Game-Theoretic Model on a Cognitive Map and its Tolerance to Errors in Input Data to Analyze a Conflict of Interests Between Russia and Norway in Barents Sea*

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Abstract Some problems of complex control in the fields connected with public life (i.e., social-economic, political and other fields) include ill-structured control object. Situation appears ill-structured if the basic parameters have qualitative (not quantitative) nature, and their values are subjective expert evaluations. Cognitive maps serve to solve control problems for ill-structured situations. Cognitive map is a model representing knowledge of the expert (or a group of experts) regarding situation; this model is described in the form of weighted directed graph. The nodes of cognitive map correspond to those concepts being employed to describe the situation. The concept may be treated as a variable (for instance, "national defence capacity") which may have different values, such as "high", "low" and so on. Weighted arc is interpreted as direct cause-effect relationship between two concepts. Suppose several decision-makers (agents) take part in the process of decision making in an ill-structured situation given that the utility of each of them depends both on his self actions and the actions of the others, than interactions of the agents can be seen as a game on the cognitive map. In the game cognitive map represents a model of ill-structured control object and clearly describe the dynamics of the situation. The use of cognitive maps in the game gives more detailed and visual simulation of the environment of the conflict in the form of simple causal links, so as to describe the goals and strategies of the agents in terms of the environment which makes it more convenient to simulate the real conflicts adequately. Since the input data for the model are expert evaluation prone to subjectiveness, it is necessary to estimate the tolerance of model results to errors in input data. Experts evaluate the "importance percentage" of a target concept compared the others target concepts and the weight of edges in cognitive map as the type of the causal links and its strength. In this paper we consider the problem of model tolerance to errors in input data and illustrate it on the material of conflict of interests between Russia and Norway in the Barents Sea.

Keywords: game, cognitive map, conflict of interests, dominant strategy, tolerance to errors.

1. Introduction

Cognitive maps were previously introduced by Axelrod (1976) to clarify and improve decision making process. A cognitive map is a weighted digraph-based mathemat-

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ical model of a decision maker belief system about some limited domain, such as a policy problem. Cognitive map nodes correspond to situation concepts. Concepts are interpreted as variables whose values may vary. Weighted edges are interpreted as direct causal links from one concept to another. Analysis of possible situation developments depending on the control (in terms of an influence on some concepts) is one of the possible applications of cognitive maps. Both direct (situation development prediction with the fixed control) and inverse (search of the appropriate control) cognitive analysis problems are considered for this purpose.
The game-theoretic model of interactions between several agents at a dynamic system in the form of a situation cognitive map was generally considered by Novikov (2008). Since the input data for the model are expert evaluation prone to subjectiveness, it is necessary to estimate the tolerance of model results to errors in input data. Experts evaluate the "importance percentage" of a target concept compared the others target concepts and the weight of edges in cognitive map as the type of the causal links and its strength. In this paper we consider the problem of model tolerance to errors in input data and illustrate it on the material of conflict of interests between Russia and Norway in the Barents Sea.

2. Description of model

2.1. General model

A linear cognitive map C is called a weighted digraph, if its nodes (concepts) and edges (causal links) meet the conditions stated below, and the undermentioned rule regarding node value dynamics is given. By $M = \{1, \ldots, m\}$ denote the set of all concepts. A causal concept is a concept where an edge starts; an effect concept is a concept where an edge ends. Thereafter let an adjacency matrix of the digraph W be a matrix with elements $w_{ji} \in R$, if elements of the matrix correspond to weights of graph edges, which define types and strengths of causal links. Strength of the causal link from the j-th causal concept to the i-th effect concept is equal to the absolute value of the edge weight $|w_{ji}|$. The sign of the edge weight corresponds to the link type: if $w_{ji} > 0$, then the causal link from the j-th concept to the i-th one is positive, if $w_{ji} < 0$, then the causal link is negative (Roberts, 1976).
All results were obtained for discrete time and the zero-time initial state. An pulse process of a cognitive map is defined by the rule (1) with the initial concept vector $x(0) = (x_1(0), x_2(0), \ldots, x_m(0))$, $x(0) \in R^m$, and the vector $p = (p_1, p_2, \ldots, p_m)$, $p \in R^m$ of an external pulse to each node at the zero time point (Roberts, 1976).

$$x_i(t+1) = x_i(t) + p_i(t), p_i(t) = \begin{cases} p_i, & \text{if } t = 0; \\ \sum_{j \in M} w_{ji} \cdot p_j(t-1), & \text{if } t = 1, 2, 3, \ldots \end{cases}. \tag{1}$$

Let us fix the discrete time point T $(T > 0)$. Then the concept vector $x(T)$ is defined by the expression:
$x(T) = x(0) + p(0) + p(1) + \cdots + p(T-1) = x(0) + p + p \cdot W + \cdots + p \cdot W^{T-1} = x(0) + p \cdot (E + W + \cdots + W^{T-1}) = x(0) + p \cdot {}_TQ$.
Where E is an identity matrix. Let a matrix ${}_TQ = E + W + \cdots + W^{T-1}$ be a matrix of an influence reachability by the time T for the adjacency matrix W. Then the sum of the consequent increments for the concept x_j is as follows:

$$\sum_{t=0}^{T} p_j(t) = \sum_{k \in M} {}_T q_{kj} \cdot p_k. \tag{2}$$

Where ${}_T q_{kj}$ are elements of the matrix ${}_T Q$. Let us consider the problem of semistructured situation control at a linear cognitive map-based model. Let control actions be external pulses to each node at the zero-time point p; where $p_j = 0$, if there is no control to the node j. A control effect is a set of all concept values at the time point :

$$x_j(T) = x_j(0) + \sum_{t=0}^{T} p_j(t), j \in M. \tag{3}$$

A control target is defined by desirable values for all or some concepts $x(T) =$ ($x_1(T)$, $x_2(T)$, $\ldots$, $x_m(T)$), $x(T) \in R^m$ (Roberts, 1976).
The agent with the number $i \in N$ has a nonempty subset of concepts $M_i \in M$ he can control. Let M_i be a set of controlled concepts of the i-th agent. For any two agents $i, j \in N : M_i \cap M_j = \emptyset$ and $\cup_{k \in N} M_k \in M$. By m_i denote the number of concepts at the set M_i.

A control action of each agent is contained in a vector of mutual control actions $p = (p_1, p_2, \ldots, p_m)$. Let the strategy s_i of the i-th agent be a vector of ordered components of the vector p with indices from the set $\{k_1, k_2, \ldots, k_{m_i}\} = M_i$: $s_i = (p_{k_1}, p_{k_2}, \ldots, p_{k_{m_i}})$. Each agent defines only "his" components of the vector p during the influence on the situation. If there are no agent who influences on the concept, then the corresponding component is null: $(\forall j \in M - \cup_{k \in N} M_k)$, $p_j = 0$. Control actions require some expense of limited resources. Let us impose the basic restrictions to control actions for each concept in the form of the interval of acceptable values: $(\forall j \in \cup_{k \in N} M_k)$ $p_j \in [-1, 1]$. Then *the set of i-th agent strategies* S_i can be represented as the Cartesian product m_i of intervals $[-1, 1]^{m_i}$. Let the hypercube $(s_1, s_2, \ldots, s_n) \in S_1 \times \cdots \times S_n$ be the set of all agent strategies $S_1 \times \cdots \times S_n$.
Let us define the utility function $f_i(x_1(T), x_2(T), \ldots, x_m(T))$ on the result set for each agent. The control target of i-th agent is the maximization the function f_i. If the i-th agent want to increase (alternatively, decrease) in the value of the concept x_j , then it is desirable for him to maximize the expression $(x_j(T) - x_j(0))$ (similarly $-(x_j(T) - x_j(0))$). If the agent i can define desirable values for several concepts, then the weighted sum should be maximized according to the above stated expressions for such concepts. Each coefficient is interpreted as an "importance percentage" of restrictions on the corresponding concept. The utility function of the i-th agent is as follows:

$$f_i(x_1(T), x_2(T), \ldots, x_m(T)) = \sum_{j \in M} \gamma_{ij} \cdot (x_j(T) - x_j(0)). \tag{4}$$

Where $|\gamma_{ij}|$ is the "importance percentage" of the j-th concept value for the i-th agent, $\gamma_{ij} \in [-1, 1]$, the sum of all $|\gamma_{ij}|$ at the right hand side of the expression (4) is equal to 1. The sign of the coefficient γ_{ij} indicates the direction of variation of the concept value (being beneficial to the agent). In particular, provided $\gamma_{ij} > 0$ the i-th agent strives for infinite increasing the j-th concept value. If $\gamma_{ij} < 0$, then the i-th agent seeks to infinitely decrease the value of the j-th concept. Finally, $\gamma_{ij} = 0$ means the i-th agent does not care about the value of the j-th concept.

Let the target concept of the i-th agent be a concept with $\gamma_{ij} \neq 0$ at the utility function (4). After the definition of all game parameters, let us represent the game at the normal form:

$$\Gamma_C = \{N, \{S_i\}_{i \in N}, \{f_i\}_{i \in N}, C, T\}. \tag{5}$$

Let substitute $x_j(T)$ with the right-hand side of the expression (2) and (3) in formula (4). Proceeding in this manner, one derives

$$f_i = \sum_{j \in M} \gamma_{ij} \cdot (x_j(T) - x_j(0)) = \sum_{k \in M} \left(\sum_{j \in M} \gamma_{ij} \cdot {}_T q_{kj} \right) \cdot p_k = \sum_{k \in M} {}_T\alpha_{ik} \cdot p_k. \tag{6}$$

Dominant strategies of the agent i are defined by:

$$p_k = sign({}_T\alpha_{ik}), k \in M_i. \tag{7}$$

The model (5) is based on the model considered in Novikov (2008) but has some difference. It has a point of control effect T (target time) that let to make analysis more detail.

2.2. A model of conflict of interests between Russia and Norway in Barents Sea

Norway and Russia have sovereign rights over shelf space in the Barents Sea, which includes: 1) the Russian continental shelf (the right of Russia), 2) the Norwegian continental shelf (the right of Norway), 3) the offshore area of Svalbard (the right is governed by the Svalbard Treaty in Paris, 1920) and 4) continental shelves space disputed zone. Disputed territory is about 175 thousand sq. km. Disputed area after 40 years of negotiations was divided into two approximately equal parts in Russian-Norwegian treaty on maritime delimitation in the Barents Sea on September 15, 2010 (hereinafter the Treaty).

There was constructed cognitive map representations of the situation surrounding the signing of the Treaty (see Fig. 1) based on the materials from open source with expert evaluations of the situation in the Barents Sea and the Treaty. During constructing the model we should took into account the proportionality of the propagation time from concept to concept along the arc. The estimated time of impact along the arcs model in about 4-5 years. Time effect of impact from concept ♯12 to ♯7 is about 10 years, so between them added a dummy concept which is not marked in Fig. 1.

Control concept for Russia – concept ♯1, for Norway – concept ♯2. The initial impact $+1$ for each of these concepts is interpreted as a desire to conclude the Treaty. The impact -1 as the absence of such aspirations, and on the contrary, his rejection. The impact value equal to zero, can be interpreted as indifference of the gamer on this issue. The target concepts for Russia will consider two: ♯3 and ♯8 (with "importance percentage" $\gamma_{1,3} = 0.5$ and $\gamma_{1,8} = 0.5$) for Norway ♯4 and ♯11 (with $\gamma_{2,4} = 0.5$ and $\gamma_{2,11} = 0.5$). The solution of the game is the equilibrium with dominant strategies. A set of solutions were found for different target times T (see Fig. 2).

We shall explain two broken lines represented on Fig. 2 in greater detail. In the model (5) the desirable variations of values of target concepts are established for the fixed point in time T in the future (target time T). Thus if the target time T

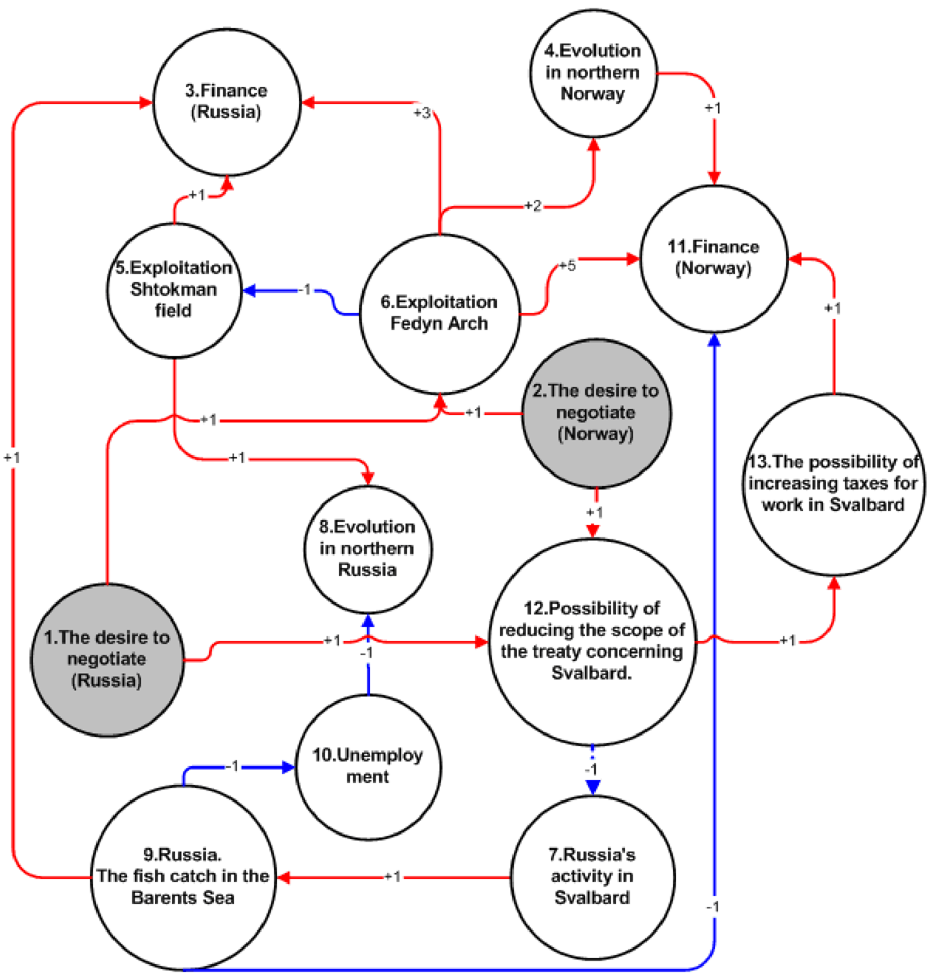

Fig. 1: Cognitive map that reflects the causal links between concepts in the problem of the disputed territory in the Barents Sea. (The target concepts Russia is ♯3 and ♯8, Norway is ♯4 and ♯11)

is small, it means, that agents are inclined to statement of short-term targets and wait for fast results from the action. Than more value so more "far-sightedness" of agent targets. There is a solution of game (5) in the form of equilibrium of dominant strategies according (7) at fixed . Different values are corresponds with different games, accordingly and different dominant strategies of agents. On axis "X" on Fig. 2 different values of target time , that is the different games in which targets of agents are changing from short term (for values 1, 2, 3, 4, 5, 6 on axis) to long run (9, 10, 11, 12, etc.). The lines in Fig. 2 shows how the "far-sightedness" agents in terms of targets, affects the optimal strategy to choose, in accordance with the targets. As can be seen from Fig. 2 division of the disputed territory in the Barents Sea in two is profitable for Norway. It is profitable without depends on target time T. In the case of Russia the situation is quite different. According to Fig. 2 the signing of the Treaty will beprofitable for Russia in the short term, but not favourable in the long run.

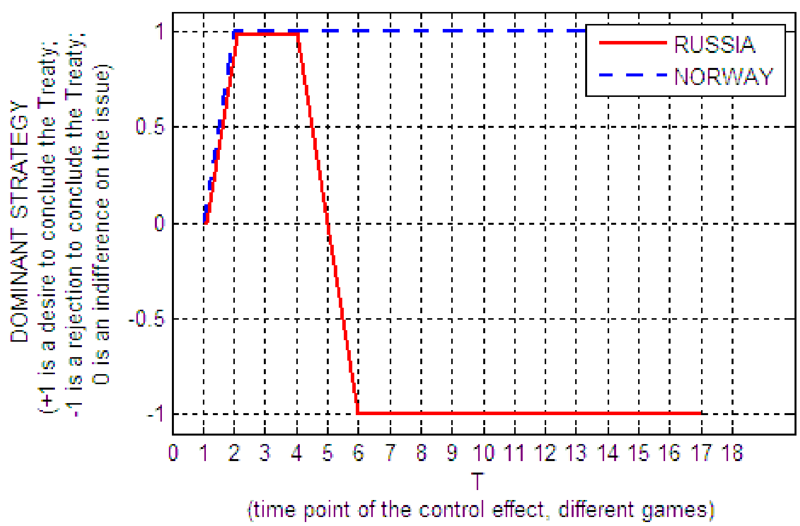

Fig. 2: The set of equilibriums in dominant strategies for games depending on the target time T. The axis "X" is the different values of the target time T, the vertical axis "Y" corresponding to the equilibrium strategies of agents: Russia and Norway.

The results of the work model (Fig. 2) were obtained with using expert evaluation of the importance of target concepts ($\gamma_{1,3}= 0.5$, $\gamma_{1,8} = 0.5$, $\gamma_{2,4} = 0.5$ and $\gamma_{2,11} = 0.5$) and expert evaluation of the weights of the arcs in the digraph of cognitive maps (Fig. 1). Let estimate the tolerance of the model results to errors in the expert evaluations.

3. Estimate of tolerance to errors in input data

3.1. Estimate of tolerance to errors in a target coefficient γ_{ij}

Let consider the situation where the expert make an error in one of the weights coefficients γ_{is} in (6). If expert did not make an error, the value of ${}_T\alpha_{ik}$ would be (8). Because of the error the value of ${}_T\alpha_{ik}$ changes to (9). It is enough to satisfy condition (10) to keep strategy (7) unchanged after making the error. From this condition we obtain an estimate of error in one coefficient in (6).

$$ {}_T\alpha_{ik} = \gamma_{i1\,T}q_{k1} + \gamma_{i2\,T}q_{k2} + \cdots + \gamma_{is\,T}q_{ks} + \cdots + \gamma_{i,m\,T}q_{k,m}. \tag{8} $$

$$ {}_T\alpha^{\varepsilon}_{ik} = \gamma_{i1\,T}q_{k1} + \gamma_{i2\,T}q_{k2} + \cdots + (\gamma_{is} \pm \varepsilon^{i}_{ks})_T q_{ks} + \cdots + \gamma_{i,m\,T}q_{k,m}. \tag{9} $$

$$ {}_T\alpha_{ik} \cdot {}_T\alpha^{\varepsilon}_{ik} = {}_T\alpha_{ik}({}_T\alpha_{ik} \pm \varepsilon^{i}_{ks} \cdot {}_Tq_{ks}) > 0 \Rightarrow \varepsilon^{i}_{ks} < \left| \frac{{}_T\alpha_{ik}}{{}_Tq_{ks}} \right|. \tag{10} $$

We claim that:

Proposition 1. *Suppose* there is an error of expert estimate in only one target coefficient γ_{is} $(s \in M_i)$ of the agent utility function f_i and it doesn't exceed the value (11) *then* the dominant strategy of the agent i is invariable.

$$ \varepsilon^{i}_{s} = \min_{k \in M_i} \left| \frac{{}_T\alpha_{ik}}{{}_Tq_{ks}} \right| \tag{11} $$

Indeed, this follows from the necessary of working the condition (10) for all control concepts of agent i. The value (11) is called the *estimate of tolerance to errors in value of* γ_{is} or *allowable error.*

Fig. 3 shows the results of calculations of (11) for the target concepts of both agents (Russia ♯ 3, ♯ 8 and Norway ♯ 4, ♯ 11). Fig. 3 shows that the least sensitive to changes (due to errors) is concept ♯ 4. The value of $\gamma_{2,4}$ is not critical to the invariance of the result when $T > 2$. The reason is probably in the causal link (♯4 $\rightarrow$ ♯11). We shall return to this fact further in analysis of allowable errors for the weights of the arcs of the digraph.

The estimates of tolerance to errors in value of γ_{ij} (11) are within the range of allowed values [-1, 1] for the other concepts. However, allowable error in γ_{ij} is quite large for the concepts ♯3 and ♯11. The allowable error in γ_{ij} is much smaller for the concept ♯8. It is clear that the correct evaluation of γ_{ij} for the concept ♯8 is the most important for choosing the optimal strategy. It requires a high confidence in the correct evaluation $\gamma_{1,8}$.

Note that the values of the allowable errors for the concepts ♯ 3 and ♯ 8 at $T = 5$ are equal to zero. It is happened because the optimal strategy for Russia at $T = 5$ is an omission (see Fig. 2). This strategy is not stable to errors in the coefficient values γ_{ij}, because (7).

A similar analysis of allowable errors for the target concepts can be used for their selecting. For example the allowable error in concept ♯4 is bigger than the high limit of the value range $\gamma_{2,4} \in [-1, 1]$. The concept ♯ 4 should not be selected as the target concept in a game with cognitive map (Fig. 1) because it is not critical for the invariance result.

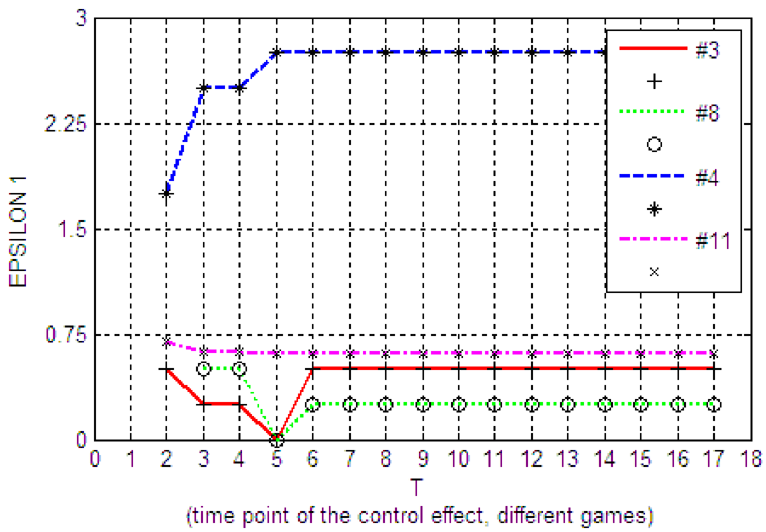

Fig. 3.

3.2. Estimate (lower bound) of tolerance to error in all target coefficients

Let consider the situation where the expert make an error in all weight coefficients in (6). If expert did not make an error, the value of ${}_T\alpha_{ik}$ would be (8). Because of the error the value of ${}_T\alpha_{ik}$ changes to (12). It is enough to satisfy condition (13) to keep strategy (7) unchanged after making the error. From this condition we obtain an estimate of error (lower bound) in all coefficients in (6).

$$ {}_T\alpha^{\varepsilon}_{ik} = (\gamma_{i1} \pm \varepsilon^i_k)_T q_{k1} + (\gamma_{i2} \pm \varepsilon^i_k)_T q_{k2} + \cdots + (\gamma_{is} \pm \varepsilon^i_k)_T q_{ks} + \cdots + (\gamma_{i,m} \pm \varepsilon^i_k)_T q_{k,m}. \tag{12} $$

We claim that:

$$ {}_T\alpha_{ik} \cdot {}_T\alpha^{\varepsilon}_{ik} = {}_T\alpha_{ik}({}_T\alpha_{ik} \pm \varepsilon^i_k \cdot \sum_{s=1}^{m} {}_T q_{ks}) > 0 \Rightarrow \varepsilon^i_k < \left| \frac{{}_T\alpha_{ik}}{\sum_{s=1}^{m} {}_T q_{ks}} \right|. \tag{13} $$

Proposition 2. *Suppose* there are some errors of expert estimates in target coefficient γ_{is} $(s \in M_i)$ of the agent utility function f_i and each of them doesn't exceed the value (14) *then* the dominant strategy of the agent i is invariable.

$$ \varepsilon^i = \min_{k \in M_i} \left| \frac{{}_T\alpha_{ik}}{\sum_{s=1}^{m} {}_T q_{ks}} \right|. \tag{14} $$

Indeed, this follows from the necessary of working the condition (13) for all control concepts of agent i. The value (14) is called the *estimate of tolerance to errors in all values of* γ_{is} or *allowable error in all values* γ_{is} $(s \in M_i)$.
Fig. 4 shows the results of calculations of (14) for both agents (Russia and Norway). Fig. 4 shows that the allowable error in all values γ_{is} is very small. It is an illustration of the unstable situation of Russia in this game (5). The optimal solution for Russia in the model is not stable for a fixed system of priorities $\gamma_{1,3} = 0.5$ and $\gamma_{1,8} = 0.5$.

The values of the target coefficients $\gamma_{2,4} = 0.5$ and $\gamma_{2,11} = 0.5$ for Norway are stable enough. The optimal solution for Norway stays the same for all γ_{ij} in range ± 0.5. Note that the values of the allowable errors for the target concepts for Russia at $T = 5$ are equal to zero. It is happened because the optimal strategy for Russia at $T = 5$ is an omission (see Fig. 2). This strategy is not stable to errors in the coefficient values γ_{ij}, because (7).

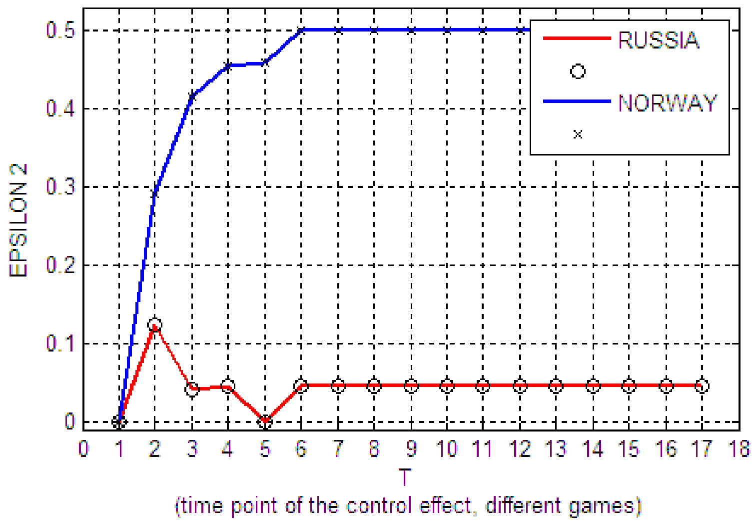

Fig. 4.

3.3. Estimate (lower bound) of tolerance to error in a weight of arc in digraph of cognitive map

Let consider the situation where the expert make an error in δ in a weight of arc w_{rs} of cognitive map. In this case, the adjacency matrix of a digraph to be the next:

$$W_\delta = \begin{pmatrix} w_{11} & \cdots & w_{1s} & \cdots & w_{1m} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ w_{r1} & \cdots & w_{rs} \pm \delta & \cdots & w_{rm} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ w_{m1} & \cdots & w_{ms} & \cdots & w_{mm} \end{pmatrix}$$

In this situation the error δ is the reason for errors in elements of *matrix of an influence reachability by the time* $T\, {}_TQ$:
${}_TQ_\varepsilon = (E + W_\delta + W_\delta^2 + \cdots + W_\delta^{T-1})$.
The elements of the matrix $({}_TQ_\varepsilon - {}_TQ)$ correspond to the changes in every element of matrix of an influence reachability by the time T caused by an error δ in adjacency matrix W. The change in one element can be represented as (15), where $P_k(w_{ks})$ the algebraic sum of products of elements of matrix W. It is necessary to estimate the error ε in the value of elements of matrix ${}_TQ$ for getting the estimation the

tolerable error δ in the matrix element w_{rs}.

$$\left|{}_Tq^{\varepsilon}_{ij} - {}_Tq_{ij}\right| = \left|\delta \cdot P_1(\{w_{ks}\}) + \delta^2 \cdot P_2(\{w_{ks}\}) + \cdots + \delta^{T-1} \cdot P_{T-1}(\{w_{ks}\})\right| < \varepsilon. \tag{15}$$

Similar to the arguments (12), (13) we obtain an estimate of error ε^i_k for the elements of ${}_TQ$ (16)-(17).

$${}_T\alpha^{\varepsilon}_{ik} = \gamma_{i1}({}_Tq_{k1} \pm \varepsilon^i_k) + \gamma_{i2}({}_Tq_{k2} \pm \varepsilon^i_k) + \cdots + \gamma_{is}({}_Tq_{ks} \pm \varepsilon^i_k) + \cdots + \gamma_{i,m}({}_Tq_{k,m} \pm \varepsilon^i_k). \tag{16}$$

$${}_T\alpha_{ik} \cdot {}_T\alpha^{\varepsilon}_{ik} = {}_T\alpha_{ik}({}_T\alpha_{ik} \pm \varepsilon^i_k \cdot \sum_{s=1}^{m} {}_T\gamma_{is}) > 0 \Rightarrow \varepsilon^i_k < \left|\frac{{}_T\alpha_{ik}}{\sum_{s=1}^{m} {}_T\gamma_{is}}\right|. \tag{17}$$

The value ε^i_k is so error in every element of matrix ${}_TQ$ that the value of the control concept k in the dominant strategy of the agent i is invariable. It is clear that if error in every element of matrix ${}_TQ$ less than (18) then the dominant strategies of every agent is invariable.

$$\varepsilon = \min_{i \in N} \min_{k \in M_i} \left|\frac{{}_T\alpha_{ik}}{\sum_{s=1}^{m} \gamma_{is}}\right| \tag{18}$$

We claim that:
Proposition 3. *Suppose* there is an error of expert estimate in a weight of arc w_{rs} of cognitive map and it doesn't exceed the value δ from (15), where ε is calculated as (18) *then* the dominant strategies of every agent is invariable.
The value δ is called the *estimate of tolerance to error in a weight of arc in digraph of cognitive map or allowable error in* $w_{rs}(r, s \in M)$.
We can calculate the allowable error δ in the weight of w_{rs} using the expression (15) with ε from (18). Note that the expression standing on the left of the inequality sign in (15) is a continuous function from δ, which always intersects the Ox-axis at 0. Consequently, if $\varepsilon > 0$ then there are values in a neighborhood of 0 witch satisfy (15).
On the basis of Symbolic Math Toolbox MATLAB were calculated the allowable error δ in all weights of arcs in digraph of cognitive map on Fig. 1 (Fig. 5).

Fig. 5 shows the dependence of the allowable error δ from target time T in the game (5). The weights of the links ($\sharp 1 \to \sharp 6$) and ($\sharp 2 \to \sharp 6$) have the most low values of allowable errors δ. This is illustration of the importance of the concept $\sharp$ 6 for agents target. The weight of the arc ($\sharp 4 \to \sharp 11$) is less critical, but important. The reason is probably in the fact that both concepts $\sharp$ 4 and $\sharp$ 11 are the target concepts for Norway. This connection provides an additional agreement between two sub-targets. The values of links ($\sharp 1 \to \sharp 12$) and ($\sharp 2 \to \sharp 12$) are also important. The arcs ($\sharp 1 \to \sharp 6$), ($\sharp 2 \to \sharp 6$), ($\sharp 1 \to \sharp 12$) and ($\sharp 2 \to \sharp 12$) are the main causal links witch providing connectivity the control concepts with all other in cognitive map.

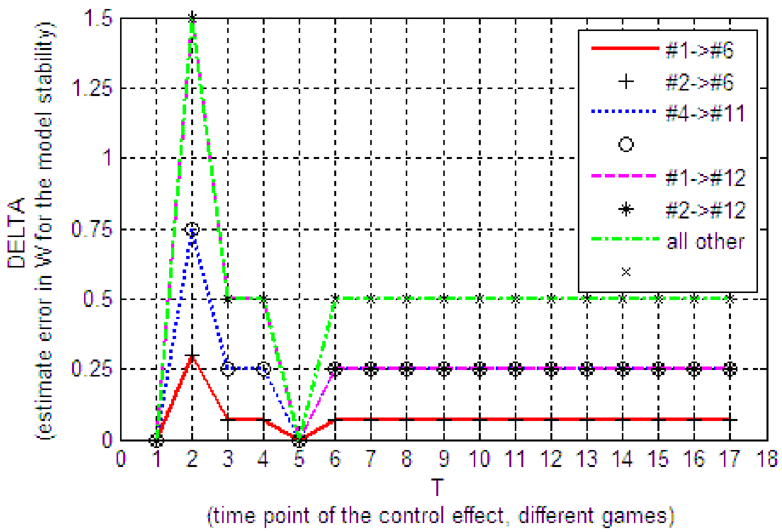

Fig. 5.

4. Conclusion

This paper deals with the issue of input data error tolerance of the game on cognitive map according to expert evaluations. The model is constructed on the basis of conflict of interests between Russia and Norway on maritime delimitation in the Barents Sea. As a result was evaluated the error in the coefficients of the utility functions of agents as well as in the weights of the arcs of the digraph of cognitive maps in this model. The estimation of errors tolerance made it possible to understand the structural properties of the model and to assess the degree of selecting target concepts feasibility.

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