

Stackelberg Strategies for Dynamic Games with Energy Players Having Different Time Durations

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Abstract We consider a system that consists of a major electrical power producer player (Public Power Corporation –PPC) playing in infinite time horizon, and minor players (power producers and consumers) remaining in the system for finite time durations, which time durations are overlapping. We study how they interact among themselves (horizontal interaction), and with the major player respectively (vertical interaction), via their decisions/strategies. We study a deterministic LQ version of the problem in discrete time. In our previous work we employed the Nash equilibrium and we studied the behavior of the system. In this paper we use the Stackelberg equilibrium with the long-term players in the role of the Leader.

Keywords: energy optimization cost, game theory, Stackelberg equilibrium.

1. Introduction

The work presented is motivated by the game between the Public Power Corporation (PPC) referred to as the major player and the many small producers/consumers referred to as the minor ones. We choose to address here the role of the time duration of the minor players (low power producers and consumers) which is small relative to the time horizon of the major player (PPC).

We study a deterministic version of the problem in discrete time. The Nash equilibrium was studied in (Kakogiannis and Papavassilopoulos, 2010) and (Kakogiannis et al., 2010). Here the Stackelberg equilibrium is employed.

We consider the LQ case and since we are interested in strategies that survive in a stochastic framework (Basar and Olsder, 1999; Papavassilopoulos, 1982) we use feedback and closed loop strategies. We provide the solution for the general case using the Riccati equations. We provide some simple numerical examples for the scalar case. We also assume that all minor players have the same cost function, act during different time periods but for the same duration T . This results to having to solve a system with $T + 1$ equations. Changing the values of the parameters involved we can easily solve each time the system of the Riccati equations and find the optimal controls-decisions and costs in every case for each player.

The Stackelberg solution we employ has a Closed Loop character for the Leader and a Feedback Stackelberg character for the Followers, since Dynamic Programming is used for deriving Followers' decisions. See (Basar and Olsder, 1999), (Simaan and Cruz, 1973a), (Simaan and Cruz, 1973b), (Fudenberg and Tirole, 1991) for more explanations of these concepts. We intend to study this case where Dynamic Programming is used for all the players and thus the Leader uses also a Feedback Stackelberg strategy, in future work.

2. Mathematical Formulation

The state equation is:

$$\begin{aligned} x_{k+1} &= Ax_k + B_0u_k + B_1u_{1k} + B_2u_{2k} + B_3u_{3k} + \\ &\quad + B_4u_{4k} + B_5u_{5k} \\ &\quad k = 0, 1, 2, 3 \dots \end{aligned} \quad (1)$$

where x_k is the state, u_k is the control of the long term player (PPC) - *Leader*, u_{ik} is the control of the minor players (clients or producers) - *Followers* at the i -th year remaining in the system ($i = 1 - 5$). For example, a minor player who enters the game at time k will use controls u_{1k} , u_{2k+1} , u_{3k+2} , u_{4k+3} , u_{5k+4} , corresponding to times k , $k+1$, $k+2$, $k+3$, $k+4$. A , B_i are given matrices of appropriate dimensions. If the players are six the state equation is

$$\begin{aligned} x_{k+1} &= Ax_k + B_0u_k + B_1u_{1k} + B_2u_{2k} + B_3u_{3k} + \\ &\quad + B_4u_{4k} + B_5u_{5k} + B_6u_{6k} \\ &\quad k = 0, 1, 2, 3 \dots \end{aligned}$$

and so on for 7 and 8 players.

The quadratic costs of the major player J_0 and the minor players (J_{1l}) who act in the interval l and $(l+4)$ are:

$$\begin{aligned} J_0 &= \sum_0^{\infty} (x_k^T Q_0 x_k + u_k^T R_0 u_k) \\ J_1 &= \sum_{k=0}^4 (x_{k+l+1}^T Q_f x_{k+l+1} + u_{(k+1)(l+k)}^T R_f u_{(k+1)(l+k)}) + x_l^T Q_f x_l \end{aligned} \quad (2)$$

The Q 's are symmetric non negative matrices and the R 's are symmetric positive defined matrices which are known. In our case we consider A , B , Q , and R constant. We will consider linear strategies for all the players, which will be derived as follows. Let $u_k = L_0 x_k$ be the Leader's strategy who is assumed to play this linear stationary strategy throughout the game. Each minor player (Follower) who acts for a period of length five, faces a Linear Quadratic problem where besides the Leader, several other minor players (Followers) are also present. We assume that they all play linear strategies in which case we can write the Ricatti equations that solve the minor player's problem. To derive the equations that provide the Li's of the minor player we proceed as follows. Consider for example the minor player who enters the calendar year 30 ($k = 30$). He sees the following system (7)-(13) where in this first equation (7) he acts as first year consumer/producer. The consumers/producers who entered earlier act with the fixed laws $L_2 x_{30}$, $L_3 x_{30}$, $L_4 x_{30}$, $L_5 x_{30}$

$$\begin{aligned} x_{k+1} &= (A + B_0 L_0 + B L_2 + B L_3 + B L_4 + B L_5) x_k + B u_{1,k} \\ &= A_1 x_k + B u_{1,k} \end{aligned} \quad (3)$$

Similarly when he is at the second year he sees the following system

$$\begin{aligned} x_{k+2} &= (A + B_0 L_0 + B L_1 + B L_3 + B L_4 + B L_5) x_k + B u_{2,k+1} \\ &= A_2 x_{k+1} + B u_{2,k+1} \end{aligned} \quad (4)$$

and the producers/consumers who entered earlier act with the fixed laws L_2x_{30} , L_3x_{30} , L_4x_{30} , L_5x_{30} and so on.

Thus the whole system of equations that the minor player (Follower) who entered the calendar year $k = 30$ and stays for five years sees, is:

$$\begin{aligned} x_{k+1} &= (A + B_0L_0 + BL_2 + BL_3 + BL_4 + BL_5)x_k + Bu_{1,k} \\ &= A_1x_k + Bu_{1,k} \end{aligned} \quad (5)$$

$$\begin{aligned} x_{k+2} &= (A + B_0L_0 + BL_1 + BL_3 + BL_4 + BL_5)x_{k+1} + Bu_{2,k+1} \\ &= A_2x_{k+1} + Bu_{2,k+1} \end{aligned} \quad (6)$$

$$\begin{aligned} x_{k+3} &= (A + B_0L_0 + BL_1 + BL_2 + BL_4 + BL_5)x_{k+2} + Bu_{3,k+2} \\ &= A_3x_{k+2} + Bu_{3,k+2} \end{aligned} \quad (7)$$

$$\begin{aligned} x_{k+4} &= (A + B_0L_0 + BL_1 + BL_2 + BL_3 + BL_5)x_{k+3} + Bu_{4,k+3} \\ &= A_4x_{k+3} + Bu_{4,k+3} \end{aligned} \quad (8)$$

$$\begin{aligned} x_{k+5} &= (A + B_0L_0 + BL_1 + BL_2 + BL_3 + BL_4)x_{k+4} + Bu_{5,k+4} \\ &= A_5x_{k+4} + Bu_{5,k+4} \end{aligned} \quad (9)$$

For this system of equations (5)-(9) and the cost

$$J_{30} = \sum_{k=0}^4 (x_{k+30+1}^T Q_f x_{k+30+1} + u_{(k+1)(30+k)}^T R_f u_{(k+1)(30+k)}) + x_{30}^T Q_f x_{30} \quad (10)$$

we derive the optimal policy by employing the Ricatti equations. It holds:

$$\begin{aligned} u_{1,k} &= L_1x_k, & u_{2,k+1} &= L_2x_{k+1}, & u_{3,k+2} &= L_3x_{k+2} \\ u_{4,k+3} &= L_4x_{k+3}, & u_{5,k+4} &= L_5x_{k+4} \end{aligned}$$

where the Li's are given by the following system of equations.

$$L_1 = -(B^T K_2 B + R)^{-1} B^T K_2 A_1 \quad (11)$$

$$K_1 = A_1^T (K_2 - K_2 B (B^T K_2 B + R)^{-1} B^T K_2) A_1 + Q_f \quad (12)$$

$$L_2 = -(B^T K_3 B + R)^{-1} B^T K_3 A_2 \quad (13)$$

$$K_2 = A_2^T (K_3 - K_3 B (B^T K_3 B + R)^{-1} B^T K_3) A_2 + Q_f \quad (14)$$

$$L_3 = -(B^T K_4 B + R)^{-1} B^T K_4 A_3 \quad (15)$$

$$K_3 = A_3^T (K_4 - K_4 B (B^T K_4 B + R)^{-1} B^T K_4) A_3 + Q_f \quad (16)$$

$$L_4 = -(B^T K_5 B + R)^{-1} B^T K_5 A_4 \quad (17)$$

$$K_4 = A_4^T (K_5 - K_5 B (B^T K_5 B + R)^{-1} B^T K_5) A_4 + Q_f \quad (18)$$

$$L_5 = -(B^T K_6 B + R)^{-1} B^T K_6 A_5 \quad (19)$$

$$K_5 = A_5^T (K_6 - K_6 B (B^T K_6 B + R)^{-1} B^T K_6) A_5 + Q_f \quad (20)$$

$$K_6 = Q_f \quad (21)$$

Since the other Followers use a similar rational, the Li's used by them and are present in the of (5)-(9) are identified with the Li's of the player under consideration derived in (11)-(21). The total cost of a minor player who entered the system at year 30 is:

$$J_30^* = x_{30}^T K_1 x_{30} \quad (22)$$

Notice that we consider linear no memory strategies. We know that may exist other solutions, which are not necessarily linear and may have memory. We know nonetheless (Selten and (Basar and Olsder, 1999)) that these solutions disappear in the presence of noise.

The Leader's cost ,can be found as follows:

$$\bar{A} = A + B_0 L_0 + B L_2 + B L_3 + B L_4 + B L_5 \quad (23)$$

$$x_k = (\bar{A})^k x_0 \quad (24)$$

$$J_0 = \sum x_0^T (\bar{A}^T)^k Q_0 (\bar{A})^k x_0 + (L_0 x_k)^T R_0 (L_0 x_k) \quad (25)$$

$$J_0 = x_0^T \sum_{k=0}^{\infty} (\bar{A}^T)^k (Q_0 + L_0^T R_0 L_0) (\bar{A})^k x_0 \quad (26)$$

The Leader's problem is to minimize (26) subject to the constraints (11)-(21) and (24). It is obviously a difficult nonlinear programming problem. It should be noted that the Leader's strategy is not derived by the Dynamic Programming Algorithm and thus it cannot be considered as a Feedback Stackelberg Strategy, as defined in (Simaan and Cruz, 1973a), (Simaan and Cruz, 1973b), (Basar and Olsder, 1999). On the other hand, the Followers' Strategies obey Dynamic Programming since they were derived using the Ricatti equations formalism , and thus can be called Feedback Stackelberg Strategies.

3. Numerical Study

In this section we present some numerical results for the scalar case and study the optimal cost of the Leader for several values of the parameters. We consider the matrices A, B, Q_0, Q_f, R as constant scalars a, b, q_0, q_f, r . We take the R's and the B's to be equal to 1. We also take the initial condition equal to 1.

After some transformations we created the following scalar equations ,where the xi's, stand for the Ki's, i=0,1,...5:

$$x_5 = q_f \quad (27)$$

$$x_4 = q_f + \bar{A}^2(x_5 + x_5^2) \quad (28)$$

$$x_3 = q_f + \bar{A}^2(x_4 + x_4^2) \quad (29)$$

$$x_2 = q_f + \bar{A}^2(x_3 + x_3^2) \quad (30)$$

$$x_1 = q_f + \bar{A}^2(x_2 + x_2^2) \quad (31)$$

$$\bar{A} = \frac{a}{1 + S + x_0} \quad (32)$$

$$S = x_1 + x_2 + x_3 + x_4 + x_5 \quad (33)$$

$$x_0 = \frac{a}{\bar{A}} - 1 - S \quad (34)$$

$$J_0 = \sum (q_0 x_k^2 + u_k^2) \quad (35)$$

$$J_0 = (q_0 + x_0^2) \frac{1}{1 - \bar{A}^2} \quad (36)$$

The quantity \bar{A} in (32) is actually the closed loop matrix of the system which has to be stable, i.e. $-1 < \bar{A} < 1$. The problem for the Leader is to minimize J_0 subject to the constrains (27)-(32) where all the $x_0, x_1, x_2, x_3, x_4, x_5, \bar{A}$ are unknowns.

A way of solving this system is to use Lagrange Multipliers and append the constrains to the cost.

We will present a quicker way based on a plot of the cost of the Leader versus the policy gain, (here it is the x_0), from where the optimal policy gain and cost of the Leader can be found. For fixed values of a, q_0, q_f and initial condition 1 we do the following. We take a value of $\bar{A} \in (-1, 1)$ for

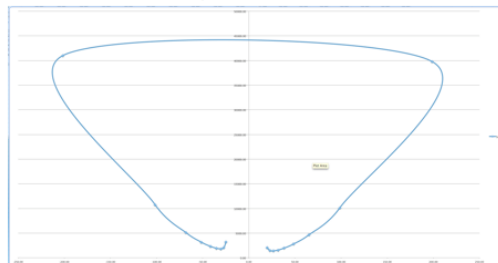
example $\bar{A} = 0.3$. For (27)-(32) we find the values for x_0 and J_0 . We do that for several values of $\bar{A} \in (-1, 1)$ and plot $J_0 - x_0$.

We present some runs and numerical results. Then we present 4 plots of J_0 (vertical) versus x_0 (horizontal) from which the best choice of the Leader's gain and his best cost are easily found. The values of a, q_0 and q_f used in these plots are given below.

Numerical Results - Plots:**Case of plot 1:**

α	20.00	20.00	20.00	20.00	20.00	20.00	20.00	20.00	20.00
q_0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
q_f	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
A	-0.90	-0.80	-0.70	-0.60	-0.50	-0.40	-0.30	-0.20	-0.10
x_1	0.54	0.33	0.22	0.17	0.14	0.12	0.11	0.10	0.10
x_2	0.39	0.28	0.21	0.17	0.14	0.12	0.11	0.10	0.10
x_3	0.28	0.23	0.19	0.16	0.14	0.12	0.11	0.10	0.10
x_4	0.19	0.17	0.15	0.14	0.13	0.12	0.11	0.10	0.10
x_5	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
x_0	-24.73	-27.11	-30.44	-35.07	-41.64	-51.58	-68.21	-101.52	-201.50
S	1.51	1.11	0.87	0.73	0.64	0.58	0.54	0.52	0.50
Jo	3224.04	2043.58	1819.38	1922.76	2313.35	3168.72	5113.81	10736.41	41015.19

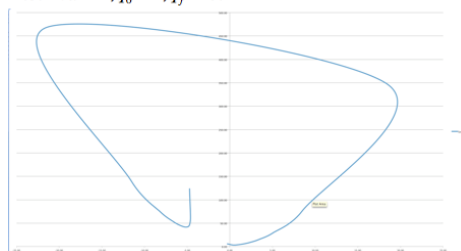
α	20.00	20.00	20.00	20.00	20.00	20.00	20.00	20.00	20.00
q_0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
q_f	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
A	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90
x_1	0.10	0.10	0.11	0.12	0.14	0.17	0.22	0.33	0.54
x_2	0.10	0.10	0.11	0.12	0.14	0.17	0.21	0.28	0.39
x_3	0.10	0.10	0.11	0.12	0.14	0.16	0.19	0.23	0.28
x_4	0.10	0.10	0.11	0.12	0.13	0.14	0.15	0.17	0.19
x_5	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
x_0	198.50	98.48	65.12	48.42	38.36	31.60	26.70	22.89	19.71
S	0.50	0.52	0.54	0.58	0.64	0.73	0.87	1.11	1.51
Jo	39799.48	10103.81	4661.63	2792.01	1963.17	1561.96	1399.58	1458.82	2050.86

Plot1 : $a = 20, q_0 = 1, q_f = 0.1$ 

Case of plot 2:

α	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
q_0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
q_f	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
A	-0.90	-0.80	-0.70	-0.60	-0.50	-0.40	-0.30	-0.20	-0.10
x1	0.54	0.33	0.22	0.17	0.14	0.12	0.11	0.10	0.10
x2	0.39	0.28	0.21	0.17	0.14	0.12	0.11	0.10	0.10
x3	0.28	0.23	0.19	0.16	0.14	0.12	0.11	0.10	0.10
x4	0.19	0.17	0.15	0.14	0.13	0.12	0.11	0.10	0.10
x5	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
xo	-4.73	-4.61	-4.73	-5.07	-5.64	-6.58	-8.21	-11.52	-21.50
S	1.51	1.11	0.87	0.73	0.64	0.58	0.54	0.52	0.50
Jo	123.01	61.69	45.84	41.65	43.77	52.77	75.16	139.24	468.12

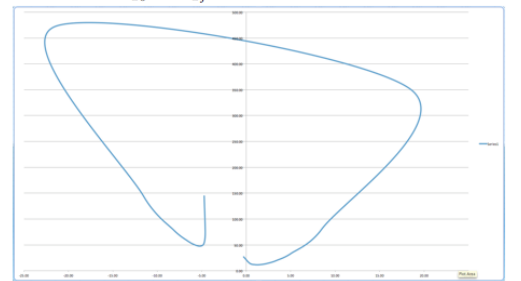
α	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
q_0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
q_f	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
A	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90
x1	0.10	0.10	0.11	0.12	0.14	0.17	0.22	0.33	0.54
x2	0.10	0.10	0.11	0.12	0.14	0.17	0.21	0.28	0.39
x3	0.10	0.10	0.11	0.12	0.14	0.16	0.19	0.23	0.28
x4	0.10	0.10	0.11	0.12	0.13	0.14	0.15	0.17	0.19
x5	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
xo	18.50	8.48	5.12	3.42	2.36	1.60	0.98	0.39	-0.29
S	0.50	0.52	0.54	0.58	0.64	0.73	0.87	1.11	1.51
Jo	346.55	75.98	29.95	15.10	8.75	5.57	3.86	3.21	5.69

Plot2 : $a = 2, q_0 = 1, q_f = 0.1$ 

Case of plot 3:

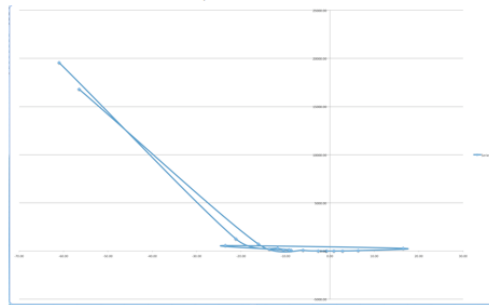
α	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
q_0	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00
q_f	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
A	-0.90	-0.80	-0.70	-0.60	-0.50	-0.40	-0.30	-0.20	-0.10
x1	0.54	0.33	0.22	0.17	0.14	0.12	0.11	0.10	0.10
x2	0.39	0.28	0.21	0.17	0.14	0.12	0.11	0.10	0.10
x3	0.28	0.23	0.19	0.16	0.14	0.12	0.11	0.10	0.10
x4	0.19	0.17	0.15	0.14	0.13	0.12	0.11	0.10	0.10
x5	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
xo	-4.73	-4.61	-4.73	-5.07	-5.64	-6.58	-8.21	-11.52	-21.50
S	1.51	1.11	0.87	0.73	0.64	0.58	0.54	0.52	0.50
Jo	144.06	72.80	53.68	47.90	49.10	57.53	79.56	143.41	472.16

α	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
q_0	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00
q_f	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
A	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90
x1	0.10	0.10	0.11	0.12	0.14	0.17	0.22	0.33	0.54
x2	0.10	0.10	0.11	0.12	0.14	0.17	0.21	0.28	0.39
x3	0.10	0.10	0.11	0.12	0.14	0.16	0.19	0.23	0.28
x4	0.10	0.10	0.11	0.12	0.13	0.14	0.15	0.17	0.19
x5	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
xo	18.50	8.48	5.12	3.42	2.36	1.60	0.98	0.39	-0.29
S	0.50	0.52	0.54	0.58	0.64	0.73	0.87	1.11	1.51
Jo	350.59	80.15	34.34	19.86	14.08	11.82	11.70	14.32	26.74

Plot3: $a = 2, q_0 = 5, q_f = 0.1$ 

Case of plot 4:

α	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
q_0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
q_f	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
A	-0.90	-0.80	-0.70	-0.60	-0.50	-0.40	-0.30	-0.20	-0.10
x_1	46.66	10.87	3.33	1.46	0.90	0.68	0.58	0.53	0.51
x_2	7.07	3.56	1.95	1.21	0.85	0.68	0.58	0.53	0.51
x_3	2.39	1.74	1.29	0.99	0.79	0.66	0.58	0.53	0.51
x_4	1.11	0.98	0.87	0.77	0.69	0.62	0.57	0.53	0.51
x_5	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
x_0	-60.94	-21.15	-11.80	-9.27	-8.73	-9.14	-10.48	-13.63	-23.53
S	57.72	17.65	7.94	4.93	3.73	3.14	2.81	2.63	2.53
Jo	19553.76	1245.37	275.05	135.73	102.87	100.59	121.78	194.49	560.29
α	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
q_0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
q_f	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
A	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90
x_1	0.51	0.53	0.58	0.68	0.90	1.46	3.33	10.87	46.66
x_2	0.51	0.53	0.58	0.68	0.85	1.21	1.95	3.56	7.07
x_3	0.51	0.53	0.58	0.66	0.79	0.99	1.29	1.74	2.39
x_4	0.51	0.53	0.57	0.62	0.69	0.77	0.87	0.98	1.11
x_5	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
x_0	16.47	6.37	2.85	0.86	-0.73	-2.60	-6.09	-16.15	-56.50
S	2.53	2.63	2.81	3.14	3.73	4.93	7.94	17.65	57.72
Jo	275.00	43.34	10.05	2.08	2.04	12.12	74.62	727.30	16806.52

Plot4 : $a = 2, q_0 = 1, q_f = 0.5$ 

4. Conclusions

In our future search we intend to use the matrix Stackelberg equilibrium model for the players and not only scalar form. Of interest is also the case where the time duration of the short time players is a random variable taking values in a certain interval. Similarly we can consider cases where the appearance of a small duration player at each instant of time is itself a random event.

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