Competition in the Logistics Market

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Abstract We consider the cases of different number of logistics firms in the market which provide service for customers. The game-theoretic model of choosing order service is constructed. The model is a n-person game with perfect information where clients defined as players. We find equilibrium strategies for clients. The existence of these equilibria is proved.

Keywords: selecting problem, logistics schemes, n-person game with full information, Nash equilibrium, fully-mixed strategies.

1. Introduction

In the modern world the role of competition in the logistics market is very significant. Thus it is logical buyers desire to purchase goods or obtain services at the lowest price in the shortest time. Therefor an important role in the selection patterns of production and distribution of goods and services is the process of selecting the best service option. In this paper we will consider three company which services to build customer orders and provides various ways to make orders. Customers, in turn, refer to the company for the service, while trying to minimize the total cost of implementing the order. At the same time customers are players competing for the best option of receiving the service. There are many publications that address the selecting problems in terms of economic analysis, inventory control theory, queuing theory, statistical evaluation, network planning and management, among which we can provide (Daganzo, 1996, Langevin and Riopel, 2005, Medonza and Ventura, 2009). In (Linke et al., 2002) the problems of the world and the main challenges of such systems, set major tasks for development of the industry are studied. Practical interest in the model presented in (Ghiani et al., 2004, Nooper and Hompel, 2009).

2. The main model

Consider the logistics market with three firms transporting goods for the customers. Each firm defines its own pricing scheme(let firms 1 and 3 serve customers in turn, firm 2 serves all customers together without queue). Customers choose firm trying to minimize net value of service casualties. The game-theoretic approach used to find optimal behavior of customers considered as players.

Denote by τ_1, τ_2, τ_3 - the time of stating in system client in selecting the firm 1 , 2 or 3, respectively, so

$$\tau_1 = \tau_{11} + \tau_{12},$$

where τ_{11} - waiting time of the order by firm 1, τ_{12} - the service time by firm 1;

$$\tau_2 = \tau_{22},$$

as waiting time of service at the firm 2 is zero, where τ_{22} - the service time by firm 2;

$$\tau_3 = \tau_{31} + \tau_{32},$$

where τ_{31} - waiting time of the order by firm 3; τ_{32} - the service time by firm 3.

The parameters τ_1, τ_2, τ_3 are random variables. Define the cost to the customer service by each firms.

Let c_1 - the cost of customer order fulfilment by firm 1, it is fixed and does not depend on the duration of the order the customer. Assume further that c_2 - the cost of customer order fulfilment by firm 2, depending on the duration of customer service by firm 2: $c_2 = c_{21} + c_{22}\tau_{22}$, where c_{21} - fixed price charged for customer order, c_{22} - the cost per unit time customer service by firm 2. And finally, c_3 - the cost of customer order fulfilment by firm 3, $c_3 = c_{32}$, where $c_{31} = 0$ - fixed price charged for customer order is equal to zero, c_{32} - the cost per unit time customer service by firm 3. In addition to the cost of order customers have losses associated with waiting for the order. Let r - specific losses incurred by the client while waiting for the order, then we can determine the total loss associated with the expectation of the order by firm 1, 2 or 3, which will be determined by the following formulas:

$$r\tau_1 = r(\tau_{11} + \tau_{12}),$$

$$r\tau_2 = r\tau_{22},$$

$$r\tau_3 = r(\tau_{31} + \tau_{32}).$$

Now it is possible to calculate the full loss of clients to service devices 1 and 2, respectively:

$$\tilde{Q}_1 = r\tau_1 + c_1,$$

$$\tilde{Q}_2 = (r + c_{22})\tau_{22} + c_{21},$$

$$\tilde{Q}_3 = r\tau_{31} + (r + c_{32})\tau_{32}.$$

Then the average loss of customers for services provided by different firms are determined by the following expectations:

$$Q_1 = E\tilde{Q}_1 = r(E\tau_{11} + E\tau_{12}) + c_1,$$

$$Q_2 = E\tilde{Q}_2 = (r + c_{22})E\tau_{22} + c_{21},$$

$$\tilde{Q}_3 = rE\tau_{31} + (r + c_{32})E\tau_{32}.$$

The problem of the system with two service devices each of which establishes its own order of service was considered in (Bure, 2002) with some adjustment.

Duration of the customer service by the firm 1, 2 and 3 are independent random variables with densities functions:

$$f_1(t) = \frac{1}{\mu_1} e^{-\frac{1}{\mu_1} t}, \;\; t > 0,$$

$$f_2(t) = \frac{1}{\mu_2} e^{-\frac{1}{\mu_2} t}, \ \ t > 0,$$

$$f_3(t) = \frac{1}{\mu_3} e^{-\frac{1}{\mu_3} t}, \ \ t > 0.$$

Assume that at the point of time group of n customers comes to service . It is known that in the service of the firm 1 are k_1 customers (of which $k_1 - 1$ are in line to order fulfillment), in the service of the firm 3 are k_3 customers (of which $k_3 - 1$ are in line to order fulfillment). Each client decides which device to choose for the ordering fulfillment. Let p_i - the probability that the client i chooses device 1, $1 - p_i$ - that the client i chooses device 2.

This model leads to the n-person game, in which customers are the players who choose the order device to implement the order.

3. The game

Define the non-antagonistic game in normal form according
(Petrosyan et al., 1998):
$\Gamma =< N, \{p_i^j\}_{i \in N}, \{H_i\}_{i \in N} >$, where
$N = \{1, \ldots, n\}$ - set of players,
$\{p_i^{(j)}\}_{i \in N}$ - set of strategies, $p_i^{(j)} \in [0, 1]$, $j = 1, 2, 3$,
$\{H_i\}_{i \in N}$ - set of payoff functions.

$$H_i = -(p_i^{(1)} Q_{1i} + (1 - p_i^{(1)} - p_i^{(3)}) Q_{2i} + p_i^{(3)} Q_{3i}) \\ = -(p_i^{(1)}(Q_{1i} - Q_{2i}) + p_i^{(3)}(Q_{3i} - Q_{2i}) + Q_{2i}),$$

where $p_i^{(1)}$ is the probability of player i choose firm 1, $p_i^{(3)}$ - is the probability of player i choose firm 3, $p_i^{(2)} = 1 - p_i^{(1)} - p_i^{(3)}$ - is the probability of player i choose firm 2. We consider the casualty functions below: $h_i = -H_i$, $i = 1, \ldots, n$.

Define customer specific loss of waiting service r.

$Q_{1i} = r(t_i^{(11)} + t_i^{(12)}) + c_1$ - player i expected loss for firm 1's service, where $t_i^{(11)}$ - mean time of waiting service by firm 1, $t_i^{(12)}$ - mean time of service by firm 1.

$Q_{2i} = (r + c_{22}) t_i^{(22)} + c_{21}$ - player i expected loss for firm 2's service, where $t_i^{(22)}$ - mean time of service by firm 2.

$Q_{3i} = r t_i^{(31)} + (r + c_{32}) t_i^{(32)}$ - player i expected loss for firm 3's service, where $t_i^{(31)}$ - mean time of waiting service by firm 3, $t_i^{(32)}$ - mean time of service by firm 3.

Firms' service times are independent exponential distributed random variables with density functions $f_1(t) = \frac{1}{\mu_1} e^{\frac{1}{\mu_1} t}$, $f_2(t) = \frac{1}{\mu_2} e^{\frac{1}{\mu_2} t}$, $f_3(t) = \frac{1}{\mu_2} e^{\frac{1}{\mu_3} t}$, $(t > 0)$ respectively.

Customers choose only one of three logistic firms. There are k_1 customers on service in the firm $1(k_1 - 1$ of them are in the queue) and k_3 customers on service in the firm $3(k_3 - 1$ of them are in the queue).

4. Main results. The point of equilibrium

Theorem 1. *There exists a unique point of equilibrium* $(p_1, \ldots, p_n)$*,* $i = 1, \ldots, n$ *in the game* Γ *defined as follows:*

1) the pure strategies $p_i = (1, 0, 0)$*,*$i = 1, \ldots, n$*,* *if:*

$$\mu_1 r((k_1+1)+\frac{1}{2}(n-1))+c_1<\mu_2(r+c_{22})+c_{21},$$

$$\mu_1 r((k_1+1)+\frac{1}{2}(n-1))+c_1<\mu_3(r(k_3+1)+c_{32}).$$

2) the pure strategies $p_i=(0,1,0)$,$i=1,\dots,n$,
if:

$$\mu_2(r+c_{22})+c_{21}<\mu_3(r(k_3+1)+c_{32}),$$

$$\mu_2(r+c_{22})+c_{21}<\mu_1(r(k_1+1))+c_1.$$

3) the pure strategies $p_i=(0,0,1)$,$i=1,\dots,n$,
if:

$$\mu_3(r(k_3+1)+\frac{1}{2}r(n-1)+c_{32})<\mu_1(r(k_1+1))+c_1,$$

$$\mu_3(r(k_3+1)+\frac{1}{2}r(n-1)+c_{32})<\mu_2(r+c_{22})+c_{21}.$$

4) the fully-mixed strategies under the choice of two firms
$p_i=\Big(\frac{\mu_2(r+c_{22})+c_{21}-\mu_1 r(k_1+1)-c_1}{\frac{1}{2}\mu_1 r(n-1)},1-$
$\frac{\mu_2(r+c_{22})+c_{21}-\mu_1 r(k_1+1)-c_1}{\frac{1}{2}\mu_1 r(n-1)},0\Big),$
$i=1,\dots,n,$
if:

$$\mu_1 r((k_1+1)+\frac{1}{2}(n-1))+c_1\le\mu_3(r(k_3+1)+c_{32}),$$

$$\mu_1(r(k_1+1))+c_1\le\mu_2(r+c_{22})+c_{21}\le\mu_1 r((k_1+1)+\frac{1}{2}(n-1))+c_1.$$

5) the fully-mixed strategies under the choice of two firms
$p_i=\Big(0,1-\frac{\mu_2(r+c_{22})+c_{21}-\mu_3(r(k_3+1)+c_{32})}{\frac{1}{2}\mu_3 r(n-1)},$
$\frac{\mu_2(r+c_{22})+c_{21}-\mu_3(r_3(k_3+1)+c_{32})}{\frac{1}{2}\mu_3 r(n-1)}\Big),$
$i=1,\dots,n,$
if:

$$\mu_3(r(k_3+1)+\frac{1}{2}r(n-1)+c_{32})\le\mu_1(r(k_1+1))+c_1,$$

$$\mu_3(r(k_3+1)+c_{32})\le\mu_2(r+c_{22})+c_{21}\le\mu_3(r(k_3+1)r+\frac{1}{2}r(n-1)+c_{32}).$$

6) the fully-mixed strategies under the choice of two firms
$p_i=\Big(\frac{\mu_3(r(k_3+1)+\frac{1}{2}r(n-1)+c_{32})-\mu_1 r(k_1+1)-c_1}{\frac{1}{2}(n-1)(\mu_1 r-\mu_3 r)},0,$
$1-\frac{\mu_3(r(k_3+1)+\frac{1}{2}r(n-1)+c_{32})-\mu_1 r(k_1+1)-c_1}{\frac{1}{2}(n-1)(\mu_1 r-\mu_3 r)}\Big),$
$i=1,\dots,n,$
if:

$$\mu_1 r((k_1+1)+\frac{1}{2}(n-1))+c_1\le\mu_2(r+c_{22})+c_{21},$$

$$\mu_1(r(k_1+1))+c_1 \le \mu_3(r(k_3+1)+\frac{1}{2}r(n-1)+c_{32}),$$

$$\mu_3(r(k_3+1)+c_{32}) \le \mu_1 r((k_1+1)+\frac{1}{2}(n-1))+c_1,$$

or

$$\mu_3(r(k_3+1)+\frac{1}{2}r(n-1)+c_{32}) \le \mu_2(r+c_{22})+c_{21},$$

$$\mu_1(r(k_1+1))+c_1 \le \mu_3(r(k_3+1)+\frac{1}{2}r(n-1)+c_{32}),$$

$$\mu_3(r(k_3+1)+c_{32}) \le \mu_1 r((k_1+1)+\frac{1}{2}(n-1))+c_1.$$

7) the fully-mixed strategies

$$p_i = \Big(\frac{\mu_2(r+c_{22})+c_{21}-\mu_1 r(k_1+1)-c_1}{\frac{1}{2}\mu_1 r(n-1)},$$

$$1-\frac{\mu_2(r+c_{22})+c_{21}-\mu_1 r(k_1+1)-c_1}{\frac{1}{2}\mu_1 r(n-1)}-$$

$$\frac{\mu_2(r+c_{22})+c_{21}-\mu_3(r(k_3+1)+c_{32})}{\frac{1}{2}\mu_3 r(n-1)}, \frac{\mu_2(r+c_{22})+c_{21}-\mu_3(r(k_3+1)+c_{32})}{\frac{1}{2}\mu_3 r(n-1)}\Big)$$

,

$i = 1, \ldots, n,$
if:

$$\mu_1(r(k_1+1))+c_1 \le \mu_2(r+c_{22})+c_{21} \le \mu_1 r((k_1+1)+\frac{1}{2}(n-1))+c_1),$$

$$\mu_3(r(k_3+1)+c_{32}) \le \mu_2(r+c_{22})+c_{21} \le \mu_3(r(k_3+1)+\frac{1}{2}r(n-1)+c_{32}),$$

$$\mu_1(r(k_1+1))+c_1 \le \mu_3(r(k_3+1)+\frac{1}{2}r(n-1)+c_{32}),$$

$$\mu_3(r(k_3+1)+c_{32}) \le \mu_1 r((k_1+1)+\frac{1}{2}(n-1))+c_1.$$

Proof. If m players including the player i choose firm 1, then player i occupy any of m places in line for service from the firm 1 with probability $\frac{1}{m}$ according (Bure, 2002). Conditional expectation waiting time before service player i without the service time players already in service by firm 1, provided that l players of the m proceed player i:

$$\sum_{l=0}^{m-1} l\mu_1\frac{1}{m} = \frac{1}{m}\mu_1\sum_{l=0}^{m-1} l = \frac{1}{m}\mu_1\frac{m(m-1)}{2} = \frac{1}{2}\mu_1(m-1) \qquad (1)$$

The expectation for firm 3 service is defined similarly.

Let $P_r^{(j)}(l)$ be the probability that r players from l choose firm j , $j = 1, 2, 3$. Then we have:

$$\sum_{m=1}^{n}\frac{1}{2}\mu_1(m-1)P_{m-1}^{(1)}(n-1) = \sum_{m=0}^{n-1}\frac{1}{2}\mu_1 m P_m^{(1)}(n-1) \qquad (2)$$

This expression for firm 3 is defined similarly.

Now we obtain expression for conditional expectation full service time by firm 1, 2 ,3 respectively:

$$t_i^{(1)} = k_1\mu_1 + \frac{1}{2}\mu_1 \sum_{m=1}^{n}(m-1)P_{m-1}^{(1)}(n-1) + \mu_1 = k_1\mu_1 + \frac{1}{2}\mu_1 \sum_{l=0}^{n-1} lP_l^{(1)}(n-1) + \mu_1,$$

$$t_i^{(2)} = \mu_2,$$

$$t_i^{(3)} = k_3\mu_1 + \frac{1}{2}\mu_3 \sum_{v=1}^{n}(v-1)P_{v-1}^{(3)}(n-1) + \mu_3 = k_3\mu_3 + \frac{1}{2}\mu_3 \sum_{h=0}^{n-1} hP_h^{(3)}(n-1) + \mu_3.$$

Expected loss for firms' 1, 2 and 3 service define as follows:

$$Q_{1i} = r(k_1\mu_1 + \frac{1}{2}\mu_1 \sum_{m\ =1, m\neq i}^{n} p_m + \mu_1) + c_1,$$

$$Q_{2i} = (r + c_{22})\mu_2 + c_{21},$$

$$Q_{3i} = \mu_3(r(k_3+1) + \frac{1}{2}r \sum_{z\ =1, z\neq i}^{n} p_z + c_{32}).$$

Then the function of expected loss is given by:

$$h_i = -H_i = p_i^{(1)}Q_{1i} + p_i^{(2)}Q_{2i} + p_i^{(3)}Q_{3i} = p_i^{(1)}(Q_{1i} - Q_{2i}) + Q_{2i} + p_i^{(3)}(Q_{3i} - Q_{2i})$$

Consider the following expressions:

$$Q_{1i} - Q_{2i} = \mu_1(r(k_1+1) + \frac{1}{2} \sum_{m\ =1, m\neq i}^{n} p_m) - \mu_2(r + c_{22}) - c_{21} + c_1$$

$$Q_{3i} - Q_{2i} = \mu_3(r(k_3+1) + \frac{1}{2} \sum_{z\ =1, z\neq i}^{n} p_z + c_{32}) - \mu_2(r_i + c_{22}) - c_{21}$$

$$Q_{3i} - Q_{1i} = \mu_3(r(k_3+1) + \frac{1}{2} \sum_{l\ =1, l\neq i}^{n} p_l + c_{32}) - \mu_1(r(k_1+1) + \frac{1}{2}(1 - \sum_{l\ =1, l\neq i}^{n} p_l)) - c_1$$

Now we ready to prove that $(p_1^*, \ldots, p_n^*)$ is really the point of equilibrium using (Feller, 1984).

The following situations are possible:

1) (1,0,0), i.e. all players except player i choose only one firm 1, then under conditions

$$Q_1 - Q_2 < 0$$

$$Q_1 - Q_3 < 0$$

player i have to choose the same strategy. So we can write condition for the first case as

$$\mu_1 r((k_1+1) + \frac{1}{2}(n-1)) + c_1 < \mu_2(r + c_{22}) + c_{21},$$

$$\mu_1 r((k_1+1)+\frac{1}{2}(n-1))+c_1<\mu_3(r(k_3+1)+c_{32}).$$

2) (0,1,0) i.e. all players except player i choose only one firm 2, then under conditions

$$Q_2-Q_1<0$$
$$Q_2-Q_3<0$$

player i have to choose the same strategy. So we can write condition for the second case as

$$\mu_2(r+c_{22})+c_{21}<\mu_3(r(k_3+1)+c_{32}),$$
$$\mu_2(r+c_{22})+c_{21}<\mu_1(r(k_1+1))+c_1.$$

3) (0,0,1) i.e. all players except player i choose only one firm 3, then under conditions

$$Q_3-Q_3<0$$
$$Q_3-Q_1<0$$

player i have to choose the same strategy. So we can write condition for the third case as

$$\mu_3(r(k_3+1)+\frac{1}{2}r(n-1)+c_{32})<\mu_1(r(k_1+1))+c_1,$$
$$\mu_3(r(k_3+1)+\frac{1}{2}r(n-1)+c_{32})<\mu_2(r+c_{22})+c_{21}.$$

4)$p_i=\Big(\frac{\mu_2(r+c_{22})+c_{21}-\mu_1 r(k_1+1)-c_1}{\frac{1}{2}\mu_1 r(n-1)},$
$1-\frac{\mu_2(r+c_{22})+c_{21}-\mu_1 r(k_1+1)-c_1}{\frac{1}{2}\mu_1 r(n-1)},0\Big)$,
$i=1,\ldots,n,$

i.e. all players except player i choose between firm 1 and firm 2, then under violation of first condition in 1) and second condition in 2) and satisfaction of second condition in 1) player i have to choose the same strategy. We can write this conditions as follows:

$$\mu_1 r((k_1+1)+\frac{1}{2}(n-1))+c_1\le\mu_3(r(k_3+1)+c_{32}),$$
$$\mu_1(r(k_1+1))+c_1\le\mu_2(r+c_{22})+c_{21}\le\mu_1 r((k_1+1)+\frac{1}{2}(n-1))+c_1.$$

5) $p_i=\Big(0,1-\frac{\mu_2(r+c_{22})+c_{21}-\mu_3(r(k_3+1)+c_{32})}{\frac{1}{2}\mu_3 r(n-1)},$
$\frac{\mu_2(r+c_{22})+c_{21}-\mu_3(r_3(k_3+1)+c_{32})}{\frac{1}{2}\mu_3 r(n-1)}\Big),$
$i=1,\ldots,n,$

i.e. all players except player i choose between firm 2 and firm 3, then under violation of first condition in 2) and second condition in 3) and satisfaction of first condition in 3) player i have to choose the same strategy. We can write this conditions as follows:

if:

$$\mu_3(r(k_3+1)+\frac{1}{2}r(n-1)+c_{32})\le\mu_1(r(k_1+1))+c_1,$$

$$\mu_3(r(k_3+1)+c_{32}) \le \mu_2(r+c_{22})+c_{21} \le \mu_3(r(k_3+1)r+\frac{1}{2}r(n-1)+c_{32}).$$

6) $p_i = \Big(\frac{\mu_3(r(k_3+1)+\frac{1}{2}r(n-1)+c_{32})-\mu_1 r(k_1+1)-c_1}{\frac{1}{2}(n-1)(\mu_1 r-\mu_3 r)}, 0,$
$1-\frac{\mu_3(r(k_3+1)+\frac{1}{2}r(n-1)+c_{32})-\mu_1 r(k_1+1)-c_1}{\frac{1}{2}(n-1)(\mu_1 r-\mu_3 r)}\Big),$
$i=1,\dots,n,$

i.e. all players except player i choose between firm 1 and firm 3, then under violation of second condition in 1) and satisfaction of first condition in 1) or second condition in 3) player i have to choose the same strategy. We can write this conditions as follows:

if:

$$\mu_1 r((k_1+1)+\frac{1}{2}(n-1))+c_1 \le \mu_2(r+c_{22})+c_{21},$$

$$\mu_1(r(k_1+1))+c_1 \le \mu_3(r(k_3+1)+\frac{1}{2}r(n-1)+c_{32}),$$

$$\mu_3(r(k_3+1)+c_{32}) \le \mu_1 r((k_1+1)+\frac{1}{2}(n-1))+c_1,$$

or

$$\mu_3(r(k_3+1)+\frac{1}{2}r(n-1)+c_{32}) \le \mu_2(r+c_{22})+c_{21},$$

$$\mu_1(r(k_1+1))+c_1 \le \mu_3(r(k_3+1)+\frac{1}{2}r(n-1)+c_{32}),$$

$$\mu_3(r(k_3+1)+c_{32}) \le \mu_1 r((k_1+1)+\frac{1}{2}(n-1))+c_1.$$

$$7) p_i = \Big(\frac{\mu_2(r+c_{22})+c_{21}-\mu_1 r(k_1+1)-c_1}{\frac{1}{2}\mu_1 r(n-1)},$$
$$1-\frac{\mu_2(r+c_{22})+c_{21}-\mu_1 r(k_1+1)-c_1}{\frac{1}{2}\mu_1 r(n-1)}-$$
$$\frac{\mu_2(r+c_{22})+c_{21}-\mu_3(r(k_3+1)+c_{32})}{\frac{1}{2}\mu_3 r(n-1)},$$
$$\frac{\mu_2(r+c_{22})+c_{21}-\mu_3(r(k_3+1)+c_{32})}{\frac{1}{2}\mu_3 r(n-1)}\Big),$$

i.e. all players except player i choose between all three firms, then under violation of all conditions in 1) - 3) player i have to choose the same strategy.

Lets prove the uniqueness of the equilibrium.

Suppose that probabilities $p_i^{(j)}$ of choosing a firm j can be different $i=1,\dots,n$, $j=1,2,3$. Value $\sum\limits_{l=0}^{n-1} lP_l^{(1)}(n-1)$ equals the sum of the expectation of success (a success we mean the choice of firm 1), then we have

$$\sum_{l=0}^{n-1} lP_l^{(1)}(n-1) = \sum_{m=1, m\neq i}^{n} p_m,$$

expected loss for firm's 1 service:

$$Q_{1i} = r(k_1\mu_1 + \frac{1}{2}\mu_1 \sum_{m\ =1, m\neq i}^{n} p_m + \mu_1) + c_1,$$

Consider the expression:

$$Q_{1i} - Q_{2i} = r(k_1\mu_1 + \frac{1}{2}\mu_1 \sum_{m\ =1, m\neq i}^{n} p_m + \mu_1) + c_1 - (r + c_{22})\mu_2 - c_{21} = 0, \quad (3)$$

assuming the sum of probabilities is unknown value.

If $Q_1 - Q_2 < 0$, then players have to choose $p_i^{(1)} = 1$. If $Q_1 - Q_2 > 0$, then players have to choose $p_i^{(1)} = 0$. If both conditions violated then is uniquely determined by solving the equation (3).

$\sum\limits_{m\ =1, m\neq i}^{n} p_m$ should be equal for all $i = 1, \ldots, n$, then $p_i = p_j,\ i \neq j$.

Case $Q_3 - Q_2$ treated similarly.

Consider the expression:

$$Q_{3i} - Q_{2i} = \mu_3(r(k_3 + 1) + \frac{1}{2} \sum_{z\ =1, z\neq i}^{n} p_z + c_{32}) - \mu_2(r_i + c_{22}) - c_{21} = 0, \quad (4)$$

assuming the sum of probabilities is unknown value.

If $Q_3 - Q_2 < 0$, then players have to choose $p_i^{(3)} = 1$. If $Q_3 - Q_2 > 0$, then players have to choose $p_i^{(3)} = 0$. If both conditions violated then is uniquely determined by solving the equation (4).

$\sum\limits_{z\ =1, z\neq i}^{n} p_z$ should be equal for all $i = 1, \ldots, n$, then $p_i = p_j,\ i \neq j$.

And in the last case $Q_3 - Q_1$ we can assume that under the choice of two firms 3 or 1 each player choose firm 3 with probability p_l then it choose firm 1 with probability $1 - p_l$. Then we have

$$Q_{3i} - Q_{1i} = \mu_3(r(k_3+1) + \frac{1}{2} \sum_{l\ =1, l\neq i}^{n} p_l + c_{32}) - \mu_1(r(k_1+1) + \frac{1}{2}(1 - \sum_{l\ =1, l\neq i}^{n} p_l)) - c_1, \quad (5)$$

If $Q_3 - Q_1 < 0$, then players have to choose $p_i^{(3)} = 1$. If $Q_3 - Q_1 > 0$, then players have to choose $p_i^{(3)} = 0$. If both conditions violated then is uniquely determined by solving the equation (5).

$\sum\limits_{l\ =1, l\neq i}^{n} p_l$ should be equal for all $i = 1, \ldots, n$, then $p_i = p_j,\ i \neq j$.

So the strategies of customers optimal behavior under competition in the logistics market are found.

5. Conclusion

In this paper we consider the market of logistics service where some firms operate. Each firm prefer its own pricing police. Customers seek for service trying to minimize its operational costs. We find the optimal behavior of customers in the logistics market under choice of three firms. The Nash equilibria was found. The existence of such equilibria was prooved.

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