# Two Models of the Influenza Epidemic

**Gubar Elena, Fotina Lidia, Nikitina Irina and Zhitkova Ekaterina**

*St.Petersburg University,*
*Faculty of Applied Mathematics and Control Processes,*
*Universitetskiy pr. 35, St.Petersburg, 198504, Russia*
*fax:+7 (812) 428 71 59*
*http://www.apmath.spbu.ru,*
*alyona.gubar@gmail.com,*
*fotina.lidia@mail.ru,*
*nikitina.ira@list.ru,*
*zhitkovakaterina@mail.ru*

**Abstract** The main purpose of current work is constructing the evolutionary model of influenza epidemic in urban population and estimation the impact of the preventive measures to the population. Also in this work we select risk-group from the population and research the epidemic process in it. Total urban population and risk group are divided into three subgroups Susceptible, Infected and Recovered and during the epidemic individuals transfer from one subgroup to the other. During the epidemic season the quantitative structure of the subgroups is changed, but these changes could be different in risk-group and in vaccinated subgroup. In the model we assume that vaccination company occurs before season epidemic of influenza begins, to avoid repeated infection of vaccinated individuals. We construct an evolution of epidemic and take into account vaccination and infection expenses, from the society point of view. Numerical simulation are also presented in the paper.

**Keywords:** Evolutionary game, vaccination problem, replicative dynamic, epidemic process, epidemic models, SIR model.

## 1. Introduction

One of the most important problem is the protection of population during annual flu epidemic season. The influenza epidemic is a fast spreading process, involving the large part of total population. To protect population and reduce sickness rate society should to focus on the organization of preventive measures. One of the most effective procedures to avoid the epidemic is vaccination. However, in addition to influenza, other forms of respiratory viral diseases circulate in the population, and individuals, vaccinated against the influenza can be infected by one of these forms, which reduces the effectiveness of vaccination. Hence vaccination can not be absolutely effective and moreover total vaccination is very expensive. Whereas complete vaccination is not effective epidemiologist offer to vaccinate only risk-group to avoid epidemic in total population. Previous research proofed that about 70 % of population should be vaccinated before the epidemic season to avoid epidemic of influenza. Also it was proofed that if society focus on some risk-groups from the total population and apply vaccination to them it is allow to reduce epidemic duration and numbers of infected humans during the epidemic.

Population can be divided in several subpopulations, i.e. "children", "social professions", "medical professions", aged person, etc. For example the most amenable

to the influenza is the risk-group "children", as far as they have numerous contacts with other subgroups in population and at the same time the weak self-control.

A large number of research show that the economic damage is reduced, when the risk-group "children" is totally vaccinated. Vaccination about 80 percent individuals in this group reduces the sickness rate in total population up to 80 - 90 percent.

Monitoring the evolution of several epidemics created the basis for a phenomenological model, describing the phases of individuals' states changing. In the works Ross (1911), Ross (1915), Ross (1916), he attempted to give the first quantitative description of an malaria epidemic, nowadays many current models are closely linked with his researches. Later Kermack and Mc Kendrick (1927) generalized this approach and built a stochastic and determine models of epidemics.

By analogy with the classical model presented by Kermack W.O., and Mc Kendrick A.G. in our work mechanism of infection is realized through meeting between Susceptible and Infected individuals. Evolution of the epidemic process is modeled as a sequential changing states from Susceptible individuals to Infected individuals and finally to Recovered individuals. Simple scheme of the epidemic process is presented in the Fig 1.

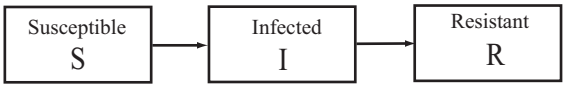



Figure1: Simple scheme of the epidemic process.

This process is described by a system of ordinary differential equations:

$$\begin{gathered} dS/dt = -aSI, \\ dI/dt = aSI - bI, \\ dR/dt = bI, \\ S + I + R = N = \text{const}, \\ S(0) = S^0, \quad I(0) = I^0, \quad R(0) = R^0 \geq 0, \end{gathered} \tag{1}$$

where $S$ – is number of Susceptible individuals, $I$ – is number of Infected individuals, $R$ – is number of Recovered individuals, $aSI$ – is infection intensity during the meeting between Susceptible and Infected individuals, $bI$ – is recovery intensity ($b = 1/T$ , $T$ – is disease's duration), $S+I+R$ – total number of individuals, involved in epidemic process (considering as a constant), $a$, $b$ – are constant nonnegative coefficients. Additionally in the model initial conditions of the epidemic beginning are used:

$$aSI - bI > 0, \quad S^0 \geq b/a. \tag{2}$$

In the paper Fu et al. (2010) the vaccination problem is considered from the point of view one individual, in the current work we focus on social aspect of vaccination problem and estimate the costs of vaccination company and the infection costs that include health care expenses, lost productivity and the possibility of pain or mortality. We assume that vaccination company occurs before the seasonal epidemic begins, because it is necessary take into account regulation immune system of individual after vaccination, because failing health after vaccination not allow to resist against another viruses. Unfortunately flu vaccines are effective only for one season owing to mutation of pathogens and waning immunity. We suppose that influenza epidemic continues until there are no more newly infected individuals.

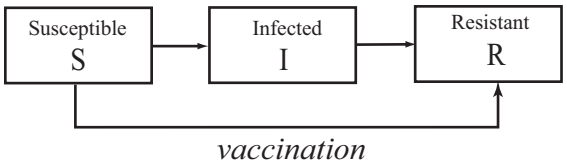



Figure2: Simple scheme of the epidemic process with vaccination company in preepidemic period.

## 2. Model of epidemic process in risk-group

In our model we construct evolutionary model of epidemic process, which are based on Susceptible-Infected-Recovered (SIR) model Zhitkova, Kolesin (2004), Fu et al. (2010) for total urban population and risk-group. The SIR model is appropriated for a large class of deceases including influenza. Complete vaccination in total urban population is considered as ineffective, hence from the total population we select $i$-th risk-group and assume that all individuals in the group have the equal rate of susceptibility. In the model we focus on one risk-group ("children") and define it as large but finite, well mixed population of individuals. Total urban population and risk-group are divided to three subpopulations: Susceptible ($S$), Infected ($I$) and Recovered ($R$). Denote as $S_i,\ I_i,\ R_i$ – the fractions of Susceptible ($S$), Infected ($I$) and Recovered ($R$), respectively. Let values ${S_i}^p,\ {I_i}^p,\ {R_i}^p$ are corresponding fractions of Susceptible ($S$), Infected ($I$) and Recovered ($R$) individuals in total urban population. The scheme of the epidemic process evolution subject to mutual infection of individuals from the risk-group and total urban population is following:

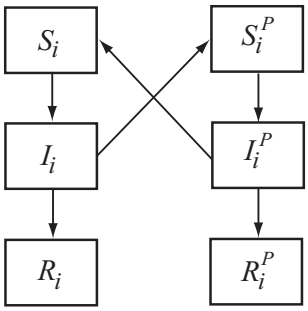



Figure3: Scheme of the epidemic process evolution in case of mutual infection of individuals

In current section we describe the process of influenza epidemic in terms of evolutionary game theory. Consider dynamic of epidemic progress as evolutionary game and suppose that state of the total urban population and risk-group in each time moment $t$ can be described in the same way by value $x(t) = (x_S(t), x_I(t), x_R(t))$. Values $x_i$ are defined by following expression

$$x_i = \frac{p_i(t)}{\sum\limits_{i=1}^{3} p_i(t)},\ i = S, I, R, \tag{3}$$

where $p_i(t) \geq 0$ – is number of individuals in $i$-th subpopulation, $p(t) = \sum_{i=1}^{3} p_i(t), i = S, I, R$ – total number of individuals in populations Susceptible, Infected and Recovered respectively.

Suppose that in total urban population and in the risk group individuals can be randomly matched and as a result they transfer from one subpopulation to another. During the transition individuals can also change their income, i.e. if susceptible become infected then he has treatment costs and his income decrease. At the same time if infected becomes recovered then his income will increase. Further we describe the changes of individuals incomes. Here we assume that individuals in risk-group are not vaccinated, then each unvaccinated individual has risk to be infected during the seasonal epidemic. Suppose that during the meeting, two Susceptible keep their conditions and do not switch over to Infected, hence we can say that Susceptible do not have any healthcare expenses. During the meeting of Susceptible and Infected, Susceptible with uniform probability can switch over to Infected or keep their own condition, therefore Susceptible may obtain some infection expenses. Infected have own infection expenses. During the meeting Susceptible and Recovered, both keep their own conditions, Susceptible do not have any healthcare expenses, but Recovered could receive their immunity in case of disease. We describe the individuals' transfers using payoff general matrix, that corresponds to the relations between the randomly matched individuals from different subgroups and it is presented below:

| | $S$ | $I$ | $R$ |
|---|---|---|---|
| $S$ | $(\beta, \beta)$ | $(\beta - c\delta, \alpha + \gamma)$ | $(\beta, \beta)$ |
| $I$ | $(\alpha + \gamma, \beta - c\delta)$ | $(\alpha + \gamma - \sigma, \alpha + \gamma - \sigma)$ | $(\alpha + \gamma, \beta)$ |
| $R$ | $(\beta, \beta)$ | $(\beta, \alpha + \gamma)$ | $(\beta, \beta)$ |

Payoff matrix for total urban population and risk-group differ by $\delta$.

In table below we present parameters of the model:

Table1: Model's parameters

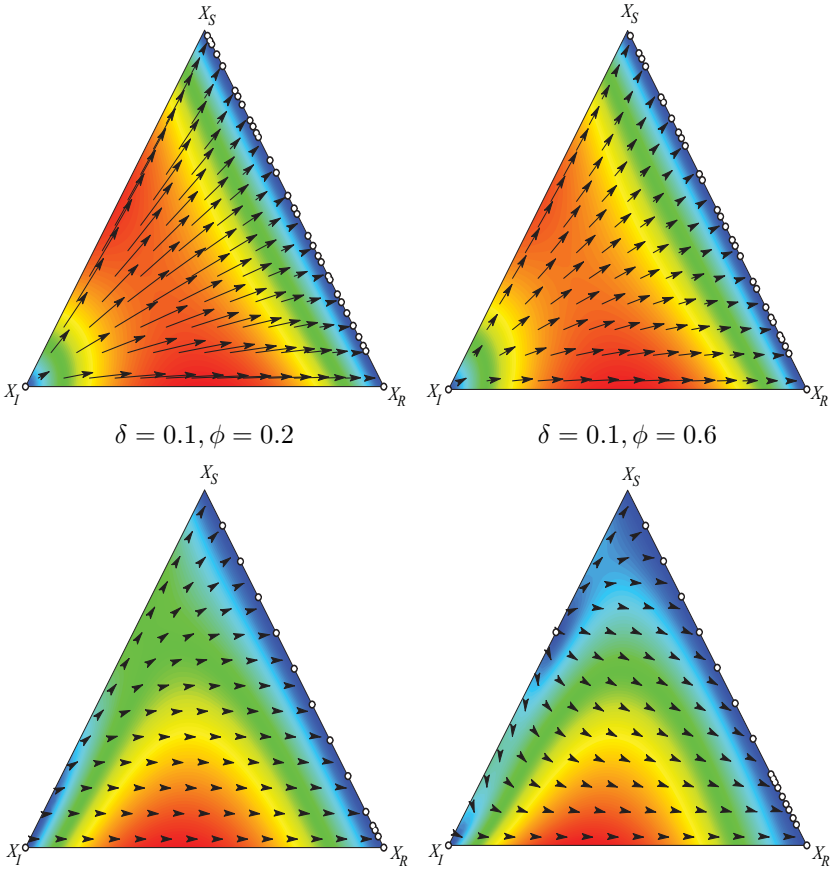
| Parameter | Definition |
|---|---|
| $\beta$ | player's payoff |
| $c$ | treatment costs |
| $\delta$ | probability of transition from a state "susceptible" to the state "infected" |
| $\alpha = (\beta - c)$ | income with treatment costst |
| $\gamma = c\phi$ | payoff growth, on conditions that "infected" transfer to "recovered" |
| $\phi$ | probability of transition from a state "infected" to the state "recovered" |
| $\sigma = c\psi$ | payoff decrease, when individuum gets worse |
| $\psi$ | probability that individuum get worse when one "infected" meet another |

During the epidemic we have a chain of changes of population states that can be described by evolutionary dynamics Weibull, (1995).

$$\dot{x}_i = [u(e^i, x) - u(x, x)]x_i = u(e^i - x, x)x_i, i = S, I, R, \tag{4}$$

where $e^i,\ i = S, I, R$ – are pure strategy of the individual, $u(x, x) = \sum_{i=1}^{k} x_i u(e^i, x)$ – is average payoff in the population. Using initial states for all subpopulations solve this system of differential equations and get distribution of Susceptible, Infected and Recovered individuals after seasonal epidemic of influenza. Initial state consists of large part of Susceptible, several Infected and a little part of Recovered individuals, randomly distributed throughout the population.

To illustrate the evolution on the population states we use special package [Sandholm (2010)] and the resulting trajectories for different values of model's parameters are presented below:




$\delta = 0.1, \phi = 0.2$ $\delta = 0.1, \phi = 0.6$

$\delta = 0.1, \phi = 0.95$ $\delta = 0.1, \phi = 0.98$

Figure4: Epidemic process in total urban population

These diagrams show that in the first case: $\delta = 0.1, \phi = 0.2$ all trajectories aspire to the boundary $x_R, x_S$ and this boundary is the set of stable states. In the second case with model's parameters $\delta = 0.1, \phi = 0.6$ all trajectories will aspire to

the vertex $x_R$ and it is stable. Here state $(0.6, 0.4, 0)$ is also equilibrium state but it is not stable.

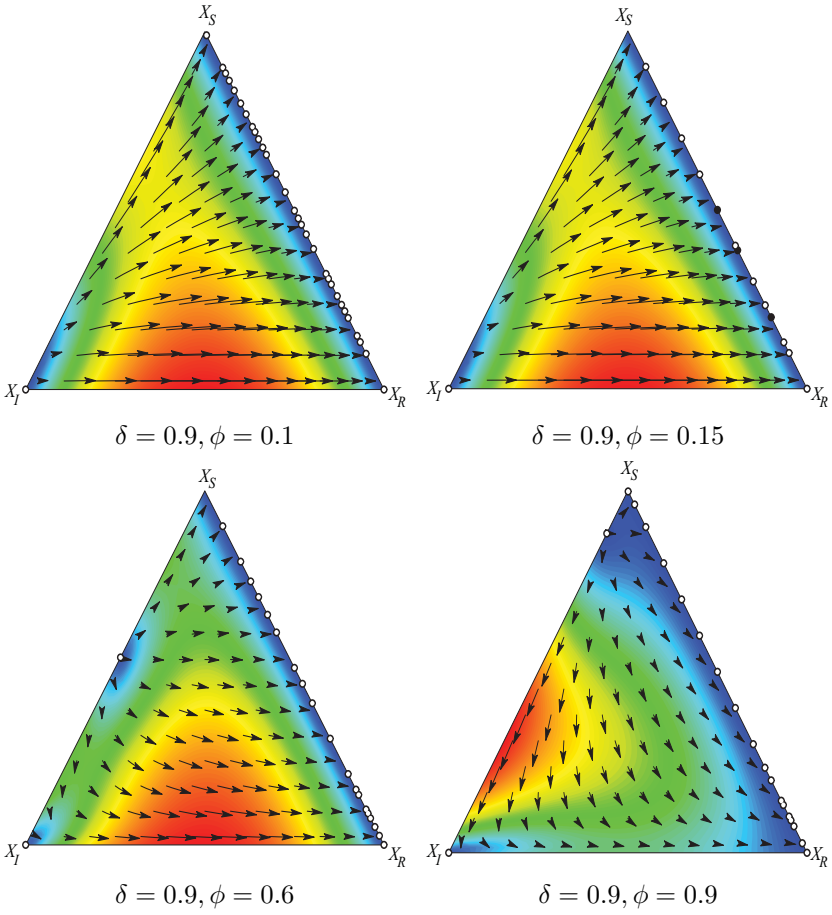



Figure5: Epidemic process in risk-group

For the risk-group we receive that with parameters $\delta = 0.9, \phi = 0.1$ and $\delta = 0.9, \phi = 0.15$ as in previous case boundary $x_R, x_S$ is the set of stationary states. For values $\delta = 0.9, \phi = 0.6$ and $\delta = 0.9, \phi = 0.9$ state $x_R$ is stationary and $(0.53, 0.47, 0)$ is equilibrium point but it is not stable.

Unfortunately this simulations are not allow us to receive a numerical strength of each subpopulation and to solve this problem in following section we use special algorithm to get a numbers of Susceptible, Infected and Recovered in population and risk-group.

### 2.1. Numerical simulation with Gillespie algorithm

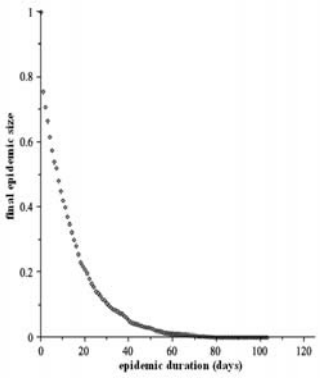
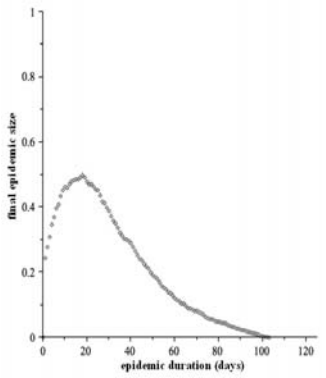
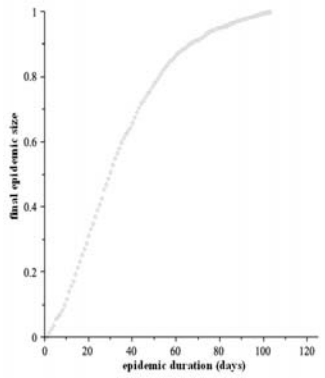
In this section we consider how epidemic progress in total population and in risk group depends on human compliance. Numerical simulations present duration and effectiveness of the epidemic process. To simulate the epidemic process we use Gillespie algorithm Gillespie (1976), Fu et al. (2010), which are presented below:

1. At time $t$ calculate transition rate $p_i(t)$, $i \in N_r$ for each susceptible and infected individual. Here $N_r$ is number of individuals in risk-group. The rate at which susceptible individuals transfer to infected is $p_i(t) = \delta l$, $l$ is number of infected neighbors. $l(t) \in (0, \ldots, N_{neib}(t))$, $N_{neib}(t) = [\eta N x_I(t - \Delta t)]$, where $\eta$ — is probability to meet infected individuum; $N$ — is total number individuals in population; $x_I(t - \Delta t)$ — is part of infected at instant moment $t - \Delta t$. The rate at which infected individuals transfer to recovered is $p_i(t) = \psi$. Total transition rate is $\lambda(t) = \sum\limits_i p_i(t)$.
2. Chose the time interval $\Delta t$ then next transition occurs in moment $t' = t + \Delta t$.
3. Verify the transition condition between subgroups for each individual $k$: $\sum\limits_{j=1}^{k-1} p_j(t)/\lambda(t) < z < \sum\limits_{j=1}^{k} p_j(t)/\lambda(t)$, $\sum\limits_{j=1}^{0} p_j(t)/\lambda(t) = 0$
4. Repeat steps 1-3 until the number of infected individuals I(t) becomes 0.

The results of the simulation we give in diagrams (Tables 2,4) and tables with parameters interpretation and values for different cases (Tables 3,5).

**Case 1:** In this case we consider risk group and total urban population with small amount of the infected neighbors. Risk-group and total urban population differ by parameter $\delta$ which is transmission rate from Susceptible to Infected or it can be interpreted as probability that susceptible human becomes infected.

Table2: Simulation results for risk-group

| Susceptible | Infected | Recovered |
|---|---|---|
|  |  |  |



We received that in total urban population epidemic duration is 106 days and epidemic peak occurs at 16-th day and in risk-group epidemic duration is 104 days and epidemic peak is at 18-th day. Shares of infected individuals in risk-group and total urban population at the epidemic peak are 0.499 and 0.446 respectively. Simulation results are presented on Tables 2-5.

Table3: Simulation results for risk-group

| Variable | Definition | Value |
|---|---|---|
| $\delta$ | Transmission rate from Susceptible to Infected | 0,95 |
| $\phi$ | Transmission rate from Infected to Recovered | 0,875 |
| $x_I(0)$ | Initial share of Infected | 0,001 |
| $T$ | Epidemic duration (days) | 104 |
| $\overline{T}$ | Epidemic peak(day) | 18 |
| $x_S(\overline{T})$ | Share of susceptible at time $\overline{T}$ | 0.230 |
| $x_I(\overline{T})$ | Share of infected at time $\overline{T}$ | 0.499 |
| $x_R(\overline{T})$ | Share of recovered at time $\overline{T}$ | 0.271 |

Table4: Simulation results for total urban population

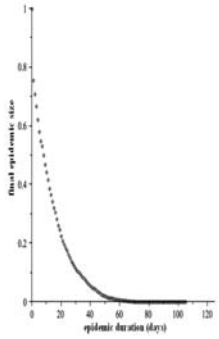
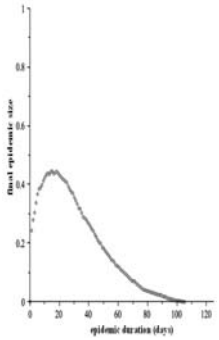
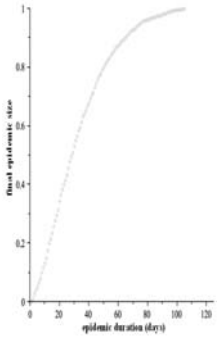
| Susceptible | Infected | Recovered |
|---|---|---|
|  |  |  |

Table5: Simulation results for total urban population

| Variable | Definition | Value |
|---|---|---|
| $\delta$ | Transmission rate from Susceptible to Infected | 0,25 |
| $\phi$ | Transmission rate from Infected to Recovered | 0,875 |
| $x_I(0)$ | Initial share of Infected | 0,001 |
| $T$ | Epidemic duration (days) | 106 |
| $\overline{T}$ | Epidemic peak(day) | 16 |
| $x_S(\overline{T})$ | Share of susceptible at time $\overline{T}$ | 0.320 |
| $x_I(\overline{T})$ | Share of infected at time $\overline{T}$ | 0.446 |
| $x_R(\overline{T})$ | Share of recovered at time $\overline{T}$ | 0.234 |

**Case 2:** In this case we consider risk group and total urban population with large amount of the infected neighbors. To define the number of infected neighbors we use initial value for $x_I(0) = 0.3$. As in **case 1** risk-group and total urban population differ by parameter $\delta$.

Table6: Results for risk-group (large number of infected neighbors)

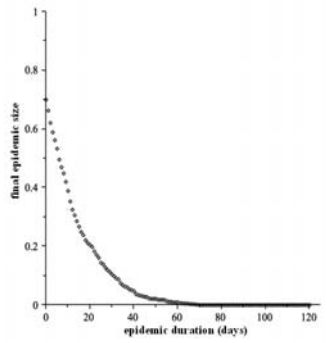
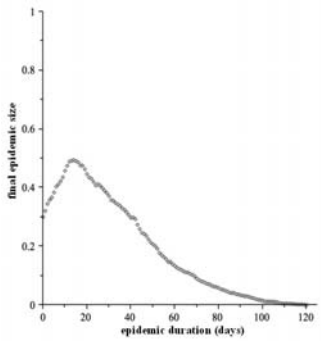
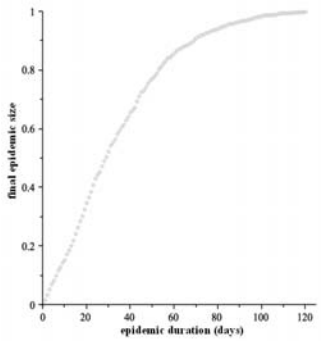
| Susceptible | Infected | Recovered |
|---|---|---|
|  |  |  |



Table7: Results for risk-group(large number of infected neighbors)

| Variable | Definition | Value |
|---|---|---|
| $\delta$ | Transmission rate from Susceptible to Infected | 0,95 |
| $\phi$ | Transmission rate from Infected to Recovered | 0,875 |
| $x_I(0)$ | Initial share of Infected | 0,3 |
| $T$ | Epidemic duration (days) | 121 |
| $\overline{T}$ | Epidemic peak(day) | 15 |
| $x_S(\overline{T})$ | Share of susceptible at time $\overline{T}$ | 0.286 |
| $x_I(\overline{T})$ | Share of infected at time $\overline{T}$ | 0.494 |
| $x_R(\overline{T})$ | Share of recovered at time $\overline{T}$ | 0.220 |

In this case we received that in total urban population epidemic duration is 117 days and epidemic peak occurs at 14-th day and in risk-group epidemic duration is 121 days and epidemic peak is at 15-th day. Also in this case we suppose that each susceptible individual has high possibility to meet infected, because in total each individual has large number of infected neighbors. As in previous simulations for the risk-group the intensity of the transition from the Susceptible to Infected is higher and hence infection spreads faster but depending of parameter $l$ epidemic lasts in risk-group longer. All results are presented in Tables 6-9.

## 3. Epidemic process in total urban population with vaccination

In this section all Susceptible ($S$) individuals are vaccinated and the society spend money on the vaccination company (this expenses include immediate monetary cost, the opportunity cost of time spent to get the vaccine, number of medical

Table8: Results for total urban population (large number of infected neighbors)

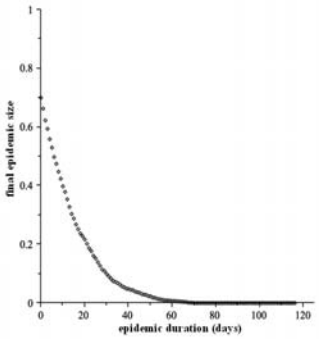
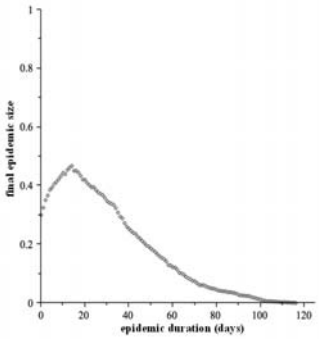
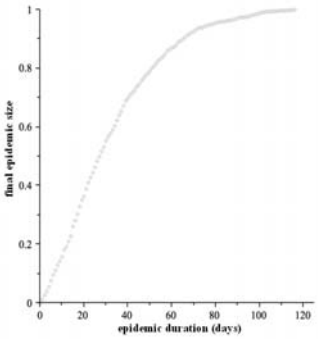
| Susceptible | Infected | Recovered |
|---|---|---|
|  |  |  |



Table9: Results for total urban population (large number of infected neighbors)

| Variable | Definition | Value |
|---|---|---|
| $\delta$ | Transmission rate from Susceptible to Infected | 0,25 |
| $\phi$ | Transmission rate from Infected to Recovered | 0,875 |
| $x_I(0)$ | Initial share of Infected | 0,3 |
| $T$ | Epidemic duration (days) | 117 |
| $\overline{T}$ | Epidemic peak(day) | 14 |
| $x_S(\overline{T})$ | Share of susceptible at time $\overline{T}$ | 0.304 |
| $x_I(\overline{T})$ | Share of infected at time $\overline{T}$ | 0.468 |
| $x_R(\overline{T})$ | Share of recovered at time $\overline{T}$ | 0.228 |

teams and any other perceived or actual effects.) However we assume that vaccination do not guarantee perfect immunity from the seasonal infection decease and Susceptible individuals keep small risk to be infected. In our work we define epidemic process as an evolutionary game with following structure.

As in section 2. let $N$ is a size of total urban population and total population is divided into three subgroups: Susceptible, Infected and Recovered. Let $K = \{S, I, R\}$ is a set of pure strategies, which correspond to the individual's state. Also denote as $x_S, x_I, x_R$ the shares of susceptible, infected and recovered individuals in total population respectively.

Then we describe the epidemic dynamics in total population over the time. Assume that during the meeting two susceptible individuals keep their conditions and do not switch over to Infected, but Susceptible individuals have some vaccination expenses. During the meeting of Susceptible and Infected, Susceptible can with uniform possibility switch over to Infected or keep their own condition, therefore Susceptible may obtain some infection expenses, but meanwhile their already had some vaccination expenses. Infected have their own healthcare expenses. Recovered could receive their immunity in case of disease or during vaccination campaign, then their expenses include monetary cost of vaccine or some healthcare costs. Payoff matrix (matrix of transition rates) for this situation is following:

| | $S$ | $I$ | $R$ |
|---|---|---|---|
| $S$ | $(\beta - \mathrm{q}, \beta - \mathrm{q})$ | $(\beta - \mathrm{q} - \mathrm{c}\theta\delta, \alpha + \gamma)$ | $(\beta - \mathrm{q}, \beta)$ |
| $I$ | $(\alpha + \gamma, \beta - \mathrm{q} - \mathrm{c}\theta\delta)$ | $(\alpha + \gamma - \sigma, \alpha + \gamma - \sigma)$ | $(\alpha + \gamma, \beta)$ |
| $R$ | $(\beta, \beta - \mathrm{q})$ | $(\beta, \alpha + \gamma)$ | $(\beta, \beta)$ |

In table below we present physical interpretation of model's parameters:

Table10: Model's parameters

| Parameter | Definition |
|---|---|
| $\beta$ | individuum's payoff |
| $q$ | vaccination cost |
| $c$ | treatment costs |
| $\theta$ | probability that vaccination is not effective |
| $\delta$ | probability of transition from a state Susceptible to the state Infected |
| $\alpha = (\beta - c)$ | income with the cost of treatment |
| $\gamma = c\phi$ | payoff growth, on conditions that Infected transfer to Recovered |
| $\phi$ | probability of transition from a state Infected to the state Recovered |
| $\sigma = c\psi$ | payoff decrease, when individuum gets worse |
| $\psi$ | probability that individuum get worse when one Infected meets another |

### 3.1. Numerical simulation with Gillespie algorithm

In this section we use modified Gillespie algorithm for the model of epidemic process with vaccination.

1. At time $t$ calculate transition rate $p_i(t)$, $i \in N$ for each susceptible and infected individual. Here $N$ is number of all individuals in population. The rate at which susceptible individuals transfer to infected is $p_i(t) = \delta m$, $m$ is number of infected neighbors. $m(t) \in (0, \ldots, I(t))$, $I(t)$ is number of infected individuals. The rate at which infected individuals transfer to recovered is $p_i(t) = \psi$. Total transition rate is $\lambda(t) = \sum\limits_i p_i(t)$.
2. Chose the time interval $\Delta t$ then next transition occurs in moment $t' = t + \Delta t$.
3. Verify the transition condition between subgroups for each individuum $k$: $\sum\limits_{j=1}^{k-1} p_j(t)/\lambda(t) < z < \sum\limits_{j=1}^{k} p_j(t)/\lambda(t)$, $\sum\limits_{j=1}^{0} p_j(t)/\lambda(t) = 0$.
4. Repeat steps 1-3 until the number of infected individuals I(t) becomes 0.

For numerical simulation we suppose that only 0.1 percent of Susceptible subgroup in total urban population is vaccinated.

We use following parameters values for numerical simulations: $\beta = 1$, $q = 0,05$, $c = 0,8$, $\theta = 0,5$, $\delta = 0,9$, $\alpha = 0,2$, $\gamma = 0,7$, $\phi = 0,875$, $\sigma = 0,04$, $\psi = 0,05$; And we present results of the simulation with Gillespie Algorithm in Table 11.

Epidemic spreading in total urban population with and without vaccination in Susceptible subpopulation. Left figures show epidemic process with vaccination and right figure show process without vaccination of susceptible part of population. After Gillespie algorithm with model parameters $\delta = 0,9$ and $\phi = 0,875$ we receive, that if Susceptible subpopulation is vaccinated then epidemic continues 117 days and epidemic peak is at 18-th day. If susceptible subpopulation is not vaccinated then epidemic peak occurs at 16-th day and epidemic process continues for 110 days. This fact we can interpret as follows, if we have vaccinated subpopulation of susceptible then individuals transfer to infected with lower intensity. The frequency of the meetings between Susceptible and Infected human is rare in this case, thus epidemics proceeds longer, but the number of infected is less and hence the treatment costs are also decrease including costs of nonworking days for the working part of urban population.

Table11: Simulation results

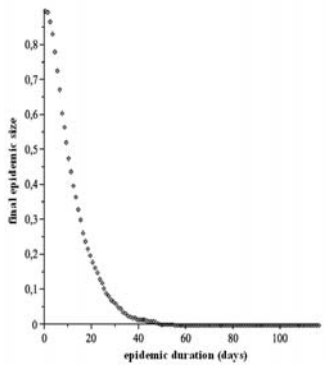
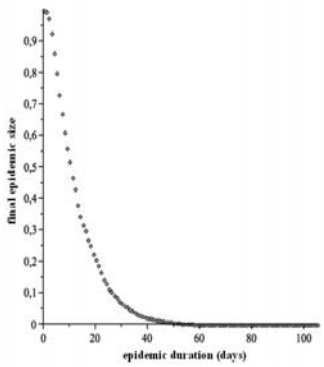
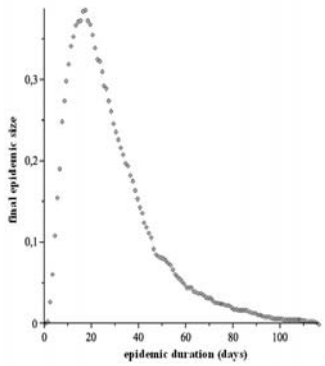
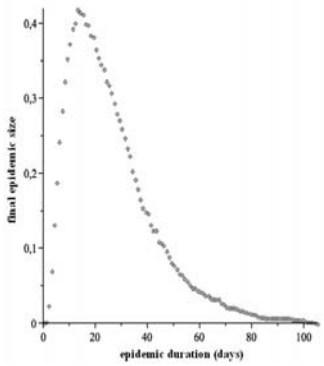
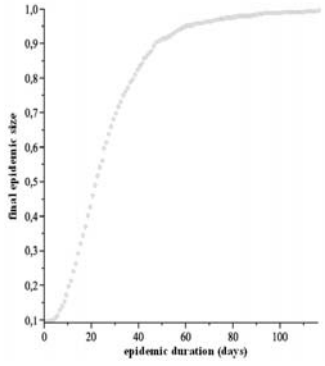
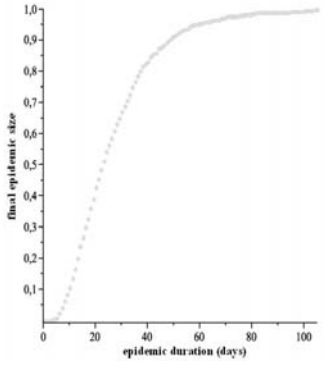
| $x_S$ — Susceptible | |
|---|---|
| Epidemic process with vaccination | Epidemic process without vaccination |
|  |  |
| $x_I$ — Infected | |
| Epidemic process with vaccination | Epidemic process without vaccination |
|  |  |
| $x_R$ — Recovered | |
| Epidemic process with vaccination | Epidemic process without vaccination |
|  |  |

Table12: Simulation results

| Epidemic process with vaccination, (initial values) | Epidemic process without vaccination, (initial values) |
|---|---|
| $x_S(0) = 0,899;$<br>$x_I(0) = 0,001;$<br>$x_R(0) = 0,1;$ | $x_S(0) = 0,999;$<br>$x_I(0) = 0,001;$<br>$x_R(0) = 0,0;$ |
| Epidemic process with vaccination, (at epidemic peak $\overline{T}$) | Epidemic process without vaccination, (at epidemic peak $\overline{T}$) |
| $x_S(\overline{T}) = 0,239;$<br>$x_I(\overline{T}) = 0,387;$<br>$x_R(\overline{T}) = 0,374;$ | $x_S(\overline{T}) = 0,323;$<br>$x_I(\overline{T}) = 0,417;$<br>$x_R(\overline{T}) = 0,26;$ |

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