Game-Theoretic Models of Collaboration among Economic Agents[1]

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Abstract In present article are considered the models explaining the mechanisms of emergence and development of situations, in which it is appropriate for economic agents to collaborate and act together despite of having independent goals. The main attention is concentrated to different approaches to definition of concept of equilibrium for model of collaboration of two agents. The work is devoted to problems in the study of economic instruments, inducing the agents, which initially have independent and uncoordinated systems of goals to commission any beneficial actions. Particularly, we consider an interaction of economic agents when each of them may take the actions, that bring benefit to other. Stimulus to "positive" behavior each agent is a waiting counter actions, that will be useful for him. To identify this class of situations it is proposed to use the term "collaboration". In a model of collaboration between two economic agents is proposed version to express of mixed strategies of players in the form of continuous distribution, which enabled us to formulate two alternative approaches of equilibrium: based on the criterion of minimizing variance of utility of participants and based on the criterion of minimizing of VaR.

Keywords: Game theory, collaboration, Nash equilibrium, value at risk (VaR), quantile.

Models that explain the mechanisms of emergence and situation development, in which it is appropriate for economic agents to collaborate and act together despite of having independent goals have become rather interesting in both theoretical and application way. An interaction of economic agents, where each of them takes actions that bring direct benefit not only to him but to other agents, can serve as the simplest example. An expectation of a beneficial counter action is an incentive for each agent to behave in this way. The most important difference between this behavior model from the "classical" models of rational economic agent's utility optimization is that here the utility of each agent depends directly on decisions made by others, whom he can indirectly influence.

We define such examples of agents' interaction with the term "collaboration". We could as well use the "indication" collaboration for that. On the other hand, such definition could create false associations with models based on cooperative games, also a further framework of the proposed model is solely based on a strategy game with complete information.

The main problem which is considered in this article is to present one possible approach based on the methods of modern game theory, which makes us able to describe and explain the mechanisms of collaborative relations between economic agents.

Obviously collaboration (in the context in which we agreed to consider it) and related issues may arise, for example, between the parties of public and private partnership, alongside with major investment projects or different schemes of financing from various levels of budget sources. Moreover, such models can also be useful in situations that go beyond "pure" economics. For instance, they can be applied to studies of intergovernmental negotiation processes aimed at achievement of agreements, which will complexly take both economic and political interests of the parties into account.

We will consider a simplified situation in order to explain the fundamental ideas of the proposed model. It describes interaction between two parties (agents, participants, players) $i \in I = \{1, 2\}$, who make a decision upon the value of their own contribution to some common project. This contribution (degree) is quantitatively characterized by some arbitrary value from 0 to 1: where "0" stands for lack of affirmative action in the project (non-collaboration, extremely selfish behavior, etc.), and "1" reflects the highest possible level of affirmative action (the maximum propensity to collaborate, ultimately constructive behavior).

If we take into consideration previously set objectives when we define the utility functions of players, we assume that the input (costs) performed by the agents reduce utility they can get, utility can increase due to inputs of his opponents. Linear relations are acceptable in model, because they reflect adequately its fundamental properties. So we define the utility function of the first player, as

$$u_1(x_1, x_2) = b_1 x_2 - a_1 x_1 \tag{1}$$

and the utility function of the second player as

$$u_2(x_1, x_2) = b_2 x_1 - a_2 x_2 \tag{2}$$

Accordingly, a is a value (score, a measure of regret) of a resource unit spent (invested in the project) by the player i and b is utility (effect, measure of satisfaction) for the i–th player, which he gets from a unit invested in the project by another party. Let's imaging such a situation as "classical" finite non-cooperative two-person game. We face the fact that it has an obvious Nash equilibrium in pure strategies

$$x_1^* = 0,\, x_2^* = 0. \tag{3}$$

Obviously our productivity functions are arranged in such a way (see Fig.2) that the best response of the first player to any second player's strategy will be to reduce his share of participation to zero.

$$\max_{x_1 \in [0,1]} \{u_1(x_1, x_2)\} = u_1(0, x_2),\, \forall x_2 \in [0, 1] \tag{4}$$

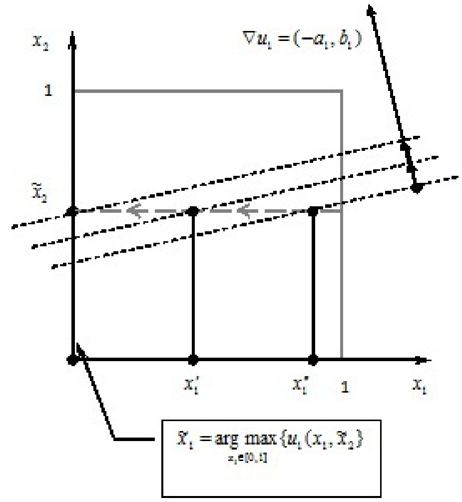

Fig. 1: Productivity functions

$$\max_{x_2 \in [0,1]} \{u_2(x_1, x_2)\} = u_2(x_1, 0), \, \forall x_1 \in [0, 1] \tag{5}$$

Thus, if we follow the concept of Nash equilibrium, we arrive to a pessimistic conclusion that the model described in the framework of collaboration between the players would not happen (the most stable situation is "mutual self–interest"). In this context, this is a particular interest to study modifications of this model in order to explain the mechanisms, which lead to the emergence of a collaborative relationship (collaboration) between economic agents.

First of all we concentrate on the approaches associated with transition from the original game to its mixed extension. Due to the fact that this scenario is based on a continuous set of pure strategies of the players, it seems obvious to set their mixed strategies as probability distributions with densities $p_1(x_1)$ and $p_2(x_2)$ on the interval $[0, 1]$, see Fig.7. According to this suggestion, a particular choice of strategies by players in a particular round of the game can be interpreted as an implementation of independent random variables $\tilde{x_1}$ and $\tilde{x_2}$.

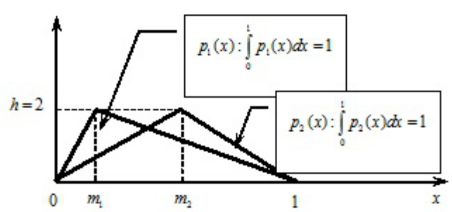

Fig. 2: Mixed strategies densities

This idea of mixed strategies of the players is a generalization of the "traditional" definition of mixed strategies in matrix and bimatrix games, which can be defined as a likelihood $(p_1, \ldots, p_k, \ldots)$ in accordance with every player implement one or another pure strategy. To follow this logic, we would have had to sample the intervals $[0, 1]$ in order to bring in traditional "discontinuous" mixed strategies. This method, however, seems to be not enough justified and reasonable in terms of reflecting the economic realities.

When mixed strategies are defined in the form of continuous distributions, a player's strategic choice is generally reduced to the choice of parameters of these distributions. Due to the fact that the number of parameters in different probability distributions classes is different, we come to a conclusion that definition of the players' strategies within the stated model will vary according to the type of distribution $p_1(x)$ or $p_2(x)$ we've chosen. Actually the value of strategies chosen by the participants in each act of the game can be viewed as a realization of independent random variables $\tilde{x}_1, \tilde{x}_2$, whose densities are known; and utilities $u_1(\tilde{x}_1, \tilde{x}_2), u_2(\tilde{x}_1, \tilde{x}_2)$ are determined as functions of random variables, which characteristics, generally spoken, can be determined with the help of $p_1(x), p_2(x)$.

We should note that specification of participants' strategic choices in the form of continuous probability distributions can be justified by the theory of evolutionary games. Namely, we can assume that we have a community consisting of groups (populations). Different populations of players have different tendencies to collaborate (collaborative behavior). These tendencies are realizations of random variables $\tilde{x}_i$ with densities $p_i(x)$. When members of different populations confront in some acts of the game, their success (or lack of success) can be expressed in terms of utility $\tilde{u}_i$. After that evolution of stochastic characteristics of propensity to cooperate takes place and these indicators reach some "benchmark" stable states, based on the experience accumulated by populations.

Of course if the strategies of participants are determined with continuous probability distributions, we can only compare them correctly if function $p_i(x)$ is restricted by some single parametric class $\mathbf{P}_i$. In this case, parameters of density functions $p_i(x)$ become "obvious" characteristics of strategies. Accordingly, the set of possible situations in a game is defined by the set of all possible combinations $p_i(x)$ of all players.

In terms of the classical Nash approach (Vorobiev, 1984), (Vorobiev, 1985), (Moulin, 1985), (Pecherskiy and Belyaeva, 2001) the equilibrium (solution) in of the described model will be characterized by such joint choice of probability distributions $(p_1^*(x), p_2^*(x))$ from which every participant in the game would not be advantageous to deviate separately from, i.e.:

$$\mathbf{E}\{u_1(\tilde{x}_1, \tilde{x}_2) \parallel p_1^*(x), p_2^*(x)\} \geqslant \mathbf{E}\{u_1(\tilde{x}_1, \tilde{x}_2) \parallel p_1(x), p_2^*(x)\} \tag{6}$$

$$\mathbf{E}\{u_1(\tilde{x}_1, \tilde{x}_2) \parallel p_1^*(x), p_2^*(x)\} \geqslant \mathbf{E}\{u_1(\tilde{x}_1, \tilde{x}_2) \parallel p_1^*(x), p_2(x)\} \tag{7}$$

for every $p_1(x) \in \mathbf{P}_1, p_2(x) \in \mathbf{P}_2$, where $\mathbf{E}\{u_i(\tilde{x}_1, \tilde{x}_2) \parallel p_1(x), p_2(x)\}$ is expected value of $u_i(\tilde{x}_1, \tilde{x}_2)$, calculated in the assumption that distribution $\tilde{x}_1$ is determined by the density function $p_1(x)$ and distribution $\tilde{x}_2$ by the density function $p_2(x)$.

Since a randomized model is being described, we cannot deny admissibility and validity of alternative approaches, which determine the equilibrium conditions with respect to other criteria. Particularly, they may be:

– minimization of variances of players' utilities (perhaps with additional restrictions on the lower levels, below which the utility expectation value cannot go);

– minimization of α–quintile values of players' utility function distributions, that is, below which the value of the utility will not fall with a probability $1-\alpha$.

1. Equilibrium based on minimization of utility variance

Let us consider the first mentioned approach in details. To some extent, the ideas of this approach are similar to the ideas in the Markowitz model of portfolio selection that minimizes risk (Binmore, 1987), (Binmore, 1988), (Cheon, 2003). In this case, we may assume that equilibrium in this model will be characterized by such joint selection of probability distributions $p_1^*(x) \in \mathbf{P}_1$ and $p_2^*(x) \in \mathbf{P}_2$, which will provide us with conditions fulfilled:

$$\mathbf{D}\{u_1(\tilde{x_1}, \tilde{x_2}) \parallel p_1^*(x), p_2^*(x)\} \leqslant \mathbf{D}\{u_1(\tilde{x_1}, \tilde{x_2}) \parallel p_1(x), p_2^*(x)\} \tag{8}$$

$$\mathbf{D}\{u_1(\tilde{x}_1, \tilde{x}_2) \parallel p_1^*(x), p_2^*(x)\} \leqslant \mathbf{D}\{u_1(\tilde{x}_1, \tilde{x}_2) \parallel p_1^*(x), p_2(x)\} \tag{9}$$

$\mathbf{D}\{u_1(\tilde{x}_1, \tilde{x}_2) \parallel p_1(x), p_2(x)\}$ — variance of $u_1(\tilde{x}_1, \tilde{x}_2)$, calculated in assumption that distribution of $\tilde{x}_1$ is determined by a density function $p_1(x)$, and distribution of $\tilde{x}_2$ is determined by a density function $p_2(x)$.

In other words, conditions (8)–(9) define the situation, in which participants make an attempts to deviate from, taken by one or another party on an individual basis, lead to an increase in the risk. Variance is used as a measure of risk. A "weak" point of this approach in determination of equilibrium is connected with the fact, that minimal risks can be achieved at an unacceptably low expected utility values. This, in turn, can be "corrected" by introducing a concept of conditional equilibrium, under which one can understand a joint choice of probability distributions $p_1^*(x) \in \mathbf{P}_1$ and $p_2^*(x) \in \mathbf{P}_2$, which provides fulfillment of conditions (8)–(9), as well as conditions

$$\mathbf{E}\{u_1(\tilde{x}_1, \tilde{x}_2) \parallel p_1(x), p_2(x)\} \geqslant \bar{u}_i,\ i = \{1, 2\} \tag{10}$$

where $\bar{u}_i$ are lower bounds on acceptable levels of expected utility of participants. Subsequent development of the approach (8)–(9) is clearly possible under condition that we specify classes of possible distributions $p_i(x_i)$. We should note that this step is substantial, moreover, it can be critical to the prospects of using this model.

Based on the general properties of solutions, which are made by real economic agents and concern issues of mutual collaboration, we can use an asymmetric triangular distribution for modeling the behavior of variables $\tilde{x}_i$, see Fig. 7[1]. On the interval $[0, 1]$ densities of asymmetric triangular distributions are uniquely determined by the choice of parameter m — the point of mode. It is known that an arbitrary random variable distributed in an asymmetric triangular law on the interval $[0, 1]$ has expected value

$$\mathbf{E}\tilde{x} = \frac{1}{3}(m+1) \tag{11}$$

and variance

$$\mathbf{D}\tilde{x} = \frac{1}{18}(m^2 - m + 1). \tag{12}$$

[1] The distribution which is used here is a generalization of an asymmetric triangular distribution (Simpson's Distribution)

On the basis of (11) and (12) we can obtain an expression for expectation of utility functions of players (1) — (2), considering them after the transition to the mixed extension of the game as functions of random variables $\tilde{x}_1, \tilde{x}_2$:

$$\mathbf{E}\{u_1(\tilde{x}_1, \tilde{x}_2)\} = \mathbf{E}\{b_1\tilde{x}_2 - a_1\tilde{x}_1\} = \frac{1}{3}[b_1(m_2+1) - a_1(m_1+1)], \tag{13}$$

$$\mathbf{E}\{u_2(\tilde{x}_1, \tilde{x}_2)\} = \mathbf{E}\{b_2\tilde{x}_1 - a_2\tilde{x}_2\} = \frac{1}{3}[b_2(m_1+1) - a_2(m_2+1)], \tag{14}$$

and their variances as well

$$\begin{aligned}\mathbf{D}\{u_1(\tilde{x}_1, \tilde{x}_2)\} = \mathbf{D}\{b_1\tilde{x}_2 - a_1\tilde{x}_1\} = b_1^2\mathbf{D}\tilde{x}_2 + a_1^2\mathbf{D}\tilde{x}_1 = \\ = \frac{b_1^2}{18}[m_2^2 - m_2 + 1] + \frac{a_1^2}{18}[m_1^2 - m_1 + 1],\end{aligned} \tag{15}$$

$$\begin{aligned}\mathbf{D}\{u_2(\tilde{x}_1, \tilde{x}_2)\} = \mathbf{D}\{b_2\tilde{x}_1 - a_2\tilde{x}_2\} = b_2^2\mathbf{D}\tilde{x}_1 + a_2^2\mathbf{D}\tilde{x}_2 = \\ = \frac{b_2^2}{18}[m_1^2 - m_1 + 1] + \frac{a_2^2}{18}[m_2^2 - m_2 + 1],\end{aligned} \tag{16}$$

Having (15) and (16) we derive that $\mathbf{D}\{u_1(\tilde{x}_1, \tilde{x}_2)\}$ and $\mathbf{D}\{u_2(\tilde{x}_1, \tilde{x}_2)\}$ are convex quadratic functions of parameters m_1 and m_2, and, consequently, they reach a global extremum at the point

$$(m_1^*, m_2^*) = (\frac{1}{2}, \frac{1}{2}), \tag{17}$$

which determines the state of equilibrium in the sense of (8) — (9) for a model of collaboration. Thus, a situation of mutual stability (in terms of minimization of risk criterion) in models constructed on basis of triangular distributions occurs when players choose their strategies relying on symmetric triangular distributions. This reflects the advantage of behavior based on the "golden mean" between extreme selfishness and willingness to maximize collaboration. If guided by the concept of conditional equilibrium (8) — (10), the global minimum point of variances $\mathbf{D}\{u_i(\tilde{x}_1, \tilde{x}_2)\}$ (17) may be outside of set of valid values m_1, m_2, defined by conditions

$$\begin{cases} -a_1m_1 + b_1m_2 \geqslant 3\bar{u}_1 - (b_1 - a_1) \\ \ \ b_2m_1 - a_2m_2 \geqslant 3\bar{u}_2 - (b_2 - a_2) \end{cases}$$

In this case, the procedure of finding conditional equilibrium in the sense of (8) — (10) reduces to solving a series of quadratic programming problems. Of course, the hypothesis, that values $\tilde{x}_i$ are distributed under the triangular law, cannot be regarded as an assumption which has non-alternative benefits. Other interesting and meaningful results for this model can also be obtained for the distributions of other classes. In particular, let us consider the model of collaboration, which is based on the assumption that the distribution of values $\tilde{x}_i$ is exponential[2] with parameters λ_i, i.e.

$$p_i(x) = \lambda_i e^{-\lambda_i x_i}$$

If we compare modifications of densities of triangular and exponential distributions, it is easy to see that the latter reflects the situation of initially low "propensity

[2] Obviously, in this case we assume the possibility of expanding the domain of x_i on the whole positive axle shaft, provided that the probability of $\tilde{x}_i > 1$ is close to zero.

to collaborate" of the economic agents more appropriately, see Fig. 3.One can also note that gamma distribution can be used for more flexible modeling of "propensity of players to collaborate" ratios.

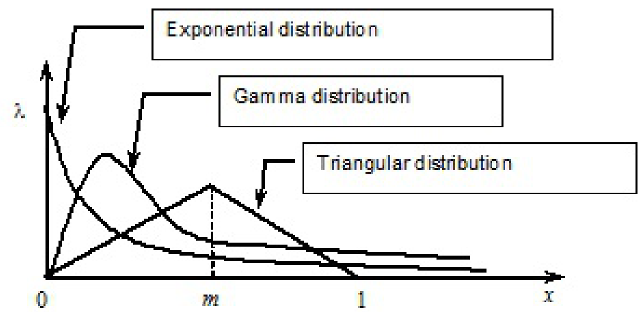

Fig. 3: Exponential, gamma and triangular distributions

If $\tilde{x}_i$ are exponentially distributed then the expected utilities of players will be

$$\mathbf{E}\{u_1(\tilde{x_1}, \tilde{x_2})\} = \mathbf{E}\{-a_1\tilde{x_1} + b_1\tilde{x_2}\} = -\frac{a_1}{\lambda_1} + \frac{b_1}{\lambda_2}, \tag{18}$$

$$\mathbf{E}\{u_2(\tilde{x_1}, \tilde{x_2})\} = \mathbf{E}\{b_2\tilde{x_1} - a_2\tilde{x_2}\} = \frac{b_2}{\lambda_1} - \frac{a_2}{\lambda_2} \tag{19}$$

In accordance with the formula for adding the variances of independent random variables, the dispersion of the utility can be expressed as

$$\mathbf{D}\{u_1(\tilde{x}_1, \tilde{x}_2)\} = \left(\frac{a_1}{\lambda_1}\right)^2 + \left(\frac{b_1}{\lambda_2}\right)^2 \tag{20}$$

$$\mathbf{D}\{u_2(\tilde{x}_1, \tilde{x}_2)\} = \left(\frac{b_2}{\lambda_1}\right)^2 + \left(\frac{a_2}{\lambda_2}\right)^2 \tag{21}$$

As appears from (21) — (22) functions $\mathbf{D}\{u_i(\tilde{x_1}, \tilde{x_2})\}$ have obvious infimums equal to 0, when $\lambda_1 \to \infty, \lambda_2 \to \infty$. This means nothing more than a repetition of "pessimistic outcome", which has been obtained earlier: the variance for exponential distributions will be as smaller, as closer they are concentrated near $x_i = 0$, which corresponds to a situation of lack of collaboration.

The above considerations are valid if $\tilde{x}_i$ are gamma distributed with some parameters κ_i, λ_i .This follows directly from the form of expectation and variance for the corresponding random variables.

$$\mathbf{E}\tilde{x}_i = \frac{\kappa_i}{\lambda_i}, \mathbf{D}\tilde{x}_i = \frac{\kappa_i}{\lambda_i^2}$$

2. Equilibrium based on the criterion of minimizing VaR utility

Let us now consider a specific approach, where players make choice about appropriate degrees of collaboration taking distribution of their utility function utility into consideration, i.e.

$$F_{u_1(\tilde{x}_1,\tilde{x}_2)}(u) = \mathbf{P}\{u_1(\tilde{x}_1,\tilde{x}_2) \leqslant u\}$$

and

$$F_{u_2(\tilde{x}_1,\tilde{x}_2)}(u) = \mathbf{P}\{u_2(\tilde{x}_1,\tilde{x}_2) \leqslant u\}$$

A similar approach can be considered as an analogue of the concept of value at risk (VaR), which is widely used in modern risk management. The behavior of the utility distribution function of a player i is a function of independent random variables $\tilde{x}_1, \tilde{x}_2$ in this model. It is presented on Fig. 8. Taking into consideration (1) and (2) we can note that for $x_i \in [0,1]$ $u_i \in [-a_i, b_i]$, and consequently

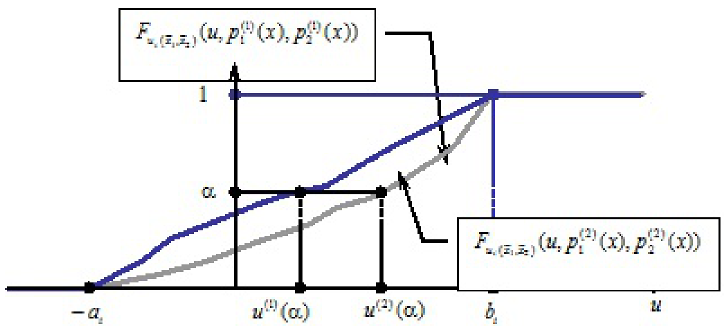

Fig. 4.

At the same time the choice of specific parameter values for $p_1(x)$ and $p_2(x)$ determines how function $F_{u_i(\tilde{x}_1,\tilde{x}_2)}(u)$ will increase on the interval $[-a_i, b_i]$. Thus, for the same level of probability quantiles of the distribution function

$$u(\alpha) = F^{-1}_{u_i(\tilde{x}_1,\tilde{x}_2)}(\alpha, p_1(x), p_2(x))$$

depends on the choice of parameters of probability distributions (densities $p_1(x)$ and $p_2(x)$) of random variables $\tilde{x}_1, \tilde{x}_2$. As we can see from Fig. 8,

$$u^{(1)}(\alpha) < u^{(2)}(\alpha),$$

i.e. α–quantile of the distribution function of the first player's utility , which we get from densities $p_1^{(1)}(x)$ and $p_2^{(1)}(x)$, is lower than the one corresponding densities $p_1^{(2)}(x)$ and $p_2^{(2)}(x)$. Thus, for the first player a strategic option defined by $p_1^{(2)}(x)$ and $p_2^{(2)}(x)$ is preferred, since its threshold below which his utility $u_i(\tilde{x}_1,\tilde{x}_2)$ wouldn't drop will be higher with probability $1-\alpha$.

In this particular approach, **the equilibrium in the model of collaboration can be defined as a set of probability distributions** $(p_1^*(x), p_2^*(x))$ **of some parametric classes** P_1 **and** P_2**, which define strategies of participants which satisfy the following conditions (with a given level of probability** α **and any other probability distributions** $(p_1(x) \in P_1$ **and** $(p_2(x) \in P_2)$

$$F^{-1}_{u_1(\tilde{x}_1,\tilde{x}_2)}(\alpha, p_1^*(x), p_2^*(x)) \geqslant F^{-1}_{u_1(\tilde{x}_1,\tilde{x}_2)}(\alpha, p_1(x), p_2^*(x)) \quad (22)$$

$$F^{-1}_{u_2(\tilde{x}_1,\tilde{x}_2)}(\alpha, p_1^*(x), p_2^*(x)) \geqslant F^{-1}_{u_2(\tilde{x}_1,\tilde{x}_2)}(\alpha, p_1^*(x), p_2(x)) \tag{23}$$

We will now pay a little bit more attention to usage of the following approach in a case when $p_i(x)$ determine random variables $\tilde{x}_i$, which are distributed under the asymmetric triangular law. As it has been already noted, the choice of the actual density $p_i(x)$ is uniquely connected to the choice of parameter m_i, which is a mode, and therefore we can consider players' utility distribution functions $u_i(\tilde{x}_1, \tilde{x}_2)$ as function of m_1 and m_2 using the notation $F_{u_i(\tilde{x}_1,\tilde{x}_2)}(u, m_1, m_2)$. We should pay our attention to the fact that even with such a simple functional form of density in the case of an asymmetric triangular distribution, functions $F_{u_i(\tilde{x}_1,\tilde{x}_2)}(u, m_1, m_2)$ do not have a "compact" analytic expression. "Method" of finding the value of $F_{u_1(\tilde{x}_1,\tilde{x}_2)}(u, m_1, m_2)$ having a particular value for $\bar{u}_1$ is shown on Fig. 2.. There is evident from Fig.5, in order to find the value

$$F_{u_1(\tilde{x}_1,\tilde{x}_2)}(\bar{u}_1, m_1, m_2) = \mathbf{P}\{u_1(\tilde{x}_1, \tilde{x}_2) \leqslant \bar{u}_1 = -a_1 x_1 + b_1 x_2\}$$

we should calculate the sum $F_I + F_{II} + F_{III} + F_{IV}$, where

$$F_I = \int\limits_0^{m_1} \frac{2x_1}{m_1} \left[\int\limits_0^{\min\left\{\frac{\bar{u}_1 + a_1 x_1}{b_1}; m_2\right\}} \frac{2x_2}{m_2} dx_2 \right] dx_1,$$

$$F_{II} = \int\limits_{m_1}^{1} \frac{2x_1 - 2}{m_1 - 1} \left[\int\limits_0^{m_2} \frac{2x_2}{m_2} dx_2 \right] dx_1,$$

$$F_{III} = \int\limits_{\frac{b_1 m_2 - \bar{u}_1}{a_1}}^{m_1} \frac{2x_1}{m_1} \left[\int\limits_{m_2}^{\frac{\bar{u}_1 + a_1 x_1}{b_1}} \frac{2x_2}{m_2 - 1} dx_2 \right] dx_1,$$

$$F_{IV} = \int\limits_{m_1}^{1} \frac{2x_1 - 1}{m_1 - 1} \left[\int\limits_{m_2}^{\frac{\bar{u}_1 + a_1 x_1}{b_1}} \frac{2x_2 - 2}{m_2 - 1} dx_2 \right] dx_1,$$

Thus, in order to find the values of distribution functions $F_{u_i(\tilde{x}_1,\tilde{x}_2)}(u, m_1, m_2)$ for arbitrary $u \in [-a_i, b_i]$ we will only have to consider all possible situations of geometry of line $u_i(x_1, x_2)$ and point (m_1, m_2).

In spite of "bad" analytical properties of functions $F_{u_i(\tilde{x}_1,\tilde{x}_2)}(u, m_1, m_2)$ we are able to describe their behavior with appropriate accuracy by using numerical methods for specific a_i and b_i. In particular the results of numerical modeling of function $F_{u_1(\tilde{x}_1,\tilde{x}_2)}(u, m_1, m_2)$ with the help of MathCAD software tools for $a_1 = 1$, $b_1 = 2$, $m_2 = 0.8$ are shown on Fig. 2.. In other words, in a situation where the first player gives value to the actions of a second player twice as much as his own costs and the strategy of a second player is determined by the asymmetric triangular distribution with mode equal to 0.8. Fig. 2. depicts graphics of the first player's utility distribution function for cases when his strategy is determined by the asymmetric triangular distribution with mode $m_1^{(1)} = 0.2$ (line **FI_1**) and $m_1^{(2)} = 0.8$ (line **FI_2**).

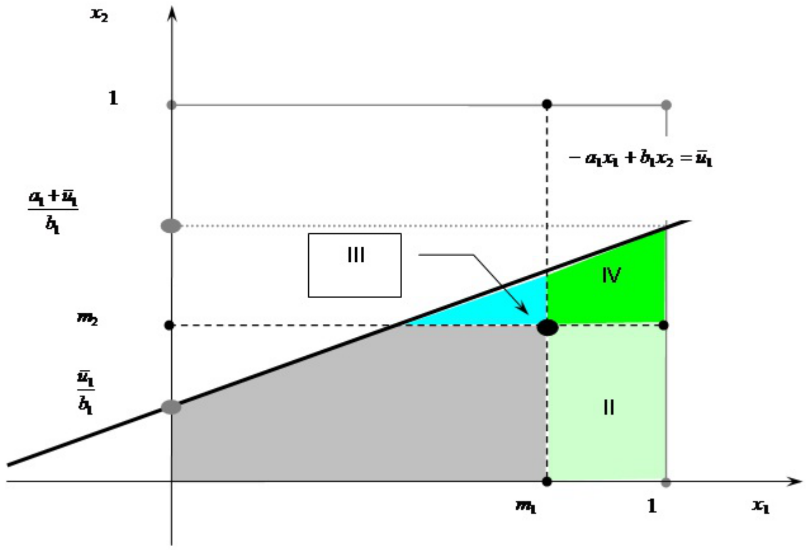

Fig. 5.

As follows from geometry of quantile lines for a level of probability α the quantiles found with respect to the distribution function **FI_2** will be less than quantiles found in respect to the distribution function **FI_1**. Thus in these conditions when the second player chooses the level of collaboration equal to $m_1^{(2)} = 0.8$, it is preferable for the first player to choose the higher level of collaboration $m_1^{(2)} = 0.8$, not $m_1^{(1)} = 0.2$, which would mean more egoistic type of behavior.

In particular - actual values.

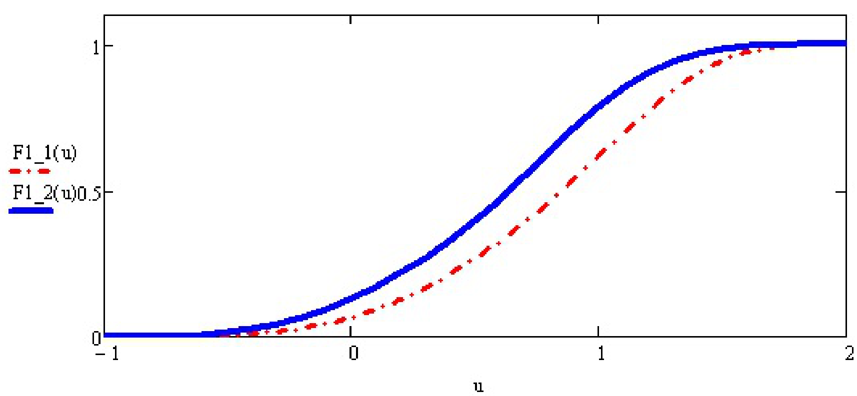

Fig. 6.

It should be admitted that from a mathematical point of view, we should modify the criterion function in respect to which the equilibrium conditions are determined in order to abandon the situation of a non-constructive equilibrium in the proposed model. Roughly speaking, if the players evaluate their results depending on the utility (or expected utility), then situation of non–collaboration becomes stable. At the same time, if they apply different criteria (variance, measure of risk, or VaR utility), the situation of collaboration is preferable.

This is what gives the approaches considered above an added significance in terms of economic meaning. In particular, applying them, we can form the principles of construction and maintenance of mechanisms for collaboration in situations that are initially characterized with selfish behavior of the parties.

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