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Coordination in Supply Networks with Combined Topology

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Abstract The paper considers the problem of supply chain coordination in supply networks using the revenue-sharing contract. The supply network has a combined topology and consists of three levels of participants: manufacturers, a distributor, and retailers. The authors consider the companies' performance improvement as the motivation of companies to operate in coordinating conditions. Simulation of contract conditions is implemented for different types of supply networks: from a simple case to a more complex one. The contract procedure is modeled as a game between two companies. The game solution is a set of contract parameters, providing Nash equilibrium, achieving the maximum of the supply network profit and Pareto-optimality of the obtained solution. The results show the application of revenue-sharing contracts in such networks has several peculiarities: the role of a coordinator in the supply network should be performed by a distributor; the distributor, by setting unified revenue shares for all retailers and manufacturers, gets a share of the expected profit of the supply network, each retailer gets a share of the supply network profit for its sales channel, and each manufacturer gets a share of the supply network profit for its product.

Keywords: supply network, motivation, performance, coordination, coordinating contract, revenue-sharing contract.

1. Introduction

The analysis of foreign and Russian studies on supply network management shows that the topic of improving companies' performance in supply networks has so far been considered by researchers mainly for supply chains consisting of two participants. Some papers (Beamon and Chen, 2014) pay attention to distribution and assembly-type supply networks, supply networks with more complex topology are considered to a lesser extent. Another shortcoming of existing studies is the consideration of predominantly centralized supply networks, while in practice supply networks are decentralized entities with a large number of independent participants, which require special coordinating mechanisms to motivate them to achieve common supply network goals (Whang, 1995; Chakraborty et al., 2015; Becker-Peth and Thonemann, 2016).

Among the coordinating mechanisms used in supply networks, coordinating

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contracts are the most common. This phenomenon can be explained by the relative ease of applying coordinating contracts in practice compared to the implementation of such approaches as collaborative planning, forecasting, and replenishment (CPFR) in supply networks, which require a high level of process automation and information exchange between participants. Attention to the topic of coordinating contracts by researchers and practitioners is also since their application allows to motivate participants in a decentralized network to achieve the best result for the entire network when they are guided only by their interests.

According to the classical approach, the supply network is a set of organizations connected by material, information, and financial flows from the sources of raw materials to the final consumers. However, the basis of intercompany relations of the organizations in a supply network is bilateral contracts defining conditions of interaction only between two participants of a supply network. In this connection, the key question of this research is, whether it is possible to coordinate supply network participants' actions so that each of them will be motivated to improve the performance of the supply network at the improvement of own performance and preservation of bilateral relations of supply network participants.

To answer the research question, the authors consider a problem of supply chain coordination with the application of the revenue-sharing contract which is widely applied in various industries, including mechanical engineering, pharmaceutics, and the film industry. Simulation of contract conditions is implemented for different types of supply networks: from a simpler case to a more complex one. The authors consider the company's performance improvement as the motivation of companies to operate in coordinating conditions.

2. Topology and Decision-making Process in Supply Networks

Among the supply network definition approaches presented in the literature we can distinguish classical, process-oriented, marketing and attitudinal approaches. The classical approach is based on logistics. According to the classical approach, a supply network is defined as a set of organizations linked by material, information and financial flows from sources of raw materials to final consumers (Ganeshan and Harrison, 1995; Lummus and Alber, 1997; Lambert and Stock, 2005; Chopra and Meindl, 2007; Christopher, 2011; Council of SCM Professionals (CSCMP), glossary; APICS Dictionary; Lu, 2011). According to the process-oriented approach, a supply network is an interconnected set of processes both inside a firm and between firms that have a commercial relationship and produce a product or service for final consumers (Krajewski et al., 2013; Slack et al., 2013; Quinn, 1997). The marketing approach views the supply network as a value chain for the final consumer (Prahalad and Ramaswamy, 2004; Krotov et al., 2008). The relational approach considers the supply network as a chain of interconnected business relationships in which each exchange relationship occurs between business partners (Braziotis et al., 2013; Anderson et al., 1994; Hanf et al., 2009).

This paper focuses on the relationships between the participants in the supply network, therefore a logistical and attitudinal approach will be used to define the supply network: a supply network is a set of firms connected by material, financial, information flows and the relationships between firms in the supply network arising from their interactions.

Graphical representation of supply network, with the same set of participants, but reflecting different flows and types of links, leads to the formation of supply networks with different topology (Hearnshaw and Wilson, 2011). For the first time, the classification of types of supply network topology is given in (Harland, 1996), in which the author distinguished a supply chain consisting of two participants, a sequential supply chain consisting of many participants interacting sequentially, and a general structure supply network. Later, Russian and foreign researchers also singled out distribution and assembly supply networks. Based on the analysis of the classifications of supply network types presented in the literature based on their topology, the following types of supply networks are proposed to be distinguished as key types: with sequential topology (supply network consisting of more than two sequentially interacting participants); distributive (supply network in which the number of participants increases in the direction of material flow, with each participant having no more than one predecessor); assembly network (supply network in which the number of participants decreases in the direction of material flow, with each participant having no more than one predecessor); combine network (a combination of distribution and assembly supply networks). (Harland, 1996; Meyr and Stadtler, 2005; Dominguez et al., 2015; Pathak et al., 2007; Meyr and Stadtler, 2005; Beamon and Chen, 2014).

In terms of the decision-making process, supply networks are divided into centralized (integrated) and decentralized (not integrated). A centralized supply network is a set of participants integrated into a group of companies or being structural subdivisions of a single company. Centralized supply networks are characterized by the presence of a single decision-making body (focal firm) (Lambert and Stock, 2005). The composition of participants in such supply networks is characterized by low variability. In decentralized network participants make decisions independently and in their interests (Christopher, 2011). Decentralized supply networks refer to networks with a high level of variability in the composition of participants, whose interaction is based on trust and is realized through the conclusion of agreements (contracts). In terms of performance efficiency, centralized networks have advantages over decentralized ones (Lee and Whang, 1999), since it is much more difficult to coordinate actions and motivate independent participants to achieve the best overall result. To solve this problem, it is necessary to use special mechanisms — coordination mechanisms.

3. Coordination in Supply Networks

The classic definition of coordination in supply networks is that presented in (Malone and Crowston, 1994). The authors define coordination as the management of interdependent actions. According to them, this definition is intuitively understandable, because "in the absence of interdependence between the actions of the participants of any process, there is nothing to coordinate". (McClellan, 2003) defines coordination as the interaction of supply network participants that ensures improved performance of the parties involved. In (Lambert and Cooper, 1998; Lambert et al., 1999) coordination is considered as a type of inter-company interaction of supply network participants, which is aimed at improving the performance of supply network participants and at the same time — the performance of the entire network.

Supply network coordination is implemented by using a certain set of techniques and tools, called coordination mechanisms, which help to manage the relationship between supply network participants (Cachon, 2003; Heydari et al., 2017), as well as motivate them to coherent actions that lead to improved performance of the supply network, taking into account the individual interests of each participant.

One of the most common tools for the coordination of decentralized networks is contracts. An analysis of the work on supply network coordination shows that researchers define a coordinating contract in different ways. (Wong et al., 2009; Leng and Parlar, 2010) define a contract as coordinating contract if its application maximizes supply network profit. (Taylor, 2002; Saha, 2013) point out that in addition to maximizing supply network profit, a win-win situation should be achieved between the participants in the supply network in which the use of the contract is beneficial to each participant (win-win situation). Other researchers use a game-theoretic approach to define the coordinating contract and model the contract procedure as a two-person game, the solution to which is a set of contract parameters that provide Nash equilibrium (Cachon, 2003). In addition, the coordinating contract should ensure that the supply network achieves maximum profit and that the resulting solution is pareto-optimal (Cachon, 2003; Heydari et al., 2017; Heydari and Asl-Najafi, 2018). In other words, when a coordinating contract is used, there is no other set of parameters for which the expected profits of the supply network participants are not less than those without the contract, and at least one of them has a greater profit (Heydari et al., 2017).

In this research, the authors use a game-theoretic approach to define a coordinating contract. The contract procedure is modeled as a game of two persons, the solution of which is a set of contract parameters, providing Nash equilibrium, achieving the maximum of the supply network profit and Pareto-optimality of the obtained solution. When a coordinating contract is used, there is no other set of parameters for which the expected profits of the supply network participants are not less than without the contract, and, at least one of them, expected profit is greater (Cachon, 2003; Heydari et al., 2017; Heydari and Asl-Najafi, 2018).

4. Coordination of the Decentralized Supply Network with Revenue-sharing Contract

To solve the problem of coordinating a decentralized supply network, we begin by considering a supply network with a consistent topology and including three participants: manufacturer, distributor, and retailer.

The manufacturer offers products to the distributor at the wholesale price per unit, w_m. Distributor offers products to the retailer at wholesale price w_d. In response to a distributor's offer, the retailer orders products in volume q. All participants interact under conditions of full and sufficient information to decide on the contract agreement and its implementation. The unit costs of the participants are as follows c_m – the manufacturer's costs of production, c_d – distributor's shipping and storage costs, c_r – the retailer's sales costs. The retailer sells products in the marketplace at retail price p. In case he cannot sell the whole volume at price p, he can sell it at residual value v, herewith, $p > v > C$, where C – unit supply network costs, $C = c_m + c_d + c_r$.

Let ξ is the continuous random variable of product demand with the density function $f_\xi(x)$, and τ is the random variable of sales volume and $\tau = g(\xi)$, where

$$\tau = g(\xi) = \begin{cases} \xi, 0 \leq q \\ q, \xi \geq q. \end{cases}$$

Denote the expectation of sales by $S(q)$:

$$S(q) = E[\tau] = q - \int_0^q F_\xi(x)dx.$$

The distributor in the supply network enters into two contracts: one to supply products to the retailer and the other to purchase products from the manufacturer. When a distributor and a retailer enter into a contract, the participants agree on shares $\varphi, (1 - \varphi)$, that each of the participants to the contract will receive from the expected revenue of the retailer $(p\text{-}v)S(q) + vq$. When a distributor and a manufacturer enter into a contract, the participants agree on shares θ,$(1 - \theta)$, that each will receive of the distributor's expected revenue: $\omega_d + (1 - \varphi)((p - v)S(q) + vq)$.

When selecting the wholesale price, the distributor takes into account that the retailer will seek to maximize his expected profit when choosing the volume of products ordered. In the conditions that the distributor has a contract for the supply of products with the manufacturer, he sets such a wholesale price, which in the conditions that the retailer will seek to maximize his profit, will maximize the profit of the whole network.

For all supply network participants to be motivated to achieve the optimal outcome for the supply network that is maximizing the supply network profit, their expected profit must depend on the performance of the entire network.

Retailer's expected profit:

$$Prof_R(q) = \varphi((p - v)S(q) + vq) - c_r q - \omega_d q. \tag{1}$$

Distributor's expected profit:

$$Prof_D(q) = \theta((1 - \varphi)((p - v)S(q) + vq) + \omega_d q) - c_d q - \omega_m q. \tag{2}$$

Manufacturer's expected profit:

$$Prof_M(q) = (1 - \theta)((1 - \varphi)((p - v)S(q) + vq) + \omega_d q) + \omega_m q - c_m q. \tag{3}$$

Supply network expected profit:

$$Prof_{SC}(q) = Prof_R(q) + Prof_D(q) + Prof_M(q) =$$

$$= (p - v)S(q) + vq - c_r q - c_d q - c_m q$$

or

$$Prof_{SC}(q) = (p - v)S(q) + vq - Cq. \tag{4}$$

Under the conditions of supply network coordination, the participants, setting shares of the expected revenue at each site of the network, receive shares of the expected profit of the supply network. To do this, let's find the coordinating relationships of the contract parameters for each section of the supply network: manufacturer — distributor, and distributor — retailer.

The coordinating relationship of the parameters of the revenue-sharing contract between a distributor and a retailer in a supply network with a consistent topology:

$$\omega_d = \varphi C - c_r. \tag{5}$$

The coordinating relationship of the parameters of the revenue-sharing contract concluded between the manufacturer and the distributor in the supply network with a consistent topology:

$$\omega_m = \theta(c_d + c_m) - c_d. \tag{6}$$

Substituting the obtained expressions (5) and (6) into the expressions for the expected profits of supply network participants (1)–(3), we get:

$$\begin{gathered} Prof_R(q) = \varphi((p-v)S(q) + vq) - c_r q - \omega_d q = \\ = \varphi((p-v)S(q) + vq) - c_r q - (\varphi C - c_r)q = \varphi((p-v)S(q) + vq) - Cq) = \\ = \varphi Prof_{SC}(q). \end{gathered} \tag{7}$$

$$\begin{gathered} Prof_D(q) = \theta((1-\varphi)((p-v)S(q) + vq) + \omega_d q) - c_d q - \omega_m q = \\ = \theta((1-\varphi)((p-v)S(q) + vq - Cq) = \\ = \theta((1-\varphi)Prof_{SC}(q). \end{gathered} \tag{8}$$

$$\begin{gathered} Prof_M(q) = (1-\theta)((1-\varphi)((p-v)S(q) + vq) + \omega_d q) + \omega_m q - c_m q = \\ = (1-\theta)((1-\varphi)((p-v)S(q) + vq - Cq) = \\ = (1-\theta)((1-\varphi)Prof_{SC}(q). \end{gathered} \tag{9}$$

From the obtained expressions (7)–(9) we can see that the distributor has the opportunity to coordinate the terms of the contracts with the supply network participants in such a way that each of them will receive a share of the expected profit of the network and will be motivated to maximize the profit of the entire supply network.

Now we add another retailer to the supply network, and the supply network becomes a distribution type. Assume that the retailers do not compete with each other. All the supply network participants continue to interact on the terms of revenue-sharing contracts. The distributor contracts with each retailer separately, but sets the amount of share it will receive from the retailers' revenues the same for both retailers. The terms of the contracts with the distributor for the first and second retailers will be as follows: (φ, ω_{d1}) and (φ, ω_{d2}). Each retailer, in response to a distributor's offer, accepts or rejects the terms of the contract.

The prices at which retailers will sell products on the market, they set independently of each other. Residual value per unit at which retailers sell products that failed to sell accordingly v_1, v_2. Unit sales costs of the first and second retailer c_{r1}, c_{r2}.

Write down the expressions for the expected profits of the first and second retailers, respectively:

$$Prof_{R1}(q_1) = \varphi((p_1 - v_1)S_1(q_1) + v_1 q_1) - c_{r1} q_1 - \omega_{d1} q_1; \tag{10}$$

$$Prof_{R2}(q_2) = \varphi((p_2 - v_2)S_2(q_2) + v_2 q_2) - c_{r2} q_2 - \omega_{d2} q_2. \tag{11}$$

Denote by Rev_1 and Rev_2 the revenues of the first and second retailers:

$$Rev_1(q_1) = (p_1 - v_1)S_1(q_1) + v_1 q_1; \tag{12}$$

$$Rev_2(q_2) = (p_2 - v_2)S_2(q_2) + v_2 q_2. \tag{13}$$

Substitute the expressions (10) and (11) in expressions for the expected profits of supply network participants (10)–(11) and (2)–(3):

$$Prof_{R1}(q_1) = \varphi Rev_1(q_1) - c_{r1} q_1 - \omega_{d1} q_1;$$

$$Prof_{R2}(q_2) = \varphi Rev_2(q_2) - c_{r2} q_2 - \omega_{d2} q_2;$$

$$Prof_D(q_1, q_2) = \theta((1 - \varphi)(Rev_1(q_1) + Rev_2(q_2)) + \omega_{d1} q_1 +$$

$$+ \omega_{d2} q_2) - c_d(q_1 + q_2) - \omega_m(q_1 + q_2);$$

$$Prof_M(q_1, q_2) = (1 - \theta)((1 - \varphi)(Rev_1(q_1) + Rev_2(q_2)) +$$

$$+ \omega_{d1} q_1 + \omega_{d2} q_2) + \omega_m(q_1 + q_2) - c_m(q_1 + q_2).$$

Total costs of the retailer for each section of the supply network:

$$C_1 = c_m + c_d + c_{r1}; \quad C_2 = c_m + c_d + c_{r2}.$$

The expression for the expected profit of the distribution supply network will be:

$$Prof_{SC}(q_1, q_2) = Rev_1(q_1) - C_1 + Rev_2(q_2) - C_2 =$$

$$Prof_{SC1}(q_1) + Prof_{SC2}(q_2).$$

Using the obtained expressions (7)–(9) we receive:

$$Prof_{R1}(q_1) = \varphi Prof_{SC1}(q_1);$$

$$Prof_{R2}(q_2) = \varphi Prof_{SC2}(q_2);$$

$$Prof_D(q_1, q_2) = \theta(1 - \varphi) Prof_{SC}(q_1, q_2);$$

$$Prof_D(q_1, q_2) = (1 - \theta)(1 - \varphi) Prof_{SC}(q_1, q_2).$$

For the distributor and the manufacturer to get a share of the expected profits of the entire supply network, and for each retailer to get a share of the expected profits of its sales channel, the distributor must coordinate separately for each section of the supply network.

Consider the situation where a manufacturer is added to a supply network with a sequential topology, the supply network becomes an assembly type with two types of products. Unit wholesale prices of the first and second manufacturers denote ω_{m1}, ω_{m2}. Manufacturers' production costs — c_{m1}, c_{m2}. The distributor's wholesale prices for the first and second manufacturer's products, at which he sells the products to the retailer ω_{d11}, ω_{d21}. The demands for products of the first and second types are denoted as τ_{11}, τ_{21}. The retailer sells each type of products at prices p_{11}, p_{21} or v_{11}, p_{21}.

The retailer's expected profit will depend on the sales of products supplied by the distributor from the first and second manufacturers:

$$Prof_R(q_{11}, q_{21}) = \varphi((p_1 - v_1)S_1(q_1) + v_1 q_1) + (p_2 - v_2)S_2(q_2) + v_2 q_2) -$$

$$-c_r(q_{11} + q_{21}) - \omega_{d11} q_{11} - \omega_{d21} q_{21}.$$

Denote the retailer's revenue from the sales of first and second types products as $Rev_{11}(q_{11})$ and $Rev_{21}(q_{21})$:

$$Rev_{11}(q_{11}) = (p_{11} - v_{11})S_{11}(q_{11}) + v_{11} q_{11};$$

$$Rev_{21}(q_{21}) = (p_{21} - v_{21})S_{21}(q_{21}) + v_{21} q_{21}.$$

The expressions for the expected profits of the supply network participants will be:

$$Prof_R(q_{11}, q_{21}) = \varphi(Rev_{11}(q_{11}) + Rev_{21}(q_{21})) - c_r(q_{11} + q_{21}) - \omega_{d11} q_{11} - \omega_{d21} q_{21};$$

$$Prof_D(q_{11}, q_{21}) = \theta((1 - \varphi)(Rev_{11}(q_{11}) + Rev_{21}(q_{21})) + \omega_{d11} q_{11} + \omega_{d21} q_{21}) -$$

$$-c_d(q_{11} + q_{21}) - \omega_{m11} q_{11} - \omega_{m21} q_{21};$$

$$Prof_{M1}(q_{11}) = (1 - \theta)((1 - \varphi)Rev_{11}(q_{11}) + \omega_{d11} q_{11}) - c_{m1} q_{11} + \omega_{m11} q_{11};$$

$$Prof_{M2}(q_{21}) = (1 - \theta)((1 - \varphi)Rev_{21}(q_{21}) + \omega_{d21} q_{21}) - c_{m2} q_{21} + \omega_{m21} q_{21}.$$

Denote the total costs of the supply network for each product as C_{11} and C_{21} , then the expressions for the total cost of the first and second product will be:

$$C_{11} = c_{m1} + c_r - c_d; \;\; C_{21} = c_{m2} + c_r - c_d.$$

The expression for the expected profit of the assembly-type supply network will be:

$$Prof_{SC}(q_{11}, q_{21}) = Rev_{11}(q_{11}) + Rev_{21}(q_{21}) - C_{11} q_{11} - C_{21} q_{21}.$$

Using the formulas (5)–(6) write the coordinating relationships of the contract parameters for the assembly supply network:

$$\omega_{d11} = \varphi C_{11} - c_r; \;\; \omega_{d21} = \varphi C_{21} - c_r.$$

$$\omega_{m1} = \theta(c_{m1} + c_r) - c_d; \;\; \omega_{m2} = \theta(c_{m2} + c_r) - c_d.$$

Substitute the expressions for ω_{d11}, ω_{d21}, ω_{m1}, ω_{m2} in expressing the expected profits of the participants of the assembly supply network:

$$Prof_R(q_{11}, q_{21}) = \varphi(Rev_{11}(q_{11}) + Rev_{21}(q_{21})) - C_{11} q_{11} - C_{21} q_{21}) =$$

$$= \varphi Prof_{SC}(q_{11}, q_{21});$$

$$Prof_D(q_{11}, q_{21}) = \theta(1 - \varphi)(Rev_{11}(q_{11}) + Rev_{21}(q_{21})) - C_{11} q_{11} - C_{21} q_{21}) =$$

$$= \theta(1 - \varphi)Prof_{SC}(q_{11}, q_{21});$$

$$Prof_{M1}(q_{11}) = (1 - \theta)(1 - \varphi)(Rev_{11}(q_{11}) - C_{11} q_{11}) =$$

$$= (1 - \theta)(1 - \varphi)Prof_{SC}(q_{11});$$

$$Prof_{M2}(q_{21}) = (1 - \theta)(1 - \varphi)(Rev_{21}(q_{21}) - C_{21} q_{21}) =$$

$$= (1-\theta)(1-\varphi)Prof_{SC}(q_{21}).$$

When coordinating ratios on the parameters of supply network contracts are met, the distributor and retailer get a share of the network's profits, and each manufacturer gets a share of the supply network section's profits for its product.

Let's combine the considered examples of distribution and assembly-type supply networks into a combined type supply network. A supply network section that includes one manufacturer, a distributor, and all retailers will be called a single-product network section. A supply network section that includes one retailer, a distributor, and all manufacturers will be referred to as a single sales channel supply network section.

The expression for the expected revenue of one retailer per product can be written as:

$$Rev_{ij}(q_{ij} = (p_{ij} - v_{ij})S(q_{ij)} + v_{ij}q_{ij},$$

$$i = \overline{1,n}; \quad j = \overline{1,m}.$$

The retailer's revenue for all the products it sells that the distributor supplies from supply network manufacturers:

$$Rev_j(q_{ij}) = \sum_{i=1}^{n} Rev_{ij}(q_{ij}).$$

$$i = \overline{1,n}; \quad j = \overline{1,m}.$$

The expression for the expected profit of a retailer in a supply network with a combined topology will be:

$$Prof_{Rj}(q_{ij}) = \varphi Rev_{ij}(q_{ij}) - \sum_{i=1}^{n}(\omega_{dij} + c_{rj})q_{ij},$$

$$i = \overline{1,n}; \quad j = \overline{1,m}.$$

The manufacturer's expected profit:

$$Prof_{Mi}(q_{ij}) = (1-\theta)\sum_{j=1}^{m}((1-\varphi)Rev_{ij}(q_{ij}) + \omega_{dij}q_{ij}) + (\omega_{mi} - c_{mi})\sum_{j=1}^{m} q_{ij},$$

$$i = \overline{1,n}; \quad j = \overline{1,m}.$$

The distributor's expected profit:

$$Prof_D(q_{ij}) = \theta\sum_{i=1}^{n}\sum_{j=1}^{m}((1-\varphi)Rev_{ij}(q_{ij}) + \omega_{dij}q_{ij}) - \sum_{i=1}^{n}\sum_{j=1}^{m}(\omega_{mi} + c_d)q_{ij},$$

$$i = \overline{1,n}; \quad j = \overline{1,m}.$$

The expected profit of supply network with combined topology:

$$Prof_{SC}(q_{ij}) = \sum_{j=1}^{m} Prof_{Mi}(q_{ij}) + Prof_D(q_{ij}) + \sum_{i=1}^{n}\sum_{j=1}^{m} Prof_{Mi}(q_{ij}) =$$

$$= \sum_{i=1}^{n}\sum_{j=1}^{m}(Rev_{ij}(q_{ij}) - C_{ij}q_{ij}),$$

where:

$$C_{ij} = c_{mi} + c_d + c_{rj},$$
$$i = \overline{1,n}; \quad j = \overline{1,m}.$$

Expressions (5) and (6), written for the coordinating relationships of the contract parameters for each pair of contracts concluded between the participants of the considered supply network will take the form:

$$\omega_{dij} = \varphi C_{ij} - c_{rj}, i = \overline{1,n}; \quad j = \overline{1,m}; \tag{14}$$

$$\omega_{mi} = \theta(c_{mi} + c_d) - c_d, i = \overline{1,n}. \tag{15}$$

Substitute (14) and (15) into the expressions of the expected profits of the supply network participants and the following expressions.

The retailer's expected profit:

$$Prof_{Rj}(q_{ij}) = \varphi Rev_{ij}(q_{ij}) - \sum_{i=1}^{n}(\varphi C_{ij} + c_{rj} + c_{rj})q_{ij} =$$
$$= \varphi\sum_{i=1}^{n}(Rev_{ij}(q_{ij}) - C_{ij}q_{ij}) = \varphi Prof_{SC}(q_{ij}), \tag{16}$$
$$i = \overline{1,n}; \quad j = \overline{1,m}.$$

The manufacturer's expected profit:

$$Prof_{Mi}(q_{ij}) = (1-\theta)\sum_{j=1}^{m}((1-\varphi)Rev_{ij}(q_{ij}) + (\varphi C_{ij} - c_{rj})q_{ij} +$$
$$+\sum_{j=1}^{m}(\theta(c_{mi} + c_d) - c_d - c_{mi})q_{ij} = (1-\theta)(1-\varphi)Prof_{SC}^{Mi}(q_{ij}), \tag{17}$$
$$i = \overline{1,n}; \quad j = \overline{1,m}.$$

The distributor's expected profit:

$$Prof_D(q_{ij}) = \theta\sum_{i=1}^{n}\sum_{j=1}^{m}((1-\varphi)Rev_{ij}(q_{ij} + (\varphi C_{ij} - c_{rj})q_{ij}) -$$
$$-\sum_{i=1}^{n}\sum_{j=1}^{m}((c_d + \theta(c_{mi} + c_d) - c_d)q_{ij}) = \theta(1-\varphi)Prof_{SC}(q_{ij}), \tag{18}$$
$$i = \overline{1,n}; \quad j = \overline{1,m}.$$

A supply network with a combined topology can be coordinated using revenue-sharing contracts concluded between the participants of the supply network, provided that the central company, in the considered network — the distributor, will coordinate and establish the equal shares received from the retailers' revenue and equal shares given to each manufacturer, from sales of its product. Supply network participants will be motivated to increase the supply network profit, thus improving their performance.

5. Revenue-sharing Contract Application in Supply Networks

Consider the application of the revenue-sharing contract using the example of the Russian electrical equipment supply network. The focus company is a distributor, the company's suppliers are manufacturing companies representing the electrical industry such as cable production, industrial equipment, lighting equipment, and security systems. We will consider small and large trading companies that have contractual relations with the distributor as a network of retailers. As an example, consider a part of the supply network that includes four manufacturers, one distributor, and nine retailers. Each manufacturer supplies only one product. Assume that the demand for this type of product is uniformly distributed. Table 1 shows retailers' unit costs, retail sales prices, and parameters β.

Table 1. Retailers' parameters

	β	c_{ri}	p_{11}	p_{12}	p_{13}	p_{14}
R_1	800	923	12 719	6 060	24 130	29 397
R_2	900	818	12 589	5 931	24 001	29 268
R_3	1000	263	11 907	5 248	23 318	28 585
R_4	850	947	12 748	6 089	24 159	29 426
R_5	950	554	12 265	5 606	23 676	28 943
R_6	900	1 060	12 887	6 228	24 298	29 565
R_7	870	1 046	12 870	6 211	24 281	29 548
R_8	1100	439	12 123	5 464	23 535	28 801
R_9	1050	583	12 300	5 642	23 712	28 979

The distributor's unit costs, c_d, are 1340 rubles. The manufacturer's unit production costs are $c_{m1} = 5\ 742$ rubles, $c_{m2} = 2\ 441$ rubles, $c_{m3} = 11\ 399$ rubles and $c_{m4} = 14\ 010$ rubles per unit for each manufacturer respectively. The distributor establishes a share of the revenue that each supply network retailer receives from the sale of all products, $\varphi = 0.7$, and the share that each manufacturer receives of the distributor's revenue for each type of product, $\theta = 0.5$.

In the first step, let's see what the results of the network participants would be, assuming that the supply network had a consistent topology and included only three participants interacting on the terms of share contracts. The retailer's expected profit (R_1) is $Prof_{R1}(q_{11}) = 489\ 197$ rubles, the manufacturer's expected profit (S_1) is $Prof_{S1}(q_{11}) = 104\ 828$ rubles and the distributor's expected profit is $Prof_D(q_{11})$= 104 828 rubles.

At the second stage, we will increase the number of retailers to nine. The supply network will become a distribution type. The expected profits of the retailers in the supply network will be:

Table 2. The expected profits of the retailers in distribution supply network

Retailers	$Prof_{Rj}(q_{1j})$, rubles
R_1	489 197
R_2	550 148
R_3	611 753
R_4	519 691
R_5	580 896
R_6	550 340
R_7	532 025
R_8	672 580
R_9	641 877

The manufacturer's expected profit is $Prof_{S1} = 1103252$ rubles, the distributor's expected profit is $Prof_D = 1103252$ rubles.

The obtained result shows that when the number of retailers increases, the distributor and the manufacturer win, and the retailer who participated in a consistent supply network does not worsen its performance. Consequently, the manufacturer and distributor will be significantly motivated to increase the number of retailers, provided they do not compete with each other.

In the third stage, let's increase the number of manufacturers in the supply network to four and see how the expected profits of the participants change. The retailers' expected profits in the supply network obtained are presented in table 3.

Table 3. The expected profits of the retailers in distribution supply network

Retailers	$Prof_{Rj}(q_{1j})$, rubles
R_1	4 328 508
R_2	4 872 377
R_3	5 434 031
R_4	4 597 982
R_5	5 151 925
R_6	4 865 293
R_7	4 703 666
R_8	5 969 743
R_9	5 692 991

The expected profits of the manufacturers are: $Prof_{S1} = 1\ 103\ 252$ rubles, $Prof_{S2} = 2\ 094\ 151$ rubles, $Prof_{S3} = 2\ 872\ 190$ rubles, $Prof_{S4} = 3\ 705\ 376$ rubles. The distributor's expected profit is $Prof_D = 9\ 774\ 968$ rubles.

The results show that when the number of manufacturers in the supply network increased, the expected profits of the retailers and distributor increased significantly, while the expected profits of the first manufacturer did not change.

Supply network participants are motivated to improve the performance of the entire supply network, as this leads to an improvement in their performance. At the same time, manufacturers are interested in increasing the number of retailers in the supply network, and retailers, on the other hand, are interested in increasing the number of manufacturers. The distributor is interested in the growth of the supply network as a whole, his expected profit increases both as the number of retailers increases and as the number of manufacturers increases. This explains the phenomenon of the supply networks formation with combined topology, which brings together multiple manufacturers, a distributor, and multiple retailers.

The use of revenue-sharing contracts in the supply network with a combined topology allows to coordinate the actions of independent participants of the decentralized supply network and to provide their motivation to achieve the best result of the entire supply network while improving their performance. The application of revenue-sharing contracts in such networks has several peculiarities. The role of a coordinator in the supply network should be performed by a distributor who, having contracts with all manufacturers and all retailers, has complete information about the costs of the supply network. The distributor, by setting a unified revenue share for all retailers and a unified revenue share for all manufacturers, gets a share of the expected profit of the whole supply network, each retailer gets a share of the supply network profit for its sales channel, and each manufacturer gets a share of the supply network profit for its product.

6. Conclusion

It has become clear that the problem of coordination of supply networks has not been fully addressed, as the vast majority of research papers focus on coordination between two participants in the chain. Given the current global environment, the logistics issue is becoming acute and the research task of decentralised supply networks with a significant number of members needs to be analysed and investigated immediately.

To answer the key research question whether it is possible, in a two-way relationship between supply network members, to coordinate their actions such that each member is incentivized to improve performance across the supply network while improving its own performance, we draw the following conclusions.

First, the use of revenue-sharing contract in a supply network with a combined topology helps coordinate the actions of the independent members in the decentralized supply network and ensure that they are motivated to achieve the best performance of the overall supply chain, while improving their own performance.

Second, the authors showed by example that supply network members are interested in increasing the number of network participants: manufacturers are interested in increasing the number of retailers, retailers are interested in increasing the number of manufacturers, and the distributor is interested in any growth in the supply chain.

Thirdly, the application of an equity contract in supply network with combined topology has a number of peculiarities. The role of coordinator should be performed by the distributor, who, having contracts with all manufacturers and all retailers in the supply chain, has full information about the chain 's costs. The distributor, by setting a single revenue share for all retailers and a single revenue share for all manufacturers, receives a share of the supply network profit, each retailer receives a share of the part of the supply network profit for its sales channel, and each producer receives a share of the part of the supply network profit for its product.

A limitation of the study is to consider a situation in which the selling price of products is fixed. Another limitation of the study is the consideration of retailers and producers who do not compete with each other. In this regard, competition between retailers or producers, an increase in the number of distributors, and the possibility of other types of coordinating contracts, such as sales-rebate or buy-back contracts, could be considered as areas for future research.

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