Optimal Strategies in the Game with Arbitrator*

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Abstract The organization of negotiations by using arbitration procedures is an actual problem in game theory. We consider a non-cooperative zero-sum game, related with an arbitration scheme, generalized well known final-offer procedure. The nash equilibrium in this game in mixed strategies is found.

Keywords: game, arbitration procedure, equilibrium, mixed strategies.

1. Introduction

The problem of some resource allocation among several participants take one of the central places in the modern theory of economical regulation. This situations occur in business (a Labour and a Manager consider the question on an improvement in the wage rate), in the market models (a Buyer, who wants to purchase some merchandise at a lower price, and the Seller, whose purpose is to sell this merchandise at a more beneficial price), insurance models, etc. This is a multicriterial problem, for which there are several solving approaches. We use game-theoretical methods of negotiation theory. In order to run the negotiations the participants call in the third independent party of one or several arbitrators participates. By the solution, we mean the Nash equilibrium in this game. The procedures with arbitrator's participation are called arbitration procedures. The problems of negotiation organization by using arbitration procedures are topical presently and in connection with of virtual enterprises appearing in the global Internet network.

There are various models of arbitration procedures. One of them is the final-offer arbitration procedure. This procedure was described in the papers (Farber,1980; Chatterjee, 1981; Kilgour, 1994).

We will find an equilibrium in the arbitration games in the terms of salary problem; however this approach may be also applied for other problems of resources allocation with arbitrator's participation.

So, we consider a non-cooperative zero-sum game in which two players L and M, called respectively the Labour and the Manager, have a dispute on an improvement in the wage rate. The player L makes an offer x, and the player M - an offer y; x and y are arbitrary real numbers. If $x \leq y$ there is no conflict, and the players agree on a payoff equal to $(x+y)/2$. If, otherwise, $x > y$, the parties call in the arbitrator A. Assume that the arbitrator's solution is a discrete random variable and denote it by z. In the final-offer arbitration scheme the arbitrator chooses the offer, which

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is closer to its solution z, i.e., the payoff function in this scheme has a form

$$H_z(x,y) = \begin{cases} \frac{x+y}{2}, & if \quad x \le y, \\ x, & if \ \ x > y, |x-z| < |y-z|, \\ y, & if \ \ x > y, |x-z| > |y-z|, \\ z, & if \ \ x > y, |x-z| = |y-z|. \end{cases} \tag{1.1}$$

Since in the function (1.1) the arbitrator's solution z is a random variable, we take for the payoff function the mathematical expectation of this function: $H(x,y) = EH_z(x,y)$.

Further, let $x \in [0,+\infty)$, $y \in (-\infty, 0]$. If $z = 0$ almost everywhere, it is evidently that the point of equilibrium in this game is the pair of pure strategies: $(0,0)$. In the papers (Mazalov et al., 2005; Mazalov et al., 2006; Mentcher, 2009) for the cases in which z is distributed in the final set of integer points the Nash equilibria in this game in mixed strategies were found.

Now we consider a generalization of the final-offer arbitration procedure. Namely, let $x \in [0,+\infty)$, $y \in (-\infty,0]$, $\alpha > 0$ and

$$H_z(x,y) = \begin{cases} x^\alpha, & if \ |x-z| < |y-z|, \\ -(-y)^\alpha, & if \ |x-z| > |y-z|, \\ z, & if \ |x-z| = |y-z|. \end{cases} \tag{1.2}$$

Let the arbitrator chooses one of the $2n+1$ numbers: $-n$, $-(n-1)$,Ě,-1, 0, 1, Ě, $n-1$, n - with equal probabilities $p = \frac{1}{2n+1}$. This game does not have a solution in pure strategies, and we will be looking for the equilibrium in mixed strategies. Denote by $f(x)$ and $g(y)$ the mixed strategies of the players L and M, respectively. We have

$$f(x) \ge 0, \ \int\limits_0^{+\infty} f(x)dx = 1; \ g(y) \ge 0, \ \int\limits_{-\infty}^{0} g(y)dy = 1.$$

Due to the symmetry, the game value is equal to zero, and the optimal strategies are symmetric in respect to the y-axis, i.e. $g(y) = f(-y)$. Hence, it suffices to construct the optimal strategy only for one player, for example L.

We find the optimal strategy for the player L in the following form:

$$f(x) = \begin{cases} 0, & if \ 0 \le x < c, \\ \varphi(x), & if \ c < x < c+2, \\ 0, & if \ c+2 < x < +\infty, \end{cases} \tag{1.3}$$

where the function $\varphi(x)$ is positive and continuously differentiable in the interval $(-(c+2),-c)$.

Denote by $H(f(x),y)$ the payoff function of the player M for the strategy $f(x)$ choosen by the player L. The function $H(f(x),y)$ is continuous on the entire semi-axis $(-\infty,0]$ and twice continuously differentiable in the interval $(c,c+2)$. The strategy (1.3) will be optimal, if $H(f(x),y) = 0$ for $y \in [-(c+2),-c]$ and $H(f(x),y) \ge 0$ for $y \in (-\infty,-(c+2)) \cup (-c,0]$.

Assume that $y \in [-(c+2),-c]$, then $-y \in [c,c+2]$, and

$$H(f(x),y) = \frac{1}{2n+1}\left[n\int_c^{c+2}(-(-y)^\alpha)f(x)dx + \int_c^{-y} x^\alpha f(x)dx + \right.$$

$$+\int_{-y}^{c+2}(-(-y)^{\alpha})f(x)dx+n\int_{c}^{c+2}x^{\alpha}f(x)dx\Bigg]. \tag{1.4}$$

If $f(x)$ is an optimal strategy, then

$$0=H(f(x),-c-0)=\frac{1}{2n+1}\left[-(n+1)c^{\alpha}+n\int_{c}^{c+2}x^{\alpha}f(x)dx\right],$$

$$0=H(f(x),-(c+2)+0)=\frac{1}{2n+1}\left[-n(c+2)^{\alpha}+(n+1)\int_{c}^{c+2}x^{\alpha}f(x)dx\right]. \tag{1.5}$$

From (1.5) we obtain the equation

$$\left(\frac{n+1}{n}\right)c^{\alpha}=\left(\frac{n}{n+1}\right)(c+2)^{\alpha}.$$

Whence we conclude that

$$c=\frac{2}{(1+\frac{1}{n})^{\frac{2}{\alpha}}-1} \tag{1.6}$$

and

$$\int_{c}^{c+2}x^{\alpha}f(x)dx=\sqrt{c^{\alpha}(c+2)^{\alpha}}. \tag{1.7}$$

For the strategy (1.3) in order to be optimal, it is necessary that $0<c\leq 2n$, whence we obtain $0<\alpha\leq 2$.

Furthere, it is necessary that $H'(f(x),y)=H''(f(x),y)=0$ in the interval $(-(c+2),-c)$. We have

$$H'(f(x,y))=\frac{1}{2n+1}\left[n\alpha(-y)^{\alpha-1}-2(-y)^{\alpha}f(-y)+\alpha(-y)^{\alpha-1}\int_{-y}^{c+2}f(x)dx\right], \tag{1.8}$$

$$H''(f(x,y))=\frac{1}{2n+1}\Big[-n\alpha(\alpha-1)(-y)^{\alpha-2}+3\alpha(-y)^{\alpha-1}f(-y)+$$

$$+2(-y)^{\alpha}f'(-y)-\alpha(\alpha-1)(-y)^{\alpha-2}\int_{-y}^{c+2}f(x)dx\Bigg]. \tag{1.9}$$

If now $H'(f(x),y)=H''(f(x),y)=0$ in the interval $(-(c+2),-c)$, then from (1.8)-(1.9) we obtain

$$(\alpha-1)(-y)^{-1}H'(f(x),y)+H''(f(x),y)=0,$$

whence

$$(\alpha+2)f(-y)-2yf'(-y)=0. \tag{1.10}$$

Assume that $x=-y$, then $x\in(c,c+2)$, $f(x)=\varphi(x)$ and

$$2x\varphi'(x)+(\alpha+2)\varphi(x)=0. \tag{1.11}$$

The solution of this equation is the function

$$\varphi(x)=\beta x^{-(\frac{\alpha}{2}+1)}. \tag{1.12}$$

Determine the constant β. From (1.8) we obtain

$$0 = H'(f(x), -c-0) = \frac{1}{2n+1}\left[\alpha(n+1)c^{\alpha-1} - 2c^{\alpha}\frac{\beta}{c^{\frac{\alpha}{2}+1}}\right],$$

whence

$$\beta = \frac{\alpha(n+1)}{2}c^{\frac{\alpha}{2}}.$$

Therefore, the function $f(x)$ from (1.3) has a form

$$f(x) = \begin{cases} 0, & \textit{if}\ \ 0 \le x < c, \\ \frac{\alpha(n+1)}{2}\cdot\frac{c^{\frac{\alpha}{2}}}{x^{\frac{\alpha}{2}+1}}, & \textit{if}\ c < x < c+2, \\ 0, & \textit{if}\ c+2 < x < +\infty, \end{cases} \tag{1.13}$$

where

$$c = \frac{2}{\left(1+\frac{1}{n}\right)^{\frac{2}{\alpha}} - 1}.$$

2. Optimal strategies

Theorem 1. *If $\alpha \in (0,2]$ and $n = 1$, then for the player L the strategy*

$$f(x) = \begin{cases} 0, & \textit{if}\ \ 0 \le x < c, \\ \frac{\alpha\cdot c^{\frac{\alpha}{2}}}{x^{\frac{\alpha}{2}+1}}, & \textit{if}\ c < x < c+2, \\ 0, & \textit{if}\ c+2 < x < +\infty, \end{cases} \tag{2.1}$$

where $c = \frac{2}{4^{\frac{1}{\alpha}}-1}$ is optimal.

Proof. Assuming in (1.13) $n = 1$, we come to the formula (2.1) with corresponding constant c. Check the fulfilment of optimal conditions.

Assume that $y \in [-(c+2), -c]$, then

$$H(f(x), y) = \frac{1}{3}\left[-(-y)^{\alpha} + \int_c^{-y}\alpha c^{\frac{\alpha}{2}}x^{\frac{\alpha}{2}-1}dx - (-y)^{\alpha}\int_{-y}^{c+2}\alpha c^{\frac{\alpha}{2}}x^{-\frac{\alpha}{2}-1}dx + 2c^{\alpha}\right] =$$

$$= \frac{1}{3}\left[-(-y)^{\alpha} + 2c^{\frac{\alpha}{2}}(-y)^{\frac{\alpha}{2}} - 2c^{\alpha} + (-y)^{\alpha} - 2c^{\frac{\alpha}{2}}(-y)^{\frac{\alpha}{2}} + 2c^{\alpha}\right] = 0. \tag{2.2}$$

Assume that $y \in (-\infty, -(c+4)]$, then

$$H(f(x), y) = \int_c^{c+2} x^{\alpha}f(x)dx = 2c^{\alpha}. \tag{2.3}$$

Assume that $y \in [-(c+4), -(c+2)]$, then $-y \in [c+2, c+4]$, $-2-y \in [c, c+2]$ and

$$H(f(x), y) = \frac{1}{3}\left[\int_c^{-2-y} x^{\alpha}f(x)dx - (-y)^{\alpha}\int_{-2-y}^{c+2} f(x)dx + 2\int_c^{c+2} x^{\alpha}f(x)dx\right] =$$

$$= \frac{2}{3}c^{\frac{\alpha}{2}}\left[(-2-y)^{\frac{\alpha}{2}} + c^{\frac{\alpha}{2}} + \frac{(-y)^{\alpha}}{(c+2)^{\frac{\alpha}{2}}} - \frac{(-y)^{\alpha}}{(-2-y)^{\frac{\alpha}{2}}}\right]. \tag{2.4}$$

We have

$$H(f(x), -(c+2)-0) = \frac{2}{3}c^{\frac{\alpha}{2}}\left[c^{\frac{\alpha}{2}} + c^{\frac{\alpha}{2}} + (c+2)^{\alpha} - 2(c+2)^{\alpha}\right] = 0. \qquad (2.5)$$

Further, assume in (2.4) $-2-y=t$, $t \in [c, c+2]$ and consider the function

$$\widetilde{H}(t) = \frac{2}{3}c^{\frac{\alpha}{2}}\left[t^{\frac{\alpha}{2}} + c^{\frac{\alpha}{2}} + \frac{(t+2)^{\alpha}}{(c+2)^{\frac{\alpha}{2}}} - \frac{(t+2)^{\alpha}}{t^{\frac{\alpha}{2}}}\right].$$

The functions $g(t) = \frac{(t+2)^{\alpha}}{(c+2)^{\frac{\alpha}{2}}}$ and

$$h(t) = t^{\frac{\alpha}{2}} - \frac{(t+2)^{\alpha}}{t^{\frac{\alpha}{2}}} = t^{\frac{\alpha}{2}}\left[1 - \left(1 + \frac{2}{t}\right)^{\alpha}\right]$$

are monotonouse increasing in the interval $[c, c+2]$. Finally, we conclude that the function $H(f(x), y)$ is monotonouse decreasing in the interval $[-(c+4), -(c+2)]$ from $2c^{\alpha}$ to 0 and therefore is positive in the interval $[-(c+4), -(c+2))$.

Assume that $y \in [-c, 0]$, then $-y \in [0, c]$, $2-y \in [2, c+2]$ and

$$H(f(x), y) = \frac{1}{3}\left[-2(-y)^{\alpha} + \int_{c}^{2-y} x^{\alpha} f(x)dx - \int_{2-y}^{c+2}(-y)^{\alpha} f(x)dx\right] =$$

$$= \frac{1}{3}\left[-(-y)^{\alpha} + 2c^{\frac{\alpha}{2}}(2-y)^{\frac{\alpha}{2}} - 2c^{\alpha} - 2c^{\frac{\alpha}{2}} - 2c^{\frac{\alpha}{2}}\frac{(-y)^{\alpha}}{(2-y)^{\frac{\alpha}{2}}}\right]. \qquad (2.6)$$

We have

$$H(f(x), -c+0) = 0, H(f(x), -0) = \frac{2}{3}c^{\frac{\alpha}{2}}(2^{\frac{\alpha}{2}} - c^{\frac{\alpha}{2}}) \geq 0. \qquad (2.7)$$

Furthere,

$$H'(f(x), y) = \frac{\alpha}{3}\left[(-y)^{\alpha-1} + c^{\frac{\alpha}{2}}\frac{-(2-y)^{\alpha} + 4(-y)^{\alpha-1} + (-y)^{\alpha}}{(2-y)^{\frac{\alpha}{2}+1}}\right]. \qquad (2.8)$$

Assume that $\alpha \in (0, 1]$, then $c \in (0, \frac{2}{3}] \subset (0, 1]$.

We have

$$-(2-y)^{\alpha} + 4(-y)^{\alpha-1} + (-y)^{\alpha} = -(2-y)^{\alpha} + (-y)^{\alpha-1}(4-y) \geq (4-y) - (2-y) = 2.$$

Therefore, $H'(f(x), y) > 0$ in the interval $(-c, 0)$ and take into consideration (2.7), we conclude that $H(f(x), y) > 0$ in this interval.

Assume that $\alpha \in (1, 2]$, then $c \in (\frac{2}{3}, 2]$. We have

$$H'(f(x), -c+0) = \frac{\alpha}{6}\frac{c^{\alpha-1}(8-c)}{c+2} > 0,$$

$$H'(f(x), -0) = -\frac{\alpha c^{\frac{\alpha}{2}}}{3 \cdot 2^{1-\frac{\alpha}{2}}} < 0. \qquad (2.9)$$

Therefore, in the interval $(-c, 0)$ exists if only one point y_0, for which $H'(f(x), y_0) = 0$. If y_0 is the unique point, then y_0 is the point of maximum for the function

$H(f(x), y)$ and take into consideration (2.7) we conclude that $H(f(x), y) > 0$ in the interval $(-c, 0)$.

Assume in (2.8) $-y = t$, $t \in [0, c]$, $y_0 = -t_0$.

Then

$$\widetilde{H}'(t) = \frac{\alpha}{3}\left[t^{\alpha-1} + c^{\frac{\alpha}{2}}\frac{t^{\alpha-1}(t+4) - (t+2)^{\alpha}}{(t+2)^{\frac{\alpha}{2}+1}}\right].$$

If now $\widetilde{H}'(t) = 0$, then we obtain the equation

$$t^{\alpha-1}(t+2)^{\frac{\alpha}{2}+1} + c^{\frac{\alpha}{2}}t^{\alpha}\left(1 - \left(1 + \frac{2}{t}\right)^{\alpha}\right) = -4c^{\frac{\alpha}{2}}t^{\alpha-1}. \qquad (2.10)$$

The function from the left part of (2.10) is monotonouse increasing in the interval $(-c, 0)$, but the function from the right part is monotonouse decreasing in the same interval. Consequently, it exists an unique point t_0, for which $\widetilde{H}'(t_0) = 0$.

□

In particular, if $\alpha = 1$, we have

$$f(x) = \begin{cases} 0, & \text{if } 0 \le x < \frac{2}{3}, \\ \sqrt{\frac{2}{3}} \cdot \frac{1}{\sqrt{x^3}}, & \text{if } \frac{2}{3} < x < \frac{8}{3}, \\ 0, & \text{if } \frac{8}{3} < x < +\infty. \end{cases} \qquad (2.11)$$

This result was published in the paper (Mazalov et al., 2005). The graf $H(f(x), y)$ has the form, presented in Fig. 1.

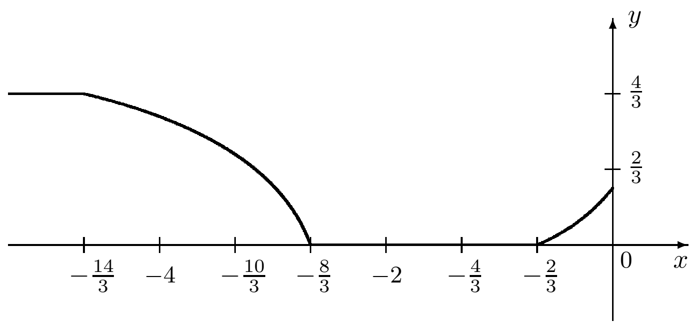

Fig. 1.

For $\alpha = 2$ we have

$$f(x) = \begin{cases} 0, & \text{if } 0 \le x < 2, \\ \frac{4}{x^2}, & \text{if } 2 < x < 4, \\ 0, & \text{if } 4 < x < +\infty. \end{cases} \qquad (2.12)$$

The graf $H(f(x), y)$ has the form, presented in Fig. 2.

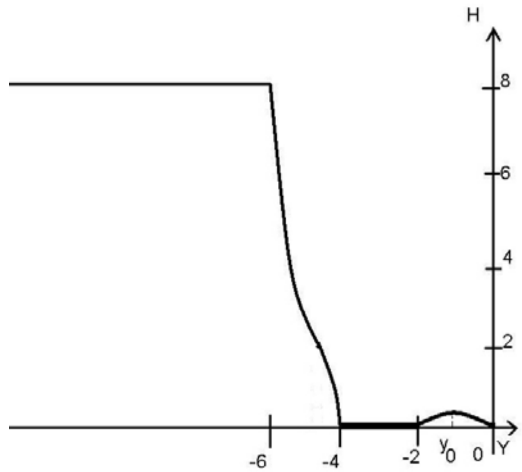

Fig. 2.

Theorem 2. *If $\alpha = 1$, then for the player L the strategy*

$$f(x) = \begin{cases} 0, & \text{if } 0 \le x < c, \\ \frac{(n+1)\sqrt{c}}{2\sqrt{x^3}}, & \text{if } c < x < c+2, \\ 0, & \text{if } c+2 < x < +\infty, \end{cases} \tag{2.13}$$

where $c = \frac{2n^2}{2n+1}$ is optimal.

Proof. Assuming in (1.13) $\alpha = 1$, we come to the formula (2.13) with corresponding constant c. Check the fulfilment of optimal conditions.

Assume that $y \in (-\infty, -(c+2) - 2n]$, then

$$H(f(x), y) = \int_c^{c+2} xf(x)dx = \sqrt{c(c+2)} = \frac{2n(n+1)}{2n+1}. \tag{2.14}$$

Furthere, let $k = 3\left[\frac{n}{2}\right] + 2$, if n is odd and $k = 3\frac{n}{2}$, if n is even. For $y \in [-(c+2) - 2n + 2r, -c - 2n + 2r]$, where $r = 0, 1, ..., n, ..., k-1$ and $y \in [-(c+2) - 2n + 2r, 0]$, if $r = k$, we find

$$H(f(x), y) = \frac{1}{2n+1}\left[ry + \left(\int_c^{-2n+2r-y} xf(x)dx + \int_{-2n+2r-y}^{c+2} yf(x)dx\right) + \right.$$
$$\left. +(2n-r)\int_c^{c+2} xf(x)dx\right] =$$
$$= \int_c^{c+2} xf(x)dx - \frac{1}{2n+1}\left[r\int_c^{c+2}(x-y)f(x)dx + \int_{-2n+2r-y}^{c+2}(x-y)f(x)dx\right]. \tag{2.15}$$

Differentiating (2.15), we obtain

$$H'(f(x),y)=\frac{1}{2n+1}\left[r+\int_{-2n+2r-y}^{c+2}f(x)dx+(2n-2r+2y)f(-2n+2r-y)\right]=$$

$$=\frac{r-n}{2n+1}\left(1+\frac{2n(n+1)}{\sqrt{2(2n+1)(-2n+2r-y)}}\right) \qquad (2.16)$$

It follows from (2.16) that in the interval $[-(c+2),-c]$, where $r=n$, the expected payoff $H(f(x),y)$ is constant and because we used the equality $H(f(x),-c-0)=0$ it yields $H(f(x),y)=0$ in the interval $[-(c+2),-c]$.

For $r<n$ (2.16) gives $H'(f(x),y)<0$ and for $r>n$ — $H'(f(x),y)>0$ in the interval $[-2n+2r-(c+2),-2n+2r-c]$.

Consequently, the function $H(f(x),y)$ is positive outside the interval $[-(c+2),-c]$. That proves the optimality of the strategy (2.13). □

Theorem 3. *If $\alpha=2$, then for the player L the strategy*

$$f(x)=\begin{cases}0, & \text{if } 0\le x<2n,\\ \frac{2n(n+1)}{x^2}, & \text{if } 2n<x<2n+2,\\ 0, & \text{if } 2n+2<x<+\infty\end{cases} \qquad (2.17)$$

is optimal.

Proof. Assuming in (1.13) $\alpha=2$, we come to the formula (2.17). Check the fulfilment of optimal conditions.

Assume then $y\in(-\infty,-(4n+2)]$, then

$$H(f(x),y)=\int_{2n}^{2n+2}x^2f(x)dx=4n(n+1). \qquad (2.18)$$

Furthere, let $y\in[-(2n+2k+2),-(2n+2k)]$, where $k=-n,-(n-1),\dots,-1,0,1,\dots,n-1,n$. Then

$$H(f(x),y)=\frac{1}{2n+1}\left[(n-k)\int_{2n}^{2n+2}(-y^2)f(x)dx+\int_{2n}^{-2k-y}x^2f(x)dx+\right.$$

$$\left.+\int_{-2k-y}^{2n+2}(-y^2)f(x)dx+(n+k)\int_{2n}^{2n+2}x^2f(x)dx\right]=$$

$$=\frac{1}{2n+1}\left[-(n-k)y^2+2n(n+1)(-2k-y-2n)+\right.$$

$$\left.+2n(n+1)y^2\left(\frac{1}{2n+2}+\frac{1}{y+2k}\right)+4n(n+1)(n+k)\right]. \qquad (2.19)$$

For $k=0$ we have $y\in[-(2n+2),-2n]$ and $H(f(x),y)=0$ in this interval.

Furthere, assume that $y=-2n-2k$. We have

$$H(f(x),-2n-2k)=\frac{4n(n+k)(k-1)}{2n+1}. \qquad (2.20)$$

From (2.20) we obtain that $H(f(x), -2n-2k) = 0$ for $k = -n$, $k = 0$ and $k = 1$; and $H(f(x), -2n-2k) > 0$ for all other considered values of k.

Besides

$$H'(f(x), y) = \frac{2k}{2n+1}\left[y - \frac{4n(n+1)k}{(y+2k)^2}\right], \tag{2.21}$$

$$H''(f(x), y) = \frac{2k}{2n+1}\left[1 + \frac{8kn(n+1)}{(y+2k)^3}\right]. \tag{2.22}$$

If now $k \geq 1$, then $H'(f(x), y) < 0$ and the function $H(f(x), y)$ is monotonous decreasing in the interval $[-(4n+2), -(2n+2)]$ from $4n(n+1)$ to 0. If $k \leq -1$, then $H''(f(x), y) < 0$ and the function $H(f(x), y)$ is concave in the interval $[-(2n+2k+2), -(2n+2k)]$.

Take into cosideration preceding arguments, we conclude that $H(f(x), y) > 0$ in the interval $(-2n, 0)$ and $H(f(x), 0) = 0$.

□

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