)|r_S\big)$ that maximizes the retailer's profit and provides the supplier with the same share of the supply chain profit as under the conditionally coordinating buyback contract with limited funding, which is greater than 0.5.

If the retailer's response is lower than (22), then supplier compares it with the order quantity q_R^* from (20) that maximizes the retailer's profit with bank loan. If they are equal, the supplier lets the retailer raise the bank loan, and they sign a wholesale-price contract with bank loan $\big(\omega(\lambda), q(\lambda)|r_B\big)$. Otherwise, if the retailer's order quantity is lower than (20), the wholesale-price contract is rejected and no contract is selected out of two alternatives (the buyback and wholesale-price contracts).

Supplier's profit share $0 < \lambda < 0.5$. If for a given value of the wholesale price the supplier's share of total profit is going to be lower than 0.5, supplier prefers the wholesale-price contract and may offer:

- wholesale-price contract $\big(\omega(\lambda), q(\lambda)|0\big)$,
- wholesale-price contract with trade credit $\big(\omega(\lambda), q(\lambda)|r_S\big)$, or
- wholesale-price contract with bank loan $\big(\omega(\lambda), q(\lambda)|r_B\big)$.

Having the value of λ calculated at the previous step, supplier calculates the value of the order quantity using the formula (18) and compares it with the order quantity that retailer responds with. If the retailer's response is equal to (18), the wholesale-price contract $\big(\omega(\lambda), q(\lambda)|0\big)$ is signed, where zero stands for the cost of borrowing as it is not needed with the retailer's sufficient funds.

If the retailer's response is lower than (18), retailer is to explain his choice, so he shares information about his need for borrowing, interest rate on the bank loan and the amount of cash at his disposal. With updated information on the retailer's funds, supplier recalculates her share of the supply chain profit using the formula (11). As proved in Section 5.1., the supplier's profit with bank financing is lower than the profit she can get with trade credit, so the supplier determines the rate on the trade credit using the formula (24) and offers it to the retailer.

Retailer reacts with a new order quantity. If it is higher than the previous one and equal to (22), the wholesale-price contract with trade credit $\big(\omega(\lambda), q(\lambda)|r_S\big)$ is signed. Otherwise, supplier has to compare the latest retailer's response with the order quantity from (20). In case the retailer's response is equal to (20), supplier refuses to provide the retailer with trade credit and allows him to take a bank loan instead. The wholesale-price contract with bank loan $\big(\omega(\lambda), q(\lambda)|r_B\big)$ is selected. However, if the retailer's response is lower than the order quantity optimal for the contract with bank loan, the wholesale-price contract is completely rejected.

5.3. Numeric Example

To demonstrate the applicability of the developed framework, a numerical case based on the example studied in (Berezinets et al., 2020) has been examined. The authors consider the case of the supply chain with the supplier and retailer involved in manufacturing and retailing of the perishable product with short shelf life. The data used in the contract negotiation is given in Table 4.

Table 4. Initial data given in the case.

Retail price	$p = \$8$
Salvage value	$\nu = \$1$
Supplier's cost	$c_S = \$3$
Retailer's cost	$c_R = \$0.3$
Parameter of the demand distribution	$\beta = 200$

Assume that the retailer's holdings of cash $K_R = \$30.65$ and are not enough to pay for any order volume in the setting given. The cost of borrowing from the bank r_B is equal to 10%.

Supplier's profit share $0.5 < \lambda < 1$. Suggest the supplier and retailer agree on the wholesale price $\omega = \$5.5$. Supplier calculates her share of total profit with buyback contract and obtains $\lambda = 0.53$, which meets the condition for the contract to be conditionally coordinating.

As the supplier wants to earn the highest profit possible, she is interested in arriving at a conditionally coordinating buyback contract that allows her to improve individual profit compared to the profit she could earn with a wholesale-price contract with the same wholesale price. For $\lambda = 0.53$ the supplier's profit with conditionally coordinating buyback contract can be \$167.25, the retailer's profit can be \$148.32, and expected profit of the supply chain can be \$315.57.

Getting this profit values is possible if the order quantity chosen by the retailer is large enough, precisely 134 units (see (9)), so the supplier is motivated to offer the buyback condition to incentivize the retailer to increase his order volume. Supplier

calculates the buyback price for the determined profit share using the formula (7) and determines the buyback price $b = \$3.71$, which she offers to the retailer.

Retailer responds with some order quantity. If it is equal to 134 units, conditionally coordinating buyback contract by Definition 2 is selected; supplier and retailer arrive at the contract $\big(\omega(\lambda) = \$5.5, b(\lambda) = \$3.71, q(\lambda) = 134|0\big)$.

However, if the retailer responds with an order volume lower than 134 units, he is supposed to explain why he wants to order so little and to share information about his need for borrowing. Retailer informs the supplier about the interest rate charged by the bank $r_B = 10\%$ and the amount of cash he has $K_R = \$30.65$.

Since the bank loan taken by the retailer will affect the allocation of total profit and the supplier's share of it, supplier recalculates λ using (11) and obtains an updated profit share $\lambda = 0.6$, which is greater than the initial $\lambda = 0.53$, but provides total supply chain profit that is lower than with trade credit, or equally with sufficient funds. Thus, the supplier needs to incentivize the retailer by offering the trade credit so that the retailer decided to increase the order quantity and the supply chain profit improved.

To do so, supplier calculates the rate she should charge for the trade credit using (17) and new buyback price as in (7). Trade credit rate is to lie within the interval $r_S \leq \min\{0.06; 0.056\}$, which means that the supplier should offer the rate $r_S = 5.6\%$. The updated buyback price should be \$4.2. These terms are offered to the retailer.

Retailer responds with a new order quantity, which should be 134 units for the retailer to get the trade credit and buyback price offered by the supplier. If the retailer arrives at the decision to order 134 units, they sign the conditionally coordinating contract with trade credit $\big(\omega(\lambda) = \$5.5, b(\lambda) = \$4.2, q(\lambda) = 134|r_S = 5.6\%\big)$. With this contract, the supply chain profit is \$315.57; the supplier's portion of total profit ($\lambda = 0.6$) is equal to \$187.62; the retailer's profit is \$127.95.

If the retailer's response does not match the optimal order quantity of 134 units, offering him the trade credit with the given wholesale price \$5,5 is not beneficial for the supplier; and the supplier turns down her offer of the trade credit. Nonetheless, the supplier still can come to a conditionally coordinating buyback contract with bank loan.

To determine the order quantity for this contract to be conditionally coordinating, the supplier calculates it with (12): $q^*_{SC} = 118 units$. If the retailer's latest response is 118 units, the supplier signs the conditionally coordinating buyback contract with bank loan $\big(\omega(\lambda) = \$5.5, b(\lambda) = \$4.2, q(\lambda) = 118|r_B = 10\%\big)$. According to its terms, total supply chain profit is \$245.87; the supplier's share of it is \$147.52; and the retailer earns \$98.35. Otherwise, if the retailer chooses to order less than 118 units, supplier turns down the buyback offer and continues with negotiating over the wholesale-price contract with trade credit.

As the retailer has already informed the supplier about the limited funding he is faced with, supplier can determine the trade credit rate – it should meet the condition (24): $r_S \leq \min\{0.06; 0.119\}$, so the supplier offers the trade credit with $r_S = 6\%$, and the retailer reacts with an updated order quantity. For the supplier to obtain 60% of the supply chain profit ($\lambda = 0.6$) with the wholesale-price contract with trade credit, the retailer's new choice of the order quantity has to be equal to 54 units (22). If the retailer's latest response matches the optimal one, the wholesale-price contract with trade credit $\big(\omega(\lambda) = \$5.5, q(\lambda) = 54|r_S = 6\%\big)$ is selected. With

this contract, the supplier obtains \$149.64; retailer obtains \$52.33; the supply chain profit is \$201.97.

But if the retailer's updated order quantity is lower than 54 units, the supplier should compare it with (20): for the bank's rate of 10% the optimal order quantity should be 47 units. If so, the wholesale-price contract $\big(\omega(\lambda) = \$5.5, q(\lambda) = 47 | r_B = 10\%\big)$ is selected. With this contract, the supplier's profit share of 60% equals to \$116.55; the retailer's profit is \$41.91; the supply chain profit is \$158.46. If the retailer chooses to order less than 47 units, the contract is rejected.

To sum up, when the supplier's negotiation leverage is greater than 0.5, the supplier's highest profit is earned with the conditionally coordinating buyback contract with trade credit (\$187.62). Retailer obtains the highest profit with the conditionally coordinating contract with sufficient funds (\$148.32). In both cases the supply chain profit is \$315.57.

With the bank loan, the supply chain profit decreases under the buyback contract; with the wholesale-price contracts both with trade credit and bank loan the supply chain profit declines more significantly – in both cases it is lower compared to that with the buyback contract with bank loan. The lowest supply chain profit is achieved with the wholesale-price with bank loan (\$158.46). It corresponds with the lowest values of the supplier's and retailer's profits – \$116.55 and \$41.91, respectively.

Supplier's profit share $0 < \lambda < 0.5$. To demonstrate how the framework can be applied to select a wholesale-price contract, suggest the supplier and retailer agree on the wholesale price $\omega = \$4.65$. Supplier calculates her share of the supply chain profit that could be earned with conditionally coordinating buyback contract using the formula (8) and obtains $\lambda = 0.35$, which is less than 0.5 and does not meet the condition for the conditionally coordinating buyback contract, so the supplier considers the wholesale-price contract.

Retailer responds with some order quantity, which the supplier needs to compare with the order quantity calculated with (18) (87 units). If the retailer responds with this order quantity, the negotiation has come to an end and the wholesale-price contract $\big(\omega(\lambda) = \$4.65, q(\lambda) = 87 | 0\big)$ is selected. According to its terms, the supplier's profit is \$143.59, the retailer's profit is \$133.33, and the supply chain profit is \$276.91.

If the retailer chooses to order less than 87 units, he informs the supplier about the interest rate charged by the bank and the amount of cash he has. Since the bank loan taken by the retailer will change the profit levels of both the supplier and retailer, supplier recalculates λ using (11) and obtains an updated value of λ: $\lambda = 0.39 \approx 0.4$, which is greater than the initial one.

With the wholesale-price contract the supplier's profit with bank loan is lower than with trade credit, and thus the supplier needs to incentivize the retailer by offering a trade credit so that the retailer increased the amount of goods he wants to order and improved the supply chain profit. To do so, supplier calculates the rate she should charge for the trade credit using (24): $r_S \leq \min\{0.04; 0.079\}$, and offers the trade credit with $r_S = 4\%$ to the retailer.

To get the trade credit, retailer should order the amount calculated with (22); in this example it should be 81 units. If he arrives at the decision to order 81 units, the wholesale-price contract with trade credit $\big(\omega(\lambda) = \$4.65, q(\lambda) = 81 | r_S = 4\%\big)$ is signed. With this contract expected profits achieve the following levels: the supply

chain profit is \$265.1, the supplier's profit is \$150.3, and the retailer's profit is \$114.8.

But if the retailer's response is lower than 81 units, supplier is not motivated to provide him with trade credit, turns down her offer and lets the retailer take the bank loan instead as long as the retailer's order quantity is equal to 72 units as in (20). In this case the wholesale-price contract with bank loan $\big(\omega(\lambda) = \$4.65, q(\lambda) = 72 | r_B = 10\%\big)$ is selected. According to its terms, total supply chain profit is \$214.94, the supplier's profit is \$121.07, and the retailer's profit is \$93.87. Otherwise, if the retailer chooses to order less than 72 units, the contract is completely rejected.

Thus, when the supplier's negotiation leverage λ is less than 0.5 and negotiated contract type is the wholesale-price contract, the highest supplier's profit is earned with trade credit (\$150.25). The highest retailer's profit is achieved in the case where he has sufficient funds (\$133.33); in this case the supply chain profit is also the highest (\$276.91). Supplier and retailer both earn their lowest profits when the retailer raises the bank loan, \$121.07 and \$93.87, respectively. The supply chain profit achieved in this case is also the lowest (\$214.94).

The case study confirms that with both buyback and wholesale-price contracts the supplier's profit is the highest with trade credit and the lowest with bank loan. This conclusion is in line with (Kouvelis, Zhao, 2012), according to which in business practice it is very common for suppliers to provide retailers with trade credit even when the retailer's funds are sufficient as it is beneficial for the supplier and positively affects her profit. The highest retailer's profit with both contract types is achieved when his funds are sufficient, which is reasonable since in this case the retailer does not run any additional cost of borrowing. The supply chain profit with trade credit is the same as with sufficient funds with the buyback contract since its profit function does not depend on λ and remains constant. Rationale for the approach to choosing the type and terms of the contract can be illustrated with Figure 2 showing the supply chain profit with both the wholesale-price and buyback contracts with two forms of credit – bank loan and trade credit.

Figure 2 shows that with the buyback contract both with sufficient funds and trade credit the supply chain profit remains steady at any value of the supplier's profit share λ, while with bank loan it slightly declines. With both buyback and wholesale-price contracts, the supply chain profits achieve similar values in the case of the retailer's sufficient funds and trade credit, thus making this type of credit more attractive in order to improve the supply chain profit. Even though with conditionally coordinating buyback contract the supply chain profit achieves higher levels in all three cases relative to the wholesale-price contract, it is important to mention that when the supplier's share of supply chain profit is lower than 0.5, buyback contract does not provide conditional coordination, hence the supplier's profit is not improved compared to the wholesale-price contract and the supplier loses profit while the retailer's profit achieves its maximal values. On the contrary, with wholesale-price contract the supply chain profit significantly goes down as λ grows, confirming that before it reaches the threshold of 0.5, the supply chain parties should turn to the wholesale-price contract and after λ passes the threshold, they should choose the buyback contract.

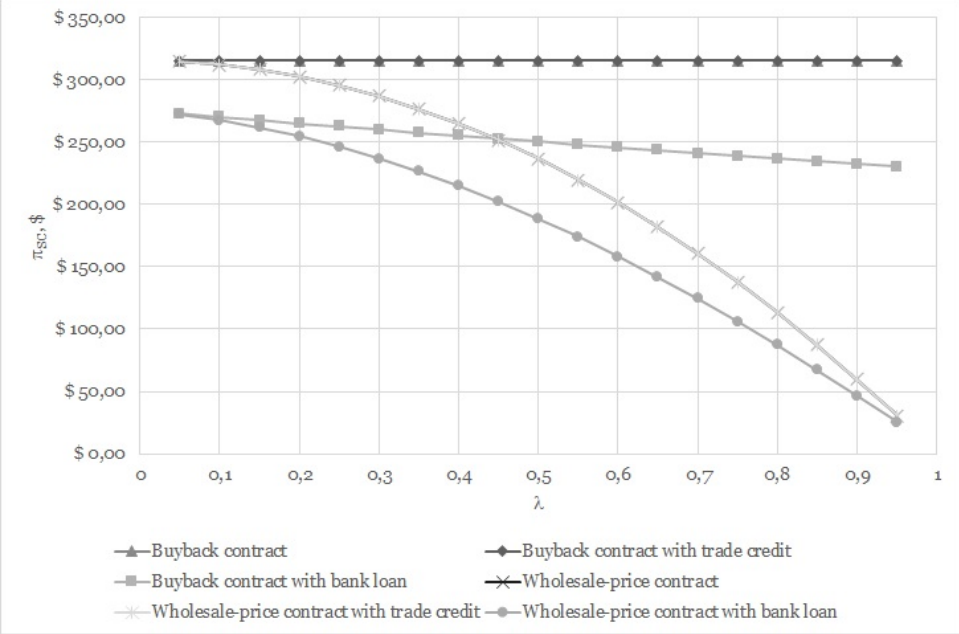

Fig. 2. Supply chain profit with the wholesale-price and buyback contracts with trade credit and bank loan

6. Conclusion

The paper considers the buyback contract with limited funding and investigates coordinating properties of this contract and aspects of its practical implementation in the case where the retailer's funds are limited and he is faced with the credit necessity. To deal with limited funding, the retailer chooses between two forms of borrowing – bank loan and trade credit.

To study supply chain coordination, the model of the buyback contract with limited funding is constructed for a supply chain consisting of one supplier and one retailer. The model analysis shows that the buyback contract does not coordinate the two-echelon supply chain either with the bank loan raised by the retailer or with the trade credit provided by the supplier. Coordination fails as the profits earned by the supply chain members do not achieve their maximum and, hence, the supply chain's profit as their sum is not maximized either.

Nonetheless, it is justified that the buyback contract with limited funding allows to achieve conditional coordination. Under conditionally coordinating buyback contract with limited funding the retailer's and supply chain's profits are to be maximized whereas the supplier's profit must exceed her profit with the wholesale-price contract. The model analysis includes the step dedicated to building the model of the wholesale-price contract with limited funding for the cases with bank loan and trade credit. It is shown that with both forms of borrowing there exist sets of conditionally coordinating buyback contracts that provide the retailer with maximal profit and improve the supplier's profit relative to the wholesale-price contract with the same form of credit; conditions that should be satisfied for these contracts to be conditionally coordinating are also identified.

To justify the supplier's motivation to offer trade credit and the retailer's choice between the supplier and bank financing, comparative analysis of both conditionally coordinating buyback and wholesale-price contracts is carried out. It shows that under both contract types the supplier's profit with trade credit is higher than with bank loan as long as the trade credit rate is lower than the rate charged by the bank and satisfies a certain constraint. The retailer's and supply chain's profits in this case also improve relative to those with the bank loan. Thus, rational supplier and retailer should always prefer trade credit to bank loan both when they enter into a conditionally coordinating buyback and wholesale-price contracts.

Based on the obtained solutions, the algorithm for selecting the parameters of conditionally coordinating buyback contract with limited funding is proposed. This algorithm allows to select the contract parameters based on a wholesale price they have agreed prior to the start of the selection process. The algorithm also helps to determine the parameters of the wholesale-price contract that improve expected profits of the supply chain members. Selection of the wholesale-price contract with limited funding concerns the case where the negotiation does not arrive at a conditionally coordinating contract (when the supplier's negotiation leverage λ is below 0.5).

The proposed model and algorithm are then applied to the case of the supply chain that is engaged in manufacturing and retailing of a perishable product with short shelf life. The case study considers the models of the buyback contract built for the cases with the bank loan and trade credit, and demonstrates that conditional coordination of the supply chain can be achieved in both cases. It also shows that profits earned by the supplier and retailer in the case where retailer utilizes a trade credit exceed those earned by them in the case where retailer takes a bank loan. Thus, it is confirmed that trade credit is preferable to bank loan as this form of financing allows the supply chain members to obtain higher supply chain profit relative to that with bank loan. Application of the selection algorithm to the numeric case illustrates how the supplier and retailer can arrive at the contracts that are beneficial for both of them and improve total gain after they have agreed on some value of the wholesale price.

References

Arshinder, A. K., S. G. Deshmukh (2008). *Supply chain coordination:perspectives, empirical studies and research directions.* International Journal of Production Economics, **115(2)**, 316–335.

Babich, V., H. Li, P. Ritchken, Y. Wang (2012). *Contracting with asymmetric demand information in supply chains.* European Journal of Operational Research, **217(2)**, 333–341.

Berezinets, I., M. Meshkova, N. Nikolchenko (2019). *The problem of supply chain profit maximization using sales rebate contract.* Contributions to Game Theory and Management, **12**, 70–99.

Berezinets, I., T. Voronova, N. Zenkevich, N. Nikolchenko (2022). *Coordinating contracts as an instrument of supply chain profit maximization under short-term financing necessity.* Contributions to Game Theory and Management, **13**, 57–94.

Berezinets, I., N. Zenkevich, A. Rucheva, N. Nikolchenko (2020). *Do the buyback contracts of undistributed stocks coordinate the supply chain?* (in Russian). Vestnik of Saint Petersburg University. Management, **19(4)**, 461–492.

Bose, I., P. Anand (2007). *On returns policies with exogenous price.* European Journal of Operational Research, **178(3)**, 782–788.

Cachon, G. P., M. Lariviere (2005). *Supply chain coordination with revenue-sharing contracts: strengths and limitations.* Management Science, **51**, 30–44.

Cachon, G. P. (2003). *Supply chain coordination with contracts.* In: Supply chain Management: Design, Coordination and Operation, (Graves, S. T. de Kok, eds.), 11, pp. 227–339.

Chen, X. (2015). *A Model of trade credit in a capital-constrained distribution channel.* International Journal of Production Economics, **159**, 347–357.

Fu, Y., F. Liu (2019). *The problem of a capital-constrained supply chain with a risk-averse retailer.* In: 2019 8th International Conference on Industrial Technology and Management (ICITM), pp. 1–6. Cambridge, United Kingdom.

Giannoccaro, I. (2018). *Centralized vs. decentralized supply chains: the importance of decision maker's cognitive ability and resistance to change.* Industrial Marketing Management, **73**, 59–69.

Granot, D., S. Yin (2005). *On the effectiveness of returns policies in the price dependent newsvendor model.* Naval Research Logistics, **52(8)**, 765–778.

Heydari, J., T. M. Choi, S. Radkhah (2017). *Pareto improving supply chain coordination under a money-back guarantee service program.* Service Science, **9(2)**, 91–105.

Kagan, J. (2022, 17 July). *Trade Credit.* Investopedia. Available from: https://www.investopedia.com/terms/t/trade-credit.asp

Kouvelis, P., W. Zhao (2012). *Financing the newsvendor: supplier vs. bank, and the structure of optimal trade credit contracts.* Operations Research, **60(3)**, 566–580.

Kouvelis P., W. Zhao (2015). *Supply chain contract design under financial constraints and bankruptcy costs.* Management Science, **62(8)**, 2341–2357.

Lee, H. L., V. Padmanabhan, S. Whang (1997). *The bullwhip effect in supply chains.* MIT Sloan Management Review, **38(3)**, 92–102.

Moon I., X.-H. Feng, K.-Y. Ryu (2015). *Channel coordination for multi-stage supply chains with revenue-sharing contracts under budget constraints.* International Journal of Production Research, **53(16)**, 4819–4836.

Pasternack, B. A. (1985). *Optimal pricing and return policies for perishable commodities.* Marketing Science, **4(2)**, 166–176.

Saha, S. (2013). *Supply chain coordination through rebate induced contracts.* Transportation Research Part E: Logistics and Transportation Review, **50(1)**, 120–137.

Taylor, T. A. (2002). *Supply chain coordination under channel rebates with sales effort effects.* Management science, **48(8)**, 992–1007.

Tsay, A. A. (2001). *Managing retail channel overstock: markdown money and return policies.* Journal of Retailing, **77(4)**, 457–492.

Tsay, A. A. (2002). *Risk sensitivity in distribution channel partnerships: implications for manufacturer return policies.* Journal of Retailing, **78(2)**, 147–160.

Tsay, A. A., S. Nahmias, N. Agrawal (1999). *Modeling supply chain contracts: a review.* In: Quantitative models for supply chain management (Tayur, S., R. Ganeshan, M. Magazine, eds.), pp. 299–336. MA: Kluwer Academic Publishers.

Wu, D. (2013). *Coordination of competing supply chains with newsvendor and buyback contract.* International Journal of Production Economics, **144(1)**, 1–13.

Xiao S., S. P. Sethi, M. Liu, S. Ma (2017). *Coordinating contracts for a financially constrained supply chain.* Omega, **72**, 71–86.

Zenkevich, N., M. Gladkova (2018). *Revenue sharing contracts for value chain coordination: the case of motion picture industry in the USA* (in Russian). Vestnik of Saint Petersburg University. Management, **17(1)**, 26–45.

Zhan, J., X. Chen, Q. Hu (2018). *The value of trade credit with rebate contract in a capital-constrained supply chain.* International Journal of Production Research, **57(2)**, 1–18.

Zhao Y., T.-M. Choi, T. C. E. Cheng, S. P. Sethi, S. Wang (2014). *Buyback contracts with price dependent demands: effects of demand uncertainty.* European Journal of Operational Research, **239(3)**, 663–673.

Zou, X., S. Pokharel, R. Piplani (2008). *A two-period supply contract model for a decentralized assembly system.* European Journal of Operational Research, **187**, 257–274.