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Owen Value for Dynamic Games on Networks*

Leon Petrosyan and Yaroslavna B. Pankratova

St. Petersburg State University,
7/9, Universitetskaya nab. ,St. Petersburg, 199034, Russia
E-mail: y.petrosyan@spbu.ru*;* y.pankratova@spbu.ru

Abstract In the presented paper, we consider dynamic network games with coalition structure in which players cooperate to get the best outcomes. As solution the Owen value is proposed. To simplify the calculations the new characteristic function is introduced based on the possibility of cutting connections by players outside the coalition. For a special case, comparison of the Owen value with other solutions is done.
Keywords: dynamic network game, Shapley value, Owen value, coalition structure.

1. Introduction

Recently, many interesting problems have been modeled with the help of dynamic games on networks. Among them the transportation problem or problem of influence in social networks. We have to mention the first researches in the field of dynamic games (Cao et al., 2008; Wie, 1995; Pai, 2010; Zhang et al., 2018; Meza, and Lopez-Barrientos, 2016; Bulgakova and Petrosyan, 2019). An obvious continuation of research in the field of dynamic games is to expand them to the class of cooperative dynamic games on networks (the following papers should be noted (Petrosyan, 2010; Gao and Pankratova, 2017), and the paper of (Yeung and Petrosyan, 2016; Petrosyan and Yeung, 2020; Tur and Petrosyan, 2022) where the new characteristic function in differential cooperative network game was introduced in a special case when the payoffs of players depend only upon their actions and actions of neighbors in the network). Different properties of the cooperative solutions of dynamic network games are investigated in (Yeung, 2010; Yeung and Petrosyan, 2004; Yeung and Petrosyan, 2018). In the paper (Petrosyan et al., 2021b), the differential games on networks with partner sets are considered. In such games, player's payoff depend upon the payoffs of players from his partner set and it is supposed that one player can belong to many partner sets. The cooperative dynamic games with two level of cooperation were considered in (Petrosyan and Sedakov, 2019) and (Petrosyan and Pankratova, 2022).

In this paper, we consider the differential network game with coalition structure in which players cooperate to get the best outcomes. As solution we take Owen value. To simplify the calculations the new characteristic function is introduced based on the possibility of cutting connections by players outside the coalition. We find the Owen value and compere it with other solution concepts.

2. Differential Network Games

Consider a class of n-person differential games on network with game horizon $[t_0, T]$. The players are connected in a network system. We use $N = \{1, 2, \cdots, n\}$

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to denote the set of players in the network. The nodes of the network are used to represent the players from the set N. We also denote the set of nodes by N and denote the set of all arcs in network N by L. The arcs in L are the *arc* $(i, j) \in L$ for players $i, j \in N$, $i \neq j$. For notational convenience, we denote the set of players connected to player i as $\tilde{K}(i) = \{j : \ arc(i, j) \in L\}$, for $i \in N$.

Let $x^i(\tau) \in R^m$ be the state variable of player $i \in N$ at time τ, and $u^i(\tau) \in U^i \subset R^k$ the control variable of player $i \in N$.

Every player $i \in N$ can cut the connection with any other player from the set $\tilde{K}(i)$ at any instant of time.

The state dynamics of the game is

$$\dot{x}^i(\tau) = f^i(x^i(\tau), u^i(\tau)), \ x^i(t_0) = x^i_0, \quad \text{for } \tau \in [t_0, T] \ \text{ and } i \in N. \tag{1}$$

The function $f^i(x^i, u^i)$ is continuously differentiable in x^i and u^i.

The payoff function of player i depends upon his state variable and the state variables of players from the sets $\tilde{K}(i)$ to which he belongs.

In particular, the payoff of player i is given as

$$H_i(x^1_0, \ldots, x^n_0, u^1, \ldots, u^n) = \sum_{j \in \tilde{K}(i)} \int_{t_0}^{T} h^j_i(x^i(\tau), x^j(\tau)) d\tau. \tag{2}$$

The term $h^j_i(x^i(\tau), x^j(\tau))$ is the instantaneous gain that player i can obtain through network links with player $j \in \tilde{K}(i)$ (note that the pair $(i, i) \notin L$). The functions $h^j_i(x^i(\tau), x^j(\tau))$, for $j \in \tilde{K}(i)$ are non-negative. For notational convenience, we use $x(t)$ to denote the vector $(x^1(t), x^2(t), \cdots, x^n(t))$.

Since the set N is finite the sum in (2) contains a finite number of summands $\leq |N|$.

3. Cooperative Differential Network Game

In this section, we use the special type of characteristic function which, at the first time, was introduced in paper (Petrosyan, 2010) and after used in (Petrosyan et al., 2021a; Tur and Petrosyan, 2022).

To achieve group optimality, the players maximize their joint payoff

$$\sum_{i \in N} \left(\sum_{j \in \tilde{K}(i)} \int_{t_0}^{T} h^j_i(x^i(\tau), x^j(\tau)) d\tau \right) \tag{3}$$

subject to dynamics (1).

We use $\bar{x}(t) = (\bar{x}^1(t), \bar{x}^2(t), \cdots, \bar{x}^n(t))$ to denote the optimal cooperative trajectory of problem of maximizing (3) subject to (1). We let the corresponding optimal cooperative trajectory of player i be denoted by $\bar{x}^i(t)$, for $t \in [t_0, T]$ and $i \in N$. The maximized joint cooperative payoff involving all players can then be expressed as

$$\sum_{i \in N} \left(\sum_{j \in \tilde{K}(i)} \int_{t_0}^{T} h^j_i(\bar{x}^i(\tau), \bar{x}^j(\tau)) d\tau \right) =$$

$$= \max_{u^1, u^2, \cdots, u^n} \sum_{i \in N} \left(\sum_{j \in \tilde{K}(i)} \int_{t_0}^{T} h^j_i(x^i(\tau), x^j(\tau)) d\tau \right) = V(N; x_0, T - t_0) \tag{4}$$

subject to dynamics (1) formulation of the worth of coalition $S \subset N$ as

$$V(S; x_0, T - t_0) = \sum_{i \in S} \sum_{j \in \tilde{K}(i) \cap S} \int_{t_0}^{T} h_i^j(\bar{x}^i(\tau), \bar{x}^j(\tau)) d\tau. \quad (5)$$

Note that the worth of coalition S is measured by the sum of the payoffs of the players in the coalition in the cooperation process with the exclusion of the gains from players outside coalition S. Thus, the characteristic function reflecting the worth of coalition S in (5) is formulated along the cooperative trajectory $\bar{x}(t)$.

For simplicity in notation, we denote

$$\alpha_{ij}(x_0, T - t_0) = \int_{t_0}^{T} h_i^j(\bar{x}^i(\tau), \bar{x}^j(\tau)) d\tau \quad (6)$$

and

$$\alpha_{ij}(\bar{x}(t), T - t) = \int_{t}^{T} h_i^j(\bar{x}^i(\tau), \bar{x}^j(\tau)) d\tau, \quad (7)$$

for $t \in [t_0, T]$.
Using the notations in (6), we can express (5) as

$$V(S; x_0, T - t_0) = \sum_{i \in S} \sum_{j \in \tilde{K}(i) \cap S} \alpha_{ij}(x_0, T - t_0), \quad (8)$$

and similarly, using (7), along the cooperative trajectory $\bar{x}(t)$

$$V(S; \bar{x}(t), T - t) = \sum_{i \in S} \sum_{j \in \tilde{K}(i) \cap S} \alpha_{ij}(\bar{x}(t), T - t), \text{ for } t \in [t_0, T].$$

4. Dynamic Owen Value

There are many different cooperative solution concepts for differential network game. For example, Core, the Shapley value (Shapley, 1953), τ-value (Tijs, 1987), etc. These solutions of cooperative dynamic games were investigated in papers (Tur and Petrosyan, 2022; Petrosyan et al., 2021a). All these solutions are based on distribution of the worth of the grand coalition. In this section, we consider the case when the players are organized in coalition structure and find the Owen value (Owen, 1977) for this type of cooperative game. After that, we give some comparison with the Shapley value.

A coalition structure on a player set N is a finite partition $P = \{P_1, \ldots, P_m\}$ of m non-empty, disjoint subsets of N, i.e. $\cup_{k=1}^{m} P_k = N$ and $P_k \cap P_l = \emptyset$ for all $k,\ l \in \{1, \ldots, m\}$, $k \neq l$. In the following the set of coalitions in the coalition structure $P = \{P_1, \ldots, P_m\}$ is denoted by $M = \{1, \ldots, m\}$ with $k \in M$ representing coalition $P_k \in P$. Furthermore, cooperative dynamic game in coalition structure P is denoted by (N, V, P). The collection of all coalition structures on N is denoted by P^N.

Using (Owen, 1977) and (van den Brink and van der Laan, 2005) define the Owen value for our characteristic function as

$$OW_i(P, x_0, T - t_0) = \sum_{\substack{L \subset M, \\ L \not\ni k}} \sum_{\substack{E \subset P_k, \\ E \ni i}} \frac{|L|!(m - |L| - 1)!}{m!} \frac{(|E| - 1)!(|P_k| - |E|)!}{|P_k|!} \times$$

$$(V(E \cup P(L); x_0, T - t_0) - V(E \backslash \{i\} \cup P(L); x_0, T - t_0)), \tag{9}$$

where $i \in P_k \in P$, $k \in M$, $P(L) = \cup_{j \in L} P_j$. We remark that the Owen value reduces to the Shapley value when $P = \{N\}$ or when $P = \{\{i\}_{i \in N}\}$. The weights of the marginal values are a product of two 'Shapley weights', reflecting the fact that first coalitions enter subsequently in a random order and that within each coalition the players enter subsequently in a random order.

Invoking (5), in our case, we can obtain the cooperative payoff of player $i \in P_k \in P$ under the Owen value as

$$OW_i(P, x_0, T - t_0) = \sum_{\substack{L \subset M, \\ L \not\ni k}} \sum_{\substack{E \subset P_k, \\ E \ni i}} \frac{|L|!(m - |L| - 1)!}{m!} \frac{(|E| - 1)!(|P_k| - |E|)!}{|P_k|!} \times$$

$$\left(\sum_{i \in E \cup P(L)} \sum_{j \in \tilde{K}(i) \cap E \cup P(L)} \int_{t_0}^{T} h_i^j(\bar{x}^i(\tau), \bar{x}^j(\tau)) d\tau - \right. \tag{10}$$

$$\left. - \sum_{i \in E \backslash \{i\} \cup P(L)} \sum_{j \in \tilde{K}(i) \cap E \backslash \{i\} \cup P(L)} \int_{t_0}^{T} h_i^j(\bar{x}^i(\tau), \bar{x}^j(\tau)) d\tau \right), \; i \in P_k \in P, \; k \in M.$$

In a dynamic game, the agreed upon optimality principle for sharing the gain has to be maintained throughout the cooperation duration (Yeung and Petrosyan, 2004; Yeung and Petrosyan, 2016) for a dynamically consistent solution. Applying the Owen value imputation in (10) to any time instance $t \in [t_0, T]$, we obtain:

$$OW_i(P, \bar{x}(t), T - t) = \sum_{\substack{L \subset M, \\ L \not\ni k}} \sum_{\substack{E \subset P_k, \\ E \ni i}} \frac{|L|!(m - |L| - 1)!}{m!} \frac{(|E| - 1)!(|P_k| - |E|)!}{|P_k|!} \times$$

$$\left(\sum_{i \in E \cup P(L)} \sum_{j \in \tilde{K}(i) \cap E \cup P(L)} \int_{t}^{T} h_i^j(\bar{x}^i(\tau), \bar{x}^j(\tau)) d\tau - \right. \tag{11}$$

$$\left. - \sum_{i \in E \backslash \{i\} \cup P(L)} \sum_{j \in \tilde{K}(i) \cap E \backslash \{i\} \cup P(L)} \int_{t_0}^{T} h_i^j(\bar{x}^i(\tau), \bar{x}^j(\tau)) d\tau \right), \; i \in P_k \in P, \; k \in M.$$

Proposition 1. *The Owen value imputation in (10)-(11) satisfies the time consistency property.*

Proof. The proof of proposition follows from direct computation by (10)- (11). Since, we have

$$OW_i(P, x_0, T - t_0) = \sum_{\substack{L \subset M, \\ L \not\ni k}} \sum_{\substack{E \subset P_k, \\ E \ni i}} \frac{|L|!(m - |L| - 1)!}{m!} \frac{(|E| - 1)!(|P_k| - |E|)!}{|P_k|!} \times$$

$$\left(\sum_{i \in E \cup P(L)} \sum_{j \in \tilde{K}(i) \cap E \cup P(L)} \int_{t_0}^{t} h_i^j(\bar{x}^i(\tau), \bar{x}^j(\tau)) d\tau - \right. \tag{12}$$

$$\left. - \sum_{i \in E \setminus \{i\} \cup P(L)} \sum_{j \in \tilde{K}(i) \cap E \setminus \{i\} \cup P(L)} \int_{t_0}^{t} h_i^j(\bar{x}^i(\tau), \bar{x}^j(\tau)) d\tau \right) + OW_i(P, \bar{x}(t), T - t),$$

$i \in P_k \in P$, $k \in M$ which exhibits the time consistency property of the Owen value imputation $OW_i(P, \bar{x}(t), T - t)$, for $t \in [t_0, T]$.

5. Example

Introduce additional notations before considering the example.

$$\alpha_{ij}(x_0, T - t_0) + \alpha_{ji}(x_0, T - t_0) = \alpha_{ij} + \alpha_{ji} = A_{ij} = A_{ji} = A(i, j) \tag{13}$$

Example. Consider the following 5 player network game with coalition structure $P = \{P_1, P_2\}$, $P_1 = \{1, 5\}$, $P_2 = \{2, 3, 4\}$ (see Figure 1).

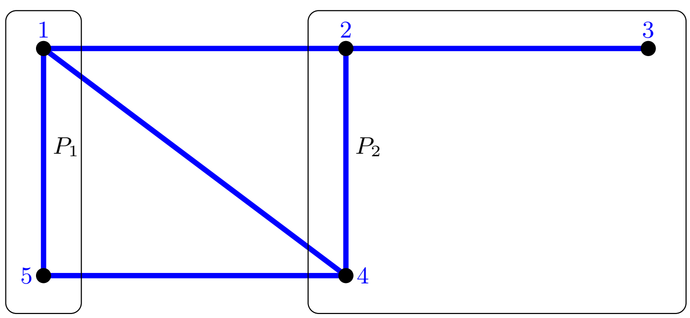

Fig. 1. 5 player network game with coalition structure $P = \{P_1, P_2\}$, $P_1 = \{1, 5\}$, $P_2 = \{2, 3, 4\}$

The values of defined above characteristic function are

$V(\{1\}) = V(\{2\}) = V(\{3\}) = V(\{4\}) = V(\{5\}) = 0,$
$V(\{1, 2\}) = \alpha_{12} + \alpha_{21} = A(1, 2)$
$V(\{1, 3\}) = V(\{2, 5\}) = V(\{3, 4\}) = V(\{3, 5\}) = 0$
$V(\{1, 4\}) = \alpha_{14} + \alpha_{41} = A(1, 4)$
$V(\{1, 5\}) = \alpha_{15} + \alpha_{51} = A(1, 5)$
$V(\{2, 3\}) = \alpha_{23} + \alpha_{32} = A(2, 3)$
$V(\{2, 4\}) = \alpha_{24} + \alpha_{42} = A(2, 4)$
$V(\{4, 5\}) = \alpha_{45} + \alpha_{54} = A(4, 5)$
$V(\{1, 2, 3\}) = \alpha_{12} + \alpha_{21} + \alpha_{23} + \alpha_{32} = A(1, 2) + A(2, 3)$
$V(\{1, 2, 4\}) = \alpha_{12} + \alpha_{21} + \alpha_{24} + \alpha_{42} + \alpha_{14} + \alpha_{41} = A(1, 2) + A(2, 3) + A(2, 4)$
$V(\{1, 2, 5\}) = \alpha_{12} + \alpha_{21} + \alpha_{15} + \alpha_{51} = A(1, 2) + A(1, 5)$
$V(\{1, 3, 4\}) = \alpha_{14} + \alpha_{41} = A(1, 4)$
$V(\{1, 3, 5\}) = \alpha_{15} + \alpha_{51} = A(1, 5)$
$V(\{1, 4, 5\}) = \alpha_{15} + \alpha_{51} + \alpha_{14} + \alpha_{41} + \alpha_{45} + \alpha_{54} = A(1, 5) + A(1, 4) + A(4, 5)$

$V(\{2,3,4\}) = \alpha_{23} + \alpha_{32} + \alpha_{24} + \alpha_{42} = A(2,3) + A(2,4)$
$V(\{2,3,5\}) = \alpha_{23} + \alpha_{32} = A(2,3)$
$V(\{2,4,5\}) = \alpha_{45} + \alpha_{54} = A(4,5)$
$V(\{3,4,5\}) = \alpha_{45} + \alpha_{54} + \alpha_{24} + \alpha_{42} = A(4,5) + A(2,4)$
$V(\{1,2,3,4\}) = \alpha_{12} + \alpha_{21} + \alpha_{23} + \alpha_{32} + \alpha_{14} + \alpha_{41} + \alpha_{24} + \alpha_{42} = A(1,2)+$
$A(2,3) + A(2,4) + A(1,4)$
$V(\{1,2,3,5\}) = \alpha_{12} + \alpha_{21} + \alpha_{23} + \alpha_{32} + \alpha_{15} + \alpha_{51} = A(1,2) + A(2,3) + A(1,5)$
$V(\{1,2,4,5\}) = \alpha_{12} + \alpha_{21} + \alpha_{14} + \alpha_{41} + \alpha_{45} + \alpha_{54} + \alpha_{15} + \alpha_{51} + \alpha_{24} + \alpha_{42} =$
$A(1,5) + A(1,4) + A(1,2) + A(2,4) + A(4,5)$
$V(\{1,3,4,5\}) = \alpha_{14} + \alpha_{41} + \alpha_{45} + \alpha_{54} + \alpha_{15} + \alpha_{51} = A(1,4) + A(1,5) + A(4,5)$
$V(\{2,3,4,5\}) = \alpha_{23} + \alpha_{32} + \alpha_{24} + \alpha_{42} + \alpha_{45} + \alpha_{54} = A(2,3) + A(2,4) + A(4,5)$
$V(\{1,2,3,4,5\}) = \alpha_{12} + \alpha_{21} + \alpha_{14} + \alpha_{41} + \alpha_{15} + \alpha_{51} + \alpha_{23} + \alpha_{32} + \alpha_{24} + \alpha_{42}+$
$\alpha_{45} + \alpha_{54} = A(1,2) + A(1,4) + A(1,5) + A(2,3) + A(2,4) + A(4,5)$

Computing the Owen value by (9) and taking into account notations (5), (13) we obtain

$$OW_1 = \frac{A(1,5) + A(1,2) + A(1,4)}{2},\ OW_2 = \frac{A(1,2) + A(2,3) + A(2,4)}{2}.$$

$$OW_3 = \frac{A(2,3)}{2},\ OW_4 = \frac{A(1,4) + A(4,5) + A(2,4)}{2},\ OW_5 = \frac{A(1,5) + A(4,5)}{2}.$$

Remark. It is not difficult to check that the Owen value coincides with the Shapley value and τ-value in this example.

6. Conclusion

Differential cooperative network games with coalition structure are considered. As optimality principle in this game we used the Owen Value introduced by Owen, 1977. The example is provided.

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