Investments in Productivity and Quality under Trade Liberalization: Monopolistic Competition Model *

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Abstract We study impact of trade liberalization on firms productivity and product quality in a monopolistic competition model. Utility has variable elasticity of substitution (VES), a producer can invest in decreasing marginal cost or in increasing quality and free entry drives profits to zero. Then in a closed economy such investments increase with the market size if and only if utility shows increasing "relative love for variety" which is elasticity of the inverse demand. Expanding these findings to international trade setting, we expect to find comparative statics of equilibria with respect to the market size and trade costs.

Keywords: investments, quality, monopolistic competition, trade liberalization, relative love for variety, country size.

1. Introduction

The cross-countries differences in productivity and quality are noticeable and there can be various explanations. Recent empirical papers on international trade show that (1) firms operating in bigger markets have lower markups, see e.g. (Syverson, 2007); (2) firms tend to be larger in larger markets, see e.g. (Campbell and Hopenhayn, 2005); (3) larger economies export higher volumes of each good, export a wider set of goods, and export higher-quality goods, see e.g. (Hummels and Klenow, 2005; (4) within an industry there can be considerable firm's heterogeneity: firms differ in efficiency, in exporting or not (associated with high/low efficiency), in wages, see review in (Reddings, 2011); (5) investment decisions are positively correlated with export status of a firm, see e.g. (Aw et al., 2008).

Modern theoretical explanation of these and other empirical regularities is based on some variations of Krugman's monopolistic competition model (Krugman, 1979)

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and its variant with heterogeneous firms, suggested by (Melitz, 2003). Unfortunately, mainly these findings are based on specific functional form of the preferences, namely utility function with constant elasticity of substitution (CES-function). This functional form has some unavoidable shortcomings broadly criticized in the literature, see e.g. (Ottaviano et al., 2002), (Behrens and Murata, 2007) and (Zhelobodko et al., 2010). Thereby we find it reasonable to put efforts in constructing related theory for utility function with *variable elasticity of substitution.*

To obtain one of possible explanations of regularities (1)-(5) mentioned, this project develops such monopolistic competition models from (Zhelobodko et al., 2010) and (Zhelobodko et al., 2011) towards endogenous choice of technology. The first part of this project models a closed economy in the spirit of (Vives, 2008) (and references within this paper). However, these papers consider partial equilibrium in oligopoly setting, whereas we take general equilibrium of monopolistic competition, which is more suitable for international trade. Our basic question is the impact of market size on equilibrium investment, prices, number of firms in the industry, total investments in the economy. We show below that this model with endogenous investments is equivalent to the model with endogenous quality. Thereby we simultaneously answer on the questions, how the market size influences not only on the mass of varieties, but also on the quality. Main idea is that when a firm sells to 1.5 billion of people (we have in mind China), it has more incentives to invest a fixed amount in decreasing its variable cost or in increasing its quality to exploit economies of scale.

Respectively, when two or more countries decrease the trade barriers, these economies of scale should work in the same direction and respective welfare gains can be as important as the increase in variety of goods (Krugman, 1979) and selection of best producers (Melitz, 2003), and the second, main part of our project is devoted to international trade. How liberalization of trade (decrease in trade costs) and countries size influence the investment decisions, the size of the firms, prices, and the number of firms in the industry? In particular, the countries of different size can choose different production technologies, thereby endogenous cross-country heterogeneity may appear and generate the Ricardian motive for trade. Here most close analogue is the paper (Tanaka, 1995) that considers the Slap's model of circular city and influence of trade liberalization on quality. The main finding is that trade liberalization decreases the product quality. Another paper (Fan, 2005) considers competition in the market of intermediate goods and studies influence of the country size on the quality of goods under specific functional forms of preferences and production functions. Several papers are devoted to influence of trade on technology adoption in the case of heterogeneous firms. In (Bustos, 2011) the classical Melitz model is supplemented by endogenous choice of technology. The set of available technologies is discrete: low marginal costs with high fixed costs or the opposite one. The finding is that trade liberalization increases the share of firms using high-investment technology. In (Yeaple, 2005) the set of technologies is also discrete. Every producer chooses not only technology, but also the quality of labour used (there are two types of the labour, skilled and unskilled). Then exporting firms are larger, employ more advanced technologies, pay higher wages, are more productive, and a reduction in trade frictions can induce firms to switch technologies, leading to an expansion of trade volumes. The main departures of this paper from those described above are: (1) our utility function is of general form; (2) our set of technologies is continuous; (3) we assume Indra-country homogeneity of producers. Of course, homogeneity of producers is a restrictive assumption and the next step would be to model the technology choice model with intra-country heterogeneity, in the spirit of (Zhelobodko et al., 2011) and similar paper (Dhingra and Morrow, 2011) (in both papers technology is fixed).

In Section 2 we introduce our model of closed economy and Section 3 gives related preliminary results. In Section 4 we introduce an open economy model and pose the questions studied.

2. Basic model: closed economy with endogenous technology

We start now modeling the closed economy with endogenous investments in technology, like in (Vives, 2008) but in general equilibrium and monopolistic competition.

Our model departures from the standard Dixit-Stiglitz monopolistic competition model (Dixit and Stiglitz, 1977) to generalize their approach in two directions: the possibility of investment in productivity and quite general utility function. We consider one sector and one production factor interpreted as labour. There are two types of agents: big number L of identical consumers and an endogenous interval $[0, N]$ of identical firms. Each firm produces her own variety of some "aggregate commodity" and the nature of substitution among these varieties determines different outcomes of competition, as we show.

2.1. Consumer

Each consumer maximizes her utility under the budget constraint through choosing an infinite-dimensional consumption vector (integrable function) $X : [0, N] \to R_+$. All consumers behave symmetrically, so consumer's index is redundant. As in (Krugman, 1979), (Vives, 1999) and (Zhelobodko et al., 2011), the preferences are described by the additive-separable utility function maximized under the budget constraint:

$$\max_X \int_0^N u(x_i)di, \qquad s.t. \quad \int_0^N p_i x_i di \leq w + \frac{\int_0^N \pi_i di}{L} = 1.$$

Here N is the endogenous mass of firms or the length of the product line, i.e, the scope (the interval) of varieties. Scalar x_i is "consumption" or amount of $i-th$ variety consumed by each consumer and $X = (x_i)_{i \leq N}$. Elementary utility function $u(\cdot)$ satisfies

$$u(0) = 0, \ u'(x_i) > 0, \ u''(x_i) < 0, \ r_{u'}(x_i) \equiv -\frac{x_i u'''(x_i)}{u''(x_i)} < 2,$$

i.e., it is everywhere increasing, strictly concave and its concavity is restricted to guarantee strict concavity of producer's profit, that ensures equilibria symmetry and uniqueness, see (Zhelobodko et al., 2010). We need not additional restrictions like homothety or CES. Instead, as in (Krugman, 1979), (Vives, 1999) and (Zhelobodko et al., 2011), we use the Arrow-Pratt measure of concavity defined for any function g as

$$r_g(z) = -\frac{zg''(z)}{g'(z)}.$$

Through varying such characteristics for utility and cost functions we classify market outcomes below, because $r_u = 1/\sigma$ is the inverse to elasticity of substitution σ among varieties and to elasticity of demand for each variety (standardly). Moreover, function r_u being the elasticity of inverse demand function, it measures "love for variety" (Zhelobodko et al., 2011).

In the budget constraint w is wage, p_i is the price for x_i and π_i is the profit of $i-th$ firm. So, the income of the each consumer consists of the unit of the labour offered nonelastically,[1] and fare share of total profit in the economy. However, actually $\pi_i = 0$ at the equilibrium due to free-entry condition. Finally, since we consider the general equilibrium model, the price level and wage can be normalized to $w \equiv 1$, therefore income amounts to 1.

The First Order Condition (FOC) for consumer's maximization program, standardly, entails the inverse demand for $i-th$ variety in the form

$$p(x_i, \lambda) = \frac{u'(x_i)}{\lambda}. \tag{1}$$

Thus, willingness to pay depends both on quantity of the individual consumption x_i and on marginal utility of income λ and increasing marginal utility of income λ leads to a decrease in demand.

2.2. Producer

On the supply side, we standardly assume one-to-one correspondence: each variety is produced by one firm that produces a single variety. However, unlike classical setting, each producer chooses the technology level. Namely, if she spends f units of labour as fixed costs, then total costs of producing y units of output is $c(f)\,y + f$ units of the labour. It is natural to assume decreasing and convex costs:

$$c'(f) < 0, \quad c''(f) > 0, \quad \lim_{f\to\infty} c(f) > c_0 > 0.$$

Thereby more expensive factory would have smaller marginal costs, investment in productivity shows decreasing returns to scale and cost cannot fade to zero, being higher than some positive c_0.[2]

Using the inverse demand function $p(x_s, \lambda)$ from (1), the profit maximization of $s-th$ producer can be formulated as[3]

$$\pi_s\left(x_s, f_s, \lambda\right) = \left(p(x_s,\lambda) - c(f_s)\right) L x_s - f_s = \left(\frac{u'(x_s)}{\lambda} - c(f_s)\right) L x_s - f_s \to \max_{x_s\geq 0, f\geq 0}.$$

Under our assumption about continuum of producers, it is standard to prove that each producer s has a negligible effect on the whole market, i.e. the Lagrange multiplier λ can be treated as (positive) constant by each s. This Lagrange multiplier is the natural aggregate statistic of the market: the bigger is marginal utility of income λ, the smaller is the demand and therefore smaller the profit of producers.

[1] It means that consumer sells her unit of labour under any wages, prices, etc.

[2] This assumption is needed for nonnegative cost function, Second Order Condition (SOC) and existence of maximum in profit maximization.

[3] Standardly, maximization of monopolistic profit w.r.t. price or quantity gives same results.

Thereby this λ can be interpreted as the degree of competition among the producers of differentiated goods, like a price index in standard Dixit-Stiglitz model, see (Zhelobodko et al., 2011).

Each producer maximizes profit w.r.t. supply x and investment f and FOC is

$$\frac{u''(x_s)x_s + u'(x_s)}{\lambda} - c(f_s) = 0, \tag{2}$$

$$c'(f_s)Lx_s + 1 = 0. \tag{3}$$

These equations are valid under SOC:

$$u'''(x_s)x_s + 2u''(x_s) < 0 \Leftrightarrow r_{u'}(x_s) < 2, \tag{4}$$

$$-\frac{\left(u'''(x_s)x_s + 2u''(x_s)\right)c''(f_s)x_s}{\lambda} - \left(c'(f_s)\right)^2 > 0. \tag{5}$$

Since for each producer the maximization profit problems are the same, further we consider only symmetric equilibria and denote $x_s = x, \ f_s = f \ \forall s$.

2.3. Equilibrium

Entry. Like in standard monopolistic competition model, we assume that firms enter into the market until their profit remains positive. Therefore free entry implies zero-profit condition

$$\frac{u'(x)}{\lambda} - c(f) = \frac{f}{Lx}. \tag{6}$$

Labour balance. Under symmetric equilibrium ($f_i = f$, $x_i = x$) the balance in labour market takes the form

$$\int_0^N \left(c(f_i)x_iL + f_i\right) di = N\left(c(f)xL + f\right) = L. \tag{7}$$

Summarizing, we define *symmetric equilibrium* as a bundle $(x^*, p^*, \lambda^*, f^*, N^*)$ satisfying the following relations:

1) rationality in consumption (1);
2) rationality in production (2)-(3) and (4)-(5);
3) free entry condition (6) and balance in labour market (7).

Now we rewrite the equilibrium equations in more convenient form, using the Arrow-Pratt measure of concavity $r_g(x)$ for any function g and excluding λ.

Proposition 1. *Equilibrium consumption/investment couple (x^*, f^*) in one-sector model with endogenous technology is the solution to the system*

$$\frac{r_u\,(x)\,x}{1 - r_u\,(x)} = \frac{f}{Lc(f)},$$

$$\left(1 - r_{\ln c}(f) + r_c(f)\right)\left(1 - r_u(x)\right) = 1,$$

under the conditions

$$r_u\,(x) < 1, \qquad \left(2 - r_{u'}\,(x)\right) r_c\,(f) > 1.$$

Subsequently equilibrium mass of firms N^* is determined by equation

$$N = \frac{L}{c(f)xL + f}.$$

2.4. Another interpretation: quality

Consider now similar setting where producer chooses investments in quality of production instead of investments in productivity. This setting is shown now to be equivalent, amounting to the same system of equilibrium equations. Therefore, the study of comparative statics of the model with endogenous productivity or endogenous quality is the *same*; these are two similar manifestations of endogenous technology.

The utility of each consumer takes the form (cf. (Tirole, 1988)) $\int_0^N u(q_i x_i)di$, whereq_i is the quality level of $i-th$ variety and x_i is related consumption volume, so that $z_i = q_i x_i$ is the satisfaction volume. Then, from FOC similar to previous one, the inverse demand function for $i-th$ variety with quality q_i is

$$p(x_i, q_i, \lambda) = \frac{q_i u'(q_i x_i)}{\lambda}.$$

Under output $y_i = Lx_i$ the cost function of $i-th$ producer is

$$\tilde{c}(q_i)y_i + \tilde{f}(q_i),$$

where, as before, $\tilde{c}(q_i), \tilde{f}(q_i)$ are marginal and fixed costs respectively. It is natural to assume that both derivatives with respect to q_i are positive, $\tilde{c}'(q_i) > 0, \tilde{f}'(q_i) > 0$ because to increase the quality of the production the producer should spend more labour per unit and also use more expensive technology. Thus, the profit maximization of $i-th$ producer amounts to

$$\left(\frac{q_i u'(q_i x_i)}{\lambda} - \tilde{c}(q_i)\right) Lx_i - \tilde{f}(q_i) \to \max_{x_i \geq 0, q_i \geq 0}.$$

Let us introduce auxiliary variables $f_i = \tilde{f}(q_i)$. Due to monotonicity of function $\tilde{f}$, there exists one-to-one correspondence between quality of production q_i and the value of fixed costs f_i :

$$q_i = q_i(f_i) = \tilde{f}^{-1}(f_i).$$

Besides, we define

$$c\left(f_i\right) = \frac{\tilde{c}(q_i(f_i))}{q_i(f_i)}, \qquad z_i = q_i(f_i)x_i.$$

Then the problem of $i-th$ producer can be rewritten in the following equivalent form:

$$\left(\frac{u'(z_i)}{\lambda} - c\left(f_i\right)\right) Lz_i - f_i \to \max_{z_i \geq 0, f_i \geq 0}.$$

Obviously, this program appears equivalent to the program with endogenous productivity introduced previously. Thus, the model with endogenous quality is mathematically equivalent to the model with endogenous investments, only quantity consumed is measured now in satisfaction z.

3. Preliminary comparative statics for closed economy

Let us study the impact of market size L on the equilibrium variables: prices p, firm size Lx and mass of firms N, investment of each firm f, and total investments in the economy (Nf). For this purpose we consider the system of equilibrium equalities as implicit function connecting (x, f, N) and L. After direct differentiation and long rearrangements we obtain elasticities of main variables (in terms of concavity of basic functions), elasticities signs, and classify the market outcomes according to increasing/decreasing $r_u(x)$ (Increasing Elasticity of the Inverse Demand – IEID or Decreasing Elasticity of the Inverse Demand – DEID).

Proposition 2. *Elasticities of the equilibrium variables w.r.t. market size L in one-sector economy are*

$$\mathcal{E}_x = \frac{(1-r_{\ln c})(1-r_u)}{(2-r_{u'})r_c-1}, \qquad \mathcal{E}_{Lx} = \frac{r_c r_u' x}{((2-r_{u'})r_c-1)r_u}$$

$$\mathcal{E}_f = \frac{r_u' x}{((2-r_{u'})r_c-1)r_u}, \qquad \mathcal{E}_{Nf} = \frac{(1-r_{\ln c})^2(1-r_u)^2}{((2-r_{u'})r_c-1)r_c} + \frac{1}{r_c} + r_u,$$

$$\mathcal{E}_N = r_u - \frac{(1-r_{\ln c})(1-r_u)^2}{(2-r_{u'})r_c-1},$$

$$\mathcal{E}_p = -\frac{r_c r_u' x}{(2-r_{u'})r_c-1}, \qquad \mathcal{E}_{\frac{p-c}{p}} = \frac{(1-r_{\ln c})r_u' x}{(2-r_{u'})r_c-1},$$

and their signs satisfy classification Table 1:

Table1: The signs of elasticities of the equilibrium variables w.r.t. market size L in one-sector economy

	DEID		CES	IEID		
	$r_u' < 0$		$r_u' = 0$	$r_u' > 0$		
	$r_{\ln c} \leq 1$	$r_{\ln c} > 1$	$r_{\ln c} \neq 1$	$r_{\ln c} > 1$	$r_{\ln c} = 1$	$r_{\ln c} < 1$
$\mathcal{E}_x$	$\nexists$	$-$	$-$	$-$	0	$+$
$\mathcal{E}_{Lx}$	$\nexists$	$-$	0	$+$	$+$	$+$
$\mathcal{E}_f$	$\nexists$	$-$	0	$+$	$+$	$+$
$\mathcal{E}_{Nf}$	$\nexists$	$+$	$+$	$+$	$+$	$+$
$\mathcal{E}_N$	$\nexists$	$+$	$+$	$+$	$+$	?
$\mathcal{E}_p$	$\nexists$	$+$	0	$-$	$-$	$-$
$\mathcal{E}_{\frac{p-c}{p}}$	$\nexists$	$+$	0	$-$	0	$+$

In Table 1, $\mathcal{E}_x$ is the elasticity of consumption of one variety, $\mathcal{E}_f$ - is the elasticity of investment per firm, $\mathcal{E}_N$ - the elasticity of mass of firms, $\mathcal{E}_{Lx}$ - the elasticity of total output of one variety, $\mathcal{E}_{Nf}$ - the elasticity of total investment, $\mathcal{E}_p$ - the elasticity of price, $\mathcal{E}_{\frac{p-c}{p}}$ - the elasticity of mark-up (we drop the proofs).

Commenting, we would say that generally these results are rather similar to conclusions in (Vives, 2008) obtained for oligopolistic model. However, our table shows in more details the influence of country/market size on related economy; Vives mainly uncovered the influence of market size on investment decisions. As we can see from the table, there can be five different types of the equilibria. For increasing/decreasing investments, utility characteristic r_u' is the criterion. Namely, standard CES case (implying constant elasticity of demand and degenerate reactions to market size) is the borderline between markets with decreasing (DEID) or increasing (IEID) elasticity of inverse demand. Main finding is that DEID *class show decreasing investments* whereas under IEID *individual investments increase in response to growing market* and investment is always positively correlated with the size of the firm. Decrease of both Lx and f happens under DEID because the mass of firms grows too fastly, excessive competition makes the output shrinking, outweighing the market-size motive to invest in marginal productivity/quality. Nevertheless, total economy investment Nf always grows because growing mass of firms dominates even when f decreases.

As to prices in this model, they respond to market size in the same way as we observed under exogenous technology (f, c) in (Zhelobodko et al., 2010); there prices also increased under DEID but decreased under IEID preferences. This discrepancy got a clear explanation. In all cases increasing market has positive effect on profits of existing firms, that invites new firms into the industry, so N increases. Then growing competition pushes marginal utility of income λ up, so consumption x of each variety decreases. Paradoxically, under IEID too convex inverse demand function (shifting down with growth of λ) pushes the price up in response. Such regularity generally remains valid under our endogenous technology, though in essence it somehow combines comparative statics w.r.t. market size L with comparative statics in costs c and f. Table 1 shows that market size effects on price generally prevail, though consumption and mass of firms may behave in a new fashion in the exotic case ($r'_u > 0$, $r_{\ln c} < 1$).

Interestingly, the nature of the cost function turns out to be the criterion only for increasing/decreasing individual consumption of each variety and related variable called markup $M = \frac{p-c}{p}$. Under sufficiently big elasticity of cost to investment expressed in condition $r_{\ln c}(f) > 1$ individual consumption decreases, otherwise it does not. The first column of the table was proved to be empty; equilibria here inexist (Table 1 also do not mention the case $r'_u = 0$, $r_{\ln c} = 1$ where equilibria are indeterminate). Existence of equilibria in the last two columns is an open question, whereas numerical examples for the middle columns are already constructed and confirm valid equilibria with all effects described by Table 1.

In the future research we plan to supplement the study of closed economy by more examples and economic interpretations. Additionally, for the version of the model with endogenous quality we plan to supplement the classification table with the behavior of more variables: $\frac{xL}{q(f)}-$ the size of the firm adjusted for quality, $pq(f)-$ the price of each variety adjusted for quality. To simplify analysis we can assume, for example, linear dependence between quality and fixed costs in the form $\tilde{f}(q) = aq$.

4. Open economy: trade model

In the previous section we have described the preliminary results for the model of closed economy. To expand this analysis to trade, now we introduce a trade model. The economy consists of two countries, "Home" (H) and "Foreign" (F), one production factor (labour) and one differentiated sector including continuum of varieties or brands. There can be a different approach: it is usual in international trade theory to assume some second sector ("agriculture") with constant returns to scale using the same labour, the trade of agricultural good going without transport costs. [4] Instead, motivated by findings in (Davis, 1998) and (Yu, 2005), we proceed with one-sector assumption, to clarify the effect of (generally non-equalized) wages on the equilibrium.

Let L be the total population of the world; $s \in [0, 1]$ is the share of population in country H (so $1-s$ is the share of population in country F). Then the population in country H equals sL, while in country F it equals $(1-s)L$. As in the case of closed economy, we assume that each consumer in each country has one unit of labour offered nonelastically. We denote wages in country H and in country F as w^H and w^F correspondingly. In what follows we normalize wages as $w^H = w$ and $w^F = 1$, without a loss of generality. Then the utility-maximization program of each consumer in country H is

$$\int_0^{N^H} u(x_i^{HH})di + \int_0^{N^F} u(x_i^{FH})di \to \max_{x^{HH}, x^{FH} \geq 0}$$

$$s.t.$$

$$\int_0^{N^H} p_i^{HH} x_i^{HH} di + \int_0^{N^F} p_i^{FH} x_i^{FH} di \leq w,$$

where N^H and N^F are masses of firms in country H and country F correspondingly, p_i^{HH}, x_i^{HH} denote price and individual consumption of variety i produced in country H and consumed in country H. Similarly, p_k^{FH}, x_k^{FH} are prices and consumption of the variety k produced in country F and consumed in country H. The program of a consumer in country F is analogous. [5]

Let f^H and f^F be fixed costs in countries H and F (chosen endogenously); $c^H = c(f^H)$ and $c^F = c(f^F)$ be the marginal costs in country H and country F (as before, we assume that $c'(f) < 0$). Besides, let us assume, standardly, that the trade incurs some transport costs of "iceberg type", i.e. to give to the consumer in the country F the unit of the goods, the producer in country H must produce $\tau > 1$ units of the production. Thus, taking into account different wages in country H and country F, the cost function in country H (in monetary terms) is

$$w\left(c(f^H)sLx_i^{HH} + \tau c(f^H)(1-s)Lx_i^{HF} + f^H\right).$$

Similarly, cost function of the producer in in country F is

$$\tau c(f^F)sLx_j^{FH} + c(f^F)(1-s)Lx_j^{FF} + f^F.$$

[4] This assumption allows to assume away the difficult question about the dependence of wages in the country upon the size of the country (see, for instance, classical model (Krugman, 1980) with identical producers and (Melitz, 2003) with heterogeneous producers). But, as shown in (Davis, 1998) and (Yu, 2005), the simplifying assumption of wage equalization is not so innocent and can crucially change the result. In (Davis, 1998) and (Yu, 2005) classical Krugman's model is supplemented with transport costs in agricultural sector, and these costs were shown to block the home market effects. However, important are not transport costs per se, but mainly the fact that these transport costs blocks the wage equalization.

[5] In country F, p_i^{HF}, x_i^{HF} mean price and consumption of variety $i$produced in country H and consumed in country F, and p_i^{FF}, x_i^{FF} relate to variety i produced in country F and consumed in country F.

Then the profit-maximizing program of a producer in countries H is

$$sL(p_i^{HH}(x_i^{HH}) - wc(f^H))x_i^{HH} + (1-s)L(p_i^{HF}(x_i^{HF}) - \tau wc(f^H))x_i^{HF} - wf^H \longrightarrow$$

$$\longrightarrow \max_{x_i^{HH},\, x_i^{HF},\, f^H},$$

and in F it is

$$sL(p_j^{FH}(x_j^{FH}) - \tau c(f^F))x_j^{FH} + (1-s)L(p_j^{FF}(x_j^{FF}) - c(f^F))x_j^{FF} - f^F \longrightarrow \max_{x_j^{FH},\, x_j^{FF},\, f^F}.$$

Here $p_i^{Hk}\left(x_i^{Hk}\right)$ is the inverse demand function for the commodity produced by $i-th$ firm in country H for consumption in country $k \in \{H,\, F\}$, and $p_j^{Fk}\left(x_j^{Fk}\right)$ is its counterpart in F.

Like in closed economy, FOC for the producer's problem, consumer's balances, free-entry (zero-profit) conditions after some calculations lead us to

Proposition 3. *Trade equilibrium* $(x^{HH},\, x^{FH},\, x^{HF},\, x^{FF},\, w,\, N^H,\, N^F,\, f^H,\, f^F)$ *is determined by equations*

$$\frac{u'(x^{HH})}{u'(x^{FH})} = \frac{wc^H}{\tau c^F} \cdot \frac{1 - r_u(x^{FH})}{1 - r_u(x^{HH})} \tag{8}$$

$$\frac{u'(x^{FF})}{u'(x^{HF})} = \frac{c^F}{\tau wc^H} \cdot \frac{1 - r_u(x^{HF})}{1 - r_u(x^{FF})} \tag{9}$$

$$N^H \cdot \frac{wc^H x^{HH}}{1 - r_u(x^{HH})} + \tau N^F \cdot \frac{c^F x^{FH}}{1 - r_u(x^{FH})} = w \tag{10}$$

$$\tau N^H \cdot \frac{wc^H x^{HF}}{1 - r_u(x^{HF})} + N^F \cdot \frac{c^F x^{FF}}{1 - r_u(x^{FF})} = 1 \tag{11}$$

$$\frac{sr_u(x^{HH})x^{HH}}{1 - r_u(x^{HH})} + \frac{\tau\,(1-s)\,r_u(x^{HF})x^{HF}}{1 - r_u(x^{HF})} = \frac{f^H}{c^H L} \tag{12}$$

$$\frac{\tau sr_u(x^{FH})x^{FH}}{1 - r_u(x^{FH})} + \frac{(1-s)\,r_u(x^{FF})x^{FF}}{1 - r_u(x^{FF})} = \frac{f^F}{c^F L} \tag{13}$$

$$c'(f^H)(sLx^{HH} + \tau(1-s)Lx^{HF}) = -1 \tag{14}$$

$$c'(f^F)(\tau sLx^{FH} + (1-s)Lx^{FF}) = -1 \tag{15}$$

$$N^H(f^H + c^H sLx^{HH} + \tau c^H(1-s)Lx^{HF}) = sL \tag{16}$$

$$N^F(f^F + \tau c^F sLx^{FH} + c^F(1-s)Lx^{FF}) = (1-s)\,L \tag{17}$$

Here equations (8) and (9) mean the optimality in consumption in countries H and F correspondingly (the ratio of marginal utilities equal the ratio of the prices, where prices are found by FOC in the producer's problem). Equations (10) and (11) are are consumer's balances in countries H and F. Equations (12) and (13) are free-entry conditions in countries H and F. Equations (14) and (15) determine optimal choice of technology in each country. Equations (16) and (17) mean balance in the labour market. Note that, as in all general equilibrium models, one of the equations can be omitted as linear dependent from others.

Now let us discuss this equations system. One can see that under absent trade costs (i.e if $\tau = 1$) there exists an equilibrium with wage and investments equalized between countries. In this case the only difference between the countries is the number of firms. This result is not very surprising since it is similar in spirit to coincidence of the integrated equilibrium and the limited-mobility equilibrium in Heckscher–Ohlin model. However, the situation changes crucially under trade cost coefficient $\tau > 1$. In this case wage and investment decisions of firms are, generally speaking, different between countries. Then an *endogenous heterogeneity of productivity/quality among firms in international trade* should appear (since f^H and f^F are different). [6]

Actually, here our aim is to study the impact of trade costs magnitude (τ) on investments per firm and total investments, and on other equilibrium variables. Is it true that when country H is bigger ($s > \frac{1}{2}$) then this country invests more per firm and has less costs per unit? What is the ratio of total investments in countries H and F? What we can say about the relation between the size of the firm, the mass of the firms and prices in these two countries? Is it true that the wage is bigger in larger country?

To answer all these questions, we plan to study comparative statics of the above equations. At the moment it seems incredible to get complete global comparative statics for these equations. At first we plan to concentrate on local comparative statics. Namely, the solution to these equations can be considered as implicit function of two parameters, s and $\frac{1}{\tau}$. Without loss of generality we can assume that couple $\left(s, \frac{1}{\tau}\right)$ belongs to the rectangle $\left[\frac{1}{2}, 1\right] \times [0, 1]$. We plan to get the answers to the questions above for the following cases: (1) $s \approx \frac{1}{2}$ and arbitrary τ - this case corresponds to the situation of the countries with similar sizes; (2) $s \approx 1$ and arbitrary τ - this case corresponds to the situation when the size of country H exceeds considerably the size of country F; (3) $\tau \approx 1$ and arbitrary $s > \frac{1}{2}$ - this case corresponds to the situation when trade barriers between countries are sufficient low; finally, (4) $\tau \approx \infty$ and arbitrary $s > \frac{1}{2}$, it corresponds to the situation of high (almost prohibitive) trade barriers. Preliminary results show that comparative statics in trade cost coefficient τ makes mass of firms N negatively correlated with investment f which is always positively connected with the size of the firm Lx. As to the market size impact, when we call elasticity relation $\mathcal{E}_{N^H/sL} > 1$ as the "Home market effect", it turns out negatively correlated with increasing individual investment, i.e., entails $\mathcal{E}_{f^H/sL} < 0$.

[6] The technological heterogeneity appearing in this model is differ from the heterogeneity from Melitz's model. In Melitz's model are per se two types of heterogeneity - intra-country and inter-country. Intra-country heterogeneity is per se self-controlled by the firms. Each firm (unimportant why - for instance, due to business ability) has different degree of efficiency, and the owner of the firm can not influence by the own decision on this heterogeneity. In our setting of the model, there is no intra-country heterogeneity, but the owner of the firm fully controls the degree of the firm's efficiency. This is the main difference of our model from the Melitz's model. In other hand, in Melitz's model there is also inter-country heterogeneity that appears due to different volume of domestic markets in countries H and F and transport costs (about the role of asymmetry in Melitz's model see, e.g. (Baldwin and Forslid, 2010)). The presence of these two factors explain different degree of efficiency of boundary firm in countries H and F. This cause of inter-country heterogeneity presents also in our model.

To better understand the equilibrium behavior, analytical results should be supplemented with computer simulations using specific functional forms of utility function and cost function.

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