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Difference Stackelberg Game Theoretic Model of Innovations Management in Universities

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Abstract We built a two-level difference game theoretic model "federal state universities" in open-loop strategies. The leading player (Principal) is the state or its representative bodies, the followers (agents) are competing a la Cournot universities. The agents assign their resources to the development of new online teaching courses which are considered as their innovative investments. An optimality principle from the point of view of agents is a set of Nash equilibria in their game in normal form, and from the point of view of the Principal it is a solution of the direct or inverse Stackelberg game "Principal-agents". The respective dynamic problems of conflict control are solved by means of the Pontryagin maximum principle and simulation modeling. The received results are analyzed, and the main conclusion is that two-level system of control of the innovative educational products promotion in the universities is necessary.

Keywords: difference Stackelberg games, economic corruption, resource allocation, simulation modeling.

1. Introduction

Problems of innovative development of the universities require new educational methods and online teaching courses that support the methods.

Cellini and Lambertini (2002,2004) consider a dynamic one-level game theoretic model of the development of new courses in the universities. They propose an equation of dynamics that describe a change of substituability between pairs of courses.

In this paper in modeling the process of innovations promotion we use an authors' concept of sustainable management in active systems (Ougolnitsky, 2016). We propose a hierarchical problem setup where the leading player (Principal) is the state or its representative bodies, and the followers (agents) are competing a la Cournot universities.

A basic model in the lower level is borrowed from (Cellini and Lambertini, 2004). The Principal exerts an economic impact (impulsion) to the payoff functionals of the agents (Basar and Olsder, 1999; Ougolnitsky, 2016; Dockner et al., 2000; Gorelov and Kononenko, 2015; Mechanism Design and Management, 2013; Geraskin, 2020).

The contribution of the paper is the following. First, we consider a dynamic model of the innovative development in universities in discrete time (a difference game). This problem formulation is more adequate to the real academic schedule in universities than a continuous one. Second, we consider a combination of an aggregative non-cooperative game of the oligopolistic agents with a Stackelberg game

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of the type "Principal-agents". Third, we proposed a description of the dependence of a parameter of the demand function on the agents' actions in the form of a difference equation.

The paper is organized as follows. In the Section 2 we give the setup of the discrete dynamic problem of hierarchical control, and describe algorithms of its solution. In the Section 3 we build a Nash equlibrium in open-loop strategies using the Pontryagin maximum principle in the case of two agents when the Principal's strategy is fixed. The Section 4 describes numerical calculations in direct and inverse Stackelberg games. In Section 5 we conduct a comparative analysis of the received results, formulate some conclusions, and sum up the investigation.

2. The Problem Setup

We consider a discrete setup of a hierarchical modification of the model from (Cellini and Lambertini, 2002, 2004), proposed in (Malsagov et al., 2020). We study a two-level discrete dynamic model that includes the Principal (a federal state or its representative bodies) and several agents (universities). The agents develop online education courses for selling. The development of courses, its differentiation by means of the modern teaching methods and information technologies are considered as innovative investments of the agents. The differentiation of the courses is treated as production of a public good, and the agents' investments as a private production of the public good (Cellini and Lambertini, 2002, 2004; Ougolnitsky and Usov, 2019). The Principal tries to increase the public good with additional consideration of her own private interests. The principal subsidizes the agents with consideration of budget constraints. Both Principal and agents use open-loop strategies. The period $[0, T]$ is equal to several years, and the discounting is not considered. The model with n agents has the following form:

- Principal's payoff functional

$$J_0(\cdot) = \sum_{t=1}^{T} \sum_{i=1}^{n} (\pi_{it} - s_{it}) + G_{0T} \to \max \tag{1}$$

- agents' payoff functionals

$$J_i(\cdot) = \sum_{t=1}^{T} (\pi_{it} - s_{it}) + G_{iT} \to \max \tag{2}$$

$$\pi_{it} = p_{it} q_{it} - c_{it} q_{it}^2 - k_{it} \tag{3}$$

- current payoff function of the i-th agent. In the Principal's payoff function a profit π_i evaluates a positive externality from the activity of universities; namely, an increase of GNP due to a greater social education level;

$$p_{it} = A - Bq_{it} - D_t \sum_{j \neq i} q_{jt} \tag{4}$$

- an inverse demand function (Cellini and Lambertini, 2002, 2004);

$$G_{iT} = (A - Bq_{iT} - D_T \sum_{j \neq i} q_{jT}) q_{iT} - c_{iT} q_{iT}^2; \quad i = 1, 2, \ldots, n$$

$$G_{0T}=\sum_{i=1}^{n}G_{iT}=\sum_{i=1}^{n}((A-Bq_{iT}-D_T\sum_{j\neq i}q_{jT})q_{iT}-c_{iT}q_{iT}^2);$$

- terminal payoffs of the Principal and agents in the moment of time T; $D_t\in[0,B]$ - a symmetrical degree of substitutability between a pair of courses. If $D_t=B$ then the courses are completely homogeneous. If $D_t=0$ then the courses are completely unique, and each agent becomes a monopolist; q_{it} - an amount of courses produced by the i-th agent (his first control variable); k_{it} - the i-th agent's individual investments to the innovative development (his second control variable), K_t - summary innovative investments of the higher education industry, $C_{it}=c_{it}q_{it}^2$; $c_{it}\in[0,A]$ - summary operation costs; functions s_{it} reflect the Principal's subsidies to the i-th agent for development of courses; this is the Principal's control strategy that is to be determined; T - a length of the game; $A>0$; $B>0$ – demand parameters.
- Principal's budget constraints

$$0\le s_{it};\quad \sum_{i=1}^{n}s_{it}\le S;\quad t=1,2,\dots,T \tag{5}$$

- agents' control constraints

$$0\le k_{it}\le K_{max};\quad 0\le q_{it}\le Q_{max};\quad i=1,2,\dots,n;\quad t=1,2,\dots,T \tag{6}$$

$K_{max}=const$ - a maximal feasible amount of one agent's investments, $Q_{max}=const$ - a maximal feasible amount of courses produced by him; S - the Principal's budget;
- difference equation of dynamics similar to the differential equation of dynamics from (Cellini and Lambertini, 2002, 2004)

$$D_{t+1}=D_t/(1+\sum_{i=1}^{n}(k_{it}+s_{it}));\quad D_0=B;\quad t=0,1,\dots,T-1. \tag{7}$$

The difference dynamics equation (7) may be interpreted as a production function. D_i is a non-increasing function of time that tends to zero when the investments tend to infinity (Cellini and Lambertini, 2004).

It is possible to analyze the model (1)-(7) from the point of view of different players. From the point of view of the agents there is a game of n persons in normal form where Nash equilibria are found. From the point of view of the Principal a direct or inverse Stackelberg game is played where the agents' best response is the Nash equilibrium in their game.

A direct Stackelberg game has the following information structure (Ugol'nitskii and Usov 2014).

1. The Principal chooses her open-loop strategies $\{s_{it}\}_{t=0;i=1}^{T-1;n}$.

2. Given the strategies $\{s_{it}\}_{t=0;i=1}^{T-1;n}$ agents play a game in normal form (2), (6), (7). Its solution is a set of Nash equilibria

$$NE(\{s_{it}\}_{t=0;i=1}^{T-1;n})=\{k_{it}(s_{it}),q_{it}(s_{it})\}_{t=0;i=1}^{T-1;n}.$$

If there are several Nash equilibria then we save the one in which the Principal's payoff is minimal.

3. The found Nash equilibrium is substituted to (1), (7). The Principal maximizes her payoff functional J_0 (1) for a non-benevolent agents' best response from the set $NE(\{s_{it}\}_{t=0;i=1}^{T-1;n})$.

4. The received set of strategies $\{s_{it}^*, k_{it}^*, q_{it}^*, \}_{t=0;i=1}^{T-1;n}$ forms a Stackelberg equilibrium.

An algorithm of solution of the inverse Stackelberg game is the following (Basar and Olsder, 1999; Ugol'nitskii and Usov 2014).

1. We found the Principal's punishment strategies if the agents refuse to cooperate:

$$\{k_{it}^P(\{s_{it}\}_{t=0}^{T-1}),\ q_{it}^P(\{s_{it}\}_{t=0}^{T-1})\} =$$

$$\arg \max_{0\le k_{it}\le K_{max};\ \ 0\le q_{it}\le Q_{max}} J_i(\{s_{it}\}_{t=0}^{T-1}, \{k_{it}\}_{t=0}^{T-1}, \{q_{it}\}_{t=0}^{T-1})$$

$$\{s_{it}^P\}_{t=0}^{T-1} = \arg \min_{0\le s_{it};\ \ \sum_{i=1}^n s_{it}=S} J_i(\{s_{it}\}_{t=0}^{T-1}, \{k_{it}^P\}_{t=0}^{T-1}, \{q_{it}^P\}_{t=0}^{T-1})$$

The guaranteed payoff of an agent if he refuse to cooperate is equal to ($i = 1, 2, \dots, n$):

$$L_i = J_i(\{s_{it}\}_{t=0}^{T-1}, \{k_{it}^P\}_{t=0}^{T-1}, \{q_{it}^P\}_{t=0}^{T-1}) =$$

$$\max_{0\le k_{it}\le K_{max};\ \ 0\le q_{it}\le Q_{max}} \min_{0\le s_{it}} J_i(\{s_{it}\}_{t=0}^{T-1}, \{k_{it}\}_{t=0}^{T-1}, \{q_{it}\}_{t=0}^{T-1})$$

2. We solve an optimal control problem (1), (5)-(7) with constraints

$$L_i < J_i(\{s_{it}\}_{t=0}^{T-1}, \{k_{it}\}_{t=0}^{T-1}, \{q_{it}\}_{t=0}^{T-1}); \quad i = 1, 2, \dots, n. \tag{8}$$

A maximum is found at the same time by three grid functions

$$\{s_{it}, k_{it}, q_{it}\}_{t=0;i=1}^{T-1;n}.$$

Denote a solution of this optimal control problem by

$$\{s_{it}^R, k_{it}^R, q_{it}^R\}_{t=0;i=1}^{T-1;n},$$

where $\{s_{it}^R\}_{t=0;i=1}^{T-1;n}$ is a reward strategy for the i-th agent if he chooses $\{k_{it}^R, q_{it}^R, \}_{t=0;i=1}^{T-1;n}$

3. The Principal reports to each agent the feedback strategy ($t = 0, 1, \dots, T - 1;\ \ i = 1, 2, \dots, n$):
$s_{it} = s_{it}^R,\ \ if\ \ k_{it} = k_{it}^R, q_{it} = q_{it}^R$ and $s_{it} = s_{it}^P$, otherwise.

The condition (8) makes for the agents the reward strategy more profitable than the punishment strategy. The solution has the form

$$(\{s_{it}^R\}_{t=0;i=1}^{T-1;n}\}; \{k_{it}^R\}_{t=0;i=1}^{T-1;n}\}; \{q_{it}^R\}_{t=0;i=1}^{T-1;n});$$

3. Nash Equilibrium

Consider an indifferent Prinipal having no her own interests. Let the Principal's strategies be fixed. Then we receive a difference game of n persons (2), (6),

(7) in which a Nash equilibrium in open-loop strategies is built by means of a discrete Pontryagin maximum principle (Pontryagin et al., 1962; Intriligator, 1971). A Hamilton function of the i -th agent in the moment of time t has the form:

$$H_{it}(k_{it}, q_{it}, \lambda_i, D_t) =$$
$$(A - Bq_{it} - D_t \sum_{j \neq i} q_{jt}) q_{it} - c_{it} q_{it}^2 - k_{it} + s_{it} + \lambda_{it} D_t / (1 + \sum_{j=1}^{n} (k_{jt} + s_{jt}))$$

where λ_{it} is a conjugate variable. From a necessary condition of extremum ($i = 1, 2, \ldots, n$)

$$\frac{\partial H_{it}}{\partial k_{it}} = 0 \text{ and } \frac{\partial H_{it}}{\partial q_{it}} = 0$$

in the case of symmetrical agents

$$c_{it} = c_t;\ H_{it} = H_t;\ s_{it} = s_t;\ k_{it} = k_t;\ q_{it} = q_t;\ \lambda_{it} = \lambda_t;\ G_{it} = G_t;\ i = 1, 2, \ldots, n$$

we receive a system of equations for determination of their optimal strategies

$$\frac{\partial H_t}{\partial k_t} = -1 - \frac{\lambda_t D_t}{(1 + n(s_t + k_t))^2} = 0; \quad \frac{\partial H_t}{\partial q_t} = A - 2(B + C) q_t - D_t (n-1) q_t = 0 \quad (9)$$

Thus,

$$k_t = (-1 + \sqrt{-\lambda_t D_t})/n - s_t; \quad q_t = A/(2(B + C) + D_t(n-1)) \quad (10)$$

Besides, we have a system of difference equations

$$D_{t+1} = D_t/(1 + n(k_t + s_t)); \quad D_0 = B; \quad t = 0, 1, \ldots, T-1$$
$$\lambda_t = (n-1) q_t^2 - \lambda_{t+1}/(1 + n(k_t + s_t)); \lambda_T = (n-1)/q_T^2 \quad (11)$$

From (9) we receive

$$\frac{\partial^2 H_t}{\partial k_t^2} = \frac{2n\lambda_t D_t}{(1 + n(s_t + k_t))^3}; \quad \frac{\partial^2 H_t}{\partial q_t^2} = -2(B + C) - D_t(n-1) < 0; \quad \frac{\partial^2 H_t}{\partial k_t \partial q_t} = 0$$

Therefore, the following proposition is proved.

Proposition. Formulas (10), (11) determine a point of maximum of the Hamilton function for a value of t if the system (10), (11) has a solution, values (10) belong to the domain of feasible solutions (6), and $\lambda_t < 0$.

If the conditions of this proposition are not satisfied for a value of t then the maximum of Hamilton function is attained on one of the ends of segments (6).

4. Numerical Results

In the numerical calculations we used the following range of variables (Table 1):

$$A \in [100, 10000];\ B \in [0.5, 500];\ c_t \in [0.01, 100];\ s_t \in [0, 3000];\ K_{max} \in [0.1, 500];$$

$$Q_{max} \in [1, 700];\ S \in [0, 10000]$$

Table 1. Input data for two agents

N	A	c_1	c_2	B	K_{max}	s_1	s_2	Q_{max}	S
1	700	3	3	3	300	200	200	50	500
2	700	10	1	3	200	200	200	50	500
3	700	5	2	3	100	100	100	100	300
4	700	5	1	1	100	300	100	100	500
5	700	5	1	0.1	100	300	100	100	500
6	700	1	2	3	100	300	100	100	500
7	700	1	0.5	0.5	300	300	100	200	500
8	300	25	15	10	100	300	100	100	500
9	300	25	15	1	100	300	100	100	500
10	500	2	1	1	100	3	1	100	10
11	1000	10	100	3	100	100	100	200	300
12	1000	1	2	3	100	100	100	200	300
13	1000	0.1	0.2	3	100	100	100	200	300
14	1500	1	2	3	100	100	100	200	300
15	1500	1	2	3	300	500	500	400	1500
16	1500	0.1	0.02	3	100	100	100	200	300
17	1000	0.1	0.02	3	100	100	100	200	300
18	1000	1	2	3	1000	1000	1000	2000	3000
19	1000	1	2	100	100	100	100	200	300
20	1000	0.1	0.02	3	1	1	1	2	10
21	1000	0.1	0.02	3	10	100	100	20	300
22	1000	0.1	0.02	5	10	500	500	20	1500
23	1000	0.1	0.02	0.05	10	100	100	20	300
24	1000	0.1	0.02	0.1	10	100	100	20	300
25	3000	0.1	0.02	3	100	100	100	200	300
26	3000	1	2	3	100	100	100	200	300
27	3000	1	2	0.3	10	1000	1000	10	3000
28	3000	1	2	3	10	10	10	20	50
29	3000	1	2	0.1	100	100	100	200	300
30	3000	1	2	6	100	100	100	200	300
31	3000	1	2	0.01	100	100	100	200	300
32	2000	1	2	3	100	100	100	100	300
33	2000	1	2	0.1	100	100	100	100	300
34	2000	1	0.1	3	100	100	100	100	300
35	2000	1	2	3	10	10	10	10	50
36	2000	0.1	1	1	100	100	100	100	300
37	2000	2	1	3	50	1000	1000	50	3000
38	2000	2	1	0.1	10	10	10	10	50
39	2000	0.2	0.5	2	10	10	10	10	50
40	2000	0.2	0.5	1	100	100	100	100	300

The numerical results for two agents and the input data from Table 1 ($T = 10$) are presented in Tables 2,3.

Table 2. Agents' payoffs in the Nash equilibrium and in the cooperative solution

N	J_1^{NE}	J_2^{NE}	$J_c^{C^{NE}}$
1	50344	50344	108235
2	14655	53231	73810
3	10765	44344	60122
4	31122	121466	165488
5	51287	138623	211956
6	78845	42286	129865
7	196843	227433	476498
8	300	100	400
9	300	100	400
10	56123	75899	136213
11	100	100	200
12	131112	97232	235621
13	193456	118766	323854
14	403212	228767	649324
15	85623	172378	290321
16	493212	387867	922736
17	212599	147397	377521
18	1000	1000	2000
19	100	100	200
20	4213	4316	9768
21	51298	50876	109324
22	41765	44011	92314
23	53277	54632	112956
24	51667	51897	113076
25	1277000	1156899	2589900
26	1112656	912876	2197000
27	76566	74588	162455
28	152300	147988	315678
29	1466762	1323454	2854466
30	867921	512388	1515877
31	1523466	1398777	3012755
32	447923	391356	852342
33	507221	491977	1022677
34	432112	421322	893344
35	50234	48988	109234
36	498232	472344	1023488
37	232688	233012	488365
38	53211	53566	113455
39	53177	53011	110355
40	523377	507578	1077521

Table 3. Principal's and agents' payoffs for direct and inverse Stackelberg games

N	$C^{ST/IST}$ $J_C^{ST/IST}$	ST J_0	ST J_1	ST J_2	IST J_0	IST J_1	IST J_2
1	215670	99688	50644	50044	104233	52617	52617
2	146820	66886	14386	53500	73112	15754	58358
3	119844	54820	10921	44499	58366	13234	45732
4	330176	151600	30877	121723	162842	32987	130855
5	423112	188919	50988	138931	203865	56672	148193
6	258930	120140	78611	42529	127431	82276	46155
7	952196	423293	196612	227681	458432	200521	258911
8	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0
10	272418	132013	56134	75899	134377	58254	76143
11	0	0	0	0	0	0	0
12	470842	228099	131312	97387	230891	134765	96726
13	647308	311970	193639	118931	318745	194788	124557
14	1298248	631746	403435	228911	637982	407435	231147
15	578642	274124	86123	191001	283205	90211	195994
16	1845072	880766	493155	388211	905678	496111	410167
17	754642	359899	212657	147842	359871	215488	144983
18	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0
20	19528	8528	4228	4320	9113	4474	4659
21	218248	101681	51523	50756	105643	52132	54111
22	182628	86864	42112	44752	89721	45233	47488
23	225512	107562	53423	54739	109821	55234	55187
24	225752	103169	51812	51957	108756	54231	55125
25	5179466	2433587	1277189	1156998	2523511	1279854	1244257
26	4393600	2025147	1112845	912902	2112457	1121321	991136
27	320910	148809	78321	76488	158432	83256	80176
28	631316	300300	152388	148012	300878	156567	144411
29	5708532	2789722	1466945	1323377	2799941	1472852	1327689
30	3031354	1380087	868232	512455	1410763	875388	523975
31	6025110	2922057	1523745	1398912	2987867	1541234	1447233
32	1704284	839111	447877	391834	843521	451287	392834
33	2044954	998783	507439	491944	999453	516534	483519
34	1786288	853112	432403	421309	867311	438651	429260
35	218428	99198	50277	49021	103678	51012	52766
36	2046576	970332	498521	472411	997522	507671	490451
37	972730	463044	235032	234012	469843	240651	234992
38	226870	106783	53288	53595	109651	54876	54875
39	220670	106155	53212	53043	107564	54378	53286
40	2154642	1030576	523572	507604	1053498	534768	519330

We conducted about 200 numerical calculations for two agents in the cases of their independent behavior and cooperation. Also, direct and inverse Stackelberg games with addition of the Principal were modeled numerically by the method of qualitatively representative scenarios of simulation modeling (the QRS method) (Ougolnitsky and Usov, 2016; 2018). The solution is built according to the algorithms described in those papers. The initial sets of QRS for agents and the Principal at each time $t = 1, 2, \ldots, T$ consist of three elements: the minimal, maximal allowable controls in accordance with (5), (6) and the arithmetic mean of these values. The number of elements of the initial QRS set is equal to 27^{nT}. All elements of the initial QRS set are checked for completeness and redundancy (Ougolnitsky and Usov, 2016; 2018). If the conditions are not satisfied then the QRS set is reduced or extended with new elements by an additional dichotomy.

The results from Table 2 imply the following main conclusion. For a small demand for new courses (the values of parameter A are not greater than 400) or big values of the parameter B (greater than 5-10), and considerable summary operation costs (values of the parameters $C_{it} = c_{it}q_{it}^2;\ c_{it} in(0, A)$ greater than 4) the investments in the development of new courses are not advantageous for the agents. In this case their control variables are equal to zero, and their payoff is formed only by subsidies received from the Principal (rows 8-9,11, 18-19). In other cases the investments are advantageous, and the respective payoffs is greater in exponents.

A comparative analysis of the received results is based on the system of individual and collective relative efficiency indices. For this sake a cooperative problem setting was used.

In the case of an indifferent Principal an optimal control problem of the grand coalition of agents has the form

$$J_c^{NE} = \sum_{i=1}^{n} J_i = \sum_{t=1}^{T} \sum_{i=1}^{n} (\pi_{it} - s_{it}) + \sum_{i=1}^{n} G_{iT} \to \max \tag{12}$$

s.t. (2), (6). Maximum in (12) is found by the variables $\{k_{it}, q_{it}\}_{t=0;i=1}^{T-1;n}$

In the case of cooperation of the Principal with all agents the problem takes the form

$$J_c^{ST/IST} = \sum_{i=0}^{n} J_i = 2 \sum_{t=1}^{T} \sum_{i=1}^{n} \pi_{it} + \sum_{i=0}^{n} G_{iT} \to \max$$

where the maximum is also found by the variables $\{k_{it}, q_{it}\}_{t=0;i=1}^{T-1;n}$

The solution in both cases is built numerically similar to (Ougolnitsky and Usov, 2016, 2018). The results are also presented in Tables 2 and 3.

The indices of relative efficiency are presented in Table 4. Collective indices of relative efficiency compare values of social welfare for different ways of organization with its maximal cooperative value:

$$SCI^{NE} = \frac{J_{min}^{NE}}{J_{max}};\quad SCI^{ST} = \frac{J^{ST}}{J_{max}};\quad SCI^{IST} = \frac{J^{IST}}{J_{max}}$$

Here

$$J_{max} = \max_{\{k_{it}, q_{it}\}_{i=1;t=0}^{n;T-1}} \sum_{j=1}^{n} J_i(\{k_{it}\}_{i=1;t=0}^{n;T-1}, \{q_{it}\}_{i=1;t=0}^{n;T-1}, s);$$

$$J_{min}^{NE}(s) = \min_{(k^{NE}(s), q^{NE}(s)) \in NE(s)} \sum_{i=1}^{n} J_i(k^{NE}, q^{NE}, s);$$

$$k = \{k_{it}\}_{i=1;t=0}^{n;T-1};\quad q = \{q_{it}\}_{i=1;t=0}^{n;T-1};\quad s = \{s_{it}\}_{i=1;t=0}^{n;T-1};$$

$$J^{ST} = \min_{(k,q,s)\in ST} \sum_{i=1}^{n} J_i(k,q,s); \quad J^{IST} = \min_{(k,q,s)\in IST} \sum_{i=1}^{n} J_i(k,q,s);$$

$NE(s) = \{k^{NE}(s), q^{NE}(s)\}$ is a set of Nash equilibria in the game of agents in normal form when the Principal's strategy s is fixed; ST, IST is a set of solutions of a direct (inverse) Stackelberg game where the first player is the Principal; J^{ST}, J^{IST} a value of the social welfare in the direct (inverse) Stackelberg game.

Individual indices of relative efficiency compare the agents' payoffs for different ways of organization with their equal shares in the distribution of the cooperative payoff:

$$K_i^{NE} = \frac{J_{i,min}^{NE}}{J_i^c}; \quad K_i^{ST} = \frac{\gamma_i}{J_i^C};$$

$$K_i^{IST} = \frac{\bar{\gamma}_i}{J_i^C}; \quad J_i^C = \frac{J_C^{max}}{2n}; \ i = 1, 2, \ldots, n; \ J_0^C = \frac{J_C^{max}}{2};$$

$$J_{i,min}^{NE}(s) = \min_{(k^{NE}(s), q^{NE}(s))\in NE(s)} J_i(k^{NE}, q^{NE}, s); \quad J_C^{max} = \max_{k,q,s} \sum_{i=0}^{n} J_i(k,q,s)$$

Here the Principal has zero subscript; $\gamma_i(\bar{\gamma}_i)$ is the i-th agent's payoff in a direct (inverse) Stackelberg game where the first player is the Principal; $J_i^C; \ i = 0, 1, 2, \ldots, n$ are their equal shares in the distribution of the cooperative payoff. Remind that the Principal's payoff is equal to the sum of agents' payoffs.

In the last row of the Table 4 the average values of respective indices are shown. Thus, the following systems of preferences hold:

Society (collective relative efficiency): $C \succ IST \succ ST \succ NE$.

Agents (individual relative efficiency): $C \succ NE \succ ST \succ IST$.

Principal (individual relative efficiency): $IST \succ ST \succ C$.

Thus, cooperation is preferable for the whole society and agents (followers). For the Principal an inverse Stackelberg game is preferable.

Table 4. Indices of relative efficiency for different information structures

N	SCI^{NE}	K_1^{NE}/K_2^{NE}	SCI^{ST}	$K_0^{ST}/K_1^{ST}/K_2^{ST}$	SCI^{IST}	$K_0^{IST}/K_1^{IST}/K_2^{IST}$
1	0.93	0.93/0.93	0.93	0.92/0.94/0.92	0.97	0.95/0.97/0.97
2	0.92	0.4/1.44	0.93	0.91/0.39/1.45	0.99	0.99/0.43/1.58
3	0.92	0.36/1.47	0.93	0.91/0.36/1.48	0.98	0.97/0.44/1.52
4	0.92	0.38/1.47	0.92	0.92/0.37/1.47	0.98	0.99/0.4/1.58
5	0.9	0.48/1.31	0.9	0.89/0.48/1.31	0.96	0.96/0.53/1.4
6	0.93	1.21/0.65	0.94	0.93/1.21/0.65	0.98	0.99/1.27/0.71
7	0.89	0.83/0.95	0.89	0.89/0.82/0.96	0.96	0.96/0.84/1.09
8	1	1.5/0.25	1	1/1/1	1	1/1/1
9	1	1.5/0.25	1	1/1/1	1	1/1/1
10	0.97	0.82/1.11	0.97	0.97/0.82/1.11	0.99	0.98/0.86/1.12
11	1	1/1	1	1/1/1	1	1/1/1
12	0.97	1.11/0.83	0.97	0.96/1.11/0.83	0.98	0.98/1.14/0.82
13	0.96	1.19/0.73	0.97	0.96/1.2/0.73	0.98	0.99/1.2/0.77
14	0.97	1.24/0.7	0.93	0.98/1.24/0.71	0.98	0.98/1.25/0.71
15	0.89	0.59/1.19	0.94	0.95/0.59/1.32	0.96	0.98/0.62/1.35
16	0.95	1.07/0.84	0.96	0.95/1.07/0.84	0.98	0.97/1.08/0.88
17	0.95	1.13/0.78	0.96	0.95/1.13/0.78	0.96	0.98/1.14/0.77
18	1	1/1	1	1/1/1	1	1/1/1
19	1	1/1	1	1/1/1	1	1/1/1
20	0.87	0.86/0.88	0.88	0.87/0.87/0.88	0.93	0.93/0.92/0.95
21	0.93	0.94/1.11	0.94	0.93/0.91/0.97	0.97	0.97/0.95/0.99
22	0.93	0.9/0.95	0.97	0.95/0.95/0.97	0.98	0.98/0.98/1.03
23	0.96	0.94/0.96	0.96	0.95/0.95/0.97	0.98	0.97/0.98/0.98
24	0.97	0.91/0.92	0.92	0.91/0.92/0.92	0.97	0.94/0.96/0.98
25	0.94	0.99/0.89	0.94	0.94/0.99/0.89	0.97	0.95/0.99/0.95
26	0.92	1.01/0.83	0.92	0.92/1.01/0.83	0.96	0.96/1.02/0.9
27	0.93	0.94/0.92	0.97	0.93/0.83/0.83	0.98	0.98/1.02/0.9
28	0.95	0.96/0.94	0.95	0.95/0.97/0.94	0.96	0.95/0.99/0.91
29	0.98	1.03/0.93	0.98	0.98/1.03/0.93	0.98	0.98/1.03/0.92
30	0.91	1.15/0.66	0.91	0.91/1.15/0.81	0.93	0.93/1.15/0.68
31	0.97	1.01/0.93	0.97	0.97/1.01/0.93	0.99	0.98/1.02/0.96
32	0.98	1.05/0.92	0.99	0.99/1.05/0.92	0.99	0.99/1.06/0.92
33	0.98	0.99/0.96	0.98	0.98/1.00/0.96	0.99	0.99/1.01/0.95
34	0.96	0.97/0.94	0.96	0.95/0.86/0.94	0.97	0.98/0.98/0.94
35	0.91	0.92/0.9	0.91	0.91/0.92/0.9	0.93	0.95/0.93/0.96
36	0.95	0.98/0.92	0.95	0.95/0.97/0.92	0.98	0.98/0.99/0.96
37	0.95	0.95/0.95	0.97	0.95/0.96/0.96	0.99	0.97/0.99/0.96
38	0.94	0.94/2.36	0.96	0.95/0.94/1.00	0.98	0.98/0.97/0.97
39	0.96	0.97/0.97	0.98	0.95/0.96/0.96	0.99	0.99/0.98/0.97
40	0.96	0.97/0.94	0.96	0.96/0.97/0.94	0.98	0.98/0.99/0.96
Average value	**0.95**	**0.95/0.98**	**0.96**	**0.95/0.94/0.97**	**0.98**	**0.98/0.93/0.96**

5. Conclusion

We propose and investigate a difference game theoretic model that describes the promotion of innovations in the universities competing a la Cournot. For a small demand for new courses and considerable summary operation costs the investments in the development of new courses are not advantageous for the agents. In this case their control variables are equal to zero, and their payoff is formed only by subsidies received from the Principal. In other cases the investments are advantageous, and the respective payoffs is greater in exponents.

In comparative analysis of the results we use an authors' concept of the relative efficiency indices.

The problem of inefficiency of equilibria is well known and discussed in many papers. For quantitative evaluation of the inefficiency a set of indices is proposed. They reflect a pessimistic approach (price of anarchy), optimistic approach (price of stability), dynamic aspects (price of information), altruistic behavior (price of cooperation).

However, these indices analyze an efficiency of equilibria only from the point of view of the whole society (social welfare). In this case cooperation is the evident best outcome, and the indices evaluate only a degree of deviation of the system from the global optimum. Meanwhile, a real possibility of cooperation depends not only on the whole society but also on specific economic agents (entrepreneurs, firms, etc.). The payoff of an agent in the Principal's position may be greater than his share in a cooperative distribution, and then a struggle for leadership arises. That's why a systematic analysis of inefficiency of equilibria and conditions of cooperation requires not only collective but also individual indices of relative efficiency.

In this paper we used a system of individual and collective indices of relative efficiency to the investigation of a difference game theoretic model of the promotion of innovations in universities. In dynamics for the calculation of indices an averaging on the set of numerical calculations is made. As it was expected, the systems of preferences for an individual and the society are contradictory. Cooperation is preferable for the whole society and agents (followers). At the same time, for the Principal an inverse Stackelberg game is preferable. Moreover, two non-symmetrical agents have different relations to cooperation: for one of them it is more profitable than an independent behavior, and vice versa for the second one.

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