Stackelberg Tariff Games between Provider, Primary and Secondary Users

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Abstract Due to the development of wireless communication technologies and the increase of integrated wireless networks, the problem of spectrum bandwidth management has become a hot research field in recent years. In this paper, we consider this problem as hierarchical game with three types of players: spectrum holder (provider), primary and secondary user. Provider assigns the price for bandwidth resource to maximize its own profit, whereas the primary user tries to find balance between using the bandwidth resource for its own needs and renting spectrum band to secondary users for profit gain. Secondary users can access to the network and transmit their signals by using the spectrum band of the primary user paying for that in proportion to the power of the transmitted signal. For this game the optimal strategies of the players are found. Numerical modelling demonstrates how the equilibrium strategies and corresponding payoffs depend of network parameters.

Keywords: spectrum market, bandwidth resource, Stackelberg game, power control.

1. Introduction

Bandwidth resource management is an important issue in wireless networks. Spectrum, which is a scarce resource, is comparatively efficiently used in wireless technologies such as WiMAX and UMTS. The quality of service (QoS) reached by a user, which is communicating a signal, heavily depends on the value of bandwidth provided by a spectrum holder.

From economic point of view, holders and users (primary, secondary, etc) form a spectrum market, within which spectrum bands are considered as a resource flowing from owners to consumers regulated by market mechanisms. Users are able to rent the spectrum band purchased from the holders. Thus, for holders it is important to find the optimal price to sell the band spectrum, whereas users try to find balance between using the bandwidth resource for its own needs and renting unused bandwidth capacity for profit gain.

The problem of such hierarchical spectrum bandwidth sharing has become a hot research field in recent years due to the development of wireless communication technologies and the increase of integrated wireless networks. In the study (Niyato and Hossain, 2007), the network model, when a WiMAX base station serves both WiMAX subscriber stations and WiFi access points/routers in its coverage area, is considered. The WiMAX and WiFi service providers try to maximize its own profits by defining the optimal price on spectrum bands. This problem is formulated as a Stackelberg leader-follower game in which the WiMAX base station and the WiFi routers are the leader and the followers, respectively. The network which is comprised by two-level architecture (WiMAX and UMTS networks serve WLAN as backbone) is considered in (Ming et al., 2009), and Vickrey-Clarke-Groves auction mechanism is used to allocate the bandwidth resource efficiently among various entities in this network. An access pricing schemes for multi-hop wireless communications are investigated in (Lam et al., 2006). The pricing is used as an incentive for the relay node to forward traffic to the gateway. A game-theoretic approach is used to analyze interactions of the access point, wireless relaying nodes, and clients from one-hop to multihop networks and when the network has an unlimited or limited channel capacity. Three level pricing scheme authority-provider-user where tariff of access to internet is proportional to throughput is studied in (Garnaev et al., 2010).

In this paper we introduce the following hierarchical game with three different types of players: spectrum holder (provider), a primary and several secondary users. The provider assigns the price for bandwidth resource to maximize its own profit. The primary user buys a license for using frequency bandwidth from the provider to transmit signals. The primary user can also earn some extra money giving unused frequencies capacity for rent to secondary users charging them by assigned tariff for interference. Each secondary user can either buy the frequency bandwidth or it can choose the service of the other provider based on comparing of the suggested QoS and prices which is set for the network access. For this game the optimal strategies of the players are found. Numerical modelling demonstrates how the equilibrium strategies and corresponding payoffs depend of network parameters.

The rest of the article is organized as follows. Section 2. presents system model for hierarchical spectrum sharing considered in this paper. The optimal strategies of the players are found in Section 3.. After that dependence of the equilibrium strategies and corresponding payoffs on network parameters is presented in Section 4.. Section 5. concludes the paper.

2. System model and assumptions

In this section, we present the market model and corresponding assumptions. Let us consider a case in which there is only one the frequency spectrum provider, one primary user which purchases a spectrum band from the provider and several secondary users which can pay for the possibility to use the spectrum of the primary user.

As a rule, in such the spectrum markets the provider is large commercical or governmental organization which possess the rights on the spectrum license and try to share this spectrum optimally. On the other hand, primary user is service distributor which provides the access to the network for end subscribers - secondary users. The provider is trying to maximize its own profit by assigning the optimal price for the bandwidth resource, whereas users are maximizing the quality of services which they provide by utilizing this resource taking into account the resource costs.

We formulate this problem as the hierarchical game with three types of players: a provider, a primary and a secondary user. The provider sets the price for bandwidth resource to maximize its own profit. Thus, the provider's payoff π^{prov} is the profit obtained by selling the bandwidth resource to the primary user:

$$\pi^{prov}(C_W) = C_W W, \tag{1}$$

where $W \geq 0$ is the spectrum band bought by the primary user and $C_W > 0$ is its price assigned by the provider. Thus, the strategy of the provider is to choose C_W, which is unbounded non-negative value.

The primary user purchases a spectrum band directly from the provider, afterwards it allows several secondary users to use the spectrum band so that to maximize its own payoff. The payoff of the primary user π^P is its throughput plus profit it obtains by giving access to the network for secondary users minus how much it costs to buy the bandwidth resource from the holder. Let us assume that the primary user charges secondary users proportional to their interfering power:

$$\pi^P(W, C_P) = \alpha W \log\left(1 + G^P \frac{h^P P^P}{\sigma^2 W + \sum_{i=1}^n h_i^S P_i^S}\right) - C_W W + C_P h_i^S P_i^S, \tag{2}$$

where $\alpha > 0$ is coefficient which shows the economic efficiency of throughput for the primary user, $G^P > 0$ is is the spreading gain of the CDMA system for the primary user, $h^P > 0$ and $h_i^S > 0$ are the fading channel gains for the primary and the i-th secondary user respectively, $\sigma^2 \geq 0$ is the background noise, $P_P \geq 0$ is the transmitting power employed by the primary user, $C_P \geq 0$ is the tariff for access to the network for the secondary user, $P_i^S \geq 0$ is the power employed by the i-th secondary user. The primary user is trying to maximize (2) by finding optimal W and C_P.

Similarly, a secondary user payoff is the throughput value provided by it minus bandwidth resource costs, i.e.

$$\pi_i^S(P^S) = \beta W \log\left(1 + G^S \frac{h_i^S P_i^S}{\sigma^2 W + h^P P^P + \sum_{j \neq i} h_j^S P_j^S}\right) - C_P h_i^S P_i^S, \tag{3}$$

where $P^S = (P_1^S, \ldots, P_n^S)$ is vector of secondary users transmitting powers, $\beta > 0$ is coefficient which shows the economic efficiency of throughput for each secondary user and $G^S > 1$ is is the spreading gain of the CDMA system for secondary users. The i-th secondary user is looking for non-negative $P_i^S \geq 0$ which maximizes its own payoff. Let us assume that maximal power which can be used by secondary users to transmit the signals is very high and in this study we will not take into account the upper limit for secondary users transmitting power.

Let us consider the problem defined as a sequence of two Stackelberg games (Romp, 1997). In the first stage, the provider maximizes its own profit by assigning the price for bandwidth resource, and the primary user buys a license for using frequency bandwidth from the provider to transmit own data without taking into account secondary users:

$$\max_{C_W} \pi^{prov}(C_W) = \max_{C_W} C_W W,$$
$$\text{subject to } C_W > 0.$$
$$\max_{W} \pi^P(W) = \max_{W} \alpha W \log\left(1 + G^P \frac{h^P P^P}{\sigma^2 W}\right) - C_W W,$$
$$\text{subject to } W \geq 0.$$

In the second stage, the primary user allows secondary users to use the spectrum band purchased in order to increase its own prifit. We assume that the spectrum band is fixed and the primary user charges secondary users proportional to their interfering power. Each secondary user transmits its own signal in such power mode that maximizes its payoff function:

$$\max_{C_P} \pi^P(W, C_P) =$$
$$= \max_{W, C_P} \alpha W \log\left(1 + G^P \frac{h^P P^P}{\sigma^2 W + \sum_{i=1}^{n} h_i^S P_i^S}\right) - C_W W + C_P \sum_{i=1}^{n} h_i^S P_i^S,$$
$$\text{subject to } C_P > 0.$$
$$\max_{P^S} \pi_i^S(P^S) = \max_{P^S} \beta W \log\left(1 + G^S \frac{h_i^S P_i^S}{\sigma^2 W + h^P P^P + \sum_{j=1, j\neq i}^{n} h_j^S P_j^S}\right) - C_P h_i^S P_i^S,$$
$$\text{subject to } P_i^S \geq 0, \forall i \in \{1, \dots, n\}.$$

3. Solution of the game

3.1. Game between the provider and the primary user

First, let us consider the case when there are only the provider and the primary user: the provider maximizes its own profit by assigning the optimal price for frequency band:

$$\begin{aligned} &\max_{C_W} \pi^{prov}(C_W) = \max_{C_W} C_W W, \\ &\text{subject to } C_W > 0. \end{aligned} \tag{4}$$

and the primary user maximizes its own payoff by purchasing the optimal amount of this band:

$$\begin{aligned} &\max_{W} \pi^P(W) = \max_{W} \alpha W \log\left(1 + G^P \frac{h^P P^P}{\sigma^2 W}\right) - C_W W, \\ &\text{subject to } W \geq 0. \end{aligned} \tag{5}$$

The following theorem allows to obtain the Nash equilibrium of the frequency spectrum game (4,5).

Theorem 1. *Under the assumptions made in the previous section, the spectrum band game between the provider and the primary user (4,5) admits a unique Nash equilibrium (NE). The corresponding to that NE optimal spectrum band price assigned by the provider C_W^* can be found as unique root of following equation:*

$$(L(-e^{-\frac{C_W}{\alpha}-1})+1)^2 = \frac{C_W}{\alpha}. \tag{6}$$

and optimal spectrum band bought by the primary user W^* can be calculated as follows:

$$W^* = -\frac{G^P h^P P^P L\left(-e^{-\frac{C_W^*}{\alpha}-1}\right)}{\sigma^2\left(1+L\left(-e^{-\frac{C_W^*}{\alpha}-1}\right)\right)}. \tag{7}$$

Proof. In order to solve for the game (4,5) we use a backward induction technique. We start with the optimization problem of the primary user and derive the best response for this user as a function of the price C_W set by the service provider.

The objective function in (5) is continuously differentiable concave and inequality constraint is continuously differentiable convex function. Therefore, Karush-Kuhn-Tucker (KKT) conditions are necessary and sufficient for optimality, i.e. the frequency band W is optimal if and only if the following conditions are satisfied:

$$C_W + \alpha\frac{G^P h^P P^P}{\sigma^2 W + G^P h^P P^P} - \alpha\log\left(1+\frac{G^P h^P P^P}{\sigma^2 W}\right)\begin{cases}\geq 0, W=0,\\ =0, W\geq 0.\end{cases} \tag{8}$$

For any finite $C_W > 0$ the optimal frequency band purchased by the primary user from the provider can be found as the following reaction function:

$$W(C_W) = -\frac{G^P h^P P^P L\left(-e^{-\frac{C_W}{\alpha}-1}\right)}{\sigma^2\left(1+L\left(-e^{-\frac{C_W}{\alpha}-1}\right)\right)}, \text{ if } C_W < +\infty, \tag{9}$$

where $L(x)$ is Lambert function which is implicit function which is defined by the following equation:

$$x = L(x)e^{L(x)}. \tag{10}$$

In the case of finite C_W, the argument of Lambert function is on the following interval $(-e^{-1}, 0)$, i.e. it is at least greater than $[-e^{-1}$. In order to satisfy the constraint $W \geq 0$, the Lambert function values have to be located on the interval $[-1, 0]$, i.e. $L\left(-e^{-\frac{C_W}{\alpha}-1}\right)$ is at least greater or equal to zero. Thus, we can restrict to single-valued Lambert function with real values.

Thus, the provider profit as a function of spectrum band price C_W can be found as follows:

$$\pi_{prov}(C_W) = C_W W(C_W) = -\frac{G^P h^P P^P C_W L\left(-e^{-\frac{C_W}{\alpha}-1}\right)}{N\left(1+L\left(-e^{-\frac{C_W}{\alpha}-1}\right)\right)}. \tag{11}$$

This function is continuous function of C_W. When $0 < C_W < \infty$, this function is increasing when $(L(-e^{-\frac{C_W}{\alpha}-1})+1)^2 \geq \frac{C_W}{\alpha}$, and it is decreasing when

$(L(-e^{-\frac{C_W}{\alpha}-1})+1)^2 \le \frac{C_W}{\alpha}$, i.e. this function reaches its maximal value when the following condition is satisfied:

$$(L(-e^{-\frac{C_W}{\alpha}-1})+1)^2 = \frac{C_W}{\alpha}. \tag{12}$$

Since $L(x)$ is strictly increasing for $x > -e^{-1}$, the left part of this equation is continuous increasing concave function on the interval $0 < C_W < \infty$. In addition, it is equal to zero at the point $C_W = 0$ and converges to 1 when $C_W \to \infty$. Thus, the equation $eq1$ can not have more than one root. We can easily check that the value of the function $(L(-e^{-\frac{C_W}{\alpha}-1})+1)^2|_{C_W=0.4\alpha} > \frac{C_W}{\alpha}|_{C_W=0.4\alpha}$. On the other side, $(L(-e^{-\frac{C_W}{\alpha}-1})+1)^2|_{C_W=\alpha} < \frac{C_W}{\alpha}|_{C_W=\alpha}$. Thus, there is a unique root C_W^* of (12) for $C_W > 0$ and it lies in the interval $0.4\alpha < C_W^* < \alpha$. This root C_W^* corresponds the maximal value of the provider profit. Let us notice that optimal price which assigned by the provider does not depend on average received power to noise power spectral density ratio. We can easily find C_W^* numerically.

The optimal spectrum band bought by the primary user W^* can be calculated according to the reaction function (9) $W^* = W(C_W^*)$. Obtained solution (W^*, C_W^*) is NE point of the game (4,5) found by backward induction. This point is unique since the reaction function (9) is single-valued and the root of the equation (12) is unique.

3.2. Game between primary user and secondary users

Let us assume that after finding optimal price C_W of frequency band, the provider does not care about how the primary user uses purchased frequency band, i.e. C_W found from previous section is fixed. The primary user allows secondary users to use purchased frequency band and assigns the price in proportion to the power of the transmitted signals of secondary users. Thus, the following two-stage game is considered. The primary user buys a license for using frequency bandwidth from the provider to transmit signals and rent frequency band to the secondary user so that to maximize its own payoff:

$$\begin{aligned} &\max_{W,C_P} \pi^P(W, C_P) = \\ &= \max_{W,C_P} \alpha W \log\left(1 + G^P \frac{h^P P^P}{\sigma^2 W + \sum_{i=1}^n h_i^S P_i^S}\right) - C_W W + C_P \sum_{i=1}^n h_i^S P_i^S, \\ &\text{subject to } \begin{cases} W \ge 0, \\ C_P > 0. \end{cases} \end{aligned} \tag{13}$$

Each secondary user tries to maximize its own payoff by using the primary user frequency band:

$$\begin{aligned} &\max_{P^S} \pi_i^S(P^S) = \max_{P^S} \beta W \log\left(1 + G^S \frac{h_i^S P_i^S}{\sigma^2 W + h^P P^P + \sum_{j=1, j\ne i}^n h_j^S P_j^S}\right) - C_P h_i^S P_i^S, \\ &\text{subject to } P_i^S \ge 0, \forall i \in \{1, \ldots, n\}. \end{aligned} \tag{14}$$

First, let us consider the optimization problem of finding optimal transmitting powers for secondary users (14). The objective functions in (14) are continuously differentiable concave and inequality constraints are continuously differentiable convex functions. Therefore, Karush-Kuhn-Tucker (KKT) conditions are necessary and sufficient for optimality, i.e. the transmitting powers P_i^S are optimal if and only if the following conditions are satisfied for any $i \in \{1, \dots, n\}$:

$$\frac{\beta W G^S h_i^S}{\sigma^2 W + h^P P^P + \sum_{j=1, j\neq i}^n h_j^S P_j^S + G^S h_i^S P_i^S} - C_P h_i^S \begin{cases} \leq 0, \text{ if } P_i^S = 0, \\ = 0, \text{ if } P_i^S \geq 0, \end{cases} \tag{15}$$

The optimal strategy of the i-th secondary user can be found from (15) as follows:

$$P_i^{S^*} = \begin{cases} \frac{1}{h_i^S} \left(\frac{\beta W}{C_P} - \frac{\sigma^2 W + h^P P^P}{G^S} - \frac{\sum_{j=1, j\neq i}^n h_j^S P_j^S}{G^S} \right), & \text{if } C_P \in I_{i1}, \\ 0, & \text{if } C_P \in I_{i0}, \end{cases} \tag{16}$$

where

$$\begin{aligned} I_{i0} &= \left[\frac{\beta W G^S}{\sigma^2 W + h^P P^P + \sum_{j=1, j\neq i}^n h_j^S P_j^S}, +\infty \right), \\ I_{i1} &= \left(0, \frac{\beta W G^S}{\sigma^2 W + h^P P^P + \sum_{j=1, j\neq i}^n h_j^S P_j^S} \right). \end{aligned} \tag{17}$$

In order for the i-th secondary user to transmit a signal, i.e. $P_i^S > 0$ the following conditions from (16) have to hold:

$$\frac{\beta W}{C_P} - \frac{\sigma^2 W + h^P P^P}{G^S} > 0 \tag{18}$$

and

$$\frac{\beta W}{C_P} - \frac{\sigma^2 W + h^P P^P}{G^S} - \frac{\sum_{j=1, j\neq i}^n h_j^S P_j^S}{G^S} > 0. \tag{19}$$

An intuitive interpretation for these conditions is the following: if the price C_P is set too high for a secondary user, this secondary user prefers not to transmit at all, depending on its channel gain, utility parameter, the spreading gain, etc. In equation (16), the i-th secondary user transmitting power depends not only on the user-specific parameters, like h_i^S, but also on the network parameter G^S, and total power level received by the primary user base station $\sum_{i=1}^n h_i^S P_i^S$.

For any equilibrium solution, the set of fixed point equations can be written in matrix form. The rows and columns corresponding to users with zero equilibrium power are deleted, and the equation below involves only the users with positive powers. It is obvious that if condition (18) does not hold none of the secondary users does not transmit any signals. Assuming that (18) is satisfied and all n users have positive power levels at equilibrium, we have the following equation for finding oprimal secondary users powers $P^{S*} = (P_1^{S*}, P_2^{S*}, \dots, P_n^{S*})$:

$$\begin{pmatrix} 1 & \frac{h_2^S}{h_1^S G^S} & \frac{h_3^S}{h_1^S G^S} & \cdots & \frac{h_n^S}{h_1^S G^S} \\ \frac{h_1^S}{h_2^S G^S} & 1 & \frac{h_3^S}{h_2^S G^S} & \cdots & \frac{h_n^S}{h_2^S G^S} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{h_1^S}{h_n^S G^S} & \frac{h_2^S}{h_n^S G^S} & \frac{h_3^S}{h_n^S G^S} & \cdots & 1 \end{pmatrix} \begin{pmatrix} P_1^{S*} \\ P_2^{S*} \\ \vdots \\ P_n^{S*} \end{pmatrix} = \begin{pmatrix} \frac{1}{h_1^S} \left(\frac{\beta W}{C_P} - \frac{\sigma^2 W + h^P P^P}{G^S} \right) \\ \frac{1}{h_2^S} \left(\frac{\beta W}{C_P} - \frac{\sigma^2 W + h^P P^P}{G^S} \right) \\ \vdots \\ \frac{1}{h_n^S} \left(\frac{\beta W}{C_P} - \frac{\sigma^2 W + h^P P^P}{G^S} \right) \end{pmatrix}. \tag{20}$$

We denote the vector in the right side as $c = (c_1, c_2, \ldots, c_n)^T = \left(\frac{a_1}{h_1^S}, \frac{a_2}{h_2^S}, \ldots, \frac{a_n}{h_n^S}\right)^T$. Let us state the following proposition by adopting the results in (Alpcan et al., 2001):

Theorem 2. *In the defined power game with n users, let the indexing be done such that $i > j$ if $a_i < a_j$, with the ordering picked arbitrarily if $a_i = a_j$. Let $m^* \le n$ be the largest integer m for which the following condition is satisfied:*

$$a_m > \frac{1}{G^S + m - 1} \sum_{i=1}^{m} a_i. \tag{21}$$

Then the power game admits a unique Nash equilibrium, which has the property that users $m^* + 1, \ldots, n$ have zero power levels, i.e.

$$P_i^S = 0, \;\; \text{if } i \in \{m^* + 1, \ldots, n\}. \tag{22}$$

The equilibrium power levels of the first m^* secondary users can be calculated uniquely from (20) as follows:

$$P_i^{S*} = \frac{G^S}{h_i^S(G^S - 1)} \left(a_i - \frac{\sum_{j=1}^{m^*} a_j}{G^S + m^* - 1} \right). \tag{23}$$

One can easily notice that in the game between the primary user and n secondary users (13,14) all variables a_i are equal to each other, i.e. $a_1 = a_2 = \ldots = a_n = \frac{\beta W}{C_P} - \frac{\sigma^2 W + h^P P^P}{G^S}$. Therefore inequality (21) can be rewritten as follows:

$$\frac{\beta W}{C_P} - \frac{\sigma^2 W + h^P P^P}{G^S} > \frac{1}{G^S + m - 1} \sum_{i=1}^{m} \left(\frac{\beta W}{C_P} - \frac{\sigma^2 W + h^P P^P}{G^S} \right). \tag{24}$$

Since we assumed that the spreading gain of the CDMA system for secondary users $G^S > 1$, the inequality (24) is satisfied for any $1 \le m \le n$, and the largest such m is equal to n. Thus, we can formulate the following corollary which defines optimal strategies of secondary users:

Corollary 3.1. *Under the assumption that $G^S > 1$, in the game between the primary user and n secondary users (13,14) secondary users have following optimal strategies depending on the price assigned by the primary user:*

$$\begin{aligned}
&\textit{If } 0 \le C_P < \frac{\beta W G^S}{\sigma^2 W + h^P P^P}, \;\textit{then } P_i^{Sopt}(C_P) = \frac{G^S \left(\frac{\beta W}{C_P} - \frac{\sigma^2 W + h^P P^P}{G^S} \right)}{h_i^S (G^S + n - 1)}, \\
&\textit{If } C_P \ge \frac{\beta W G^S}{\sigma^2 W + h^P P^P}, \;\textit{then } P_i^{Sopt}(C_P) = 0,
\end{aligned} \tag{25}$$

for any $i \in \{1, \ldots, n\}$.

The i-th secondary user payoff can be calculated as follows:

$$\begin{aligned}
\pi_i^{S*}(C_P) &= \beta W \log \left(\frac{\beta W (G^S + n - 1)}{C_P(\sigma^2 W + h^P P^P) + (n-1)\beta W} \right) - \\
&- \frac{G^S \beta W - C_P(\sigma^2 W + h_P P_P)}{G^S + n - 1}, \text{if } 0 \le C_P < \frac{\beta W G^S}{\sigma^2 W + h^P P^P}, \\
\pi_i^{S*}(C_P) &= 0, \text{if } C_P \ge \frac{\beta W G^S}{\sigma^2 W + h^P P^P}.
\end{aligned} \tag{26}$$

If secondary users apply their optimal strategies the payoff of the primary user can be rewritten in the following form:

$$\begin{aligned}
&\pi^{P*}(C_P, W) = \alpha W \log\left(1 + \frac{(G^S+n-1)G^P h^P P^P}{\sigma^2 W(G^S-1) - nh^P P^P + \frac{1}{C_P} nG^S\beta W}\right) - \\
&- C_P \frac{n(\sigma^2 W + h^P P^P)}{G^S+n-1} + W\left(\frac{nG^S\beta}{G^S+n-1} - C_W\right),\ \text{if } 0 \le C_P < \frac{\beta W G^S}{\sigma^2 W + h^P P^P}, \\
&\pi^{P*}(C_P, W) = \alpha W \log\left(1 + G^P \frac{h^P P^P}{\sigma^2 W}\right) - C_W W,\ \text{if } C_P \ge \frac{\beta W G^S}{\sigma^2 W + h^P P^P}.
\end{aligned} \tag{27}$$

Let us introduce the following notations:

$$\begin{aligned}
&a := (G^S+n-1)G^P h^P P^P, \quad b := \sigma^2 W(G^S-1) - nh^P P^P, \quad c := nG^S\beta W, \\
&d := \frac{n(\sigma^2 W + h^P P^P)}{\alpha W(G^S+n-1)}, \quad \bar{C}_P = \frac{\beta W G^S}{\sigma^2 W + h^P P^P}, \\
&\bar{P}_i^S(C_P) = \frac{G^S\left(\frac{\beta W}{C_P} - \frac{\sigma^2 W + h^P P^P}{G^S}\right)}{h_i^S(G^S+n-1)}, \forall i \in \{1,\dots,n\}, \\
&\bar{\pi}^P(C_P) = \alpha W \log\left(1 + \frac{(G^S+n-1)G^P h^P P^P}{\sigma^2 W(G^S-1) - nh^P P^P + \frac{1}{C_P} nG^S\beta W}\right) - \\
&- C_P \frac{n(\sigma^2 W + h^P P^P)}{G^S+n-1} + W\left(\frac{nG^S\beta}{G^S+n-1} - C_W\right),\ \text{if } 0 \le C_P < \frac{\beta W G^S}{\sigma^2 W + h^P P^P}.
\end{aligned} \tag{28}$$

The following theorem defines the conditions when the game between the primary user and secondary users (13,14) has inner NE point, i.e. $C_P > 0$ and $P_i^S > 0, \forall i \in \{1,\dots,n\}$.

Theorem 3. *If parameters defined in (28) satisfy one of the following sets of conditions:*

$$\left\{\begin{array}{l}
b < 0, \\
4b^2 + 4ab + acd > 0, \\
0 < \frac{\sqrt{acd(4b^2+4ab+acd)} - (a+2b)}{2bd(a+b)} < \bar{C}_P, \\
\bar{\pi}^P\left(\frac{\sqrt{acd(4b^2+4ab+acd)} - (a+2b)}{2bd(a+b)}\right) < \bar{\pi}^P\left(\bar{C}_P\right),
\end{array}\right. \tag{29}$$

$$\left\{\begin{array}{l}
b = 0, \\
0 < \frac{1}{d} - \frac{c}{a} < \bar{C}_P,
\end{array}\right. \tag{30}$$

or

$$\left\{\begin{array}{l}
b > 0, \\
0 < \frac{\sqrt{acd(4b^2+4ab+acd)} - (a+2b)}{2bd(a+b)} < \bar{C}_P,
\end{array}\right. \tag{31}$$

then the game between the primary user and n secondary users (13,14) admits a unique inner NE point (C_P^*, P^{S*}) *such that*

$$\begin{aligned} &\text{If conditions (30) hold, then } \begin{cases} C_P^* = \frac{1}{d} - \frac{c}{a}, \\ P_i^{S*} = \bar{P}_i^S(C_P^*), \forall i \in \{1,\dots,n\} \end{cases} \\ &\text{If conditions (29) or (31) hold, then } \begin{cases} C_P^* = \frac{\sqrt{acd(4b^2+4ab+acd)}-(a+2b)}{2bd(a+b)}, \\ P_i^{S*} = \bar{P}_i^S(C_P^*), \forall i \in \{1,\dots,n\} \end{cases} \end{aligned} \tag{32}$$

Proof. The proof of the theorem is deferred to the Appendix.

Corollary 3.2. *If the following set of conditions*

$$\begin{cases} b < 0, \\ 4b^2 + 4ab + acd > 0, \\ 0 < \frac{\sqrt{acd(4b^2+4ab+acd)}-(a+2b)}{2bd(a+b)} < \bar{C}_P, \\ \bar{\pi}^P\left(\frac{\sqrt{acd(4b^2+4ab+acd)}-(a+2b)}{2bd(a+b)}\right) = \bar{\pi}^P\left(\bar{C}_P\right), \end{cases} \tag{33}$$

is fulfilled, then

$$C_P^* = 0 \text{ or } \frac{\sqrt{acd(4b^2+4ab+acd)}-(a+2b)}{2bd(a+b)},$$

and each secondary user transmission power is $P_i^{S*} = P_i^{Sopt}(C_P^*)$.

Corollary 3.3. *If none of the conditions (29), (30), (31) or (33) is fulfilled, then the optimal price* C_P^* *assigned by the primary user can be calculated as follows*

$$\begin{cases} C_P^* = 0, \text{ if } \frac{nG^S\beta}{G^S+n-1} > \alpha\log\left(1+G^P\frac{h^PP^P}{\sigma^2W}\right), \\ C_P^* = C_P^{**}, \text{ if } \frac{nG^S\beta}{G^S+n-1} < \alpha\log\left(1+G^P\frac{h^PP^P}{\sigma^2W}\right), \\ C_P^* = 0 \text{ or } C_P^{**}, \text{ if } \frac{nG^S\beta}{G^S+n-1} = \alpha\log\left(1+G^P\frac{h^PP^P}{\sigma^2W}\right), \end{cases}$$

where C_P^{**} *is any* C_P *from the interval* $[\bar{C}_P, +\infty)$. *In this case, each secondary user transmitting power is* $P_i^{S*} = P_i^{Sopt}(C_P^*)$.

Let us conclude that the Theorem 3 and its corollaries allow to calculate the optimal price assigned by the primary user. Thus, after buying optimal spectrum band from the provider, the primary user allows secondary users to transmit their signals in the same spectrum band and assigns the price C_P^*. The solution $C_P = 0$ means that provider aims to assign the minimum possible price whereas secondary users transmission power values are maximum possible, i.e. $P_i^{S*} \to +\infty$. When $C_P^* \geq \bar{C}_P$, the primary user in fact forbids secondary users to transmit signals in the same spectrum band.

4. Numerical examples

The provider payoff $\pi^{prov}(C_W)$ for $\alpha = 1$ and different values of average received power to noise power spectral density ratio are shown in Fig. 1. The optimal value of C_W is equal to 0.468. One can notice that the payoff of the provider does not depend on network parameters but depends only on the primary user willingness to pay factor. The function π^P for different values of average received power to noise power spectral density ratio when C_W is equal to optimal is shown in Fig. 2. Thus, the primary user grows when its SINR increases. Figure 3 depicts how the

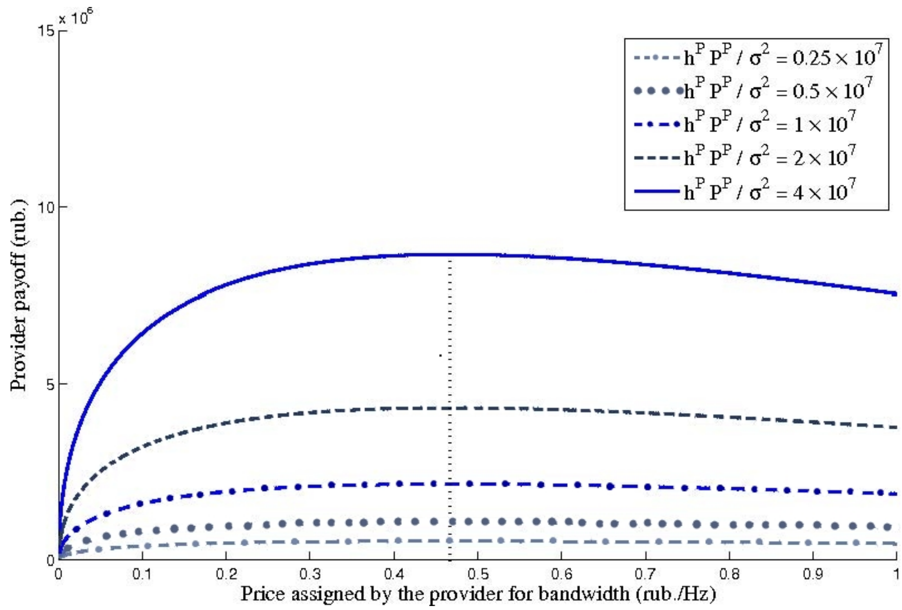

Figure1: Provider profit for different values of average received power to noise power spectral density ratio.

primary user payoff depends on the price assigned by him as a fee for secondary users to transmit signals in the same spectrum band. For all examples considered, the optimal strategy of the primary user is to assign very high price so that secondary users prefer not to transmit their signals in the primary user frequency band.

5. Conclusion

In this paper, we have introduced the hierarchical game between provider, a primary and several secondary users. The provider assigns the price for bandwidth resource to maximize its own profit, whereas the primary user buys a license for using frequency bandwidth from a provider to transmit signals. The primary user can also earn some extra money giving unused frequencies capacity for rent to secondary users charging them by assigned tariff for interference and each secondary user can either stay inactive or transmit a signal using the frequency band rented from the primary user. We have considered this hierarchical game as a sequence of two Stackelberg games: one between the provider and the primary user, and one between the primary user and secondary users. For each such game optimal strategies of the players have been found. Numerical modelling has demonstrated how the equilibrium strategies and corresponding payoffs depend of network parameters.

In the future, we are planning to consider also the case when the provider takes into account activity of secondary users when it sells the spectrum band to the

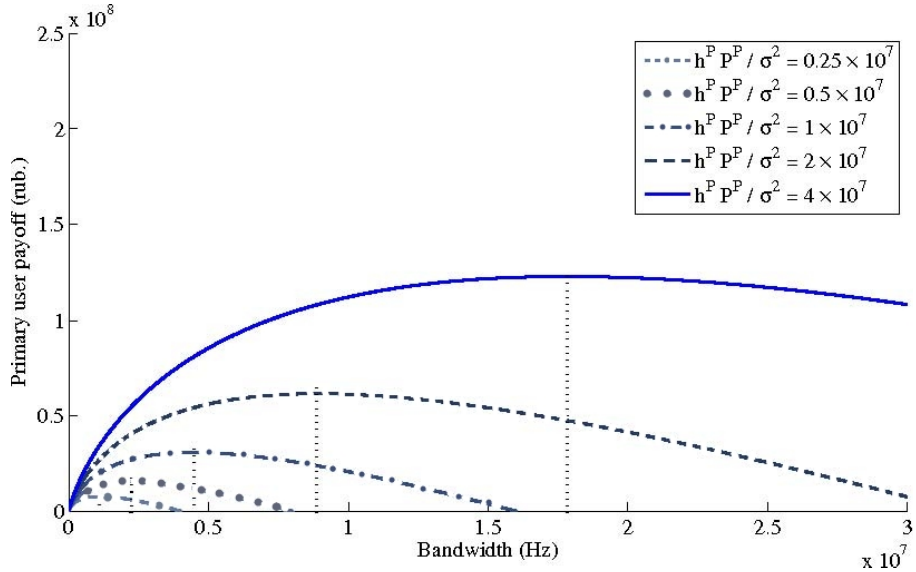

Figure2: Primary user payoff for different values of average received power to noise power spectral density ratio when C_W is optimal.

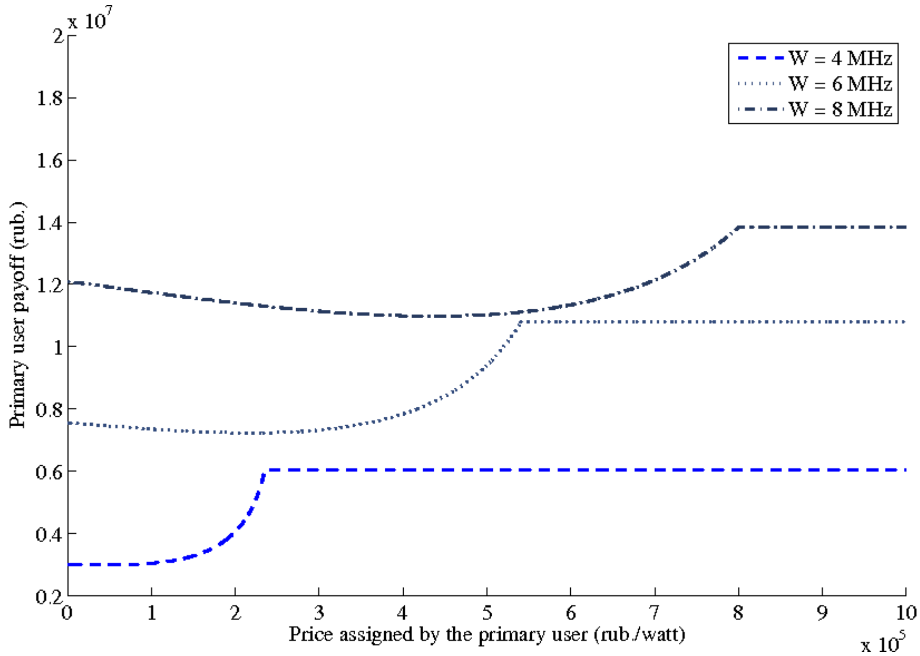

Figure3: Primary user payoff for different values of the spectrum band purchased.

primary user. There is possibility when it is beneficial for the primary user to inform the provider that secondary users transmit their signals in the same spectrum band.

Appendix

In this section we give a proof of Theorem 3:

Proof. Based on the assumptions listed in Section 2. we can easily see that

$$\begin{aligned} & a>0, \quad c>0, \quad d>0, \quad a+b>0, \\ & \text{If } b<0, \text{ then } -\frac{c}{b}<\frac{c(a+2b)}{2b(a+b)}<-\frac{c}{a+b}, \\ & \text{If } b>0, \text{ then } -\frac{c}{a+b}<\frac{c(a+2b)}{2b(a+b)}<-\frac{c}{b}. \end{aligned} \tag{34}$$

Let us notice that functon $\bar{\pi}^P(C_P)$ is continuous function of C_P and equal to $\pi^P(C_P, W)$ when $C_P \in (0, \bar{C}_P)$. Further in this proof we consider function $\bar{\pi}^P(C_P)$ and focus on the behavior of this function in the interval $C_P \in (0, \bar{C}_P)$.

When $C_P \to -\frac{c}{b}$ function $\bar{\pi}^P(C_P) \to +\infty$ and when $C_P \to -\frac{c}{a+b}$ then $\bar{\pi}^P(C_P) \to -\infty$. The first and second order derivatives of function $\bar{\pi}^P(C_P)$ with respect to C_P can be found as follows:

$$\begin{aligned} \frac{\partial \bar{\pi}^P(C_P)}{\partial C_P} &= \frac{ac}{(c+bC_P)(c+aC_P+bC_P)} - d, \\ \frac{\partial^2 \bar{\pi}^P(C_P)}{\partial^2 C_P} &= -\frac{ac(ac+2bc+2b^2C_P+2abC_P)}{(c+bC_P)^2(c+(a+b)C_P)^2}. \end{aligned} \tag{35}$$

First we consider the case when $b<0$. Let us notice that in this case $-\frac{c}{a+b}<0$ and $\bar{C}_P < -\frac{c}{b}$. If $4b^2+4ab+acd \le 0$ then function $\bar{\pi}^P(C_P)$ is increasing in the interval $C_P \in (-\frac{c}{a+b}, -\frac{c}{b})$ and the optimal C_P for the primary user is equal to $\bar{C}_P$. Otherwise, since function $\bar{\pi}^P(C_P)$ is concave when $C_P \le \frac{c(a+2b)}{2b(a+b)}$ and convex when $C_P \ge \frac{c(a+2b)}{2b(a+b)}$, and the point $C_P = \frac{c(a+2b)}{2b(a+b)}$ belongs to the interval $(-\frac{c}{b}, -\frac{c}{a+b})$, there is one local maximum at the point $C_P = \frac{\sqrt{acd(4b^2+4ab+acd)}-(a+2b)}{2bd(a+b)}$ and one local minimum at the point $C_P = \frac{-\sqrt{acd(4b^2+4ab+acd)}-(a+2b)}{2bd(a+b)}$. We can conclude that function $\bar{\pi}^P(C_P)$ is strictly increasing in the interval $C_P \in (-\frac{c}{a+b}, \frac{\sqrt{acd(4b^2+4ab+acd)}-(a+2b)}{2bd(a+b)}]$, strictly decreasing when $C_P \in [\frac{\sqrt{acd(4b^2+4ab+acd)}-(a+2b)}{2bd(a+b)}, \frac{-\sqrt{acd(4b^2+4ab+acd)}-(a+2b)}{2bd(a+b)}]$ and strictly increasing in the interval $C_P \in [\frac{-\sqrt{acd(4b^2+4ab+acd)}-(a+2b)}{2bd(a+b)}, -\frac{c}{b})$. Thus, if local maximum $C_P = \frac{\sqrt{acd(4b^2+4ab+acd)}-(a+2b)}{2bd(a+b)}$ belongs to the interval $(0, \bar{C}_P)$ and the value of function $\bar{\pi}^P(C_P)$ at this point is greater than in the case when $C_P = \bar{C}_P$, then the optimal price assigned by the primary user C_P^* is equal to $\frac{\sqrt{acd(4b^2+4ab+acd)}-(a+2b)}{2bd(a+b)}$.

If $b=0$, then $\frac{\partial^2 \bar{\pi}^P(C_P)}{\partial^2 C_P} < 0$ and therefore the function $\bar{\pi}^P(C_P)$ is strictly concave in the interval $C_P \in (-\frac{c}{a+b}, +\infty)$. One can notice that $-\frac{c}{a+b} < 0$ and function $\bar{\pi}^P(C_P)$ is strictly increasing in the interval $C_P \in (-\frac{c}{a+b}, \frac{1}{d} - \frac{c}{a}]$ and strictly decreasing in the interval $C_P \in [\frac{1}{d} - \frac{c}{a}, +\infty)$. Thus, if $\frac{1}{d} - \frac{c}{a}$ belongs to the interval $(0, \bar{C}_P)$, then the optimal price assigned by the primary user $C_P^* = \frac{1}{d} - \frac{c}{a}$.

Finally, in the case $b > 0$ we can see that $-\frac{c}{b} < -\frac{c}{a+b} < 0$. Function $\bar{\pi}^P(C_P)$ is convex when $C_P \leq \frac{c(a+2b)}{2b(a+b)}$ and concave when $C_P \geq \frac{c(a+2b)}{2b(a+b)}$, and point $C_P = \frac{c(a+2b)}{2b(a+b)}$ belongs to the interval $C_P \in (-\frac{c}{b}, -\frac{c}{a+b})$. One can notice that $4b^2 + 4ab + acd > 0$ when $b > 0$, and therefore function $\bar{\pi}^P(C_P)$ has one local minimum $C_P = \frac{-\sqrt{acd(4b^2+4ab+acd)}-(a+2b)}{2bd(a+b)}$ in the interval $C_P \in (-\infty, -\frac{c}{b}$ and one local maximum $C_P = \frac{\sqrt{acd(4b^2+4ab+acd)}-(a+2b)}{2bd(a+b)}$ in the interval $C_P \in (-\frac{c}{a+b}, +\infty)$. In particular it means that function $\bar{\pi}^P(C_P)$ is strictly increasing when $C_P \in (-\frac{c}{a+b}), \frac{\sqrt{acd(4b^2+4ab+acd)}-(a+2b)}{2bd(a+b)}]$ and strictly decreasing when $C_P \in (\frac{\sqrt{acd(4b^2+4ab+acd)}-(a+2b)}{2bd(a+b)}], +\infty$. Thus, if local maximum $C_P = \frac{\sqrt{acd(4b^2+4ab+acd)}-(a+2b)}{2bd(a+b)}$ belongs to the interval $(0, \bar{C}_P)$, then the optimal price assigned by the primary user C_P^* is equal to $\frac{\sqrt{acd(4b^2+4ab+acd)}-(a+2b)}{2bd(a+b)}$.

One can easily notice that in each case described above optimal price C_P^* assigned by the primary user is unique, i.e. there is no such $\hat{C}_P \in (0, \bar{C}_P)$ for which $\bar{\pi}^P(\hat{C}_P) \geq \bar{\pi}^P(C_P^*)$ if one of the conditions (29), (30) or (31) is satisfied. Conversely, if none of the conditions (29), (30) or (31) is fulfilled, then C_P^* is not optimal or not unique. In order to maximize own payoff, the i-th secondary user applies its best response strategy $\bar{P}_i^S(C_P)$ and therefore $P_i^{S*} = \bar{P}_i^S(C_P^*)$.

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