Tax Auditing
Using Statistical Information about Taxpayers

Suriya Sh. Kumacheva

St.Petersburg University,
Faculty of Applied Mathematics and Control Processes,
Bibliotherapy pl. 2, St.Petersburg, 198504, Russia
E-mail: s_kumach@mail.ru

Abstract A model of tax auditing in assumption, that tax authority have some statistical information about the distribution of income among population, is considered.

It is supposed that the true tax liability of each taxpayer takes its value from finite set. Dividing the range of possible tax payments on intervals, we make a correspondence between each interval and some group of taxpayers. The reported tax liability is the function of the true tax liability, which takes its values from the set of its argument's values. If the evasion was revealed, the taxpayer must pay the level of his evasion and penalty (marginal penalty rate assumed to be a constant). Tax evasions of the groups of taxpayers with the same level of income and the same level of rationality are investigated.

The tax authority assumed to get some statistical information, which can be considered as an indicator of the existing tax evasion. The information, mentioned above, is called a signal.

A taxpayers strategy is to make a decision to evade or not to evade, i.e. to declare his income level less or equal to his true income. A tax authority's strategy is to choose the audit probability.

The following results were obtained: the optimal audit strategies for each income-level group; the optimal audit strategies with consideration of the signals; the proposition about the optimal budget for tax auditing.

Keywords: tax auditing, tax evasion, statistical information, income distribution, optimal audit strategies, optimal budget.

1. Introduction

This paper is devoted to a consideration of a probability model of tax auditing. This model is constructed in assumption, that tax authority have some statistical information about the distribution of income among population. The mentioned assumption was used in previous models of tax control, such as (Sanchez and Sobel, 1993), (Chander and Wilde, 1998), (Vasin and Vasina, 2002) and (Vasin and Panova, 1999). The model of "Auditing with signal" (Macho-Stadler and Perez-Castrillo, 2002) is based on the assumption of the authority's access to the information, well-correlated with the true level of the taxpayer's income. The models, considered in (Boure and Kumacheva, 2005) and (Boure and Kumacheva, 2010), are also oriented on using of indirect information about taxpayers, but differ from the models, mentioned above, by the another mathematical interpretation of the problem.

2. The Base Model

As in previous models (Macho-Stadler and Perez-Castrillo, 2002) and (Boure and Kumacheva, 2005) there is an assumption in the model that the true tax liability i of each taxpayer takes its value from finite set. The tax liability is a non-dimensional relative value, which can be defined as some possible payment of one taxpayer, measured in money units. Dividing the range of possible tax payments on N intervals, we make a correspondence between each interval and some group of taxpayers.

Let $H_0, H_1, \ldots, H_{N-1}$ be the average income levels in this groups, measured in money units, thus, $M = \{m_0, m_1, \ldots, m_{N-1}\}$ is the set of corresponding values of the true tax liability, defined from the following equalities:

$$m_l = \frac{H_l}{H_{N-1}}, \tag{1}$$

where $l = \overline{0, N-1}$, $0 \leq m_0 < m_1 < \ldots < m_{N-1} = 1$.

Along with i the reported tax liability r is considered. The function $r(i)$ and its argument (true tax liability) take values from the same set, $r \in M$, $0 \leq r(i) \leq i$. For measuring in the unified system (money units) tax liabilities can be multiplied on money coefficient $h = t \cdot H_{N-1}$, where t is the fixed tax rate.

In this model the audit is supposed to be effective, i. e. it reveals the existing evasion. As it was done in (Chander and Wilde, 1998) and (Macho-Stadler and Perez-Castrillo, 2002), let's consider the simplest (proportional) way of penalty taking. If the evasion was revealed, the taxpayer must pay the level of his evasion and penalty $(1+\pi)(i-r)h$. The marginal penalty rate π assumed to be a constant.

There are different levels of rationality: risk averse, risk neutral and risk preferred (disposed to risk). First, let's consider risk neutral players. They are rational, thus, they evade only when their economic benefits are greater than the expected tax payments. Therefore, tax evasions of the groups of taxpayers with the same level of income are investigated.

To make the model more obvious it is investigated in assumption, that an evasion is possible only when the next income-level group of the taxpayers evade. Rejection of this assumption will not change studying of this model in general, but will sophisticate the model. There will be much more cases of evasion to consider. They can be analyzed by the analogical way.

2.1. The Profiles of Compliance

Under the assumptions, made above, there exist N different situations related to the values of the declared tax liability r for different fixed values of i. Each of these situations is called **a profile of compliance** of declared income with its true level:

Profile 1: $r(m_1) = m_0$; $r(m_2) = m_1$; $\ldots$; $r(m_{l-1}) = m_{l-2}$; $r(m_l) = m_{l-1}$; $r(m_{l+1}) = m_l$; $\ldots$; $r(m_{N-1}) = m_{N-2}$;
Profile 2: $r(m_1) = m_1$; $r(m_2) = m_1$; $\ldots$; $r(m_{l-1}) = m_{l-2}$; $r(m_l) = m_{l-1}$; $r(m_{l+1}) = m_l$; $\ldots$; $r(m_{N-1}) = m_{N-2}$;

. . . .

Profile l: $r(m_1) = m_1$; $r(m_2) = m_2$; $\ldots$; $r(m_{l-1}) = m_{l-1}$; $r(m_l) = m_{l-1}$; $r(m_{l+1}) = m_l$; $\ldots$; $r(m_{N-1}) = m_{N-2}$;

Profile $l+1$: $r(m_1) = m_1$; $r(m_2) = m_2$; $\ldots$; $r(m_{l-1}) = m_{l-1}$; $r(m_l) = m_l$; $r(m_{l+1}) = m_l$; $\ldots$; $r(m_{N-1}) = m_{N-2}$;

. . . .

Profile N: $r(m_1) = m_1$; $r(m_2) = m_2$; $\ldots$; $r(m_{l-1}) = m_{l-1}$; $r(m_l) = m_l$; $r(m_{l+1}) = m_{l+1}$; $\ldots$; $r(m_{N-1}) = m_{N-1}$.

The profiles are presented in order of compliance arising. The first profile is the less compliant: there are tax evasions of all groups of taxpayers. In the second profile the group with true tax liability m_1 does not evade, in the next profile the groups with tax liabilities m_1 and m_2 do not, and so on. In the l-th profile there are evasions of the groups with true tax liability $i \geq m_l$, $l = \overline{1, N-1}$. The most compliant profile is the profile N, in which each group of taxpayers declares its true tax liability $r(i) = i$.

Let A be an event, that the tax audit passed. So $P(A|r = m_{l-1})$ is the probability of auditing those who declared $r = m_{l-1}$.

A taxpayer's strategy is to make a decision to evade or not to evade, i. e. to declare his income level less or equal to his true income ($r(i) < i$ or $r(i) = i$). A tax authority's strategy is to choose the audit probability $P(A|r = m_{l-1})$.

A taxpayer's aim is to maximize his payoff. In the considered model the tax authority's aim is to achieve the highest compliance of declared tax liability with its true value, i. e. to realize the most compliance profile.

2.2. The Analysis of Compliance

Let's consider a situation when every taxpayer evades of taxation and declares his tax liability one level less its true value. This situation corresponds to the first profile. The exit from this profile is possible when the tax evader's profit is less or equal to his expected post-audit payments, even just for the group with the true tax liability m_1. It means that

$$\begin{gathered} P(A|r = m_0)(1+\pi)(m_1 - m_0)h \geq (m_1 - m_0)h \Leftrightarrow \\ \Leftrightarrow P(A|r = m_0) \geq \tfrac{1}{1+\pi}. \end{gathered} \tag{2}$$

Mathematical meaning of probability demands that the value in the right side of (2) be less or equal to one:

$$\frac{1}{1+\pi} \leq 1.$$

This condition is satisfied when penalty rate π takes every nonnegative value.

Let the condition (2) be satisfied. For exit from the next compliance profile the condition

$$P(A|r = m_1) \geq \frac{1}{1+\pi} \tag{3}$$

must also be satisfied.

Using the similar reasoning about the taxpayers with greater values of the declared tax liability, the conditions of the exit from other compliance profiles can be obtained:

$$\begin{array}{l} P(A|r=m_0) \geq \frac{1}{1+\pi}, \\ P(A|r=m_1) \geq \frac{1}{1+\pi}, \\ \dots\dots\dots\dots\dots \\ P(A|r=m_l) \geq \frac{1}{1+\pi}, \\ \dots\dots\dots\dots\dots \\ P(A|r=m_{N-2}) \geq \frac{1}{1+\pi}. \end{array} \tag{4}$$

In this way the next theorem was proofed.

Theorem 1. *The most compliance profile realizes when and only when the conditions (4) fulfilled jointly.*

There is one easy and obvious corollary of the Theorem 1.

Corollary 1. *The satisfying of the first* $(l+1)$ *conditions of (4),* $l = \overline{0, N-2}$*, is necessary and sufficient for exit from the* $(l+1)$*-th to the next compliance profile.*

2.3. The Compliance Cost Estimation

In the model considered the tax authority's aim is to achieve the highest compliance of the declared tax liability with its true value, i. e. to minimize the number of tax evasions. It is necessary to take into consideration that an increasing of compliance does not increase the net tax income. In real economy the most compliant profile is too expensive for the tax authority. The auditing of all groups of taxpayers can lead the tax authority to their bankruptcy and make the control function of a fiscal system impossible. That's why it is necessary to estimate the cost of compliance.

Let the whole taxable population be 1. Let's denote by k_l the part of taxpayers, who declared their tax liability as $r = m_l$ for $l = \overline{0, N-1}$, therefore,

$$\sum_{l=0}^{N-1} k_l = 1.$$

Let's define the net tax income in the next way:

$$R = T + P - C, \tag{5}$$

where summand P is penalties, C is the cast of auditing, T is taxes, and, besides,

$$T = T_{preA} + T_{postA}, \tag{6}$$

where T_{preA} is pre-audit payments and T_{postA} is post-audit payments. Thereby,

$$T_{preA} = \sum_{l=0}^{N-1} k_l \; m_l \; h, \tag{7}$$

$$C = n_a c, \tag{8}$$

where $c > 0$ is the cast of one audit and n_a is the part of audited taxpayers among the whole population:

$$n_a = \sum_{l=0}^{N-2} k_l P(A|r = m_l). \tag{9}$$

For the net tax income estimation it is necessary to investigate a problem of estimation of post-audit payments. The tax authority doesn't know, what part of taxpayers, who declared m_l, $l = \overline{0, N-2}$, evades of taxation. But the tax authority assumed to have some statistical information about a distribution of income levels of taxpayers.

Let's consider the parameters

$$\gamma_0 = P(<< m_0 >>), \gamma_1 = P(<< m_1 >>), \dots, \gamma_l = P(<< m_l >>), \dots, \\ \gamma_{N-1} = P(<< m_{N-1} >>), \tag{10}$$

which are the probability distribution of the income among taxpayers (γ_l is the part of the taxpayers, who have true tax liability m_l, $l = \overline{0, N-1}$, in correspondence with the statistical information, mentioned above). In this model the information about such distribution assumed to be highly reliable, that is, the values of parameters (10) are close to reality.

Let's consider the value

$$\delta(m_l, m_{l-1}) = P(<< m_l >> |A, r = m_{l-1}), \tag{11}$$

which is a part of the taxpayers, who have true tax liability $i = m_l$, declared $r = m_{l-1}$ and were audited, $l = \overline{1, N-1}$. Considering the estimations of values (11), we obtain, that in each profile they depend only on the interrelation of the parameters (10):

$$\delta(m_l, m_{l-1}) = \begin{cases} 1, \ in \ 1 \\ 1, \ in \ 2 \\ \dots\dots\dots, \\ 1, \ in \ l-1, \\ \frac{\gamma_l}{\gamma_{l-1}+\gamma_l}, \ in \ l, \\ 0, \ in \ l+1, \\ \dots\dots\dots, \\ 0, \ in \ N. \end{cases}$$

The next equality fulfilled for each $\delta(m_l, m_{l-1})$:

$$\delta(m_l, m_{l-1}) \ P(A|r = m_{l-1}) = P(<< m_l >> \cap A|r = m_{l-1}),$$

where $P(<< m_l >> \cap A \,|\, r = m_{l-1})$ is a probability of revelation of true tax liability $i = m_l$ in the result of auditing taxpayers, who declared $r = m_{l-1}$, where $l = \overline{1, N-1}$.

Then, post-audit payments can be defined in the next way:

$$P + T_{postA} = \\ = \sum_{l=0}^{N-2} k_l \delta(m_{l+1}, m_l) \ P(A|r = m_l) \ (m_{l+1} - m_l) \ (1 + \pi) \ h. \tag{12}$$

Let's estimate the most compliant profile N. The audit probabilities satisfies the Theorem 1. There is no evaders, therefore, the summands of (12) are equal to zero. Then, the net tax income is

$$R = T_{preA} - C. \tag{13}$$

Let's estimate the cost of the tax auditing for this case. With (8) and (9) we obtain:

$$C \geq \sum_{l=0}^{N-2} k_l \cdot \frac{c}{1+\pi}.$$

Therefore, the minimum cost of the tax audit, which guarantees the most compliant profile, is

$$C_{\min} = \frac{c}{1+\pi}(1 - k_{N-1}). \qquad (14)$$

In real economy the tax authority often has no possibility to realize the most compliant profile, because its budget is strongly restricted. Let B be the budget of the tax authority. If the inequality

$$B \geq \frac{c}{1+\pi}(1 - k_{N-1}), \qquad (15)$$

fulfilled, the resources of the tax authority allow to realize the most compliant profile. If condition (15) is violated, the tax authority can estimate possibilities of realization of lower-compliant profiles using the relations (5) – (15).

2.4. Numerical Simulation

Making some assumptions about interrelation between parameters (10) and parts of those, who declared different levels of income, let's make a numerical simulation, which allows to study the next questions:

1. What are the minimal values of the audit probabilities $P(A|r = m_l)$, $l = \overline{0, N-2}$, which ensure the most compliant profile for different values of the penalty rate π?
2. How get estimations of the net tax revenue in different profiles for fixed π?
3. How realization of the most compliant profile correlates with the task of maximizing the net tax revenue in the conditions of the restricted budget of the tax authority?

Table1: The distribution of income among taxpayers

group	income in group (rubles in month)	statistical information
0	$\leq$ 4200	0,026
1	4200 – 10600	0,360
2	10600 – 20200	0,336
3	20200 – 30000	0,146
4	30000 – 40000	0,063
5	40000 – 50000	0,028
6	50000 – 75000	0,026
7	$\geq$ 75000	0,015

Let's consider the model in a case when the number of income levels $N = 8$. The distribution of income among the population of the Russian Federation (The website of the Russian Federation State Statistics Service, 2010), presented in Table 1, was considered as numerical data for the simulation.

To define an average income $H_0, H_1, H_2, H_3, H_4, H_5, H_6, H_7$ in the groups of taxpayers with corresponding levels of the true tax liability let's make the next assumptions:

1. In the groups 0–6 a homogeneous distribution of income is considered;
2. in the 7-th group a Pareto distribution is considered.

We should remind that the density $f(x)$ and function $F(x)$ of a homogeneous distribution of the value X on the interval $(b-a, b+a)$ are defined as

$$f(x) = \begin{cases} \frac{1}{2a}, \ if \ |x-b| \leq a, \\ 0, \ if \ |x-b| > a, \end{cases}$$

$$F(x) = \begin{cases} 0, \ if \ x < b-a, \\ \frac{1}{2a}(x-b+a), \ if \ |x-b| \leq a, \\ 1, \ if \ x > b+a, \end{cases}$$

and the expectation is $MX = b$.

A Pareto distribution is often used in the modeling and prediction of an income distribution and has the next density

$$f(x) = \begin{cases} \frac{ab^a}{x^{a+1}}, \ if \ x \geq b, \\ 0, \ if \ x < b, \end{cases}$$

and function

$$F(x) = \begin{cases} 1 - \left(\frac{b}{x}\right)^a, \ if \ x \geq b, \\ 0, \ if \ x < b, \end{cases}$$

and the expectation $MX = \frac{a}{a-1} \cdot b$. In this experiment we consider $a = 2$ as the most suitable for our simulation. The considered value of the money coefficient is $h = 0,13 \cdot H_7$.

There are the next main results of held numerical simulation.

The First Result. As tax policy gets softer (in sense of the penalty rate's value), the bigger part of taxpayers must be audited in each group. This result is presented in Table 2.

Table2: Values of the audit probability in each group

penalty rate	audit probability
0,10	0,91
0,25	0,80
0,50	0,67
0,75	0,57
1,00	0,50
2,00	0,33
3,00	0,25
5,00	0,17

The Second Result. The most compliant profile is not always optimal (due to maximizing the net tax revenue). When the most compliant profile can not be realized, the tax authority should audit every possible part of evaded taxpayers. If even 1% of evaders was audited, the net tax revenue can induce at the expense of post-audit payments. These results are presented in Table 3.

Table3: The net tax revenue in different profiles

the profile number	case of add. audit of 25% (mln. rub.)	case of add. audit of 10% (mln. rub.)	case of add. audit of 5% (mln. rub.)	case of add. audit of 1% (mln. rub.)	case of no add. audit (mln. rub.)
1	498937,8415	268728,8889	191992,5714	130603,5173	115256,2538
2	322827,0067	211471,1716	174352,5600	144657,6706	136562,4874
3	268025,2788	202342,0431	180447,6312	162932,1017	158553,2193
4	166582,6996	166563,9323	166557,6765	166552,6719	166551,4208
5	170824,2497	170433,9353	170303,8305	170199,7467	170173,7257
6	173384,5936	172363,9679	172023,7593	171751,5924	171683,5507
7	178576,8403	176812,0522	176223,7895	175753,1793	175635,5268
8	189134,8716				

The Third Result. When the tax authority's budget is strongly restricted, the more compliance leads the less revenue. And vice versa: the rational distribution of resources for auditing makes the less compliant profile be more profitable then the most compliant profile. The result is shown in Table 4.

Table4: Cases of the budget distribution (mln. rub.)

group	portions of audited taxpayers			
0	0,5	0,5	0,5	0,4
1	0,5	0,5	0,5	0,4
2	0,5	0,5	0,32	0,4
3	0,135	0,1	0,32	0,4
4	0,135	0,1	0,49	0,4
5	0,2	0,4	0,49	0,45
6	0,2	0,4	0,49	0,45
budget	19996,11	19949,85	19930,95	19987,15
revenue	166814,98	167702,42	366158,66	785298,44

3. The Model of Auditing with Signals

To make the considered model more realistic let's modify it by making some assumptions.

Postulating the rationality of the taxpayers, we should consider the tax evasions of the whole groups of taxpayers with the same level of income. But the term "profile of compliance" should be modified by the rejection of the assumption, that an evasion is possible only when the next-level group of taxpayers evades. The relations (1) – (11) and (5) – (9) remains valid for modified model. Analogically to the base model, the taxpayers don't evade if their expected tax payments are greater than their profit, obtained in the case of the evasion:

$$P(A|r = m_l)(1 + \pi)(m_{l+1} - m_l)h \geq (m_{l+1} - m_l)h,$$

that is equivalent to inequality (3), which is correct for each nonnegative value of the penalty rate π.

The issue of differentiation of the evaders and honest taxpayers among the part of audited taxpayers (the probability $P(A|r = m_l)$) remains unresolved. Let's consider this problem in details.

In economy practice the tax authority has no reliable information about the taxpayers' income. But officials can use some indirect statistical information about every taxpayer. In (Macho-Stadler and Perez-Castrillo, 2002) this information was called a signal. Let's follow the mentioned terminology.

In fact, signal s can be considered as an indicator of the existing tax evasion. It can take two values $s \in \{d, u\}$, where u is a signal, that the taxpayer has an income higher, than he declared, and d is an absence of the information, that the taxpayer's income is higher, than he declared. Let's consider conventional probability

$$P\{s = d|r = m_l\} = \frac{|\{r = m_l\} \cap \{s = d\}|}{|\{r = m_l\}|},$$

which is a part of those, on whom the signal $s = d$ was got, among those, who declared their tax liabilities $r = m_l$. In the same way $P\{s = u|r = m_l\}$ is defined, $l = \overline{0, N-2}$.

Let's also consider $P(A|r = m_l, s = u)$. This value is a probability of auditing the taxpayers, who declared $r = m_l$, and got a signal $s = u$.

From the total probability formula it is clear (for $l = \overline{0, N-2}$) that

$$\begin{gathered} P(A|r = m_l) = P(A|r = m_l, s = d)\, P\{s = d|r = m_l\} + \\ + P(A|r = m_l, s = u)\, P\{s = u|r = m_l\}. \end{gathered} \tag{16}$$

For $l = \overline{0, N-2}$ let's take into consideration a signal's reliability coefficient

$$\lambda = \frac{P(A|r = m_l, s = u)}{P(A|r = m_l, s = d)}. \tag{17}$$

This coefficient shows in what times the audit of those taxpayer, who got the signal $s = u$, is more probable, than the audit of a taxpayer with the same declared income and the signal $s = d$. It is naturally to assume that $P(A|r = m_l, s = u) \geq$ $\geq P(A|r = m_l, s = d)$. Thus, $\lambda \geq 1$. Putting (17) in (16), we obtain:

$$\begin{gathered} P(A|r = m_l) = \\ = P(A|r = m_l, s = u)\left(\tfrac{1}{\lambda} P\{s = d|r = m_l\} + P\{s = u|r = m_l\}\right). \end{gathered} \tag{18}$$

Due to the binary character of the signal s, it is obviously that

$$P\{s = d|r = m_l\} + P\{s = u|r = m_l\} = 1.$$

From this fact and the equation (18), the next relation can be obtained:

$$\begin{gathered} P(A|r = m_l) = \\ = \tfrac{1}{\lambda} \cdot P(A|r = m_l, s = u)\left(1 + (\lambda - 1)P\{s = u|r = m_l\}\right). \end{gathered} \tag{19}$$

Getting the relation (19) to the inequality (3), we can estimate the probability $P(A|r = m_l, s = u)$ in the next way:

$$P(A|r = m_l, s = u) \geq \frac{\lambda}{1+\pi} \cdot \frac{1}{1 + (\lambda - 1)P\{s = u|r = m_l\}}. \tag{20}$$

Obtained inequality is correct, when the value in the right part, is not bigger than one. In the case, when $P\{s = u|r = m_l\} \in [\frac{1}{1+\pi}; 1]$, this condition fulfilled for every nonnegative λ. If $P\{s = u|r = m_l\} < \frac{1}{1+\pi}$, the inequality is correct, when

$$\lambda \leq (1+\pi) \cdot \frac{1 - P\{s = u|r = m_l\}}{1 - (1+\pi)P\{s = u|r = m_l\}} \tag{21}$$

fulfilled.

Let's generalize the above reasoning by the next theorem.

Theorem 2. *For non-evading of the taxpayers from the true tax liability $i = m_{l+1}$ to the declared tax liability $r = m_l$ it is necessary and sufficient to satisfy condition (20). Moreover, if $P\{s = u|r = m_l\} < \frac{1}{1+\pi}$, the signal's reliability coefficient λ should satisfy (21).*

As far as s is closer to reality, the value $P\{s = u|r = m_l\}$ is closer to $\frac{\gamma_{l+1}}{k_l}$. Thus, the value $P\{s = u|r = m_l\}$ can have four different interpretations depending on that, which situation is realized.

1. The taxpayers with the true tax liability m_l evade ($r(m_l) = m_{l-1}$), and taxpayers with the true tax liability m_{l+1} don't evade ($r(m_{l+1}) = m_{l+1}$). In this case the value $k_l = 0$ (none of taxpayers declared $r = m_l$), therefore, $P\{s = u|r = m_l\} = 0$.
2. Both of the mentioned groups evade: $r(m_{l+1}) = m_l$, $r(m_l) = m_{l-1}$. If the signal is reliable, then $P\{s = u|r = m_l\} = \frac{\gamma_{l+1}}{k_l}$. In this case $k_l = \gamma_{l+1}$, thus, $P\{s = u|r = m_l\} = 1$.
3. The taxpayers with the true tax liability m_{l+1} evade ($r(m_{l+1}) = m_l$), and taxpayers with the true tax liability m_l don't evade $r(m_l) = m_l$. Therefore,
$$P\{s = u|r = m_l\} = \frac{\gamma_{l+1}}{\gamma_l + \gamma_{l+1}}. \tag{22}$$
4. Both of the groups don't evade: $r(m_{l+1}) = m_{l+1}$, $r(m_l) = m_l$. Then, in the case of the reliable signal, $P\{s = u|r = m_l\} = 0$.

Let B_l be a budget, which allows to realize the optimal tax audit in the l-th group, $l = \overline{0, N-2}$. Therefore,

$$B_l = k_l P(A|r = m_l)c,$$

or, with (19):

$$B_l = \frac{k_l\, c}{\lambda} \cdot P(A|r = m_l, s = u)\left(1 + (\lambda - 1)P\{s = u|r = m_l\}\right).$$

The obtained results can be formulated in the next theorem.

Theorem 3. *The optimal strategy and the budget B_l of auditing the taxpayers, declared $r = m_l$, depend on the part of those, on whom the signal $s = u$ was got:*

1. *If $P\{s = u|r = m_l\} = 1$, then $P(A|r = m_l) = \frac{1}{1+\pi}$, $B_l = \frac{\gamma_{l+1} c}{1+\pi}$;*
2. *If $P\{s = u|r = m_l\} = \frac{\gamma_{l+1}}{\gamma_l+\gamma_{l+1}}$, then*

$$P(A|r = m_l) = \frac{(\gamma_l + \gamma_{l+1})}{(1+\pi)(\gamma_l + \lambda\gamma_{l+1})} \cdot \left(1 + (\lambda - 1) \cdot \frac{\gamma_{l+1}}{\gamma_l + \gamma_{l+1}}\right),$$

$$B_l = \frac{(\gamma_l + \gamma_{l+1})^2 c}{(1+\pi)(\gamma_l + \lambda\gamma_{l+1})} \cdot \left(1 + (\lambda - 1) \cdot \frac{\gamma_{l+1}}{\gamma_l + \gamma_{l+1}}\right);$$

   moreover, if $\gamma_l > \pi\gamma_{l+1}$, the signal's reliability coefficient λ should be less or equal to $\frac{(\pi+1)\gamma_l}{\gamma_l - \pi\gamma_{l+1}}$;
3. *If $P\{s = u|r = m_l\} = 0$, then $P(A|r = m_l) = 0$, $B_l = 0$.*

Having restricted resources, the tax authority can utilize them, considering the indirect information about each taxpayer (signals) and the probability distribution of income among taxpayers.

4. Generalization

The model, investigated above, was considered in assumption that the taxpayers' evasion is restricted by the previous level of tax liability. This assumption is related to understanding that “daring” evasions are easier to reveal by the tax authority. An evasion also supposed to be possible only when the next group of taxpayers evades. It is necessary to notify that only risk neutral taxpayers were considered.

Now let's generalize the model, studied above, by the rejection of these assumptions.

4.1. Rejection of the Assumptions about Restricted Evasion

First, this rejection modifies the term of “profile of compliance”. The set of possible situations considerably grows.

In the most compliant profile taxpayers evade from the l-th level ($l = \overline{1, N-1}$) to every possible level of income and, therefore, we can consider the evasion to the tax liability m_0. For exit from this profile it is necessary to make an audit with the probability $P(A|r = m_0)$, which should satisfies (2). If the mentioned conduction fulfilled, the next compliant profile realizes. For exit from this profile the condition, analogical to (2), must be fulfilled for the probability $P(A|r = m_1)$. And so on.

These reasoning leads to the realization of the conditions (4). Thus, the Theorem 1 fulfilled even if we reject of the assumptions about restricted evasion.

4.2. Different Types of Rationality

Let's consider different types of taxpayers' rationality:

1. Risk neutral (their behaviour was considered in the previous models);
2. Risk averse (“honest” or “cowards”);
3. Risk preferred (“cheats”).

Let's assume that risk averse taxpayers evade never. They always declare their true tax liability and pay their taxes. Opposite, risk preferred taxpayers always evade, trying to win their profits.

In order to make this model closer to reality it is rational to consider every rationality group of taxpayers with different weighting coefficients. The structure of the profiles of compliance are modified, thus, the basic results will be modified, correspondingly to these coefficients.

5. Conclusion

A model of tax auditing in assumption, that tax authority has some statistical information about the distribution of income among population, is considered.

The following results were obtained: the optimal audit strategies for each income-level group; the optimal audit strategies with consideration of the signals; the proposition about the optimal budget for tax auditing.

Some generalizations of this model were also considered. The main results were illustrated by the numerical simulation.

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