On a Mutual Tracking Block for the Real Object and its Virtual Model-Leader

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Abstract The research is devoted to a feedback control problem of stochastic stable mutual tracking for motions of a real dynamical object, and some virtual computer simulated model-leader, under dynamical and informational disturbances. The control and disturbance actions in the model are determined by proposed random tests. To obtain solution to the considered problem we apply the so-called extremal minimax and maximin shift conditions. Theoretical results are illustrated by numerical simulations.

Keywords: feedback control, nonlinear system, extremal shift.

1. Introduction

The investigations in this work are based on the approaches, methods and constructions from the theory of stochastic processes, theory of stability, theory of optimal control and differential games, tracing and observation of the processes and so on, which proposed and are developed in the works of Bellman (1957), Isaacs (1965), Krasovskii and Subbotin (1974), Kurzhanski (1977), Mishchenko (1972), Osipov and Kryazhimskii (1995), Pontryagin, Boltyanskii, Gamkrelidze and Mishchenko (1962) and many other authors. This work uses the ideas of the books (Krasovskii and Subbotin, 1974; Krasovskii and Krasovskii, 1994). The stochastic process for the solution of considered problem is based on the appropriate constructions of the so-called extremal shift (Krasovskii, 1980) of given controlled x-object to its virtual w-model-leader.

2. Mutual tracking block. Extremal shift.

The dynamics of x-object is described by the vector ODE – nonlinear in controls u and disturbances v:

$$\dot{x} = A(t)x + f(t,u,v) + h_{din}(t), \quad t_0 \le t \le \theta, \tag{1}$$

subject to restrictions:

$$u \in P = \{u^{[1]}, \dots, u^{[M]}\}, \quad v \in Q = \{v^{[1]}, \dots, v^{[N]}\}. \tag{2}$$

Here symbols M and N are given numbers. Symbol $h_{din}(t)$ denotes a random vector-function restricted by the following constrains:

$$|h_{din}(t)| \le H, \quad E\{h_{din}(t)\} \le \delta_{din},\ t \in [t_0, \theta], \tag{3}$$

where H stands for a sufficiently large constant, δ_{din} is a small constant, where $E\{\cdots\}$ is the mathematical expectation (Liptser and Shiryaev, 1974).

Let us consider the case, when the *saddle point condition for the small game* (McKinsey, 1952), i.e.:

$$min_{u\in P}\, max_{v\in Q}\, \langle l\cdot f(t,u,v)\rangle = max_{v\in Q}\, min_{u\in P}\, \langle l\cdot f(t,u,v)\rangle, \tag{4}$$

where l is any n-dimensional vector and the symbol $\langle l\cdot f(t,u,v)\rangle$ denotes the inner product in R^n,*is not satisfied* for the function $f(t,u,v)$.

Let us choose a partition $t_k \in \Delta\{t_k\} = \{t_0, t_1, \ldots, t_k < t_{k+1}, \ldots, t_K = \theta\}$, where K is a large number, and consider the finite-difference equation for x-object:

$$x[t_{k+1}] = x[t_k] + (A(t_k)x[t_k] + f(t_k,u,v) + h_{din}(t_k))(t_{k+1} - t_k). \tag{5}$$

Together with a real x-object we consider the motion of an abstract w-model:

$$w[t_{k+1}] = w[t_k] + (A(t_k)w[t_k] + \sum_{i=1}^{M}\sum_{j=1}^{N} f(t_k, u^{[i]}, v^{[j]})p_i q_j + h_{din}(t_k))(t_{k+1} - t_k). \tag{6}$$

Here numbers $p_i, i = 1,\ldots,M$ and $q_j, j = 1,\ldots,N$ satisfy conditions:

$$p_i \geq 0, i = 1,\ldots,M, \sum_{i=1}^{M} p_i = 1, \quad q_j \geq 0, j = 1,\ldots,N, \sum_{j=1}^{N} q_j = 1. \tag{7}$$

We assume that the motion of w-model is simulated by a computer, implemented in a regulator, and considered as the "leader" (or "pilot") for the motion of x-object.

Further, we consider the case, when position $\{t_k, x[t_k]\}, k = 0,\ldots,K$, of x-object is estimated with some informational error $\Delta_{inf}[t_k]$, such that at each time moment $t_k \in \Delta t_k$ only the distorted position $\{t_k, x^*[t_k]\}$ is known, where:

$$x^*[t_k] = x[t_k] + \Delta_{inf}[t_k]. \tag{8}$$

Here $\Delta_{inf}[t_k]$ is a random vector.

Control actions for x-object and w-model, which provide mutual tracking in the combined process x-object, x-model-leader, are constructed as follows.

At the moment $t_k, k = 0,\ldots,K-1$, a vector of actions $u^0[t] = u^0[t_k] \in P,\ t \in [t_k, t_{k+1})$, for the real x-object is chosen by probability test:

$$P(u^0[t_k] = u^{[i]} \in P) = p_i^o, \quad i = 1,\ldots,M. \tag{9}$$

Here symbol P denotes probability (Liptser and Shiryaev, 1974) and probabilities $p_i^o : p_i^o \geq 0,\ i = 1,\ldots,M,\ \sum_{i=1}^{M} p_i^0 = 1$, are chosen from the so-called *Extremal Minimax Shift Condition*:

$$min_p\, max_q\, \langle l^*[t_k], \sum_{i=1}^{M}\sum_{j=1}^{N}\ f(t_k, u^{[i]}, v^{[j]})p_i q_j\rangle = \langle l^*[t_k], \sum_{i=1}^{M}\sum_{j=1}^{N}\ f(t_k, u^{[i]}, v^{[j]})p_i^0 q_j^*\rangle, \tag{10}$$

under restrictions (7). Here $l^*[t_k] = x^*[t_k] - w[t_k]$.

Let the "control action" $q^0[t_k]$ for the virtual w-model be chosen from the *Extremal Maxmin Shift Condition*:

$$max_p \; min_q \; \langle l^*[t_k], \sum_{i=1}^{M}\sum_{j=1}^{N} f(t_k, u^{[i]}, v^{[j]})p_i q_j\rangle = \langle l^*[t_k], \sum_{i=1}^{M}\sum_{j=1}^{N} f(t_k, u^{[i]}, v^{[j]})p_i^* q_j^0\rangle, \tag{11}$$

Probabilities $\{q_j\}$ that define the stochastic disturbances $v[t_k] \in Q$ on x-object, and "actions" $\{p_i\}$ for w-model may take arbitrary values subject to conditions (7).

Theorem 1. *Under described above choices (10) and (11) of the random actions* $u^0[t_k]$ *for* x*-object and "actions"* $q^0[t_k]$ *for* w*-model, for any chosen beforehand numbers* V^* *and* $0 < \beta < 1$*, there exist sufficiently small numbers* $\delta_0 > 0$*,* $\delta_{inf} > 0$*,* $\delta_{din} > 0$*,* $\delta > 0$*, such that the following inequality holds:*

$$P(V(t, l[t]) \leq v^*, \quad \forall t \in [9, \theta]) \geq 1 - \beta, \tag{12}$$

if $l[t_0] \leq \delta_0$*,* $E\{|l(t) - l^*(t)|\ l(t)\} \leq \delta_{inf}$*, for any admissible* $l[t] = x[t] - w[t]$*,* $t \in [0, \theta]$*,* $E\{h_{din}\{t\}\}$*, and* $\Delta t = t_{k+1} - t_k \leq \delta$*. Here:*

$$V(t, l[t]) = V(t, x[t], w[t]) = |x[t] - w[t]|^2 e^{\lambda t}. \tag{13}$$

Presented results are illustrated by a model example and its numerical simulation.

Example 1. In this section we apply the elaborated algorithms to computer for tracing of a motions x-object and w-model for the concrete 2-dimensional system. Let us consider the model problem such that the control x-object (1) is described by the finite-differential equation (5), where in our concrete case we assume:

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, A(t) = \begin{pmatrix} 0 \; 1 \\ 1 \; 0 \end{pmatrix}, \tag{14}$$

$$u \in P = u^{[1]} = -1, u^{[2]} = 1, \quad v \in Q = v^{[1]} = -1, v^{[2]} = 1, \tag{15}$$

and the function $f(t, u, v)$ has the form:

$$f(t, u, v) = \begin{Bmatrix} 0,5u + (u+v)^2 + v & for & t \in [0, \frac{\vartheta}{4}) \cup [\frac{\theta}{2}, \frac{3\vartheta}{4}), \\ u + (u+v)^2 + 0,5v & for & t \in [\frac{\vartheta}{4}, \frac{\theta}{2}) \cup [\frac{3\vartheta}{4}, \vartheta]. \end{Bmatrix} \tag{16}$$

As it was described above, $h_{din}(t_k)$ in (1) is a dynamical error (1) that has a random character. And we used the positional stochastic feedback control scheme in which the informational image $x^*[t_k]$ at the current moment $t_k \in \Delta t_k$ satisfies the condition (8). Here the w-model (6), (7) that corresponds to x-object (5),(14),(16) has the form:

$$w[t_{k+1}] = w[t_k] + (A(t_k)w[t_k] + \widetilde{f_{pq}}(t_k) + h_{din}(t_k))(t_{k+1} - t_k), \tag{17}$$

where:

$$\widetilde{f_{pq}}(t_k) = \sum_{i=1}^{2}\sum_{j=1}^{2} f(t_k, u^{[i]}, v^{[j]})p_i q_j. \tag{18}$$

Here $f(t_k, u^{[i]}, v^j)$ is a function (16), and: $p_i \geq 0, i = 1, 2,\ p_1 + p_2 = 1, \quad q_j \geq 0, j = 1, 2,\ q_1 + q_2 = 1$.

Under the values of parameters of the $\{x, w\}$ system (5),(14)–(18): $x_1[0] = -1.0$, $x_2[0] = 1.0$, $w_1[0] = -0.95$, $w_2[0] = 1.05$, $\vartheta = 4.0$, $\Delta t = t_{k+1} - t_k = \vartheta = 0.01$, $E|h_{inf}| \leq \delta_{inf} = 0.01$, $E|h_{din}(t)| \leq \delta_{din} = 0.01$, we obtain the results of the computer simulation for the motions of the x-object (solid line) and w-model (dashed line) presented at the figure 1.

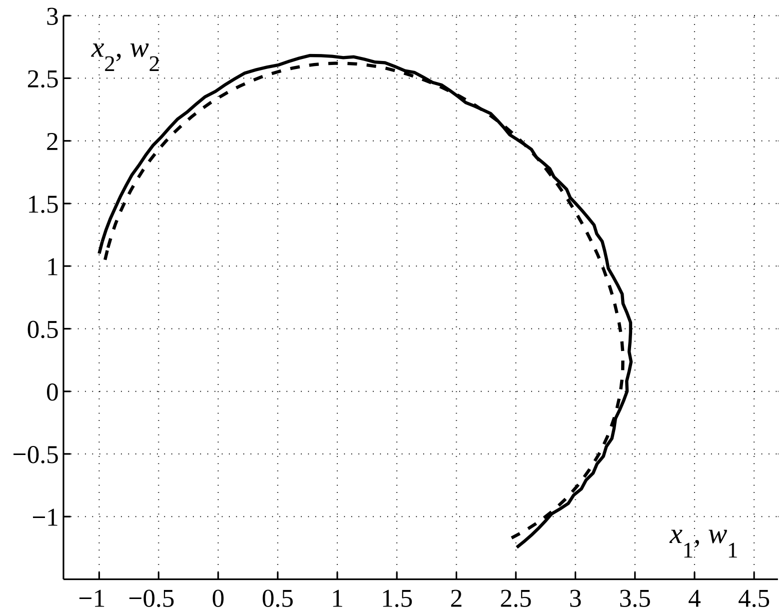

Fig. 1

At this Fig. 1 we have the phase portrait of the motion of x-object and w-model. In this case we chose the control actions $u^0[t] = u^0[t_k] \in P, \quad t_k \leq t < t_{k+1}$ for x-object and "actions" $q^0[t_k]$ for w-model under the algorithms (Extremal Minimax and Maximin Shifts Conditions) from section 2 . The control actions $v[t] = v[t_k] \in Q, \quad t_k \leq t < t_{k+1}$ for x-object and "actions" $p[t_k]$ for w-model we constructed by some random mechanism.

3. Conclusion

By the Theorem 1 the solution of the considered problem of the mutual tracing of the motions of the real controlled object and its virtual model-leader is established. The illustrative example and its computer simulation is given.

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