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Modeling of the City's Transport Network Using Game-Theoretic Methods on the Example of Petrozavodsk *

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Abstract The paper presents the results of modeling of the city's transport network. The effectiveness of the game-theoretic method for estimating the centrality of graph vertices using the Myerson value is demonstrated on the transport graph. Correspondences in the given graph are found with gravitational and entropy approaches, using the information about citizens and companies distributed by vertices in the graph. The results of computer calculations are represented on the transport network of the city of Petrozavodsk.

Keywords: transport network, correspondence matrix, centrality measures, equilibrium flows.

1. Introduction

One of the important factors in the socio-economic development of the region is the state of its transport system, which connects all the city resources, residential areas, companies, stores, public transport, etc. Improving the transport system enhances the quality of life, reduces the price of goods transportation, reduces traffic accidents, enables the economic efficiency of the region.

There are various ways to improve the transport system. This can be done through the upgrade of the road surface, the construction of new interchanges, bypasses, bridges, and pedestrian crossings, increasing the traffic lanes, addition of new traffic lights. Furthermore, improvement can be achieved by changes to the traffic regulations, the restriction of entry to the city center, implementing one-way traffic on certain streets and special lanes for public transport, rational determination of public transport routes.

Resolving these kinds of issues requires the mathematical modeling of the city's transport system (Sheffy, 1985; Nagurney, 1999; Kerner, 2009; Gasnikov, 2013; Nesterov and Palma, 2003; Nurminsky and Shamray, 2010). The modeling of the transport system consists of several stages. The first stage is the construction of a road network graph. The flows on the road segments of the transport network are then defined. This scheme is based on the correspondence matrix that contains the information of citizens movement from one vertex of the graph to another. The correspondence matrix allows to find equilibrium transport flows (Wardrop, 1952), create

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optimal public transport schedules, determine the optimal locations of the public transport stops, identify rational routes for public transport, as well as cycle lanes (Ford et al., 2019), estimate the number of passengers transported and the revenue of public transport companies.

The correspondence information can be obtained from population surveys or using public street cameras. However, this information is generally difficult to obtain or may be inaccurate. Traditionally, gravitational and entropy models are used to construct a correspondence matrix (Gasnikov, 2013; Shvetsov, 2003; Shvetsov, 2009). Clearly, they must be combined with the results of the population survey.

This paper presents the results of computer modeling of the city's transport network on the example of Petrozavodsk. In section 2 the transport network model construction is described as well as the distribution of citizens and companies by city districts. Section 3 contains the results of the computation based on the gravitational and entropy models. Section 4 analyses the structure of the transport network graph. Based on the well-known PageRank method and the game-theoretic approach proposed by the authors (Mazalov and Trukhina, 2014; Avrachenkov et al., 2018; Mazalov and Khitraya, 2021) the centrality of vertices in a graph was found.

2. Transport Network Model Construction

The road graph presented in the OpenStreetMap, a non-profit project aimed at creating a free geographical world map by Internet users, was used to build the basis of the transport network model of Petrozavodsk.

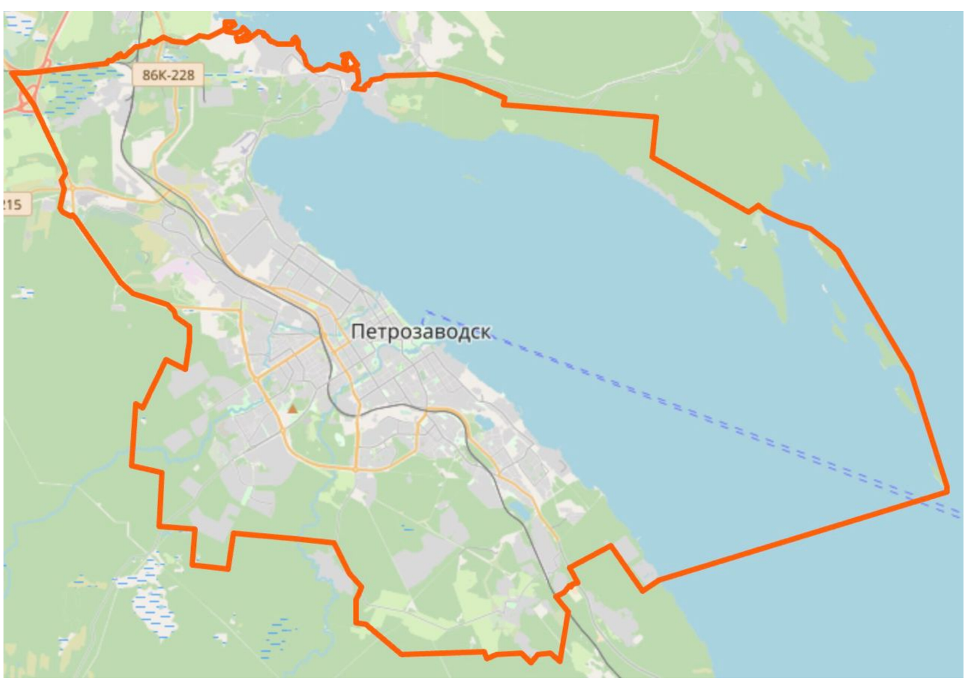

Fig. 1. Petrozavodsk urban district

The class of roads "roads on which road traffic is possible" was chosen to be significant. The intersections of these roads are selected as vertices of the graph.

According to the summary report on the results of the monitoring of the performance efficacy of the local government bodies of the urban districts and municipal areas of the Republic of Karelia in 2021[1], permanent average population of the Petrozavodsk urban district was 280 801. To distribute the city's residents according to the nodes of the graph, information was required on the number of living quarters located on the territory of Petrozavodsk. Web portal "Housing reform (Reforma ZHKKH)" presents information about the houses of the Republic of Karelia that is provided under government decree #731 of 23 September 2010. The exported data contains the address of each house according to the federal information address system, as well as the technical specifications of the building.

In the first stage, residents were distributed evenly over all living quarters, based on information from open sources. If the number of living quarters for the house was not specified, the parameter "total number of rooms" was chosen.

$$\frac{\text{total\_ppl}}{\text{total\_quarters}} = \text{residents\_per\_quarter},$$

where total_ppl is the total city population,
total_quarters is the total number of living quarters,
residents_per_quarter is the number of residents per quarter.

The number of the house residents is calculated as the product of the number of residents per quarter and the number of living quarters in the house:

$$\text{residents\_per\_quarter} \cdot \text{living\_quarters\_count},$$

where living_quarters_count is the number of living quarters in the house.

Next, for each house by the address received on the portal, geographic coordinates were obtained using the Yandex Maps service. These coordinates are used to find the nearest node of the graph to which the house was attached. According to this binding, the weight of each vertex represents the total number of residents living in houses in the vicinity of the graph node, i.e. the intersection of urban roads.

At this stage, the transport network graph included 1615 vertices and 2185 edges. Since some of the vertices had negligible weight, it was decided to combine closely spaced vertices with more significant vertices to simplify the model. In addition, the vertices of the graph, obtained from processing complex intersections, bilateral roads and entrances to the yards, were combined. Thus, a graph with 875 vertices and 1181 edges was obtained (Figure 2).

Figure 3 shows the distribution of the city residents by the graph vertices. A larger vertex size corresponds to a larger weight value. The largest size of the nodes corresponds to the suburbs and places with new dense buildings.

The next stage of work was to search for data on organizations located in the territory of the Petrozavodsk City District. Data from the OpenStreetMap project

[1]Summary report on the results of the monitoring of the performance efficacy of the local government bodies of the urban districts and municipal areas of the Republic of Karelia in 2021, available at: https://gov.karelia.ru/upload/medialibrary/7d2/inatqynw9blv2ib412inu0u9lwcijshk/SVODNYY-DOKLAD.pdf (accessed 13.11.2022).

Fig. 2. Transport network graph

were used first. All objects described as, for instance, an office, an educational institution, a shopping center, etc. have been selected. For each type of facility, a weight was chosen based on both the approximate number of employees typically employed by that type of establishment and the attendance of such establishments by visitors. Thus, the number of staff and the visitor flow to the city clinic or school will exceed the same for the trade organization. The weight value was chosen from the half-interval $(0, 1]$.

Organizations were bound to the graph nodes according to the geographical coordinates, just like the residential buildings before. Each node was assigned an additional characteristic "weight of organizations", which is a type-weighted sum of the number of organizations associated with this vertex.

Figure 4 visualizes the distribution of organizations over the vertices of the transport graph. The larger vertex size corresponds to the larger weighted sum of organizations. The figure shows that a large number of organizations are concentrated in the city center.

To construct the correspondence matrices, the vertices of the original transport graph were subdivided into disjoint subsets corresponding to the city districts. The information on the city districts is presented in the Table 1.

Figure 5 shows the transport graph, with the city districts as vertices. Here the vertex labels correspond to the numbering in the first column of the table 1. If two regions are directly connected by roads, then there is an edge between the corresponding vertices. The lengths of the edges are proportional to the lengths of the corresponding shortest paths in the original graph.

Fig. 3. Distribution of the city residents by the graph vertices

Table 1. Petrozavodsk city districts

№	City districts	Number of inhabitants	Weight of organizations	Number of vertices
1	Golikovka	25723.8	69.3	60
2	Drevlyanka	51134.7	123.7	98
3	Zheleznodorozhny	3414.4	6.9	14
4	Zareka	15482.9	36.1	44
5	Kamenny bor	5057.4	10.0	16
6	Kirpichnny zavod	1006.1	0.5	21
7	Klyuchevaya	26689.4	46.2	64
8	Kukkovka	36887.5	77.5	86
9	Oktyabrsky	35685.6	80.1	44
10	Pervomaisky	17684.1	91.0	42
11	Perevalka	26275.2	41.5	113
12	Peski	0.0	0.8	7
13	Ptizefabrika	850.8	0.7	16
14	Rybka	4017.6	4.6	37
15	Sainavolok	582.9	0.0	7
16	Severnaya promzona	747.2	19.4	18
17	Solomennoe	2419.6	5.3	27
18	Sulazhgora	4665.8	18.7	44
19	Teplichny	326.4	4.0	12
20	Tomici	47.3	0.6	15
21	Center	21526.2	264.1	89

Fig. 4. Distribution of organizations over the vertices of the transport graph

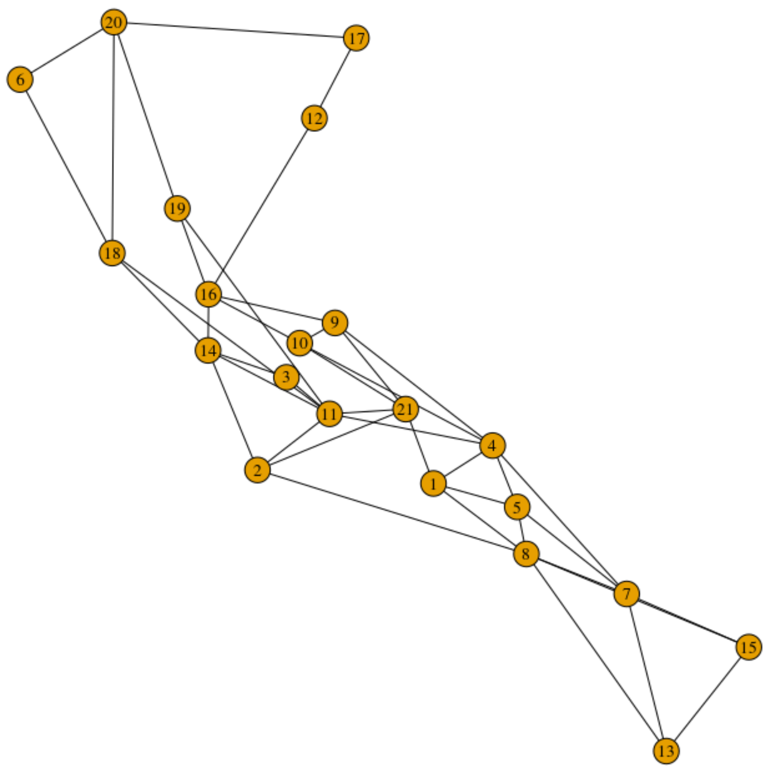

Fig. 5. City districts transport graph

3. Correspondence Matrix

3.1. Gravity model

A gravity model of transportation flows (Gasnikov, 2013; Shvetsov, 2003; Shvetsov, 2009) is based on an analogy with Newton's law of universal gravitation: the force of gravitational attraction between two material points with masses m_1 and m_2, separated by a distance r, is proportional to both masses and inversely proportional to the square of the distance. In the gravity model modified in accordance with the specifics of traffic flows (Wilson, 1967; Arrowsmith, 1973), the elements of the correspondence matrix are calculated as

$$T_{ij} = s_i d_j f(c_{ij}) \left(\sum_j V_j f(c_{ij}) \right)^{-1}, \tag{1}$$

$$\sum_j T_{ij} = s_i, \tag{2}$$

$$\sum_i T_{ij} = d_j. \tag{3}$$

Here, s_i is the total number of trips starting in i, d_j is the total number of trips ending in j, c_{ij} is the cost of a trip between i and j. Constraints (2) and (3) ensure the adequacy of the mathematical model: the sum of each row of the correspondence matrix must match the number of trips starting at the corresponding point i; the sum of each column must match the number of trips ending at the corresponding point j.

The function $f(c_{ij})$ is some generalization of the distance between the points i and j. It was shown in (Wilson, 1967) that the best fit for this model is the function $f(c_{ij}) = \exp(-\beta\, c_{ij})$, where β is the calibration coefficient. In the literature, when modeling work trips, $\beta = 0.065$ is often used.

The algorithm for calculating the correspondence matrix according to the gravitational model is presented in (Arrowsmith, 1973). To implement the algorithm in relation to the transport graph of the city of Petrozavodsk, we will consider the cost of a trip between the vertices i and j to be directly proportional to the distance between i and j along the shortest path in the graph, taking into account the weights of the edges (lengths of road sections). We use the value of the calibration coefficient $\beta = 0.065$.

With a given partition of the transport graph vertices into city districts, we define the travel costs as follows: $c_{R_i R_j}$ is the average distance between all possible pairs of vertices i and j from the districts R_i and R_j: $i \in R_i$, $j \in R_j$; $c_{R_i R_i}$ is the average distance between all possible pairs of vertices i and j inside the region R_i: $i, j \in R_i$.

Let the vector s contain the distribution of residents over the vertices of the city, and the vector d be the total weight of organizations. If the correspondence matrix is calculated between districts, then the R_i-th component of the vector s is the sum of the number of residents at the vertices of the district R_i, the R_i-th component of the vector d is the sum of the weights of organizations at the vertices of the district R_i.

To calculate the correspondence matrix between 21 districts of the city based on the gravity model, 6 iterations of the algorithm were required. The resulting correspondence matrix is shown in Figure 6.

3.2. Entropy model

An entropy model of transportation flows (Gasnikov, 2013) is based on the second law of thermodynamics, which states that any closed physical system tends to reach a stable equilibrium state, which is characterized by the maximum entropy of this system. A correspondence matrix should maximize the entropy. A correspondence matrix depends on priori probabilities v_{ij} wich is a probability of choosing the communication ij by an individual. Furthermore balance restriction on incoming and outcoming flow should be satisfied: $s_i = \sum_{j=1}^{N} T_{ij}$ and $d_j = \sum_{i=1}^{N} T_{ij}$. It is shown [4] that a correspondence matrix can be calculated by the following iterative scheme.

$$T_{ij}^{k} = T_{ij}^{k-1} s_i \left[\sum_{j=1}^{N} T_{ij}^{k-1}\right]^{-1}, \quad T_{ij}^{k+1} = T_{ij}^{k} d_j \left[\sum_{i=1}^{N} T_{ij}^{k}\right]^{-1}$$

The matrix of a priori probabilities is used as the initial matrix $T^0 = v$, then at each iteration the fulfillment of the balance constraints for incomes and outcomes is alternately achieved.

The input parameters for the entropy model are: s_i, the outgoing flow for each source i; d_j, the incoming flow for each sink j; the probability matrix v_{ij} which is a priori probability of choosing the communication ij. In a transport graph, each vertex acts as both a source and a sink. s_i is equal to the number of people living at i, d_j is equal to the number of people working at j.

If we take into account only information about the outgoing and incoming flows at the vertices, and consider a priori probability of a trip between any pair of graph vertices to be the same, i.e. $v_{ij} = \frac{1}{N^2}$, where N is the number of vertices, then the entropy model gives the following correspondence matrix:

$$T_{ij} = \frac{s_i d_j}{P},$$

where $P = \sum s_k = \sum d_t$ is the total number of individuals making the trip.

Let us construct a matrix of a priori probabilities based on the poll results. For each respondent, we can determine which vertices correspond to the addresses of work and residence indicated by them (provided that they were filled in correctly). But to estimate v_{ij} directly from the poll results, the number of respondents should be much more than N^2, which is practically impossible. Therefore, we construct a matrix Q, the rows and vertices of which correspond to city districts, and the element Q_{hw} is equal to the proportion of respondents whose residential address is in district h, and whose work address is in district w. Let $R(i)$ be the district where i is located. We define the matrix of a priori probabilities as follows:

$$v_{ij} = \frac{Q_{R(i)R(j)}}{\sum_{k:R(k)=R(i)} s_k \sum_{t:R(t)=R(j)} d_t}$$

Simply put, the proportion of respondents who move between a pair of districts is evenly distributed among all possible pairs of vertices from those districts.

The correspondence matrix was calculated according to the iterative scheme (Gasnikov, 2013). The calculation of the correspondence matrix for the entire graph was completed in 12 iterations. The resulting matrix is needed for further modeling, but for a visual presentation of the results, we group the rows and columns by districts. Let us construct a matrix:

$$D_{kp} = \sum_{i:R(i)=k} \sum_{j:R(j)=p} T_{ij}$$

The resulting correspondence matrix is shown in Figure 7.

3.3. Comparison of the correspondence matrices

0.78	1.18	0.07	0.40	0.11	0.00	0.48	0.83	0.78	0.89	0.41	0.01	0.01	0.04	0.00	0.19	0.05	0.17	0.04	0.01	2.75
1.34	3.07	0.15	0.66	0.18	0.01	0.83	1.50	1.53	1.79	0.92	0.01	0.01	0.10	0.00	0.37	0.10	0.40	0.08	0.01	5.19
0.09	0.17	0.01	0.04	0.01	0.00	0.05	0.09	0.11	0.13	0.06	0.00	0.00	0.01	0.00	0.03	0.01	0.03	0.01	0.00	0.36
0.46	0.68	0.04	0.26	0.07	0.00	0.31	0.51	0.47	0.52	0.24	0.00	0.00	0.03	0.00	0.11	0.03	0.10	0.02	0.00	1.66
0.15	0.22	0.01	0.09	0.02	0.00	0.11	0.18	0.15	0.17	0.08	0.00	0.00	0.01	0.00	0.04	0.01	0.03	0.01	0.00	0.53
0.02	0.05	0.00	0.01	0.00	0.00	0.01	0.02	0.04	0.04	0.02	0.00	0.00	0.00	0.00	0.01	0.00	0.01	0.00	0.00	0.10
0.78	1.19	0.06	0.43	0.13	0.00	0.74	1.00	0.76	0.85	0.39	0.01	0.01	0.04	0.00	0.18	0.05	0.16	0.04	0.01	2.70
1.09	1.75	0.09	0.58	0.17	0.01	0.81	1.36	1.07	1.21	0.56	0.01	0.01	0.06	0.00	0.25	0.07	0.23	0.05	0.01	3.78
0.91	1.59	0.10	0.48	0.13	0.01	0.55	0.95	1.35	1.47	0.59	0.01	0.01	0.07	0.00	0.33	0.09	0.29	0.06	0.01	3.74
0.46	0.81	0.05	0.23	0.06	0.00	0.27	0.47	0.64	0.73	0.30	0.01	0.00	0.03	0.00	0.16	0.04	0.15	0.03	0.00	1.86
0.69	1.39	0.08	0.35	0.09	0.01	0.40	0.72	0.85	0.98	0.49	0.01	0.01	0.05	0.00	0.21	0.05	0.20	0.04	0.01	2.76
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.02	0.04	0.00	0.01	0.00	0.00	0.02	0.03	0.02	0.03	0.01	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.09
0.10	0.21	0.01	0.05	0.01	0.00	0.06	0.10	0.13	0.16	0.07	0.00	0.00	0.01	0.00	0.04	0.01	0.04	0.01	0.00	0.41
0.02	0.03	0.00	0.01	0.00	0.00	0.02	0.02	0.02	0.02	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.06
0.02	0.03	0.00	0.01	0.00	0.00	0.01	0.02	0.03	0.03	0.01	0.00	0.00	0.00	0.00	0.01	0.00	0.01	0.00	0.00	0.08
0.06	0.10	0.01	0.03	0.01	0.00	0.04	0.06	0.09	0.10	0.04	0.00	0.00	0.00	0.00	0.03	0.01	0.02	0.01	0.00	0.24
0.11	0.24	0.01	0.06	0.02	0.00	0.06	0.12	0.17	0.19	0.08	0.00	0.00	0.01	0.00	0.05	0.01	0.06	0.01	0.00	0.46
0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.58	0.97	0.06	0.30	0.08	0.00	0.35	0.61	0.67	0.76	0.34	0.01	0.01	0.04	0.00	0.16	0.04	0.15	0.03	0.00	2.51

Fig. 6. Correspondence matrix based on the gravity model
(% of the total population of the city).

0.63	1.95	0.03	0.26	0.15	0.05	0.39	0.37	0.78	1.1	0.35	0.02	0	0.03	0	0.08	0.16	0.21	0.02	0.02	2.57
2	0.88	0.03	0.47	0.14	0.05	0.35	1.35	1.06	2.98	0.32	0.02	0	0.09	0	0.6	0.14	0.19	0.01	0.01	7.55
0.09	0.14	0.01	0.07	0.04	0.01	0.11	0.11	0.11	0.16	0.1	0.01	0	0.01	0	0.02	0.05	0.06	0	0	0.09
0.95	0.49	0.03	0.26	0.15	0.05	0.38	0.74	0.39	0.55	0.35	0.02	0	0.03	0	0.08	0.16	0.21	0.02	0.02	0.64
0.14	0.21	0.01	0.11	0.06	0.02	0.17	0.16	0.17	0.24	0.15	0.01	0	0.01	0	0.04	0.07	0.09	0.01	0.01	0.14
0.03	0.04	0	0.02	0.01	0	0.03	0.03	0.03	0.05	0.03	0	0	0	0	0.01	0.01	0.02	0	0	0.03
0.54	1.66	0.03	0.22	0.13	0.04	0.33	0.95	1.32	0.47	0.91	0.02	0	0.03	0	0.07	0.14	0.18	0.01	0.01	2.46
0.83	2.14	0.03	0.91	0.13	0.04	0.34	0.33	1.36	0.48	0.31	0.02	0	0.03	0	0.22	0.14	0.19	0.01	0.01	5.63
0.46	2.14	0.05	0.75	0.22	0.07	0.56	0.54	1.13	0.8	0.52	0.03	0.01	0.05	0	0.36	0.47	0.31	0.02	0.02	4.21
1.2	1.23	0.04	0.16	0.09	0.03	0.24	0.94	0.24	0.35	0.22	0.01	0	0.02	0	0.05	0.1	0.13	0.01	0.01	1.21
0.81	1.87	0.04	0.66	0.19	0.07	0.49	0.48	1.49	0.7	0.45	0.03	0.01	0.04	0	0.32	0.2	0.27	0.02	0.02	1.23
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.02	0.04	0	0.02	0.01	0	0.03	0.03	0.03	0.04	0.03	0	0	0	0	0.01	0.01	0.02	0	0	0.02
0.11	0.17	0.01	0.09	0.05	0.02	0.13	0.13	0.13	0.19	0.12	0.01	0	0.01	0	0.03	0.05	0.07	0.01	0.01	0.11
0.02	0.02	0	0.01	0	0	0.01	0.01	0.01	0.04	0.01	0	0	0	0	0	0.01	0.01	0	0	0.05
0.02	0.03	0	0.02	0.01	0	0.02	0.02	0.02	0.03	0.02	0	0	0	0	0.01	0.01	0.01	0	0	0.02
0.06	0.09	0.01	0.05	0.03	0.01	0.07	0.07	0.07	0.2	0.06	0	0	0.01	0	0.01	0.03	0.04	0	0	0.06
0.13	0.19	0.01	0.1	0.06	0.02	0.15	0.15	0.15	0.22	0.14	0.01	0	0.01	0	0.03	0.06	0.08	0.01	0.01	0.13
0.01	0.01	0	0.01	0	0	0.01	0.01	0.01	0.02	0.01	0	0	0	0	0	0	0.01	0	0	0.01
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.33	1.01	0.03	0.27	0.16	0.05	0.4	0.39	0.4	1.71	0.37	0.02	0	0.04	0	0.09	0.17	0.22	0.02	0.02	2

Fig. 7. Correspondence matrix based on the entropy model
(% of the total population of the city).

The average difference between the elements of the correspondence matrices calculated on the basis of the gravity and entropy models (Figure 6 and 7) was 0.1%, the maximum difference was 2.35% of the total population of the city. Thus, the correspondence matrices are in good agreement with each other and represent similar patterns of distribution of work trips.

Figure 8 represents the difference between the correspondence matrices calculated on the basis of the gravity and entropy models, normalized row-wise by the population of the city districts.

The maximal difference between the elements of the correspondence matrices is observed for city districts with a relatively large population or the weight of organizations, which is explained by the specifics of each mathematical model and the

0.016	-0.084	0.004	0.015	-0.004	-0.005	0.010	0.049	0.000	-0.023	0.006	-0.001	0.000	0.001	0.000	0.011	-0.012	-0.004	0.002	-0.001	0.020
-0.037	0.120	0.006	0.011	0.002	-0.002	0.026	0.008	0.026	-0.066	0.033	0.000	0.001	0.001	0.000	-0.013	-0.003	0.011	0.004	0.000	-0.129
-0.001	0.026	0.002	-0.024	-0.026	-0.012	-0.049	-0.013	-0.001	-0.024	-0.034	-0.004	0.000	-0.002	0.000	0.003	-0.032	-0.026	0.001	-0.003	0.219
-0.088	0.034	0.001	0.001	-0.014	-0.009	-0.013	-0.042	0.015	-0.004	-0.021	-0.003	0.000	-0.002	0.000	0.005	-0.023	-0.020	0.001	-0.002	0.184
0.008	0.007	-0.001	-0.014	-0.022	-0.012	-0.030	0.009	-0.009	-0.038	-0.042	-0.004	0.000	-0.004	0.000	0.000	-0.032	-0.033	0.000	-0.003	0.219
-0.007	0.026	0.001	-0.027	-0.026	-0.011	-0.053	-0.019	0.007	-0.016	-0.036	-0.003	0.000	-0.001	0.000	0.011	-0.030	-0.014	0.004	-0.003	0.198
0.025	-0.050	0.004	0.022	0.000	-0.004	0.043	0.005	-0.059	0.040	-0.055	-0.001	0.001	0.001	0.000	0.011	-0.009	-0.002	0.002	-0.001	0.025
0.019	-0.030	0.005	-0.025	0.003	-0.003	0.036	0.078	-0.023	0.055	0.019	-0.001	0.001	0.002	0.000	0.003	-0.005	0.004	0.003	0.000	-0.140
0.036	-0.043	0.004	-0.021	-0.007	-0.005	-0.001	0.032	0.017	0.053	0.005	-0.001	0.000	0.001	0.000	-0.003	-0.030	-0.002	0.003	-0.001	-0.037
-0.117	-0.067	0.001	0.011	-0.005	-0.005	0.004	-0.074	0.063	0.061	0.012	-0.001	0.000	0.002	0.000	0.017	-0.009	0.002	0.004	-0.001	0.102
-0.012	-0.052	0.004	-0.033	-0.010	-0.006	-0.010	0.026	-0.068	0.030	0.004	-0.002	0.000	0.001	0.000	-0.012	-0.016	-0.007	0.002	-0.001	0.163
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.006	0.013	-0.001	-0.017	-0.022	-0.012	-0.017	0.016	-0.013	-0.040	-0.043	-0.004	0.001	-0.004	0.000	-0.001	-0.033	-0.033	0.000	-0.003	0.207
-0.004	0.032	0.002	-0.026	-0.026	-0.011	-0.051	-0.015	0.001	-0.021	-0.034	-0.004	0.000	-0.001	0.000	0.006	-0.032	-0.022	0.002	-0.003	0.209
-0.018	0.049	0.001	0.005	-0.010	-0.008	0.018	0.047	0.018	-0.083	-0.015	-0.002	0.001	-0.001	0.000	0.005	-0.020	-0.017	0.001	-0.002	0.031
-0.005	0.008	0.000	-0.025	-0.026	-0.011	-0.051	-0.016	0.013	-0.012	-0.038	-0.003	0.000	-0.002	0.000	0.011	-0.030	-0.023	0.003	-0.003	0.210
0.003	0.017	0.001	-0.017	-0.022	-0.010	-0.040	-0.006	0.027	-0.114	-0.030	-0.002	0.000	-0.001	0.000	0.014	-0.020	-0.017	0.003	-0.002	0.216
-0.007	0.025	0.001	-0.027	-0.026	-0.011	-0.053	-0.018	0.007	-0.015	-0.036	-0.003	0.000	-0.001	0.000	0.009	-0.030	-0.016	0.003	-0.003	0.201
-0.005	0.013	0.000	-0.026	-0.026	-0.011	-0.052	-0.016	0.009	-0.014	-0.037	-0.003	0.000	-0.002	0.000	0.011	-0.029	-0.020	0.004	-0.003	0.208
-0.007	0.019	0.001	-0.027	-0.026	-0.011	-0.053	-0.018	0.008	-0.015	-0.038	-0.003	0.000	-0.001	0.000	0.011	-0.028	-0.016	0.004	-0.002	0.202
0.033	-0.005	0.003	0.004	-0.010	-0.006	-0.006	0.028	0.035	-0.123	-0.003	-0.002	0.000	0.000	0.000	0.010	-0.016	-0.009	0.002	-0.001	0.066

Fig. 8. The difference between the correspondence matrices calculated on the basis of the gravity and entropy models, normalized row-wise by the population of the city districts.

absence in the gravitational model (and the presence in the entropy one) of individual preferences of people. Thus, in the gravity model, more trips are concentrated within the densely populated Drevlyanka district (matrix element $T_{2,2}$), more trips are directed to the Center district with the maximum weight of organizations (column 21), with the exception of the Drevlyanka district ($T_{2.21}$), a remote district Klyuchevaya ($T_{8.21}$) and the densely populated district Kukkovka ($T_{9.21}$).

4. Centrality Measures

Centrality measure describes how well a considered vertex is located on the paths connecting other vertices and, accordingly, how important the node is for the whole transport system.

As an approach to calculate the centrality of graph vertices, we can consider ranking of vertices using the PageRank method. This method can be related to the random walk process. On the set n of web pages, a hyperlink is followed with some probability α. At the same time, with probability $1-\alpha$, the process can go to a random web page. Then, the stationary distribution of the process can be interpreted as the final probability of being in vertices of the graph. The greater the probability, the more important the vertex for the system, and the greater its centrality value.

The matrix of the transition probabilities $\widetilde{P}$ is calculated as

$$\widetilde{P} = \alpha P + (1-\alpha)(\frac{1}{n}\mathbb{E}),$$

where the value α is selected from $(0,1)$, $\mathbb{E}_{n\times n}$ is the unity matrix, $P_{n\times n}$ is the matrix of elements:

$$p_{ij} = \begin{cases} \frac{1}{k} & \text{vertex } i \text{ has } k > 0 \text{ outgoing links and } j \text{ is one of them,} \\ 0 & \text{vertex } j \text{ is not an outgoing link for } i, \\ \frac{1}{n} & \text{vertex } i \text{ does not have outgoing links } k = 0. \end{cases}$$

Figure 9 presents the results of ranking graph vertices using the PageRank method ($\alpha = 0.85$). A larger centrality value corresponds to a larger node size on the map.

Fig. 9. Vertices centralities by the PageRank method

Also, the PageRank method can be used for a weighted graph with a weight matrix W and a degree matrix D, then the matrix is $P = D^{-1}W$. In this study, we select road lengths as the edge weight values. Figure 10 shows the results of ranking the vertices of a weighted graph.

Fig. 10. Vertices centralities by the PageRank method taking into account edge weights

A game-theoretic approach can be used to determine vertex centralities in a graph. Consider a graph $G = (V, E)$ where V is the set of vertices and E is the set of edges. Define a cooperative game $\Gamma =< V, v >$, $|V| = n$ on the graph as follows. The vertices are the players, and the characteristic function $v(K), K \subset V$, is defined as the number of simple paths of length m in the subgraph corresponding to the coalition K. The number $m = 1, 2...$ is fixed. Obviously, the function $v(K)$ is monotonic, i.e., $v(K_1) \leq v(K_2)$ for $K_1 \subset K_2$. Then the graph vertices can be ranked using the solution of the cooperative game in the form of the Shapley-Myerson value.

The papers (Mazalov and Trukhina, 2014; Avrachenkov et al., 2018) showed that if the characteristic function $v(K), K \subset V$ is defined as the number of simple paths of length m in the subgraph K, then the Myerson value for player i can be calculated as

$$\varphi_i^m = \frac{a_m(i)}{m+1}, \tag{4}$$

where $a_m(i)$ denotes the number of simple paths of length m passing through vertex i. Paths $i_1, i_2, \ldots, i_k$ and $i_k, \ldots, i_2, i_1$ are considered to be the same.

To calculate the value using the formula (4), you need to find the number of all simple paths of a certain length passing through a given vertex. However, in the general case, it's not an easy problem. To calculate the value using formula (4), one needs to find the number of all simple paths of a certain length passing through a given vertex. This is not a simple computational task. A modification of the Myerson value (Mazalov and Khitraya, 2021) was proposed, which is easier to calculate. The idea of this representation is that each simple path contains the vertex i, for which the Myerson vector is calculated, only once. Then $a_m(i)$ can be treated as the number of occurrences of the vertex i in all simple paths of length m. In (Mazalov and Khitraya, 2021), a modification of Myerson centrality is described, where the centrality of the kth order of a vertex i is the number of occurrences of the vertex i in paths of length k, including cycles. The vector $\sigma(k)$ is the vector of vertex centralities of the graph G whose i-th component is equal to

$$\sigma_i(k) = \frac{s_i(k)}{k+1}, \ i = 1, \ldots, n,$$

where $s_i(k)$ is the total number of appearances of vertex i in the paths of length k calculated by formula

$$s_i(k) = \sum_{j=1}^{n} a_{ij}^{(k)} + \sum_{l=1}^{n} \left[a_{li} \sum_{j=1}^{n} a_{ij}^{(k-1)} + a_{li}^{(2)} \sum_{j=1}^{n} a_{ij}^{(k-2)} + \ldots + a_{li}^{(k)} \right].$$

Here $a_{ij}^{(k)}$ are the elements of the adjacency matrix raised to the corresponding power k.

Figure 11 shows a visual representation of vertex centrality values of the road network of the city of Petrozavodsk. A larger vertex size corresponds to a larger Myerson centrality value.

Centrality values were also calculated for a graph whose vertices are city districts (Figure 5). Figure 12 shows a comparison diagram of the centrality values of city districts. The district numbers correspond to the numbering in Table 1. The resulting Myerson centrality values are multiplied by 10^{-7}; the PageRank values are multiplied by 10^2 for ease of comparison. Myerson centrality values were calculated for paths of length $k = 10$ (M on the diagram). When ranking the vertices of a weighted graph using the PageRank method and choosing the edge weight equal to the length of the corresponding road segment, large centrality values arise for remote regions (PR_w on the diagram). In this regard, it was decided to choose the values reciprocal of the road lengths ($PR_{\frac{1}{w}}$ in the diagram) as the edge weights. This made it possible to balance the obtained values.

Fig. 11. Myerson centrality

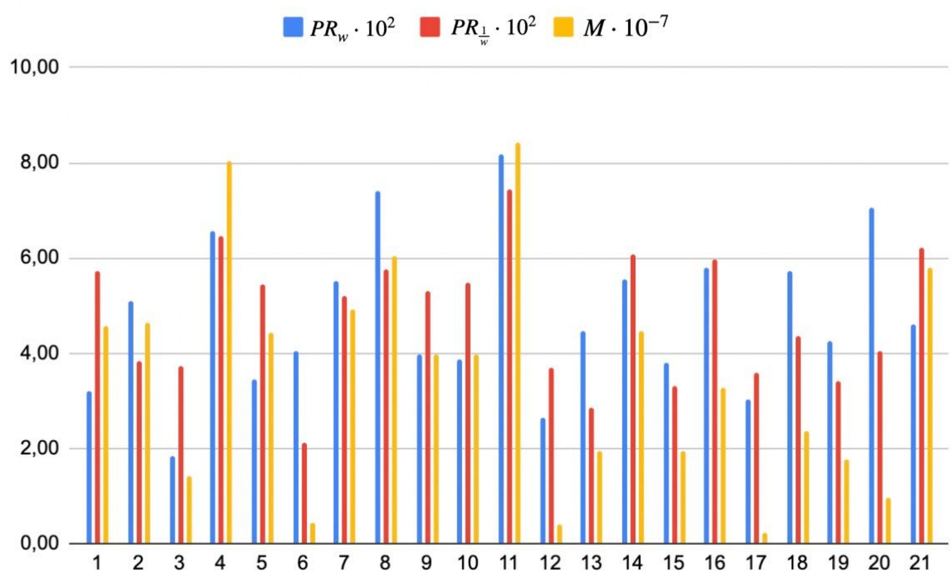

Fig. 12. Comparison of centrality values for the graph of city districts.

5. Conclusion

The paper presents the results of numerical calculations for the analysis of the transport network of a city on the example of Petrozavodsk. To determine the traffic flows in the city, a poll of residents was conducted, which provided preliminary information about the correspondence matrix. These results were taken into account when using the entropy model to evaluate working trips in the city. Correspondence matrices calculated by the entropy model and the gravity model were compared.

The structure of the transport graph was analyzed using the PageRank methods and the Myerson value. The obtained results were analyzed and interpreted in relation to the transport network of the city of Petrozavodsk.

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