# A Model of Coopetitive Game and the Greek Crisis

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**Abstract** In the present work we propose an original economic coopetitive model applied to the Greek crisis. This model is based on normal form game theory and conceived at a macro level. We aim at suggesting feasible solutions in a super-cooperative perspective for the divergent interests which drive the economic policies of the countries in the euro area.

**Keywords:** Greek crisis; euro area; trade imbalances; coopetition; coopetitive games; normal form games; Kalai-Pareto solutions.

## 1. Introduction

In this contribution we focus on the Greek crisis, because Greece, which is a EU member and a country that is part of the euro area, since the end of 2009 has entered in a deep financial and economic crisis. Although Greeces GDP reaches only 2 per cent of total GDP of the whole euro area (IMF, 2011), the Greek crisis is creating many troubles to the euro area and all over the world. The risk of insolvency of Greece, mainly due to its public finance mismanagement, has represented the extreme situation of a general sovereign debt crisis which has hit the southern countries of the eurozone (PIIGS) and that has interested the whole euro area in the last three years. The Greek economy, after its accession to the euro, has lost competitiveness, due to its generous wage increases and high domestic prices induced also by ECBs monetary policy . The lack of competitiveness has created an heavy and increasing current account imbalance. Financial aid programs have been devised to help Greece by the euro area authorities and IMF in May 2010 (EU Council, 2010) and again in July 2011 (EU Council, 2011). These financial aid programs have unfortunately proved belated and insufficient. The causes of these errors are certainly of political and institutional nature and relate to the governance of the euro area, which we do not discuss in this work. However, the success of any support program is conditioned to the capacity of Greek government to meet the fiscal adjustment targets and also by the ability of the Greek economy of triggering the growth (Darvas, Pisani-Ferry, Sapir, 2011; Schilirò, 2011). Germany, on the other hand, is the most competitive economy of the euro area, it is heavily export-oriented, in fact it is the second world's biggest exporter, with exports accounting for more than one-third of national output (IMF, 2011). Thus Germany has a large current account surplus with Greece and other euro partners; hence significant trade imbalances occur within the euro area. The main purpose of our contribution is to explore win-win solutions for Greece and Germany, adopting an appropriate game theory model in which we assume Germanys increasing demand of Greek exports. In this work we do not analyze the causes of the financial crisis in Greece and, more generally, the sovereign debt crisis of the euro area with its relevant economic, financial and institutional effects on the European Monetary Union. Rather, we concentrate on the problem of the current account imbalances of Greece providing a coopetitive model which shows the possible win-win solutions. So we look to the stability and growth of the Greek economy. Such targets, in fact, should drive the economic policy of Greece and other countries of the euro area.

Organization of the paper. The work is organized as follows:

1. section 2 examines the Greek crisis, suggesting a possible way out to reduce the intra-eurozone imbalances through coopetitive solutions within a growth path;

2. sections from 3 to 6 provides an original model of coopetitive game applied to the Eurozone context, showing the possible coopetitive solutions;

3. conclusions end up the paper.

Introduction and Section 2 of this paper are written by D. Schilirò, sections from 3 to 6 are written by D. Carfì conclusions are written by the two authors, however the whole paper is written in strict joint cooperation.

**Note.** Baldwin and Gros (2010, p.4) maintain that in the period 2000-2007 The one-size monetary policy plainly failed to fit all (the euro countries). Booming economic performance in Greece, Ireland, and Spain was accompanied by prices that rose much more than average. The cumulative excess inflation was 10 percentage points for Ireland, and 8 points for Greece and Spain. The asymmetric development of output and competitiveness produced massive current account imbalances. The total current account balance of Germany has been over 5 per cent of GDP in 2011 (IMF, 2011).

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## 2. The Greek Crisis and the coopetitive solution

The severe financial and economic crisis of Greece has revealed the weaknesses of Greek economy, particularly the mismanagement of the public finance, the difficulties of the banking sector, but above all the lack of competitiveness.

### 2.1. The Greek economy and the global crisis

With the outbreak of the global crisis of 2008-2009, Greece relied on state spending to drive growth, thus the country has accumulated a huge public debt, which in 2010 amounted to 328 billion euros, that is a Debt/GDP ratio equal to 142 per cent Ð according to IMF (2011) Ð and the debt situation has worsened in 2011. This has created deep concerns about fiscal sustainability of the Greek economy, whereas its financial exposition has prevented the Greek government to find capitals in the financial markets. In addition, since joining the European monetary union, Greece has lost competitiveness especially compared to France and Germany, due to the sharp increase of unit labor costs and higher domestic prices (Boone, Johnson, 2012). The austerity measures implemented by the Greek government, although insufficient, have hit hard the Greek economy, since its rate of growth has been negative in 2010 and 2011, with an unemployment rate soaring from 12. 4 per cent in 2010 to 16.5 per cent in 2011 (IMF, 2011), making the financial recovery very problematic, as Mussa (2010) had already envisaged. Furthermore, Greek exports are much less than imports, so the current account balance has been 10.45 per cent of GDP in 2010 and 8.37 per cent of GDP in 2011 (IMF, 2011). Therefore, taking for granted the need of a fiscal consolidation, the focus of economic policy of Greece should become its productive system and growth must be the major goal for the Greek economy in a medium term perspective. However, a policy-solution that implies a greater amount of exports from Greece towards the euro countries could help its re-equilibrium process.

### 2.2. The soundest European economy: Germany

Germany, on the other hand, is considered the soundest European economy. It is the second world's biggest exporter, its wide commercial surplus is partly originated by the exports in the euro area, that accounts just above 40 per cent of its total exports, even if this share is declining (IMF, 2011). In fact, during the last twenty years from 1991 ( when the freshly unified country still traded in its own quite strong currency, the Deutsche Mark), to 2011 its export share has gradually increased vis-Ĺ-vis industrial countries, but it has also showed a changing trend, which reflects the shifting economic powers on a global scenario (see the note). Thus Germany's growth path has been driven by exports. We do not discuss in this work the factors explaining Germany's increase in export share, but we observe that its international competitiveness has been improving, with the unit labor cost which has been kept fairly constant, since wages have essentially kept pace with productivity. Consequently, the prices of the German products have been relatively cheap, favoring the export of German goods towards the euro countries, but even more towards the markets around the world, especially those of the emerging economies (China, India, Brasil, Russia). Moreover, since 2010 Germany has recovered very well from the 2008-2009 global crisis and it is growing at a higher rate than the others euro partners. Therefore we share the view that Germany in particular (but also the other surplus countries of the euro area), should contribute to overcome the Greek crisis by stimulating its demand of goods from Greece, since Germany Đ as some economists as Posen (2010) and Abadi (2010) underlined Đ has benefited from being the anchor economy for the euro area over the last 12 years.

**Note.** See also the article: EuropeÕs Economic Powerhouse Drifts East, on New York Times July 18, 2011, that highlights the shift of German exports and investments outside the euro area in the recent years (2006-2010).

### 2.3. A win-win solution for Greece and Germany

The Fiscal Compact or Fiscal Stability Treaty, the intergovernmental treaty recently signed by almost all of the member states of the European Union in March 2012 (the treaty will enter into force on 1 January 2013, if by that time 12 members of the euro area have ratified it) is probably too much focused on the budget discipline.We believe, instead, that a correct economic policy for Greece (and the other southern countries) should aim not only at adjusting government budget but also current account imbalances and, at the same time, at improving the growth path of its real economy in the medium and long term. This more complex policy , which requires a set of instruments and actions to reform the Greek economy, is probably the more suitable, although not easy to implement, for assuring a sustainable path to

Greece over time and also to contribute to the stability of whole eurozone (Schilirò, 2011). As we have just argued, Germanys relatively modest wage increases and weak domestic demand favored the export of German goods towards the euro countries and all over the world. In this context, we suggest, in accordance with Posen (2010), to look for a win-win solution (a win-win solution is the outcome of a game which is designed in a way that all participants can profit from it in one way or the other), which entails that Germany, which still represents the leading economy in Europe, should contribute to re-balance its trade surplus within the euro-area and thus ease the pressure on the southern countries of the euro area, particularly Greece. Obviously, we are aware that this is a mere hypothesis and that our framework of coopetition represents a normative model. However, we believe that a coopetitive behavior, that implies a cooperative attitude, despite the diverging interests, is the most sensible and convenient strategy that the members of the euro area should follow. A coopetitive behavior, in fact, is different form a purely cooperative attitude and it also avoid to transform the euro area into a sort of transfer union. Finally, our model does not represent a test to see whether it is convenient for Greece leave the euro or not. Therefore, we pursue our hypothesis and suggest an economic coopetive model as an innovative instrument to analyze possible outcomes to obtain a win-win solution involving Greece and Germany.

### 2.4. Our coopetitive model

The two strategic variables of our model are investments and exports for Greece, since this country must concentrate on them to improve the structure of production and its competitiveness, but also shift its aggregate demand towards a higher growth path in the medium term. Thus Greece should focus on innovative investments, specially investments in knowledge (Schilirò, 2010), to change and improve its production structure and to increase its production capacity and productivity. As a result of that its competitiveness will improve. These investments should be supported by the private investors and the government should make easier this process; moreover, in an open economy this innovative investments could come from abroad. An economic policy that focuses on investments and exports, instead of consumptions, will address Greece towards a sustainable growth and, consequently, its financial reputation and economic stability will also improve. On the other hand, the strategic variables of our model for Germany are private consumption and imports. While the coopetitive variable (or shared variable) in our model is represented by the export of Greek goods to Germany (or, if you like, by the import of Greek goods in Germany). Thus, the idea which is driving our model to contribute to overcome the economic crisis in Greece is based on a notion of coopetition where the cooperative aspect is very important, since both Germany and Greece belong to an economic and monetary union. Therefore, we are not considering a scenario in which Germany and Greece are competing in the same European market for the same products, rather we are assuming a situation in which Germany stimulates its domestic demand and, in doing so, will create also a larger market for products coming from abroad. In this situation Germany agrees to purchase a certain amount of goods imported from Greece, consequently Greece will increase its exports by selling more products to Germany. This shared variable, decided together by Greece and Germany, becomes the main instrumental variable of the model. The final result will be that Greece find itself in a better position, but also Germany will get an economic advantage determined by the higher growth in the two countries.

In addition, there is the important (indirect) advantage of a greater stability within the euro area. Finally, our model will provide a new set of tools based on the notion of coopetition, that could be fruitful for the setting of the euro area economic policy issues.

### 2.5. The coopetition in our model

The concept of coopetition was essentially devised at micro-economic level for strategic management solutions by Brandenburger and Nalebuff (1995, 1996), who suggest, given the competitive paradigm (Porter, 1985), to consider also a cooperative behavior to achieve a win-win outcome for both players. Brandenburger and Nalebuff maintains that coopetition means that Ç you have to compete and cooperate at the same time. The combination makes for a more dynamic relationship than the words competition and cooperation suggest individually (1996, pp.4-5). Therefore, coopetition becomes, in our model, a complex theoretical construct and it is the result of the interplay between competition and cooperation, since it represents the synthesis between the competitive paradigm (Porter, 1985) and the cooperative paradigm (Gulati, Nohria, Zaheer, 2000). We have already devised a coopetitive model at a macroeconomic level (Carfì, Schilirò, 2011). In this model (2011), that adopted the same variables of the present one (consumption and imports for Germany and innovative investments and exports for Greece), we have developed a coopetitive game by excluding the mutual influence of the actions (or strategies) for the two players. In other words, we excluded the dependence of the payoff functions of each player on the strategies of other players. This choice has allowed us to greatly simplify the model, secondly it has highlighted the coopetitive aspect, although at the expense of the classical feature of game theory. In the present model, instead, we continue to highlight the coopetitive strategy in its cooperative dimension, represented by the shared variable (identified in the export of Greek goods to Germany), but, in addition, we reintroduce the classical strategic interaction between the two players. Furthermore, this generalization of the model allows us to reach to competitive solutions or, better still, to a family of competitive solutions Ĺ la Nash from which to choose the win win solution. Also note that in this generalized model, competitive solutions Ĺ la Nash are not equivalent to the prisoner's dilemma solutions, because our solutions are optimal (maximum) and not minimal as in the case of the prisoner's dilemma. Therefore, our new model of coopetitive games aims at offering possible solutions to the partially divergent interests of Germany and Greece in a perspective of a cooperative attitude that should drive their policies.

## 3. Coopetitive games

### 3.1. Introduction

In this paper we develop and apply the mathematical model of a *coopetitive game* introduced by David Carfì in (Carfì and Schilirò, 2011 and Carfì, 2010). The idea of coopetitive game is already used, in a mostly intuitive and non-formalized way, in Strategic Management Studies (see for example Brandenburgher and Nalebuff).

**The idea.** A coopetitive game is a game in which two or more players (participants) can interact *cooperatively and non-cooperatively at the same time*. Even Brandenburger and Nalebuff, creators of coopetition, did not define, precisely, a *quantitative way to implement coopetition* in the Game Theory context.

The problem to implement the notion of coopetition in Game Theory is summarized in the following question:

- *how do, in normal form games, cooperative and non-cooperative interactions can live together simultaneously, in a Brandenburger-Nalebuff sense?*

In order to explain the above question, consider a classic two-player normal-form gain game $G = (f, >)$ - such a game is a pair in which $f$ is a vector valued function defined on a Cartesian product $E \times F$ with values in the Euclidean plane $\mathbb{R}^2$ and $>$ is the natural strict sup-order of the Euclidean plane itself (the sup-order is indicating that the game, with payoff function $f$, is a gain game and not a loss game). Let $E$ and $F$ be the strategy sets of the two players in the game $G$. The two players can choose the respective strategies $x \in E$ and $y \in F$

- cooperatively (exchanging information and making binding agreements);
- not-cooperatively (not exchanging information or exchanging information but without possibility to make binding agreements).

The above two behavioral ways are mutually exclusive, at least in normal-form games:

- the two ways cannot be adopted simultaneously in the model of normal-form game (without using convex probability mixtures, but this is not the way suggested by Brandenburger and Nalebuff in their approach);
- there is no room, in the classic normal form game model, for a simultaneous (non-probabilistic) employment of the two behavioral extremes *cooperation* and *non-cooperation*.

**Towards a possible solution.** David Carfì (Carfì and Schilirò, 2011 and Carfì, 2010) has proposed a manner to pass this *impasse*, according to the idea of coopetition in the sense of Brandenburger and Nalebuff. In a Carfì's coopetitive game model,

- the players of the game have their respective strategy-sets (in which they can choose cooperatively or not cooperatively);
- there is a common strategy set $C$ containing other strategies (possibly of different type with respect to those in the respective classic strategy sets) that *must be chosen cooperatively*;
- the strategy set $C$ can also be structured as a Cartesian product (similarly to the profile strategy space of normal form games), but in any case the strategies belonging to this new set $C$ must be chosen cooperatively.

### 3.2. The model for $n$-players

We give in the following the definition of coopetitive game proposed by Carfì in (Carfì and Schilirò, 2011 and Carfì, 2010).

**Definition (of $n$-player coopetitive game).** Let $E = (E_i)_{i=1}^n$ be a finite $n$-family of non-empty sets and let $C$ be another non-empty set. We define **$n$-player coopetitive gain game over the strategy support** $(E, C)$ any pair $G = (f, >)$, where $f$ is a vector function from the Cartesian product ${}^{\times}E \times C$ (here ${}^{\times}E$ denotes the classic strategy-profile space of $n$-player normal form games, i.e. the Cartesian product of the family $E$) into the $n$-dimensional Euclidean space $\mathbb{R}^n$ and $>$ is the natural sup-order of this last Euclidean space. The element of the set $C$ will be called **cooperative strategies of the game**.

A particular aspect of our coopetitive game model is that any coopetitive game $G$ determines univocally a family of classic normal-form games and vice versa; so that any coopetitive game could be defined as a family of normal-form games. In what follows we precise this very important aspect of the model.

**Definition (the family of normal-form games associated with a coopetitive game).** Let $G = (f, >)$ be a coopetitive game over a strategic support $(E, C)$. And let

$$g = (g_z)_{z \in C}$$

be the family of classic normal-form games whose member $g_z$ is, for any cooperative strategy $z$ in $C$, the normal-form game

$$G_z := (f(., z), >),$$

where the payoff function $f(., z)$ is the section

$$f(., z) : {}^{\times}E \to \mathbb{R}^n$$

of the function $f$, defined (as usual) by

$$f(., z)(x) = f(x, z),$$

for every point $x$ in the strategy profile space ${}^{\times}E$. We call the family $g$ (so defined) **family of normal-form games associated with (or determined by) the game** $G$ and we call **normal section** of the game $G$ any member of the family $g$.

We can prove this (obvious) theorem.

***Theorem.*** *The family $g$ of normal-form games associated with a coopetitive game $G$ uniquely determines the game. In more rigorous and complete terms, the correspondence $G \mapsto g$ is a bijection of the space of all coopetitive games - over the strategy support $(E, C)$ - onto the space of all families of normal form games - over the strategy support $E$ - indexed by the set $C$ .*

*Proof.* This depends totally from the fact that we have the following natural bijection between function spaces:

$$\mathcal{F}({}^{\times}E \times C, \mathbb{R}^n) \to \mathcal{F}(C, \mathcal{F}({}^{\times}E, \mathbb{R}^n)) : f \mapsto (f(., z))_{z \in C},$$

which is a classic result of theory of sets. □

Thus, the exam of a coopetitive game should be equivalent to the exam of a whole family of normal-form games (in some sense we shall specify).

In this paper we suggest how this latter examination can be conducted and what are the solutions corresponding to the main concepts of solution which are known in the literature for the classic normal-form games, in the case of two-player coopetitive games.

### 3.3. Two players coopetitive games

In this section we specify the definition and related concepts of two-player coopetitive games; sometimes (for completeness) we shall repeat some definitions of the preceding section.

**Definition (of coopetitive game).** Let $E$, $F$ and $C$ be three nonempty sets. We define **two player coopetitive gain game carried by the strategic triple** $(E, F, C)$ any pair of the form $G = (f, >)$, where $f$ is a function from the Cartesian product $E \times F \times C$ into the real Euclidean plane $\mathbb{R}^2$ and the binary relation $>$ is the usual sup-order of the Cartesian plane (defined component-wise, for every couple of points $p$ and $q$, by $p > q$ iff $p_i > q_i$, for each index $i$).

**Remark (coopetitive games and normal form games).** The difference among a two-player normal-form (gain) game and a two player coopetitive (gain) game is the fundamental presence of the third strategy Cartesian-factor $C$. The presence of this third set $C$ determines a total change of perspective with respect to the usual exam of two-player normal form games, since we now have to consider a normal form game $G(z)$, for every element $z$ of the set $C$; we have, then, to study an entire ordered family of normal form games in its own totality, and we have to define a new manner to study these kind of game families.

### 3.4. Terminology and notation

**Definitions.** Let $G = (f, >)$ be a two player coopetitive gain game carried by the strategic triple $(E, F, C)$. We will use the following terminologies:

- the function $f$ is called the **payoff function of the game** $G$;
- the first component $f_1$ of the payoff function $f$is called **payoff function of the first player** and analogously the second component $f_2$ is called **payoff function of the second player**;
- the set $E$ is said **strategy set of the first player** and the set $F$ the **strategy set of the second player**;
- the set $C$ is said the **cooperative (or common) strategy set of the two players**;
- the Cartesian product $E \times F \times C$ is called the (**coopetitive) strategy space of the game** $G$.

**Memento.** The first component $f_1$ of the payoff function $f$ of a coopetitive game $G$ is the function of the strategy space $E \times F \times C$ of the game $G$ into the real line $\mathbb{R}$ defined by the first projection

$$f_1(x, y, z) := \mathrm{pr}_1(f(x, y, z)),$$

for every strategic triple $(x, y, z)$ in $E \times F \times C$; in a similar fashion we proceed for the second component $f_2$ of the function $f$.

**Interpretation.** We have:

- two players, or better an ordered pair $(1, 2)$ of players;
- anyone of the two players has a strategy set in which to choose freely his own strategy;
- the two players can/should *cooperatively* choose strategies $z$ in a third common strategy set $C$;
- the two players will choose (after the exam of the entire game $G$) their cooperative strategy $z$ in order to maximize (in some sense we shall define) the vector gain function $f$.

### 3.5. Normal form games of a coopetitive game

Let $G$ be a coopetitive game in the sense of above definitions. For any cooperative strategy $z$ selected in the cooperative strategy space $C$, there is a corresponding normal form gain game

$$G_z = (p(z), >),$$

upon the strategy pair $(E, F)$, where the payoff function $p(z)$ is the section

$$f(., z) : E \times F \to \mathbb{R}^2,$$

of the payoff function $f$ of the coopetitive game - the section is defined, as usual, on the competitive strategy space $E \times F$, by

$$f(., z)(x, y) = f(x, y, z),$$

for every bi-strategy $(x, y)$ in the bi-strategy space $E \times F$.

Let us formalize the concept of game-family associated with a coopetitive game.

**Definition (the family associated with a coopetitive game).** Let $G = (f, >)$ be a two player coopetitive gain game carried by the strategic triple $(E, F, C)$. We naturally can associate with the game $G$ a family $g = (g_z)_{z \in C}$ of normal-form games defined by

$$g_z := G_z = (f(., z), >),$$

for every $z$ in $C$, which we shall call **the family of normal-form games associated with the coopetitive game** $G$.

**Remark.** It is clear that with any above family of normal form games

$$g = (g_z)_{z \in C},$$

with $g_z = (f(., z), >)$, we can associate:

- a family of payoff spaces

  $$(\mathrm{im} f(., z))_{z \in C},$$

  with members in the payoff universe $\mathbb{R}^2$;

– a family of Pareto maximal boundary

$$(\partial^* G_z)_{z \in C},$$

with members contained in the payoff universe $\mathbb{R}^2$;

– a family of suprema

$$(\sup G_z)_{z \in C},$$

with members belonging to the payoff universe $\mathbb{R}^2$;

– a family of Nash zones

$$(\mathcal{N}(G_z))_{z \in C};$$

with members contained in the strategy space $E \times F$;

– a family of conservative bi-values

$$v^{\#} = (v_z^{\#})_{z \in C};$$

in the payoff universe $\mathbb{R}^2$.

And so on, for every meaningful known feature of a normal form game.

Moreover, we can interpret any of the above families as *set-valued paths* in the strategy space $E \times F$ or in the payoff universe $\mathbb{R}^2$.

It is just the study of these induced families which becomes of great interest in the examination of a coopetitive game $G$ and which will enable us to define (or suggest) the various possible solutions of a coopetitive game.

## 4. Coopetitive games for Greek crisis

Our first hypothesis is that Germany must stimulate the domestic demand and to re-balance its trade surplus in favor of Greece. The second hypothesis is that Greece, a country with a declining competitiveness of its products and a small export share, aims at growth by undertaking innovative investments and by increasing its exports primarily towards Germany and also towards the other euro countries.

The coopetitive model that we propose hereunder must be interpreted as normative model, in the sense that:

– *it imposes some clear a priori conditions to be respected, by binding contracts, in order to enlarge the possible outcomes of both countries;*
– *consequently, it shows appropriate win-win strategy solutions, chosen by considering both competitive and cooperative behaviors, simultaneously;*
– *finally, it proposes appropriate fair divisions of the win-win payoff solutions.*

The strategy spaces of the model are:

– the strategy set of Germany $E$, set of all possible consumptions of Germany, in our model, given in a conventional monetary unit; we shall assume that the strategies of Germany directly influence only Germany pay-off;

- the strategy set of Greece $F$, set of all possible investments of Greece, in our model, given in a conventional monetary unit (different from the above Germany monetary unit); we shall assume that the strategies of Greece directly influence only Greece pay-off;
- a shared strategy set $C$, whose elements are determined together by the two countries, when they choose their own respective strategies $x$ and $y$, Germany and Greece. Every strategy $z$ in $C$ represents an amount - given in a third conventional monetary unit - of Greek exports imported into Germany, by respecting a binding contract.

Therefore, in the model, we assume that Germany and Greece define the set of coopetitive strategies.

## 5. The model

**Main Strategic assumptions.** We assume that:

- any real number $x$, belonging to the unit interval $U := [0, 1]$, can represent a consumption of Germany (given in an appropriate conventional monetary unit);
- any real number $y$, in the same unit interval $U$, can represent an investment of Greece (given in another appropriate conventional monetary unit);
- any real number $z$, again in $U$, can be the amount of Greek exports which is imported by Germany (given in conventional monetary unit).

In this model, we consider a linear affine mutual interaction between Germany and Greece, more adherent to the real state of the Euro-area.

Specifically, in opposition to the above first model:

- we consider an interaction between the two countries also *at the level of their non-cooperative strategies*;
- we assume that *Greece also should import (by contract) some German production*;
- we assume, that the German revenue, given by the exportations in Greece of the above production, *is absorbed by the Germany bank system* - in order to pay the Greece debts with the German bank system - so that this money does not appear in the payoff function of Germany (as possible gain) but only in the payoff function of Greece (as a loss).

**Main Strategic assumptions.** We assume that:

- any real number $x$, belonging to the interval $E := [0, 3]$, represents a possible consumption of Germany (given in an appropriate conventional monetary unit);
- any real number $y$, in the same interval $F := E$, represents a possible investment of Greece (given in another appropriate conventional monetary unit);
- any real number $z$, again in the interval $C = [0, 2]$, can be the amount of Greek exports which is imported by Germany (given in conventional monetary unit).

### 5.1. Payoff function of Germany

We assume that the payoff function of Germany $f_1$ is its Keynesian *gross domestic demand*:

– $f_1$ is equal to the private consumption function $C_1$ plus the gross investment function $I_1$ plus government spending (that we shall assume equal 2, constant in our interaction) plus export function $X_1$ minus the import function $M_1$, that is

$$f_1 = 2 + C_1 + I_1 + X_1 - M_1.$$

We assume that:

– the German private consumption function $C_1$ is the first projection of the strategic coopetitive space $S := E^2 \times C$, that is defined by

$$C_1(x, y, z) = x,$$

for every possible german consumption $x$ in $E$, this because we assumed the private consumption of Germany to be the first strategic component of strategy profiles in $S$;

– the gross investment function $I_1$ is constant on the space $S$, and by translation we can suppose $I_1$ equal zero;

– the export function $X_1$ is defined by

$$X_1(x, y, z) = -y/3,$$

for every Greek possible investment $y$ in innovative technology; so we assume that the export function $X_1$ is a strictly decreasing function with respect to the second argument;

– the import function $M_1$ is the third projection of the strategic space, namely

$$M_1(x, y, z) = z,$$

for every cooperative strategy $z \in 2U$, because we assume the import function $M_1$ depending only upon the cooperative strategy $z$ of the coopetitive game $G$, our third strategic component of the strategy profiles in $S$.

**Recap.** We then assume as payoff function of Germany its Keynesian gross domestic demand $f_1$, which in our model is equal, at every triple $(x, y, z)$ in the profile strategy set $S$, to the sum of the strategies $x$, $-z$ with the export function $X_1$, viewed as a reaction function to the Greece investments (so that $f_1$ is the difference of the first and third projection of the strategy profile space $S$ plus the function export function $X_1$).

Concluding, the payoff function of Germany is the function $f_1$ of the set $S$ into the real line $\mathbb{R}$, defined by

$$f_1(x, y, z) = 2 + x - y/3 - z,$$

for every triple $(x, y, z)$ in the space $S$; where the reaction function $X_1$, defined from the space $S$ into the real line $\mathbb{R}$ by

$$X_1(x, y, z) = -y/3,$$

for every possible investment $y$ of Greece in the interval $3U$, is the export function of Germany mapping the level $y$ of Greece investment into the level $X_1(x, y, z)$ of German export, corresponding to the Greece investment level $y$.

The function $X_1$ is a strictly decreasing function in the second argument, and this monotonicity is a relevant property of $X_1$ for our coopetitive model.

### 5.2. Payoff function of Greece

We assume that the payoff function of Greece $f_2$ is again its Keynesian gross domestic demand - private consumption $C_2$ plus gross investment $I_2$ plus government spending (assumed to be 2) plus exports $X_2$ minus imports $M_2$), so that

$$f_2 = 2 + C_2 + I_2 + X_2 - M_2.$$

We assume that:

- the function $C_2$ is irrelevant in our analysis, since we assume the Greek private consumptions independent from the choice of the strategic triple $(x, y, z)$ in the space $S$; in other terms, we assume the function $C_2$ constant on the space $S$ and by translation we can suppose $C_2$ itself equal zero;
- the function $I_2 : S \to \mathbb{R}$ is defined by

  $$I_2(x, y, z) = y + nz,$$

  for every $(x, y, z)$ in $S$ (see above for the justification);
- the export function $X_2$ is the linear function defined by

  $$X_2(x, y, z) = z + my,$$

  for every $(x, y, z)$ in $S$ (see above for the justification);
- the function $M_2$ is now relevant in our analysis, since we assume the import function, by *coopetitive contract with Germany*, dependent on the choice of the triple $(x, y, z)$ in $S$, specifically, we assume the import function $M_2$ defined on the space $S$ by

  $$M_2(x, y, z) := -2x/3,$$

  so, Greece too now, must import some German product, with value $-2x/3$ for each possible German consumption $x$.

So, the payoff function of Greece is the linear function $f_2$ of the space $S$ into the real line $\mathbb{R}$, defined by

$$f_2(x, y, z) = 2 - 2x/3 + (y + nz) + (z + my) = \\ = 2 - 2x/3 + (1 + m)y + (1 + n)z,$$

for every pair $(x, y, z)$ in the strategic Cartesian space $S$.

We note that the function $f_2$ depends now significantly upon the strategies $x$ in $E$, chosen by Germany, and that $f_2$ is again a linear function.

We shall assume the factors $m$ and $n$ non-negative and equal respectively (only for simplicity) to 0 and 1/2.

### 5.3. Payoff function of the game

We so have build up a coopetitive gain game with payoff function $f : S \to \mathbb{R}^2$, given by

$$f(x, y, z) = (2 + x - y/3 - z, 2 - 2x/3 + (1 + m)y + (1 + n)z) = \\ = (2, 2) + (x - y/3, -2x/3 + (1 + m)y) + z(-1, 1 + n),$$

for every $(x, y, z)$ in $[0, 3]^2 \times [0, 2]$.

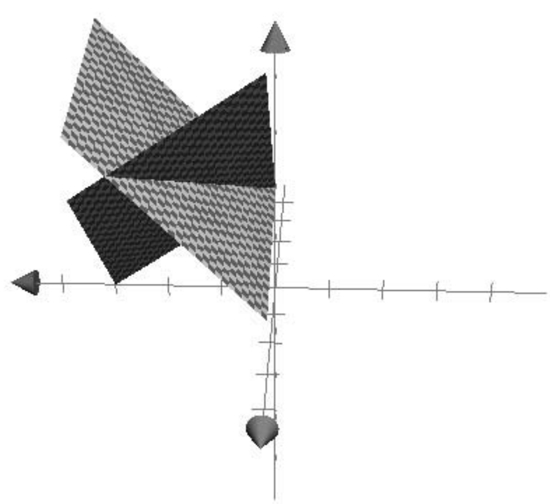

Fig. 1: 3D representation of $(f, <)$.

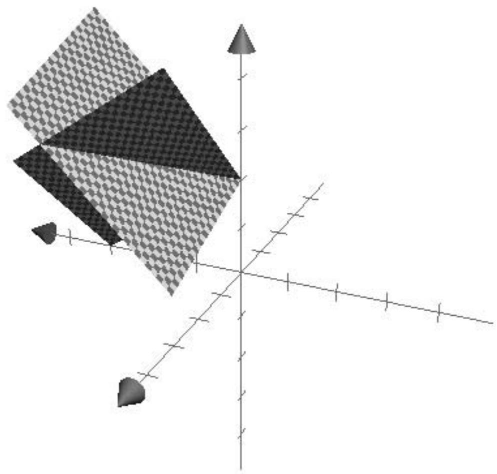

Fig. 2: 3D representation of $(f, <)$.

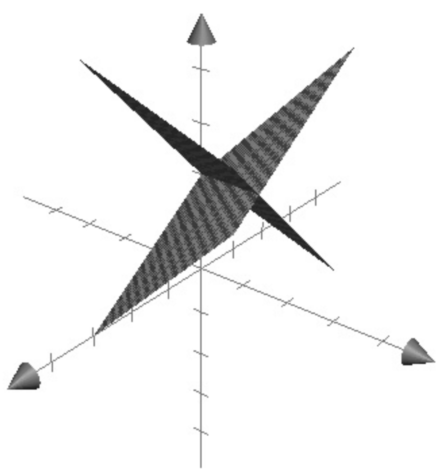

Fig. 3: 3D representation of $(f, <)$.

### 5.4. Study of the second game $G = (f, >)$

Note that, fixed a cooperative strategy $z$ in $2U$, the section game $G(z) = (p(z), >)$ with payoff function $p(z)$, defined on the square $E^2$ by

$$p(z)(x, y) := f(x, y, z),$$

is the translation of the game $G(0)$ by the "cooperative" vector

$$v(z) = z(-1, 1 + n),$$

so that, we can study the initial game $G(0)$ and then we can translate the various informations of the game $G(0)$ by the vectors $v(z)$, to obtain the corresponding information for the game $G(z)$.

So, let us consider the initial game $G(0)$. The strategy square $E^2$ of $G(0)$ has vertices $0_2$, $3e_1$, $3_2$ and $3e_2$, where $0_2$ is the origin of the plane $\mathbb{R}^2$, $e_1$ is the first canonical vector $(1, 0)$, $3_2$ is the vectors $(3, 3)$ and $e_2$ is the second canonical vector.

### 5.5. Topological Boundary of the payoff space of $G_0$

In order to determine the the payoff space of the linear game it is sufficient to transform the four vertices of the strategy square (the game is an affine invertible game), the critical zone is empty.

**Payoff space of the game $G(0)$.** So, the payoff space of the game $G(0)$ is the transformation of the topological boundary of the strategy square, that is the parallelogram with vertices $f(0, 0)$, $f(3e_1)$, $f(3, 3)$ and $f(3e_2)$. As we show in the below figure 4.

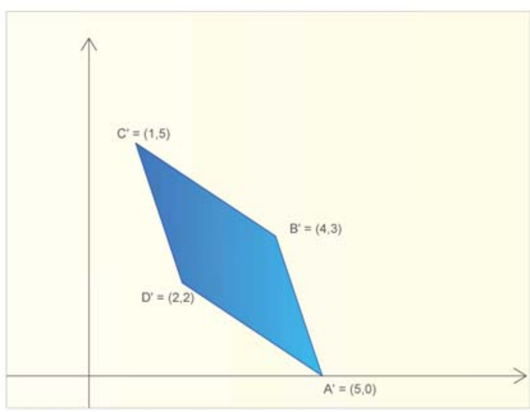


Fig. 4: Initial payoff space of the game $(f, <)$.

**Nash equilibria.** The unique Nash equilibrium is the bistrategy $(3, 3)$. Indeed, the function $f_1$ is linear increasing with respect to the first argument and analogously the function $f_2$ is linear and increasing with respect to the second argument.

### 5.6. The payoff space of the coopetitive game $G$

The image of the payoff function $f$, is the union of the family of payoff spaces

$$(\mathrm{im} p_z)_{z \in C},$$

that is the convex envelope of the union of the image $p_0(E^2)$ and of its translation by the vector $v(2)$, namely the payoff space $p_2(E^2)$: the image of $f$ is an hexagon with vertices $f(0, 0)$, $f(3e_1)$, $f(3, 3)$ and their translations by $v(2)$. As we show below.

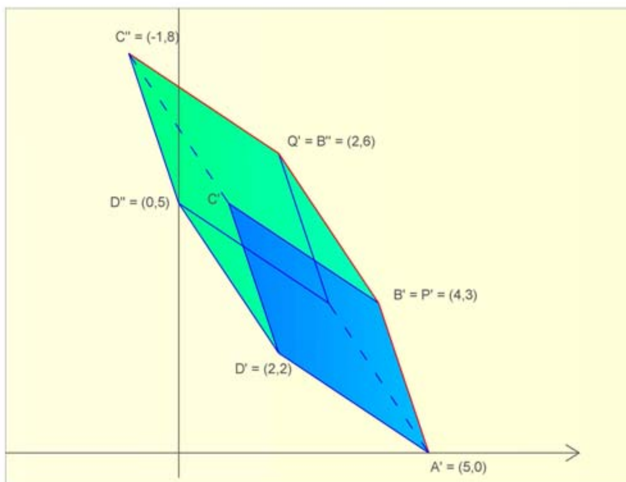


Fig. 5: Payoff space of the game $(f, <)$.

### 5.7. Pareto maximal boundary of the payoff space of $G$

The Pareto sup-boundary of the coopetitive payoff space $f(S)$ is the union of the segments $[A', B']$, $[P', Q']$ and $[Q', C'']$, where $P' = f(3, 3, 0)$ and

$$Q' = P' + v(2).$$

**Possibility of global growth.** It is important to note that the absolute slopes of the segments $[A', B']$, $[P', Q']$ of the Pareto (coopetitive) boundary are strictly greater than 1. Thus the collective payoff $f_1 + f_2$ of the game is not constant on the Pareto boundary and, therefore, the game implies the possibility of a transferable utility global growth.

**Trivial bargaining solutions.** The Nash bargaining solution on the entire payoff space, with respect to the infimum of the Pareto boundary and the Kalai-Smorodinsky bargaining solution, with respect to the infimum and the supremum of the Pareto boundary, are not acceptable for Germany: they are collectively (TU) better than the Nash payoff of $G_0$ but they are disadvantageous for Germany (it suffers a loss!): these solutions could be thought as rebalancing solutions, but they are not realistically implementable.

### 5.8. Transferable utility solutions

In this coopetitive context it is more convenient to adopt a transferable utility solution, indeed:

– the point of maximum collective gain on the whole of the coopetitive payoff space is the point $Q' = (2, 6)$.

**Rebalancing win-win solution relative to maximum gain for Greece in *G***
Thus we propose a rebalancing win-win coopetitive solution relative to maximum gain for Greece in $G$, as it follows (in the case $m = 0$):

1. we consider the portion $s$ of transferable utility Pareto boundary
$$M := Q' + \mathbb{R}(1, -1),$$
obtained by intersecting $M$ itself with the strip determined (spanned by convexifying) by the straight lines $P' + \mathbb{R}e_1$ and $C'' + \mathbb{R}e_1$, *these are the straight lines of Nash gain for Greece in the initial game* $G(0)$ *and of maximum gain for Greece in* $G$*, respectively.*
2. we consider the Kalai-Smorodinsky segment $s'$ of vertices $B'$ - Nash payoff of the game $G(0)$ - and the supremum of the segment $s$.
3. our best payoff rebalancing coopetitive compromise is the unique point $K$ in the intersection of segments $s$ and $s'$, that is the best compromise solution of the bargaining problem $(s, (B', \sup s))$.

Figure 6 below shows the above extended Kalai-Smorodinsky solution $K$ and the Kalai-Smorodinsky solution $K'$ of the classic bargaining problem $(M, B')$. It is evident that the distribution $K$ is a rebalancing solution in favor of Greece with respect to the classic solution $K'$.

**Rebalancing win-win solution relative to maximum Nash gain for Greece**
We propose here a more realistic rebalancing win-win coopetitive solution relative to maximum Nash gain for Greece in $G$, as it follows (again in the case $m = 0$):

1. we consider the portion $s$ of transferable utility Pareto boundary
$$M := Q' + \mathbb{R}(1, -1),$$
obtained by intersecting $M$ itself with the strip determined (spanned by convexifying) by the straight lines $P' + \mathbb{R}e_1$ and $Q' + \mathbb{R}e_1$, *these are the straight lines of Nash gain for Greece in the initial game* $G(0)$ *and of maximum Nash gain for Greece in* $G$*, respectively.*

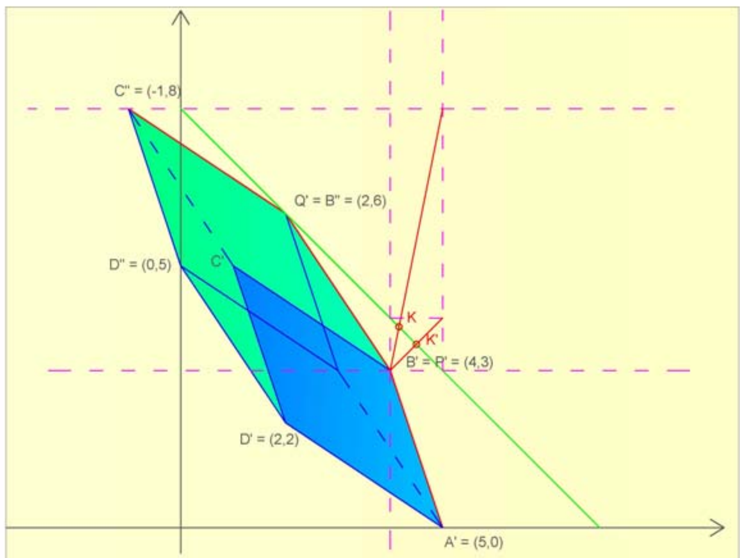


Fig. 6: Two Kalai win-win solutions of the game $(f, <)$, represented with $n = 1/2$.

2. we consider the Kalai-Smorodinsky segment $s'$ of vertices $B'$ - Nash payoff of the game $G(0)$ - and the supremum of the segment $s$.
3. our best payoff rebalancing coopetitive compromise is the unique point $K$ in the intersection of segments $s$ and $s'$, that is the best compromise solution of the bargaining problem $(s, (B', \sup s))$.

Figure 7 below shows the above extended Kalai-Smorodinsky solution $K$ and the Kalai-Smorodinsky solution $K'$ of the classic bargaining problem $(M, B')$. The new distribution $K$ is a rebalancing solution in favor of Greece, more realistic than the previous rebalancing solution.

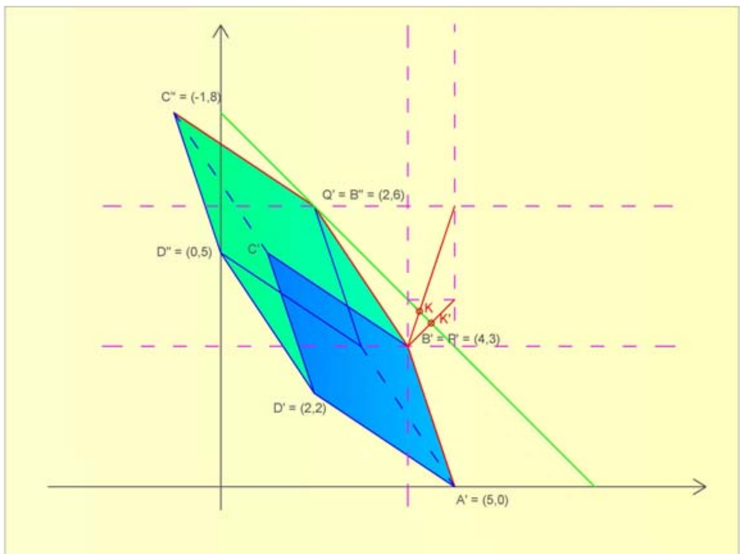


Fig. 7: Two Kalai win-win solutions of the game $(f, <)$, represented with $n = 1/2$.

### 5.9. Win-win solution

The payoff extended Kalai-Smorodinsky solutions $K$ represent win-win solutions, with respect to the initial Nash gain $B'$. So that, as we repeatedly said, *also Germany can increase its initial profit from coopetition.*

**Win-win strategy procedure.** The win-win payoff $K$ can be obtained in a **properly transferable utility coopetitive fashion**, as it follows:

- 1) the two players agree on the cooperative strategy 2 of the common set $C$;

– 2) the two players implement their respective Nash strategies in the game $G(2)$, so competing *à la Nash*; the unique Nash equilibrium of the game $G(2)$ is the bistrategy $(3,3)$;
– 3) finally, they share the "social pie"

$$(f_1 + f_2)(3,3,2),$$

in a **transferable utility cooperative fashion** (by binding contract) according to the decomposition $K$.

## 6. Conclusions

In conclusion, we desire to stress that:

– the model of coopetitive game, provided in the present contribution, is essentially a *normative model.*
– our model of coopetition has pointed out the strategies that could bring to win-win solutions, in a **transferable utility and properly cooperative perspective**, for Greece and Germany.

In the paper, we propose:

– transferable utility and properly coopetitive solutions, which are convenient for Greece and also for Germany.
– a new extended Kalai-Smorodinsky method, appropriate to determine rebalancing partitions, for *win-win solutions*, on the transferable utility Pareto boundary of the entire coopetitive game.

The solutions offered by our coopetitive model aim at "enlarging the pie and sharing it fairly"; more specifically:

– our model is a growth model, in the sense that it suggests solutions which imply the increase of the GDP of Greece due to the actions of the variables: exports (the shared variable) and investments. It also allows to find "fair" amounts of Greek exports which Germany must cooperatively import.
– in our analytical model, the enlargement of the "pie", which is represented in figure 5 as the coopetitive payoff space $f(E \times F \times C)$, shows the set of all possible payoff shares determining reasonable (in an extended Kalai-Smorodinsky sense) win-win solutions for both Greece and Germany.

**Acknowledgements.** The authors wish to thank ***Dr. Eng. Alessia Donato*** for her valuable help in the preparation of the figures.

## 7. Appendix: Solutions of a coopetitive game

### 7.1. Introduction

The two players of a coopetitive game $G$ - according to the general economic principles of *monotonicity of preferences* and of non-satiation - should choose the cooperative strategy $z$ in $C$ in order that:

- the reasonable Nash equilibria of the game $G_z$ are $f$-preferable than the reasonable Nash equilibria in each other game $G_{z'}$;
- the supremum of $G_z$ is greater (in the sense of the usual order of the Cartesian plane) than the supremum of any other game $G_{z'}$;
- the Pareto maximal boundary of $G_z$ is higher than that of any other game $G_{z'}$;
- the Nash bargaining solutions in $G_z$ are $f$-preferable than those in $G_{z'}$;
- *in general, fixed a common kind of solution for any game $G_z$, say $S(z)$ the set of these kind of solutions for the game $G_z$, we can consider the problem to find all the optimal solutions (in the sense of Pareto) of the set valued path $S$, defined on the cooperative strategy set $C$. Then, we should face the problem of* ***selection of reasonable Pareto strategies*** *in the set-valued path $S$ via proper selection methods (Nash-bargaining, Kalai-Smorodinsky and so on).*

Moreover, we shall consider the maximal Pareto boundary of the payoff space $\mathrm{im}(f)$ as an appropriate zone for the bargaining solutions.

The payoff function of a two person coopetitive game is (as in the case of normal-form game) a vector valued function with values belonging to the Cartesian plane $\mathbb{R}^2$. We note that in general the above criteria are multi-criteria and so they will generate multi-criteria optimization problems.

In this section we shall define rigorously some kind of solution, for two player coopetitive games, based on a bargaining method, namely a Kalai-Smorodinsky bargaining type. Hence, first of all, we have to precise what kind of bargaining method we are going to use.

### 7.2. Bargaining problems

In this paper, we shall propose and use the following original *extended* (and quite general) definition of bargaining problem and, consequently, a natural generalization of Kalai-Smorodinsky solution. In the economic literature, several examples of *extended bargaining problems* and *extended Kalai-Smorodinski solutions* are already presented. The essential root of these various extended versions of bargaining problems is the presence of *utopia points* not-directly constructed by the disagreement points and the strategy constraints. Moreover, the Kalai-type solution, of such extended bargaining problems, is always defined as a Pareto maximal point belonging to the segment joining the disagreement point with the utopia point (if any such Pareto point does exist): we shall follow the same way. In order to find suitable new win-win solutions of our realistic coopetitive economic problems, we need such new kind of versatile extensions. For what concerns the existence of our new extended Kalai solutions, for the economic problems we are facing, we remark that conditions of compactness and strict convexity will naturally hold; we remark, otherwise, that, in this paper, we are not interested in proving general or deep mathematical results, but rather to find reasonable solutions for new economic coopetitive context.

**Definition (of bargaining problem).** Let $S$ be a subset of the Cartesian plane $\mathbb{R}^2$ and let $a$ and $b$ be two points of the plane with the following properties:

- they belong to the small interval containing $S$, if this interval is defined (indeed, it is well defined if and only if $S$ is bounded and it is precisely the interval $[\inf S, \sup S]_{\leq}$);
- they are such that $a < b$;
- the intersection
$$[a, b]_{\leq} \cap \partial^* S,$$
among the interval $[a, b]_{\leq}$ with end points $a$and $b$ (it is the set of points greater than $a$ and less than $b$, **it is not** the segment $[a, b]$) and the maximal boundary of $S$ is non-empty.

In these conditions, we call **bargaining problem on $S$ corresponding to the pair of extreme points** $(a, b)$, the pair

$$P = (S, (a, b)).$$

Every point in the intersection among the interval $[a, b]_{\leq}$and the Pareto maximal boundary of $S$ is called **possible solution of the problem** $P$. Some time the first extreme point of a bargaining problem is called **the initial point of the problem** (or **disagreement point** or **threat point**) and the second extreme point of a bargaining problem is called **utopia point** of the problem.

In the above conditions, when $S$ is convex, the problem $P$ is said to be convex and for this case we can find in the literature many existence results for solutions of $P$ enjoying prescribed properties (Kalai-Smorodinsky solutions, Nash bargaining solutions and so on ...).

**Remark.** Let $S$ be a subset of the Cartesian plane $\mathbb{R}^2$ and let $a$ and $b$ two points of the plane belonging to the smallest interval containing $S$ and such that $a \leq b$. Assume the Pareto maximal boundary of $S$ be non-empty. If $a$ and $b$ are a lower bound and an upper bound of the maximal Pareto boundary, respectively, then the intersection

$$[a, b]_{\leq} \cap \partial^* S$$

is obviously not empty. In particular, if $a$ and $b$ are the extrema of $S$ (or the extrema of the Pareto boundary $S^* = \partial^* S$) we can consider the following bargaining problem

$$P = (S, (a, b)),\ (or\ P = (S^*, (a, b)))$$

and we call this particular problem a *standard bargaining problem on* $S$ (or *standard bargaining problem on the Pareto maximal boundary* $S^*$).

### 7.3. Kalai solution for bargaining problems

Note the following property.

**Property.** If $(S, (a, b))$ is a bargaining problem with $a < b$, then there is at most one point in the intersection

$$[a, b] \cap \partial^* S,$$

where $[a, b]$ is the **segment joining the two points** $a$ and $b$.

*Proof.* Since if a point $p$ of the segment $[a, b]$ belongs to the Pareto boundary $\partial^* S$, no other point of the segment itself can belong to Pareto boundary, since the segment is a totally ordered subset of the plane (remember that $a < b$). □

**Definition (Kalai-Smorodinsky).** We call **Kalai-Smorodinsky solution (or best compromise solution) of the bargaining problem** $(S, (a, b))$ the unique point of the intersection

$$[a, b] \cap \partial^* S,$$

if this intersection is non empty.

So, in the above conditions, the Kalai-Smorodinsky solution $k$ (if it exists) enjoys the following property: there is a real $r$ in $[0, 1]$ such that

$$k = a + r(b - a),$$

or

$$k - a = r(b - a),$$

hence

$$\frac{k_2 - a_2}{k_1 - a_1} = \frac{b_2 - a_2}{b_1 - a_1},$$

if the above ratios are defined; these last equality is the *characteristic property of Kalai-Smorodinsky solutions.*

We end the subsection with the following definition.

**Definition (of Pareto boundary).** We call **Pareto boundary** every subset $M$ of an ordered space which has only pairwise incomparable elements.

### 7.4. Nash (proper) solution of a coopetitive game

Let $N := \mathcal{N}(G)$ be the union of the Nash-zone family of a coopetitive game $G$, that is the union of the family $(\mathcal{N}(G_z))_{z\in C}$ of all Nash-zones of the game family $g = (g_z)_{z\in C}$ associated to the coopetitive game $G$. We call *Nash path of the game* $G$ the multi-valued path

$$z \mapsto \mathcal{N}(G_z)$$

and Nash zone of $G$ the trajectory $N$ of the above multi-path. Let $N^*$ be the Pareto maximal boundary of the Nash zone $N$. We can consider the bargaining problem

$$P_{\mathcal{N}} = (N^*, \inf(N^*), \sup(N^*)).$$

**Definition.** If the above bargaining problem $P_{\mathcal{N}}$ has a Kalai-Smorodinsky solution $k$, we say that $k$ is the properly coopetitive solution of the coopetitive game $G$.

The term "properly coopetitive" is clear:

– *this solution $k$ is determined by cooperation on the common strategy set $C$ and to be selfish (competitive in the Nash sense) on the bi-strategy space $E \times F$.*

### 7.5. Bargaining solutions of a coopetitive game

It is possible, for coopetitive games, to define other kind of solutions, which are not properly coopetitive, but realistic and sometime affordable. These kind of solutions are, we can say, *super-cooperative*.

Let us show some of these kind of solutions.

Consider a coopetitive game $G$ and

- its Pareto maximal boundary $M$ and the corresponding pair of extrema $(a_M, b_M)$;
- the Nash zone $\mathcal{N}(G)$ of the game in the payoff space and its extrema $(a_N, b_N)$;
- the conservative set-value $G^\#$ (the set of all conservative values of the family $g$ associated with the coopetitive game $G$) and its extrema $(a^\#, b^\#)$.

*We call:*

- ***Pareto compromise solution of the game*** $G$ *the best compromise solution (K-S solution) of the problem*

$$(M, (a_M, b_M)),$$

  *if this solution exists;*
- ***Nash-Pareto compromise solution of the game*** $G$ *the best compromise solution of the problem*

$$(M, (b_N, b_M))$$

  *if this solution exists;*
- ***conservative-Pareto compromise solution of the game*** $G$ *the best compromise of the problem*

$$(M, (b^\#, b_M))$$

  *if this solution exists.*

### 7.6. Transferable utility solutions

Other possible compromises we suggest are the following.

Consider *the transferable utility Pareto boundary $M$ of the coopetitive game $G$*, that is the set of all points $p$ in the Euclidean plane (universe of payoffs), between the extrema of $G$, such that their sum

$$+(p) := p_1 + p_2$$

is equal to the maximum value of the addition $+$ of the real line $\mathbb{R}$ over the payoff space $f(E \times F \times C)$ of the game $G$.

**Definition (TU Pareto solution).** We call **transferable utility compromise solution of the coopetitive game** $G$ the solution of any bargaining problem $(M, (a, b))$, where

- $a$ and $b$ are points of the smallest interval containing the payoff space of $G$
- $b$ is a point strongly greater than $a$;
- $M$ is the transferable utility Pareto boundary of the game $G$;
- the points $a$ and $b$ belong to different half-planes determined by $M$.

Note that the above fourth axiom is equivalent to require that the segment joining the points $a$ and $b$ intersect $M$.

### 7.7. Win-win solutions

In the applications, if the game $G$ has a member $G_0$ of its family which can be considered as an "initial game" - in the sense that the pre-coopetitive situation is represented by this normal form game $G_0$ - the aims of our study (following the standard ideas on coopetitive interactions) are

- to "enlarge the pie";
- to obtain a win-win solution with respect to the initial situation.

So that we will choose as a threat point $a$ in TU problem $(M,(a,b))$ the supremum of the initial game $G_0$.

**Definition (of win-win solution).** Let $(G, z_0)$ be a **coopetitive game with an initial point**, that is a coopetitive game $G$ with a fixed common strategy $z_0$ (of its common strategy set $C$). We call the game $G_{z_0}$ as **the initial game of** $(G, z_0)$. We call **win-win solution of the game** $(G, z_0)$ any strategy profile $s = (x, y, z)$ such that the payoff of $G$ at $s$ is strictly greater than the supremum $L$ of the **payoff core** of the initial game $G(z_0)$.

**Remark 1.** The payoff core of a normal form gain game $G$ is the portion of the Pareto maximal boundary $G^*$ of the game which is greater than the conservative bi-value of $G$.

**Remark 2.** From an applicative point of view, the above requirement (to be strictly greater than $L$) is very strong. More realistically, we can consider as win-win solutions those strategy profiles which are strictly greater than any reasonable solution of the initial game $G_{z_0}$.

**Remark 3.** Strictly speaking, a win-win solution could be not Pareto efficient: it is a situation in which the players both gain with respect to an initial condition (and this is exactly the idea we follow in the rigorous definition given above).

**Remark 4.** In particular, observe that, if the collective payoff function

$$^{+}(f) = f_1 + f_2$$

has a maximum (on the strategy profile space $S$) strictly greater than the collective payoff $L_1 + L_2$ at the supremum $L$ of the payoff core of the game $G_{z_0}$, the portion $M(> L)$ of Transferable Utility Pareto boundary $M$ which is greater than $L$ is non-void and it is a segment. So that we can choose as a threat point $a$ in our problem $(M,(a,b))$ the supremum $L$ of the payoff core of the initial game $G_0$ *to obtain some compromise solution.*

**Standard win-win solution.** A natural choice for the utopia point $b$ is the supremum of the portion $M_{\geq a}$ of the transferable utility Pareto boundary $M$ which is upon (greater than) this point $a$:

$$M_{\geq a} = \{m \in M : m \geq a\}.$$

**Non standard win-win solution.** Another kind of solution can be obtained by choosing $b$ as the supremum of the portion of $M$ that is bounded between the minimum and maximum value of that player $i$ that gains more in the coopetitive interaction, in the sense that

$$\max(\mathrm{pr}_i(\mathrm{im}f)) - \max(\mathrm{pr}_i(\mathrm{im}f_0)) > \max(\mathrm{pr}_{3-i}(\mathrm{im}f)) - \max(\mathrm{pr}_{3-i}(\mathrm{im}f_0)).$$

**Final general remark** In the development of a coopetitive game, we consider:

- a first virtual phase, in which the two players make a binding agreement on what cooperative strategy $z$ should be selected from the cooperative set $C$, in order to respect their own rationality.
- then, a second virtual phase, in which the two players choose their strategies forming the profile $(x, y)$ to implement in the game $G(z)$.

Now, in the second phase of our coopetitive game $G$ we consider the following 4 possibilities:

1. the two players are non-cooperative in the second phase and they do or do not exchange info, but the players choose (in any case) Nash equilibrium strategies for the game $G(z)$; in this case, for some rationality reason, the two players have devised that the chosen equilibrium is the better equilibrium choice in the entire game $G$; we have here only one binding agreement in the entire development of the game;
2. the two players are cooperative also in the second phase and they make a binding agreement in order to choose a Pareto payoff on the coopetitive Pareto boundary; in this case we need two binding agreements in the entire development of the game;
3. the two players are cooperative also in the second phase and they make two binding agreements, in order to reach the Pareto payoff (on the coopetitive Pareto boundary) with maximum collective gain (first agreement) and to share the collective gain according to a certain subdivision (second agreement); in this case we need three binding agreements in the entire development of the game;
4. the two players are non-cooperative in the second phase (and they do or do not exchange information), the player choose (in any case) Nash equilibrium strategies; the two players have devised that the chosen equilibrium is the equilibrium with maximum collective gain and they make only one binding agreement to share the collective gain according to a certain subdivision; in this case we need two binding agreements in the entire development of the game.

## References

Abadi, C. (2010). *Germany Debates Euro Instability Fault*. Global Post, March 19.

Baglieri, D., Carfì, D., Dagnino, G. (2010). *Profiting from Asymmetric R&D Alliances: Coopetitive Games and Firms' Strategies*, paper presented at the 4th Workshop on Coopetition Strategy "Coopetition and Innovation", Montpellier, June 17–18, 2010.

Blanchard, O., Dell'Ariccia, G., Mauro, P. (2010). *Rethinking Macroeconomic Policy.* IMF SPN Working Paper, February: 1–18.

Brandenburger, A. M., Nalebuff, B. J. (1995). *The Right Game: Use Game Theory to Shape Strategy.* Harvard Business Review, **64**, 57–71, July-August.

Carfì, D. (2009). *Payoff space in $C^1$-games.* Applied Sciences (APPS), **11**, 1–16. http://www.mathem.pub.ro/apps/

Carfì, D. (2010). *A Model for Coopetitive Games*, paper presented at Sing6, Palermo, July 6–9 2010

Carfì, D., Schilirò, D. (2011). *Crisis in the Euro area: coopetitive game solutions as new policy tools.* TPREF-Theoretical and Practical Research in Economic Fields, summer issue 2011, pp. 1–30. ISSN 2068 Đ 7710 http://www.asers.eu/journals/tpref.html

Carfì, D., Ricciardello, A. (2010). *An Algorithm for Payoff Space in C1-Games.* Atti della Accademia Peloritana dei Pericolanti, Classe di Scienze Fisiche Matematiche e Naturali, **LXXXVIII(1)**, 1–19. http://antonello.unime.it/atti/

Eurostat (2011). News Release, Euro Indicators, Bruxelles: pp. 1–14, April.

Gulati, R., Nohria, N., Zaheer, A. (2000). *Strategic Networks.* Strategic Management Journal, **21**, 203–215.

Henning, C. R. (2006). *The Exchange Rate Weapon and Macroeconomicn Conflict.* In International Monetary Power, ed. D.M. Andrews, Cornell University Press, Ithaca, 177–138.

Mussa, M. (2010). *Beware of Greeks Bearing Debts.* Working Paper, Peterson Institute for International Economics, May 17. http://www.piie.com.

Porter, M. (1985). *Competitive Advantage. Creating and Sustaining Superior Performance.* Free Press, New York.

Posen, A. S. (2010). *Having a Large Euro Area Is an Advantage for Germany.* Die Welt, February 19. http://www.piie.com.

Schilirò, D. (2010). *Investing in Knowledge: Knowledge, Human Capital and Institutions for the Long Run Growth.* In Governance Of Innovation, edited by M. J. Arentsen, W. van Rossum, A. E. Steenge, Edward Elgar, Cheltenham, 33–50.

Schilirò, D. (2011). *A new governance for the EMU and the economic policy framework.* MPRA Paper No. 30237, April, 1–18.

Stiles, J. (2001). *Strategic Alliances.* In Rethinking Strategy, edited by H. W. Volberda, T. Elfring, Sage Publications, London.