The Strategy of Tax Control in Conditions of Possible Mistakes and Corruption of Inspectors

Suriya Sh. Kumacheva

St.Petersburg State University,
Faculty of Applied Mathematics and Control Processes,
Bibliotherapy pl. 2, St.Petersburg, 198504, Russia
E-mail: s_kumach@mail.ru

Abstract A generalization of the game-theoretical model of tax control adjusted for possible corruption and inspectors mistakes is considered. The hierarchical model has a three-level structure: at the highest level of a hierarchy is an administration of tax authority, in the middle is an inspector, subordinated to tax administration, and at the lowest level are n taxpayers. It is supposed, that an interaction between risk-neutral players of different levels of a hierarchy corresponds to scheme "principal-to-agent".
The model is studied for the case when the penalty is proportional to the level of evasion. It is supposed that a tax inspector may turn out a bribetaker or make ineffective tax audit, i.e. make a mistake and don't reveal an existing tax evasion.
In the case of corruption a tax control supposed to be effective, i.e. reveals existing tax evasions always. As in previous models, it is supposed that fact of corruption is very difficult to reveal and an inspector is punished only for negligent audit.
In the case of ineffective auditing it is assumed that the tax inspector can mistake and miss an existing evasion with the probability, which can be considered as a part of negligent inspectors of their total number.
For every possible situation the players profit functions and optimal strategies are found.

Keywords: tax auditing, tax evasion, corruption, ineffective auditing.

1. Introduction

One of the most important aspects of modeling of taxation is the tax control. Mathematical models of tax inspection considering a corruption earlier were studied in (Chander and Wilde, 1992), (Hindriks and Keen and Muthoo, 1999) and (Vasin and Panova, 1999). Due to the mathematical tradition, founded in these works the game-theoretical model of tax audit adjusted for possible corruption and inspectors' mistakes is considered.

In the basis of this model there is a hierarchical game, described in (Kumacheva and Petrosyan, 2009). In the mentioned game the tax authority (high level of the hierarchy) and the finite number of taxpayers (low level of the hierarchy) are players, is considered.

To investigate the case of corruption let's consider an improved hierarchical model, which has a three-level structure: at the highest level of a hierarchy is an administration of tax authority, in the middle is an inspector, subordinated to tax administration, and at the lowest level are n taxpayers. As in the previous models, such as (Chander and Wilde, 1998) and (Vasin and Morozov, 2005), it is supposed that the interaction between the tax authority and each taxpayer corresponds to the scheme "principal-to-agent". The players' behaviour is supposed to be risk neutral.

For studying the case of inspectors' mistake let's suppose that the auditing is not 100%-effective and consider a probability of an inspector's mistake as a parameter of the model.

2. The Base Model

In the studied model a set of n taxpayers is considered; each of them has income level equal to i_k, where $k = \overline{1,n}$. The income of the taxpayer r_k is declared at the end of a tax period, where $r_k \le i_k$ for each $k = \overline{1,n}$. Let t be the tax rate, π be the penalty rate. These rates are assumed to be constant.

As in (Kumacheva and Petrosyan, 2009) and (Boure and Kumacheva, 2010), it is considered here that the audit of the k-th taxpayer is made by the tax authority with the probability p_k $(0 \le p_k \le 1)$. Model is constructed following the assumption, that the taxpayers are aware of these probabilities.

If the evasion is revealed as the result of the tax audit, then the evaded taxpayer should pay the penalty, which depends on the evasion's level. In (Boure and Kumacheva, 2010) the model was studied in four cases of penalties, which are known from (Vasin and Morozov, 2005):

1. the net penalty is proportional to evasion;
2. the penalty is proportional to difference between true and payed tax;
3. the penalty is restricted by the given level of the agent's minimal income in the case of his nonoptimal behaviour;
4. the post-audit payment is proportional to the revealed evaded income.

Let's consider the first case, when the penalty is proportional to evasion. In other words, if the evasion is revealed, the taxpayer should pay the underpaid tax and the penalty, both of which depend on the evasion's level.

Without consideration of possible corruption the expected tax payment of the k-th taxpayer in this case of penalty is defined from the equation

$$u_k = tr_k + p_k(t + \pi)(i_k - r_k), \tag{1}$$

where the first summand is always paid by the taxpayer (pre-audit payment), and the second – as the result of the tax auditing, made with probability p_k (post-audit payment).

Let's make the model more sophisticated by upgrading to a three-level game. The tax authority is divided on the administration and a subordinated inspector, who may be a corruptionist. As earlier, it is supposed, that the interaction between risk-neutral players of the different levels of the hierarchy corresponds to the scheme "principal-to-agent".

The tax authority sends an inspector for the tax audit with the probability p_k, which costs c_k, $k = \overline{1,n}$. For the bribe b_k audit inspector can agree not to inform his administration about the evasion revealed. With the probability $\tilde{p_k}$ the tax administration makes corruption-free re-auditing of this taxpayer, which costs $\tilde{c_k}$. Both of the audits are supposed to be effective, i. e. they reveal the existing evasion.

If a result of re-auditing is the revelation of the fact that the evasion was concealed by the inspector, the taxpayer must pay $(t + \pi)(i_k - r_k)$ (as earlier) and the inspector must pay a fine $f \cdot (i_k - r_k)$, where f is an inspector's penalty coefficient. Following (Hindriks and Keen and Muthoo, 1999), it is supposed, that the fact of corruption is very difficult to reveal and an inspector is punished only for negligent audit.

2.1. The Condition of an Evasion

The k-th taxpayer evades, if his expected payments in the case of evasion are less than the tax, which he pays, declaring his true income, i. e. the inequality

$$tr_k + p_k(t + \pi)(i_k - r_k) < ti_k,$$

which holds or, that is equivalent to:

$$p_k(t + \pi)(i_k - r_k) < t(i_k - r_k). \tag{2}$$

The condition (2) is violated, if the probability of audit $p_k = p^*$ for each $k = \overline{1, n}$, where

$$p^* = \frac{t}{t + \pi}. \tag{3}$$

2.2. The Condition of a Bribe Existence

Let's suppose, that there was an audit and an inspector identified an evasion of the k-th taxpayer. Let's define the condition, in which it is more profitable for a taxpayer to pay a bribe to an inspector, then to pay a post-audit payment. Expected payments of a taxpayer in a case, when a bribe was given, but the tax evasion was revealed whatever as a result of re-audit, is $\tilde{p_k}(t + \pi)(i_k - r_k) + b_k$. The bribe is profitable for a taxpayer, when

$$\tilde{p_k}(t + \pi)(i_k - r_k) + b_k < (t + \pi)(i_k - r_k). \tag{4}$$

It follows from this inequality, that the value of profitable bribe for the k-th taxpayer should be less than his tax payments when there was no re-audit:

$$b_k < (1 - \tilde{p_k})(t + \pi)(i_k - r_k). \tag{5}$$

It is profitable for an inspector to take a bribe, if it is more than an expected penalty, which an inspector should pay in the case of re-auditing. That is

$$b_k > \tilde{p_k} f(i_k - r_k). \tag{6}$$

Thus, we can obtain a mutually an inspector and a taxpayer beneficial bribe condition:

$$\tilde{p_k} f(i_k - r_k) < b_k < (1 - \tilde{p_k})(t + \pi)(i_k - r_k). \tag{7}$$

It means, that a bribe is possible only, when an interval

$$(\tilde{p_k} f(i_k - r_k); (1 - \tilde{p_k})(t + \pi)(i_k - r_k))$$

exists. It doesn't exist, if the probability of re-auditing takes value

$$\tilde{p}^* = \frac{t + \pi}{t + \pi + f}, \tag{8}$$

which is defined as the solution of the equation, which is got as a marginal case of the inequality (7).

3. The Stages of the Game

The studied hierarchical game can be divided on the next stages.

On the first stage a tax inspector is not considered as a separate level of hierarchy and an interaction between the tax authority and a taxpayer is studied. Being the high-level player, the tax authority makes the first move, choosing a pair of vectors: $p = (p_1, \ldots, p_n)$ and $\tilde{p} = (\tilde{p_1}, \ldots, \tilde{p_n})$. Components of vector p are values of probabilities of audits of each taxpayer, and components of vector $\tilde{p}$ are values of probabilities of the re-auditings of activities of tax inspectors. The second move is made by taxpayers: they make decisions to evade or to pay their taxes honestly, that is to declare $r_k < i_k$ or $r_k = i_k$, $k = \overline{1, n}$. If there was no tax audit, the game can be considered as finished on this stage.

If the tax authority sends an inspector for auditing of the k-th taxpayer, the second stage of the game begins. There is an interaction between an inspector and a taxpayer on this stage. Let the first taxpayer's move is a choice of the strategy of evasion. Then the strategy of his second step is a decision whether to give a bribe to an inspector or not. The strategy of the inspector, who revealed evasion, is the choice whether to take a bribe or not.

The third stage is an interaction between the administration of the tax authority and both of the subordinated levels of hierarchy. The realization of this stage does not depend on the results of the previous stage and happens, if the tax authority makes a re-audit of an inspector's activity.

Thus, players' strategies are the following. For each $k = \overline{1, n}$ the administration of the tax authority chooses probabilities p_k and $\tilde{p_k}$ of auditing of a taxpayer and re-auditing of an inspector's activity correspondingly. On the first move the taxpayer makes a decision to evade or not, on the second move – to give a bribe to an inspector or not. The strategy of the inspector, who revealed evasion, is the choice whether to take a bribe or not.

4. Possible Situations

In the model considered there are three possible situations:

1. there were an evasion and a given bribe;
2. a taxpayer evaded, but there was no corruption;
3. an honest payment due to a declaration.

Furtheron, let's consider each of them separately.

4.1. An Evasion with a Corruption

The situation of an evasion with a bribe is possible in the following cases:

1. Let conditions (2) and (7) be fulfilled, i. e. an evasion is profitable for a taxpayer and a bribe is profitable for both sides (the taxpayer and the inspector).
2. Let the condition (2) isn't fulfilled, i. e. there is a big risk of revelation of an evasion of a taxpayer. But the interval, defined in (7), exists, therefore, it is possible to reach an agreement about a bribe.

As in the previous models (Kumacheva and Petrosyan, 2009) and (Boure and Kumacheva, 2010), let's consider expected tax payments of the k-th taxpayer, k =

$= \overline{1,n}$. Let's change (1), assuming a possibility of a bribe. Then we obtain that in both cases expected tax payments are

$$u_k = tr_k + p_k[\tilde{p_k}(t+\pi)(i_k - r_k) + b_k]. \tag{9}$$

The expected payoff w_k of the k-th taxpayer is:

$$w_k = i_k - tr_k - p_k[\tilde{p_k}(t+\pi)(i_k - r_k) + b_k].$$

The inspector takes a bribe, but can be audited and fined, therefore, his expected payoff, got from auditing of the k-th taxpayer, (over his wages), is

$$J_k = p_k(b_k - \tilde{p_k} f(i_k - r_k)).$$

The tax authority's profit function in this case has the form

$$R_k = tr_k + p_k\left[\tilde{p_k}\left((t+\pi+f)(i_k - r_k) - \tilde{c_k}\right) - c_k\right]. \tag{10}$$

It should be noted that the tax authority's net income, a taxpayer's declared income and, therefore, his expected payoff in general depend on the strategy of the tax authority. Thus, the functions $R_k(p_k, \tilde{p_k})$, $r_k(p_k, \tilde{p_k})$ and $w_k(p_k, \tilde{p_k})$ will be considered further.

Choosing a strategy $r_k(p_k, \tilde{p_k})$ at the first stage of the game, the taxpayer analyzes the possibility of both an evasion and a fact of corruption. Some combination of the mentioned distortions can be realized when the expected post-audit payments of the taxpayer (the second summand of (9)) is less then his underpaid taxes:

$$p_k[\tilde{p_k}(t+\pi)(i_k - r_k) + b_k] < t(i_k - r_k).$$

Taking into account, that a bribe, which the taxpayer means to give, should satisfy the inspector ((6) is fulfilled), the last inequality takes a form:

$$p_k\tilde{p_k}(t+\pi+f) < t.$$

If this condition isn't fulfilled, the probabilities p_k and $\tilde{p_k}$ relate as follows:

$$p_k\tilde{p_k} = \frac{t}{t+\pi+f}. \tag{11}$$

A fact of choosing strategies, that satisfied (11), by the tax authority, does not let the simultaneous implementation of the evasion and bribe. Let's consider the next situations.

4.2. An Evasion without Corruption

Let p_k satisfies the condition (2). The k-th taxpayer evaded, made his declared income lower than his true level. However, he risked vainly, and the tax authority send an inspector, who revealed the tax evasion. Negotiations about a bribe are doomed to failure, because the probability of re-audit is chosen by the tax administration correspondingly to (11).

In this situation the k-th taxpayer's profit function is

$$w_k = i_k - tr_k - p_k(t+\pi)(i_k - r_k);$$

the tax authority's payoff is defined as

$$R_k = tr_k + p_k\left[((t+\pi)(i_k - r_k) - c_k) - \tilde{p_k}\tilde{c_k}\right] =$$
$$= tr_k + p_k\left((t+\pi)(i_k - r_k) - c_k\right) - \frac{t}{t+\pi+f}\,\tilde{c_k}.$$

The inspector gets nothing over his usual wages.

4.3. The Honest Payment Corresponding to the Declaration

Let's suppose, that the condition (2) is violated. If p_k and $\tilde{p_k}$ relate as in (11), a taxpayer will not risk to evade, understanding that if an evasion is revealed it will be impossible to reach an agreement with an inspector about mutually beneficial bribe.

In the considered situation the expected tax authority's net income R_k (profit function), got from the taxation of the k-th taxpayer is

$$R_k = t\,i_k - p_k(c_k + \tilde{p_k}\tilde{c_k}),$$

the taxpayer's declared income $r_k^* = i_k$, the bribe $b_k = 0$. I. e., the taxpayer's payoff is $w_k = i_k - ti_k$ (his true income level less honestly paid tax), an inspector's benefit over his usual wages is $J_k = 0$ (he does not get a bribe).

Proposition 1. *The maximum tax authority's income, got from the taxation of the k-th taxpayer, is reached when audit probability $p_k = p^*$ and re-audit probability $\tilde{p_k} = \tilde{p}^*$:*

$$\max_{p_k,\tilde{p_k}} R_k(p_k,\tilde{p_k}) = R_k(p^*,\tilde{p}^*) = ti_k - \frac{t}{t+\pi}\,c_k - \frac{t}{t+\pi+f}\,\tilde{c_k}. \tag{12}$$

Herewith the taxpayer's maximum payoff $w_k(p^*,\tilde{p}^*) = i_k - ti_k$ is reached when his declared income $r_k^* = r_k(p^*,\tilde{p}^*) = i_k$; when the taxpayer and the tax authority have such strategies, the inspector's benefit over his usual wages is $J_k = 0$.

Proof. At the first stage the tax authority's optimal strategy is a choice of the minimum value of the probability of audit, that guarantees violation of the condition (2), that is, $p_k = p^*$, where p^* is defined from the equality (3). From the results, given in (Boure and Kumacheva, 2010) it follows that this strategy is the tax authority's optimal strategy in order to maximize its income.

The k-th taxpayer will declare $r_k^* = i_k$, if on the second stage of his interaction with the tax authority the possibility of corruption is excluded, i. e., $\tilde{p_k}$ relates with $p_k = p^*$ by the condition (11). If p^* from (3) is put in (11), the minimum value of the probability of re-audit, which guarantees violation of the condition (7), will be obtained. It means that $\tilde{p_k} = \tilde{p}^*$, where $\tilde{p}^*$ is defined from (8), for each $k = \overline{1,n}$.□

4.4. Cases, Allowing Unprofitable Activities of the Tax Authority

The case of unprofitable activities of the tax authority, when the parameters t, π and c_k relate so as inequality

$$(t+\pi)i_k < c_k \tag{13}$$

holds was considered in (Boure and Kumacheva, 2010).

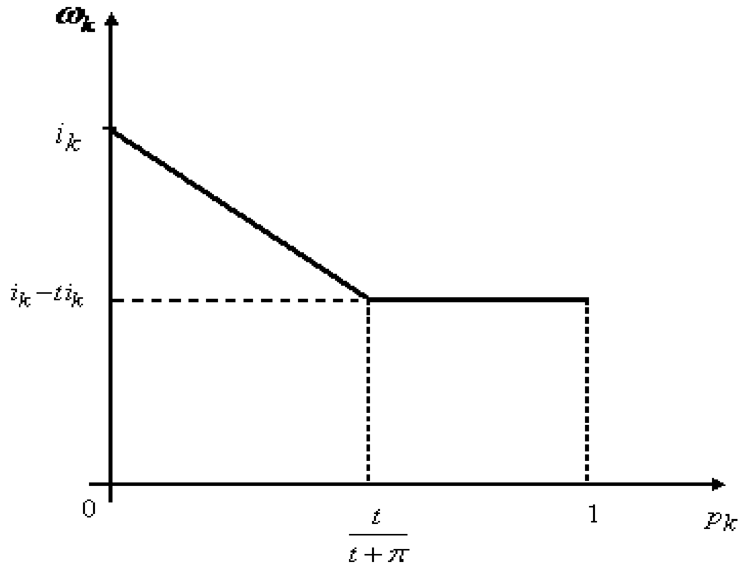

Fig. 1: Dependence the k-th taxpayer's expected profit w_k on the probability of audit p_k

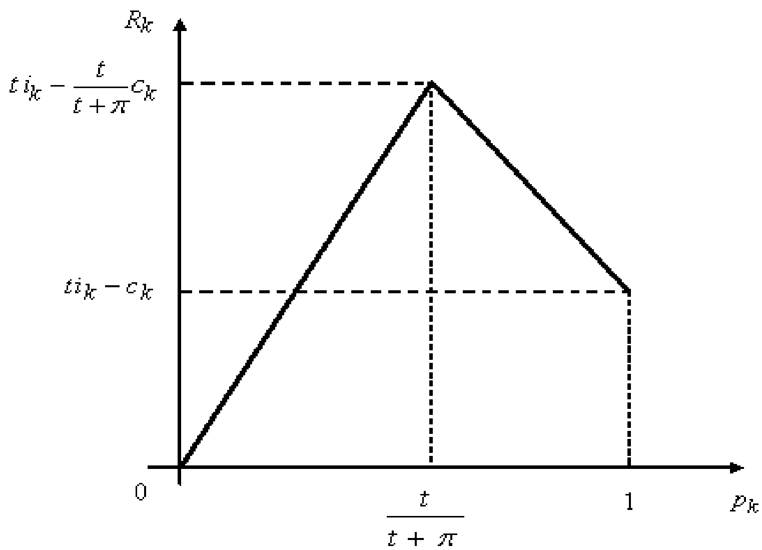

Fig. 2: Dependence the tax authority's expected profit R_k on the probability of audit p_k

The proposition was formulated for this case. This proposition implies that for the tax authority it is optimal (in order to maximize its profit function) not to audit the k-th taxpayer, because each value of the probability of the audit $p_k > 0$ gives only dead losses, i. e. $R_k \leq 0$.

If in the case of unprofitable activities of the tax authority the players act optimally (corresponding to the mentioned proposition), the game is finished on the first stage, as an inspector is not sent for auditing. Thus, there is no sense in the further interaction between players of different levels of the hierarchy.

If the inequality

$$(t+\pi)i_k \geq c_k \tag{14}$$

is fulfilled for the parameters t, π and c_k, the second stage of the game is implemented. Re-auditing with any value of the probability will be unprofitable for the tax authority, if the relations of the parameters t, π, f, c_k and $\tilde{c_k}$ such, as following inequality

$$\tilde{c_k} > (t+\pi+f)\left(i_k - \frac{c_k}{t+\pi}\right) \tag{15}$$

holds.

In this case the optimal (in order to maximize its net tax income) strategy is $\tilde{p_k} = 0$. But then for any value of p_k the taxpayer and the inspector have an opportunity to reach an agreement about bribe b_k, and, thus, the taxpayer can evade with impunity. In this case the optimal audit strategy is $p_k = 0$.

It is obvious that if (6) holds, the right side of inequality (15) becomes a negative and, therefore, it is fulfilled for any relation of $\tilde{c_k}$, π and f. Thus, inequality, opposite to (15), implies (4).

5. The Optimal Player's Strategies

Following (Chander and Wilde, 1998) and (Vasin and Morozov, 2005), let's notice, that the tax authority's strategy in general is some optimal contract (Vasin and Morozov, 2005) or optimal scheme (Chander and Wilde, 1998) $(t, \pi, p, \tilde{p}, f)$, where t, π and f are the parameters of long-term tax control, and $p = (p_1, \ldots, p_n)$ and $\tilde{p} = (\tilde{p_1}, \ldots, \tilde{p_n})$ are the strategies, chosen by the tax authority in each tax period for the k-th taxpayer, $k = \overline{1,n}$.

As it was in (Boure and Kumacheva, 2010), the tax authority's net income is defined as a sum of the payoffs R_k, $k = \overline{1,n}$. It's obvious, that

$$\max_{p,\tilde{p}} R = \sum_{k=1}^{n} R_k(p^*, \tilde{p}^*).$$

Correspondingly to the previous proposition, the maximum value of the net tax income from taxation of the k-th taxpayer is reached on a restricted class of strategies of the tax authority, which fulfills (11). The taxpayer's best reply on the tax authority's activity (due to the mentioned optimal strategies) is defined in the same proposition.

The generalization of the considered reasonings is formulated in the next theorem.

Theorem 1. *1. If a relation of parameters t, π, f, c_k and $\tilde{c_k}$ allows to make a profitable audit of the k-th taxpayer (the inequality, opposite to (15), holds), the* *maximum of the tax authority's income, got from taxation of the k-th taxpayer, is reached when the strategy of auditing (3)*

$$p_k = p^* = \frac{t}{t+\pi}$$

and the strategy of re-auditing (8)

$$\tilde{p_k} = \tilde{p}^* = \frac{t+\pi}{t+\pi+f}$$

and has a form (12). In conditions of such strategy of the tax authority the k-th taxpayer's optimal strategy (in order to maximize his payoff) is $r_k^*(p^*, \tilde{p}^*) = i_k$*; his payoff is* $w_k(p^*, \tilde{p}^*) = i_k - ti_k$.

2. *In the case, when for parameters* t, π, f, c_k *and* $\tilde{c_k}$ *holds (15), the maximum of the tax authority's income is reached when the strategy of auditing* $p^* = 0$ *and the strategy of re-auditing* $\tilde{p}^* = 0$*; its value is* $R_k = 0$*. In this case the k-th taxpayer optimal strategy is* $r_k^*(0,0) = 0$*; his payoff is* $w_k(0,0) = i_k$.
The inspector's payoff is $J_k = 0$ *in both cases.*

Thus, taxpayers' and the tax authority's optimal strategies are found in conditions of possible corruption.

6. Possible Mistakes of Inspectors

Let's suppose that the auditing is not 100%-effective. It means that tax inspectors can make unintentional mistakes and miss an existing evasion.

Let's consider a parameter μ, which has two different meanings.

On the one hand μ is the probability of an inspector's mistake. Then, from the probabilistic point of view we obtain that the value $(1-\mu)$ is the effectiveness of auditing. Therefore it can be included as an additional specifying multiply of the probability of auditing p_k in every equality.

On the other hand μ can be considered as a part of negligent inspectors of their total number. Then, the probability of re-auditing $\tilde{p_k}$ depends on μ. As in (Hindriks and Keen and Muthoo, 1999), it is considered that there is no way to identify if the auditing was negligent or the inspector was corrupted. So, as in the case of corruption, the negligent inspector pays a fine $f \cdot (i_k - r_k)$ and the tax evader pays penalty $(t+\pi)(i_k - r_k)$. To construct the optimal strategy the tax administration needs to obtain an estimation $\widehat{\mu}$ of the probability μ.

7. Conclusion

In this paper the game-theoretical model of tax control, based on the hierarchical game with a three-level structure and adjusted for possible corruption and inspectorsŠ mistake, is considered. The playersŠ profit functions and optimal strategies are found considering two mentioned features.

In the previous papers with the familiar problems (Chander and Wilde, 1992) and (Hindriks and Keen and Muthoo, 1999) and (Vasin and Panova, 1999) a binary distribution of taxpayers' income was considered. The game-theoretical model, presented in this paper, differs from the mentioned models by the assumption about a nonuniformity of taxpayers not only on the income level, but on the costs of auditing for the tax authority. Another specific feature, on which this hierarchical model was constructed, is the assumption that a strategy of the tax authority doesn't depend on the taxpayer's income, declared in given tax period.

However, it should be noted, that results, obtained for this model, highly correlate with previous conclusions, published in the papers (Chander and Wilde, 1992) and (Vasin and Panova, 1999), which are devoted to problems of taxation.

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