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Online-Offline Competition with Heterogeneous Consumers: An Example for No Existence of Pure Strategy Nash Equilibrium

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Abstract Existing literature on competition between online and offline firms has focused on market conditions that guarantee the existence of a pure strategy Nash equilibrium. In this note, by constructing a concrete example, we provide a first attempt to show that the equilibrium existence result does not necessarily hold when consumers' preferences are heterogeneous. Specifically, we consider the competition between one online firm and several offline firms in a market organized as a Salop model, where consumers' preferences have a binary distribution. We identify a boundary scenario where the type distribution is binary with one type of consumer loyal to online shopping and the other type loyal to offline shopping. We show that there is no pure strategy Nash equilibrium for this boundary scenario, which indicates that the market may not be stable under such conditions. Our study contributes to a better understanding of the equilibrium existence conditions for the online versus offline retail competition.

Keywords: online versus offline, price competition, salop model, heterogeneous consumers, existence of equilibrium.

1. Introduction

The competition between online shopping and offline shopping has long been a hot research topic in the academic field. With the impact of the COVID-19 pandemic, online shopping is becoming more and more popular while offline shopping has taken a hit, as consumers could purchase from online firms without taking the risk of epidemic transmission. In real life, we can see that there are both online firms and offline firms in many industries. So the study of online versus offline competition not only helps us better understand these industries but also provides some implications for regulators to design policies to improve social welfare.

The literature on online versus offline market competition has typically focused on market conditions that guarantee the existence of a pure strategy Nash equilibrium (Balasubramanian, 1998; Ford, Li, Li, and Zheng, 2019; Wang and Zheng, 2020; Ford, Li, and Zheng, 2021, which are partially privatized). In a recent study by Han, Lien, Lien, and Zheng (2022), an examination of full parameter space is

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conducted in a circular market structure with both online and offline firms, which can be partially accessible, and their result shows that there always exists a pure strategy Nash equilibrium for the price competition among firms. Given the existing result in the literature, one may wonder whether a pure strategy Nash equilibrium always exists for such a market structure, and if not, under which conditions this is not the case. In this note, we provide a first attempt to address this issue. By departing from the standard model via only one dimension, that is by introducing consumer heterogeneity, we show that a pure strategy Nash equilibrium does not always exist.

Specifically, we consider a circular market, where an online firm is located at the center of the circle, several offline firms are evenly distributed along the circle, and a continuum of consumers is uniformly distributed along the circle. All consumers have a sufficiently high valuation for making a purchase but there is no valuation for the second unit of consumption. Consumers' transport costs and firms' production costs are both linear. First, all firms simultaneously and independently set their prices; then each consumer decides where to make the purchase. Such a setting is standard in the literature and well reflects the reality in many situations.[1] In addition, we allow consumers to be heterogeneous in our model. In the simplest setting, a consumer can be either type 1 or type 2, where the type measures a consumer's relative preference between online and offline shopping. Consumers' type distribution and location distribution are independent. Such a consumer heterogeneity assumption may better capture the reality, as in real life consumers' preferences are indeed different from one another. However, through rigorous analysis, we show that such a simple departure from the standard homogeneous preference setting may lead to the no existence of pure strategy Nash equilibrium.

Our work belongs to the large literature on online versus offline competition, which includes (Balasubramanian, 1998; Chen, Hu, and Li, 2017; Ford, Li, Li, and Zheng, 2019; Forman, Ghose, and Goldfarb, 2009; Guo and Lai, 2017; Liu, Gupta, and Zhang, 2006; Loginova, 2009; Wang and Zheng, 2020), among many others. Our work is also closely related to the literature on spatial competition, see Boyer and Moreaux (1993), Fleckinger and Lafay (2010), Hernandez (2011), Hotelling (1990), Kats (1995), Madden and Pezzino (2011), and Salop (1979). In particular, our work is an extension of the basic framework in (Balasubramanian, 1998), which was the first study to consider online versus offline competition, under the assumption that all consumers have the same preference between online shopping and offline shopping. It is worth noting that most studies in the literature focus on the case where consumers are homogeneous except for their location difference. However, here we take into account that consumers may also have different preferences on shopping online versus offline. Such a difference in setting leads to the difference in result: While most studies pay attention to the existence and properties of pure strategy Nash equilibrium, we differ from the literature by showing that such an equilibrium may not exist under consumer preference heterogeneity.[2]

[1] For example, the competition between MOOCs and regular universities can be treated as an example of online versus offline competition (Han, Lien, Lien, and Zheng, 2022). Balasubramanian (1998), and Ford, Li, and Zheng (2021) provide more related examples and further discussion of such markets.

[2] A recent work by Han, Lian, and Zheng (2022) also studies the existence of pure strategy Nash equilibrium in the market competition between online and offline stores,

We contribute to the literature in two aspects. Firstly, we provide a simple example with no existence of pure strategy Nash equilibrium for competition between online and offline firms with heterogeneous consumers, which contributes to the literature by offering a better understanding of equilibrium existence conditions. Secondly, our study provides some insights into consumers' behavior in the presence of preference heterogeneity. Specifically, when one group of consumers prefers shopping online very much while the rest are loyal customers of offline firms, the former type of consumers will only purchase from the online firm and the latter will only shop offline in any equilibrium (if exist). That is to say, in the market, there is no neutral consumer (i.e., customers who are likely to shop online or offline, depending on their distance from the online and offline firms). We also conclude that under such conditions, the market can't reach a pure strategy equilibrium. To be precise, we find that when all consumers are classified as loyal users of either online or offline shopping, the market cannot be stable.

The rest of our paper proceeds as follows. In Section 2 we present the model setup. In Section 3 we perform the analysis and present our main result. We conclude and discuss in Section 4.

2. The Model

Suppose the market is organized as a Salop circle (Salop, 1979). A unit measure of utility-maximizing consumers is uniformly distributed along the circle. Every consumer wants to buy one unit of the product and the demand is inelastic, which could be ensured by assuming that consumers' valuation of the product v is sufficiently high. Each product is indivisible. So a consumer can only purchase from one firm at a time. There are only two types of firms in the market, online firm (denoted by o) and offline firm (denoted by r). Consistent with Balasubramanian (1998), we assume that online firms are located at the center of the circle, as online shopping does not require consumers to pay any transport cost. Offline firms are evenly spaced on the circumference. Any consumer who travels a distance x incurs a transport cost tx, where t is the transport cost per unit distance. For online shopping, there is no transport cost, but instead, consumers have to bear a cost μ which measures the relative preference between online shopping and offline shopping (Han, Lien, Lien, and Zheng, 2022). μ can also be interpreted as the degree of fitness of the products from offline firms compared with online firms (e.g., Balasubramanian, 1998; Ford, Li, and Zheng, 2020). A negative μ means that compared with offline shopping, online shopping is so convenient and fast that consumers prefer to shop online if all firms sell homogeneous goods. A positive μ, on the contrary, means that consumers have to bear a higher opportunity cost during online shopping than during offline shopping.

As we mentioned in the introduction, consumers' preferences for online shopping μ have two possible values, μ_1 and μ_2, where $\mu_1 < \mu_2$. We refer to a consumer with μ_i as type i consumer ($i \in \{1, 2\}$). There are γ_1 fraction of type 1 consumers and γ_2 fraction of type 2 consumers in the market ($\gamma_1 + \gamma_2 = 1$). $\gamma_1 > 0$ and $\gamma_2 > 0$, otherwise this model will degenerate into the homogeneous preferences model which is well studied in the literature mentioned above. In addition, here we only consider

but they consider homogeneous preference and quadratic transport cost while we consider heterogeneous preference and linear transport cost.

the boundary scenario that μ_1 is very small and μ_2 is very large.[3] The values of μ_1 and μ_2 are such that in equilibrium, if $\gamma_1 = 1$, consumers will only purchase from the online firm, while if $\gamma_2 = 1$, consumers only purchase from offline firms. The consumers corresponding to each possible value of μ are still uniformly distributed along the Salop circle. For a consumer with location x and preference μ, the utility of purchasing from an online firm or an offline firm i with location x_i is

$$\begin{aligned} u^r &= v - |x - x_i|\, t - p_i^r, \\ u^o &= v - \mu - p^o. \end{aligned}$$

u^r and u^o denote the utility of online shopping and offline shopping, respectively. v is high enough to make sure that no one will exit the market. p_i^r is the price set by the offline firm, while p^o is the price set by the online firm.

For the sellers, we assume that there are N offline firms and one online firm.[4] Each firm has the same constant marginal cost c, and zero fixed cost. There is no outside option. We refer to s_i^r as the market share of an offline firm i and s^o as the market share of the online firm. Thus the profit functions are

$$\begin{aligned} \pi_i^r &= p_i^r s_i^r - c s_i^r, \forall i = 1, \ldots, N, \\ \pi^o &= p^o s^o - c s^o. \end{aligned}$$

The market shares can be divided into two parts, type 1 market share (denoted $s_{i,1}^r$ and s_1^o) and type 2 market share (denoted $s_{i,2}^r$ and s_2^o), which are shown below.

$$\begin{aligned} s_i^r &= \gamma_1 s_{i,1}^r + \gamma_2 s_{i,2}^r, \forall i = 1, \ldots, N, \\ s^o &= \gamma_1 s_1^o + \gamma_2 s_2^o. \end{aligned}$$

The timing of the game is as follows. In the first stage, each profit-maximizing firm sets its price simultaneously and independently. In the second stage, each consumer decides where to purchase the product. The whole market structure is illustrated by Figure 1, in which we use blue color to denote the type 1 market and red color to denote the type 2 market.

3. Analysis and Result

Based on the model setup, we first consider the range of parameters, μ_1 and μ_2. In theory, μ_1 and μ_2 can be any real numbers, but in this study, we are particularly interested in the boundary scenarios where μ_1 and μ_2 are very different from each other. The boundary condition for μ under one type of consumer has already been derived in the literature, and the readers are referred to Balasubramanian (1998), and Ford, Li, and Zheng (2020) for details.[5] So here we focus on such a range for

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[3] A complete analysis of the full parameter space is available upon request.

[4] The analysis can be easily conducted for the case of more than one online firm where all online firms charge a price equal to the marginal cost in any equilibrium, because if there are two or more online firms, they are competing in a Bertrand game, where the competitor with the lowest price will get all online users. So all online firms will continue to cut prices until they reach the marginal cost.

[5] Balasubramanian (1998) only considers the case that $\mu \geq 0$, while Ford, Li, and Zheng (2020) allows μ to be negative. However, in neither study, μ is allowed to be different for different consumers.

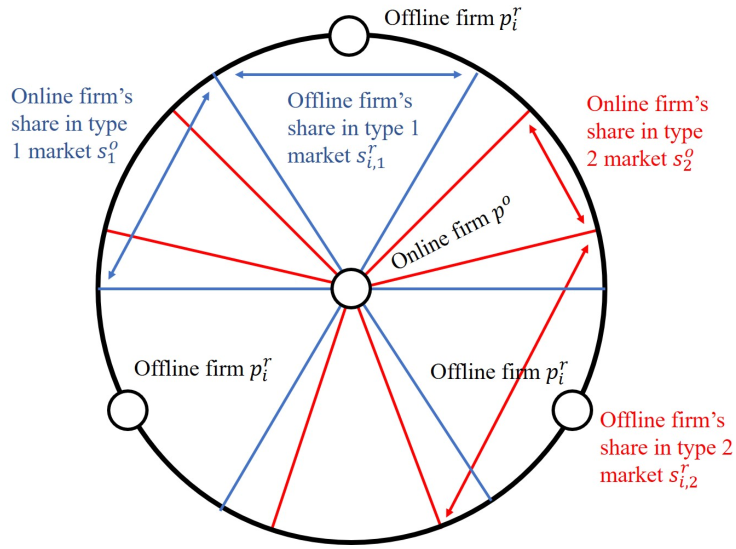

Fig. 1. An Illustration of the Market Structure with Two Types of Consumers

μ_1 and μ_2 directly, where type 1 consumers have a very strong preference for online shopping and type 2 consumers have a very strong preference for offline shopping (see Condition 1).

$$\frac{\mu_1}{t} \leq -\frac{1}{2N} < \frac{1}{N} \leq \frac{\mu_2}{t}. \tag{1}$$

Following the same approach in (Balasubramanian, 1998), we focus on the symmetric equilibrium where marginal consumers are indifferent between online shopping and offline shopping. With the presence of different types of consumers, there should also be type 1 marginal consumers and type 2 marginal consumers. It should be noted that in the type 1 market the competition is only between online shopping and offline shopping, but not within offline firms. This is because when the online firm sets the price to be c, there will always be some consumers preferring online shopping, which means that in any equilibrium the online firm will at least attract some type 1 users. Due to the symmetry, we only need to consider the competition between the online firm and one offline firm. The distance between a type 1 marginal consumer and the nearest offline firm i is $x_{i,1}$. In a symmetric equilibrium, all offline firms charge the same equilibrium price and have the same market shares. Furthermore, in a Salop circle, firms are able to attract consumers from both sides, which means $s_{i,1}^r = 2x_{i,1}$ in equilibrium. As there will always be some type 1 consumers who prefer to shop online, we do not consider the case where $x_{i,1}$ is greater than $\frac{1}{2N}$, and $s_{i,1}^r$ should be less than or equal to $\frac{1}{N}$. We have

$$v - tx_{i,1} - p_i^r = v - \mu_1 - p^o \Longrightarrow x_{i,1} = \begin{cases} 0, \ if \ p^o - p_i^r + \mu_1 < 0 \\ \frac{p^o - p_i^r + \mu_1}{t}, \ if \ 0 \leq \frac{p^o - p_i^r + \mu_1}{t} \leq \frac{1}{2N} \end{cases},$$

$$i = 1, \ldots, N.$$

In real life, it is hard to perceive that a firm's market share is negative, so here we assume that $x_{i,1}$, $x_{i,2}$, $s_{i,1}$, and $s_{i,2}$ are all non-negative. When $p^o - p_i^r + \mu_1$

is negative, all consumers prefer the online firm to offline firm i, thus leading to $x_{i,1} = 0$.

Similarly, for type 2 market consumers, we assume that the distance between this consumer and the nearest offline firm i is $x_{i,2}$. In a symmetric equilibrium, $s_{i,2} = 2x_{i,2}$. We have

$$v - tx_{i,2} - p_i^r = v - \mu_2 - p^o \Longrightarrow x_{i,2} = \begin{cases} \frac{p^o - p_i^r + \mu_2}{t}, \; if \; 0 \leq \frac{p^o - p_i^r + \mu_2}{t} \leq \frac{1}{2N} \\ \frac{1}{N} + \frac{p_{i+1}^r + p_{i-1}^r}{2t} - \frac{p_i^r}{t}, \; if \; \frac{p^o - p_i^r + \mu_2}{t} > \frac{1}{2N} \end{cases},$$
$$i = 1, \ldots, N.$$

Similar to the previous analysis, each offline firm will at least attract some type 2 consumers, so we omit the case where $p^o - p_i^r + \mu_2 < 0$. On the other hand, if $p^o - p_i^r + \mu_2$ is too large (i.e., p^o is large or p_i^r is small), the indifference condition for a type 2 marginal consumer will change (that is to say, now the marginal consumer has to choose between two offline firms, instead of an offline firm and an online firm), which is shown below.

$$\begin{aligned} &v - tx_{i,2} - p_i^r = v - t\left(\frac{1}{N} - x_{i,2}\right) - p_{i+1}^r > v - \mu_2 - p^o, x_{i,2} \in [0, \frac{1}{N}], \\ &v - tx'_{i,2} - p_i^r = v - t\left(\frac{1}{N} - x'_{i,2}\right) - p_{i-1}^r > v - \mu_2 - p^o, x'_{i,2} \in [0, \frac{1}{N}] \\ &\Longrightarrow s_i^r = x_{i,2} + x'_{i,2} = \frac{1}{N} + \frac{p_{i+1}^r + p_{i-1}^r}{2t} - \frac{p_i^r}{t}, \; i = 1, \ldots, N. \end{aligned}$$

$x_{i,2}$ measures the type 2 consumers who are between the offline firms, i and $(i+1)$, and purchase from firm i. While $x'_{i,2}$ measures the type 2 consumers who are between the offline firms, i and $(i-1)$, and purchase from firm i. Regarding the market share of the online firm and offline firms, our findings are summarized in Lemma 1.

Lemma 1. *In a circular market, suppose there are N offline firms and one online firm with linear transport cost (parameter t) among offline firms and vertical differentiation (parameter μ) between online and offline firms. Suppose all firms have the same constant marginal cost, and the vertical differentiation parameter follows a binary distribution with a lower limit μ_1 and an upper limit μ_2. If $\frac{\mu_1}{t} \leq -\frac{1}{2N} < \frac{1}{N} \leq \frac{\mu_2}{t}$ and assume a pure strategy Nash equilibrium exists, then consumers with μ_1 will only purchase from the online firm, while consumers with μ_2 will only purchase from the offline firms.*

Proof. On the premise that the difference between μ_2 and μ_1 is larger than $\frac{3}{2N}$, only when $\frac{p^o - p_i^r + \mu_1}{t} < -\frac{1}{N}$ is it possible for the online firm to attract some type 2 consumers.

$$\frac{p^o - p_i^r + \mu_2}{t} = \frac{p^o - p_i^r + \mu_1}{t} + \frac{\mu_2 - \mu_1}{t} < \frac{1}{2N} \Longrightarrow \frac{p^o - p_i^r + \mu_1}{t} < \frac{1}{2N} - \frac{\mu_2 - \mu_1}{t},$$
$$i = 1, \ldots, N.$$

Suppose the online firm could earn some profits from the type 2 market, then its own profit is

$$\pi^o = \gamma_1(p^o - c)s_1^o + \gamma_2(p^o - c)s_2^o = \gamma_1(p^o - c) + \gamma_2(p^o - c)\left(1 - 2N\frac{p^o - p_i^r + \mu_2}{t}\right).$$

As we mentioned above, we only focus on the case where $\gamma_1 > 0$ and $\gamma_2 > 0$. By differentiating π^o with respect to p^o, we get

$$\begin{aligned}
&\frac{d\pi^o}{dp^o} = \gamma_1 + \gamma_2\left(1 - 2N\frac{p^o - p_i^r + \mu_2}{t} - 2N\frac{p^o - c}{t}\right) = 0 \\
&\Longrightarrow 2p^o = c + p_i^r + \frac{\gamma_1 + \gamma_2}{\gamma_2}\cdot\frac{t}{2N} - \mu_2.
\end{aligned} \tag{2}$$

Similarly, for any offline firm, we can obtain the profit and the first order condition with respect to p_i^r.

$$\begin{aligned}
&\pi_i^r = 2\gamma_2(p_i^r - c)\frac{p^o - p_i^r + \mu_2}{t} \\
&\Longrightarrow \frac{d\pi_i^r}{dp_i^r} = 2\gamma_2\left(\frac{p^o - p_i^r + \mu_2}{t} - \frac{p_i^r - c}{t}\right) = 0 \Longrightarrow 2p_i^r = c + p^o + \mu_2.
\end{aligned} \tag{3}$$

From the Equations 2 and 3, we could derive the equilibrium prices, which are shown below.

$$\begin{cases} p_i^r = c + \frac{\gamma_1+\gamma_2}{\gamma_2}\cdot\frac{t}{6N} + \frac{\mu_2}{3}, \\ p^o = c + \frac{\gamma_1+\gamma_2}{\gamma_2}\cdot\frac{t}{3N} - \frac{\mu_2}{3}. \end{cases}$$

$$\Longrightarrow \frac{p^o - p_i^r + \mu_1}{t} = \frac{\gamma_1 + \gamma_2}{6N\gamma_2} - \frac{\frac{2\mu_2}{3} - \mu_1}{t} > \frac{1}{6N} + \frac{\mu_2}{3t} - \frac{\mu_2 - \mu_1}{t} \geq \frac{1}{2N} - \frac{\mu_2 - \mu_1}{t}.$$

So we can see that the online firm will not reduce the price to attract some type 2 consumers. In equilibrium, type 2 consumers only purchase from offline firms. The type 2 market degenerates into competition among offline firms. Thus in the symmetric equilibrium all offline firms share the type 2 market equally (i.e., $x_{i,2} = \frac{1}{2N},\ i = 1, \ldots, N$). The online firm's profit only comes from the transaction in the type 1 market, as shown below.

$$\pi^o = \gamma_1(p^o - c)s_1^o = \gamma_1(p^o - c)\left(1 - 2N\frac{p^o - p_i^r + \mu_1}{t}\right).$$

Consider the first order condition with respect to p^o, and we have

$$\frac{d\pi^o}{dp^o} = \gamma_1\left(1 - 2N\frac{p^o - p_i^r + \mu_1}{t} - 2N\frac{p^o - c}{t}\right) \leq 0, x_{i,1} = \frac{p^o - p_i^r + \mu_1}{t} \in [0, \frac{1}{2N}). \tag{4}$$

From Equation (4), we could conclude that when $\frac{\mu_1}{t} \leq -\frac{1}{2N}$, the online firm will keep reducing price until all type 1 consumers choose online shopping, which implies that $x_{i,1} = 0$ (i.e., $p^o = p_i^r - \mu_1$).

Lemma 1 implies that when $p^o - p_i^r + \mu_1 < 0$, the online firm will increase price to maximize profit without losing any market shares in type 1 market. So we have $p^o = p_i^r - \mu_1$ in any pure strategy equilibrium (if exist).

Next, we solve the offline firms' profit-maximization problem. The profit function of a typical offline firm is

$$\begin{aligned}
\pi_i^r &= 2\gamma_1(p_i^r - c)x_1 + \gamma_2(p_i^r - c)\left(\frac{1}{N} + \frac{p_{i+1}^r + p_{i-1}^r}{2t} - \frac{p_i^r}{t}\right) \\
&= 2\gamma_1(p_i^r - c)\frac{p^o - p_i^r + \mu_1}{t} + \gamma_2(p_i^r - c)\left(\frac{1}{N} + \frac{p_{i+1}^r + p_{i-1}^r}{2t} - \frac{p_i^r}{t}\right).
\end{aligned}$$

As we mentioned before, for all offline firms, $x_{i,1}$ is equal to zero. Given the online firm's price being fixed, if an offline firm reduces the price by a little bit, it can attract some type 1 consumers from the online firm. However, if an offline firm raises its price, it cannot earn any profit from the type 1 market. So in this critical state the right derivative (denoted by $\lim_{\Delta p\to 0+} d\pi_i^r/dp_i^r$) may not be equal to the left derivative (denoted by $\lim_{\Delta p\to 0-} d\pi_i^r/dp_i^r$) with respect to the price p_i^r. Since an offline firm maximizes its profit, the equations of derivative concerning the price are

$$\begin{aligned}
&\lim_{\Delta p\to 0-}\frac{d\pi_i^r}{dp_i^r}=\\
&\quad 2\gamma_1\left(\frac{p^o-p_i^r+\mu_1}{t}-\frac{p_i^r-c}{t}\right)+\gamma_2\left(\frac{1}{N}+\frac{p_{i+1}^r+p_{i-1}^r-2p_i^r}{2t}-\frac{p_i^r-c}{t}\right)\\
&\Longrightarrow \lim_{\Delta p\to 0-}\frac{d\pi_i^r}{dp_i^r}=-2\gamma_1\frac{p_i^r-c}{t}+\gamma_2\left(\frac{1}{N}-\frac{p_i^r-c}{t}\right)=-(2\gamma_1+\gamma_2)\frac{p_i^r-c}{t}+\gamma_2\frac{1}{N}.
\end{aligned} \tag{5}$$

$$\lim_{\Delta p\to 0+}\frac{d\pi_i^r}{dp_i^r}=\gamma_2\left(\frac{1}{N}+\frac{p_{i+1}^r+p_{i-1}^r-2p_i^r}{2t}-\frac{p_i^r-c}{t}\right)=-\gamma_2\frac{p_i^r-c}{t}+\gamma_2\frac{1}{N}. \tag{6}$$

Notice that the right derivative (Equation 6) is always greater than the left derivative (Equation 5), indicating that the offline firms can never set an optimal price, and thus there is no pure strategy symmetric Nash equilibrium in this market. This is because given $x_1=0$ by Lemma 1 if an offline firm raises its price, the change in its profit only comes from the type 2 market. While if this offline firm cuts prices, then it will attract some type 1 consumers, which also contributes to its profit. Thus offline firms cannot charge a price where raising or lowering the price will both reduce profits.[6] Our findings are summarized in Proposition 1.

Proposition 1. *In a circular market, suppose there are N offline firms and one online firm with linear transport cost among offline firms and vertical differentiation between online and offline firms. Suppose all firms have the same constant marginal cost, and the vertical differentiation parameter follows a binary distribution with a large enough upper limit and a small enough lower limit. If* $\frac{\mu_1}{t}\le-\frac{1}{2N}<\frac{1}{N}\le\frac{\mu_2}{t}$, *then there is no pure strategy Nash equilibrium in this market.*

It is noted that the prices set by offline firms should satisfy the following constraint.

$$c+\frac{\gamma_2 t}{(2\gamma_1+\gamma_2)N}<p_i^r<c+\frac{t}{N},\ i=1,\dots,N.$$

On the one hand, p_i^r cannot be higher than $c+\frac{t}{N}$. Otherwise, the right derivative and the left derivative are both negative and it is profitable for offline firms to reduce prices. On the other hand, p_i^r cannot be lower than $c+\frac{\gamma_2 t}{(2\gamma_1+\gamma_2)N}$, as the derivatives

[6] However, offline firms can set prices where raising or lowering the prices both increase the profits.

are both positive if $p_i^r < c + \frac{\gamma_2 t}{(2\gamma_1+\gamma_2)N}$, which means offline firms should increase prices until a derivative turns from positive to negative. The parameter range of p_i^r changes according to the value of γ_1, which is shown in Figure 2, where the curve of blue color represents the lower bound of p_i^r.

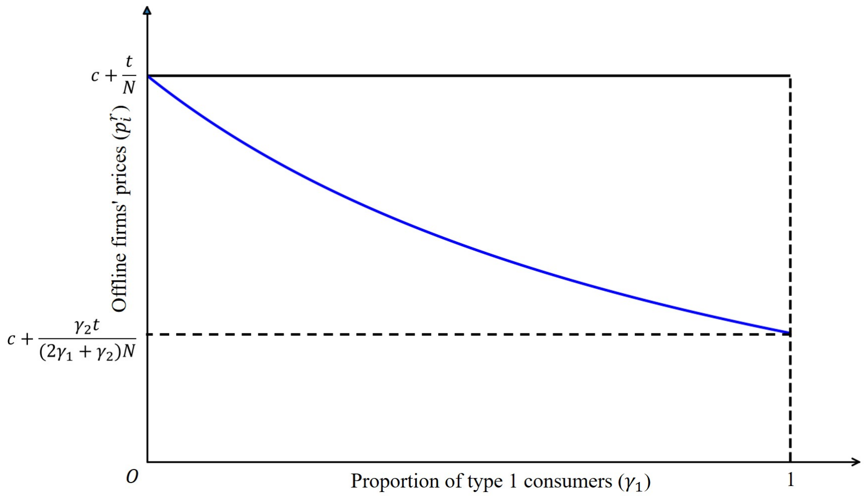

Fig. 2. Constraint for Prices of offline Firms

When γ_1 is small, the range of p_i^r is narrow as the lower bound has a high value and approaches the upper bound. When γ_1 is large, the lower bound has a low value and thus leads to a wider range of p_i^r. Such a result implies that although all type 1 consumers purchase from the online firm, the proportion of type 1 consumers can still affect the prices of offline companies. In a market, the higher the fraction of loyal consumers of the online firm, the more inclined offline firms are to reduce prices.

Some interesting inferences could be drawn from the results. First, we can see that in a market where the online firm and offline firms sell the homogeneous product, when some consumers like online shopping very much while the rest prefer offline shopping in particular, then there will be no neutral consumers who swing between the online firm and offline firms, depending on the distance between consumers and the nearest offline firms. That is to say, all type 1 consumers are loyal to online shopping while all type 2 consumers are loyal to offline shopping. Second, there is no pure strategy Nash equilibrium under such conditions, indicating that when the difference between different consumers is too large (i.e., μ_1 is too small and μ_2 is too large), the market may not be stable.

4. Conclusion

When different consumers have different preferences for online shopping and offline shopping, the online versus offline competition may be very different compared with the case where all consumers have homogeneous preferences. In our paper, we reconsider the results of Balasubramanian (1998) under the setting of heterogeneous consumers' preferences. We find that when some consumers prefer online shopping while others prefer offline shopping, all consumers become either loyal to online or offline shopping, and no consumer chooses to change the way they shop depending on their location. Furthermore, there is no pure strategy Nash equilibrium under such conditions, as the offline firms cannot reach a global optimum.

Based on this paper some further research questions could be explored. For example, future work could consider the comprehensive characterization of the market when the consumers' preferences follow a general distribution. It is also of interest to consider the mixed strategy Nash equilibrium when a pure strategy Nash equilibrium does not exist. We hope such studies will provide more meaningful insights into the online versus offline competition.

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