Dynamic Models of Corruption in Hierarchical Control Systems*

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Abstract Dynamic game theoretic models of corruption in two- and three-level control systems as well as optimal control problems and their applications to the optimal exploitation of bioresources and water quality control are considered. Several model examples are investigated analytically.

Keywords: corruption, hierarchical control systems, dynamic Stackelberg games, optimal control.

1. Introduction

Corruption is a social-economic phenomenon that exerts an essential negative influence to the society. The number of papers on dynamic models of corruption in hierarchical control systems is quite small. Basically they are multistage game theoretic models in which the dynamics of controlled system is not described explicitly. In one of the first papers of this class (Basu et al., 1992) a recursive setting is considered in which it is supposed that if the controller makes the collusion with the controlled person then he could be caught and should pay a bribe himself. The chain of corruption could be both finite and infinite. The authors of (Basu et al., 1992) have shown that in some conditions an increase of the probability of punishment has a greater effect in the fight with corruption than the penalty enlargement. In the paper (Olsen and Torsvik, 1998) a two-stage model of the type principal - controller - agent is considered in which the principal uses an illegal character of transactions between the controller and the agent and can get a greater payoff in a long-term period than in a short-term one. In the paper (Yang, 2005) a structural analysis of corruption in Chinese enterprises licensing is presented on the base of a repeated bargaining model. It demonstrates that once relative bargaining powers are correctly accounted then certain institutional features of the Chinese licensing system lead to bribery as a robust outcome. Exercises in comparative statics reveal that certain conventional anti-corruption measures may have counterintuitive effects. If overlapping jurisdictions are introduced, the resulting bureaucratic competition could help to fight with corruption. The model of "petty corruption" in (Lambert-Mogiliansky et al., 2007) describes the structure of bureaucratic "tracks", and the information among the participants. Entrepreneurs apply, in sequence, to a "track" of two or more bureaucrats in a prescribed order for approval of their projects. The first result establishes that in a one-shot situation no project ever

* This work is supported by RFBR, project No.12-01-00017=E0

gets approved, so a repeated interaction setting is used. In that context the trigger-strategy equilibria are characterized that minimize the social loss due to the system of bribes, and those that maximize the expected total bribe income of the bureaucrats. The results are used to shed some light on two much advocated anti-corruption policies: the single window policy and rotation of bureaucrats. In the paper (Bhattacharya and Hodler, 2010) it is studied how natural resources can feed corruption and how this effect depends on the quality of the democratic institutions. The game-theoretic model predicts that resource rents lead to an increase in corruption if the quality of the democratic institutions is relatively poor, but not otherwise. The paper (Balafoutas, 2011) investigates the role of guilt aversion for corruption in public administration. Corruption is modeled as the outcome of a psychological game played between a bureaucrat, a lobby, and the public. It is studied how the behavior of the lobby and the bureaucrat depend on perceived public beliefs, when these are constant and when they are allowed to vary over time. The papers (Blackburn and Forgues-Puccio, 2010, Cerqueti and Coppier, 2011) should be noticed separately in which the models of economic growth with corruption are investigated.

This paper logically continues the paper (Antonenko et al., 2012) where the static models of corruption were considered, and is concerned with the dynamic models in this domain. In the frame of our conception the description of corruption and the methods of struggle with it in dynamics are based on the following principles.

1. The basic modeling pattern is a hierarchical structure "principal - supervisor - agent - object" in different modifications and its investigation by means of the optimal control theory and dynamic Stackelberg games theory. The supervisor may be corrupted, while the principal is not corrupted and fights with corruption. So the elements of the above structure are bribe-fighter, bribe-taker, bribe-giver and object of impact respectively. In some cases a simplified model "bribe-taker - bribe-giver - object" (supervisor - agent - object) is used in which the non-corrupted principal is considered implicitly (parametrically).
2. The leading player of any level (principal or supervisor) uses methods of compulsion (administrative or legislative impacts) or impulsion (economic impacts) for achievement of his/her objectives. The mathematic formalization of compulsion means an impact of the leader to the set of admissible strategies of the follower (as a rule, without a feed back), and impulsion means the impact to the follower's payoff function (as a rule, with a feed back) (Ougolnitsky, 2011).
3. The cases of administrative corruption when administrative requirements or constraints are weakened for a bribe and the economic corruption when the economic ones are weakened are differentiated. The model of administrative corruption is the compulsion with a feed back on bribe, and the model of economic corruption is the impulsion with an additional feed back on bribe.
4. Corruption exists in the form of capture and extortion. In the case of capture a basic set of administrative or economic services is guaranteed while additional indulgences are provided for a bribe. In the case of extortion a bribe is required already for the basic set of services.
5. For studying corruption in hierarchical control systems with consideration of the requirements of sustainable development both descriptive and normative approaches are applicable. In the case of descriptive approach the functions of administrative

and economic corruption are given, and the main problem is to identify their parameters on statistical data. In the case of normative approach the corruption (bribery) function is found as the solution of an optimization or game theoretic problem.
6. The investigation of corruption in the system "principal - supervisor - agent - object" is possible from three points of view. If the bribery function is known then from the point of view of the agent the corruption can be described by an optimal control model. From the supervisor's point of view a hierarchical parametrical Germeyer game of the class Γ_{2t} arises which solution in the form of bribery function with a feedback on the value of bribe is known in a general form (Gorelik et al., 1991). From the point of view of the principal the problem of fight with corruption consists in seeking of such values of control parameters that the found optimal strategy of the supervisor satisfies the requirements of homeostasis for the controlled dynamic system (object).
7. It makes sense to build "genetic" series of sequentially complicated models that more and more precisely describe the real phenomena of corruption in hierarchical control systems. The principal logical pattern of this sequential complication has a form "optimal control models - dynamic hierarchical two-person games - dynamic hierarchical three-person games". With consideration of the possible modifications of the models of each type the "series" become the "genetic networks".

It is the last principle that determines the structure of the paper.

2. The system of dynamic models of corruption

Let's begin from the "genetic series" of the models of economic corruption. For the convenience of interpretation and without loss of generality we will speak about the problems of optimal exploitation of bioresources. In the case of harvesting strategy the equation of dynamics of the controlled system with initial conditions has a form

$$\dot{x} = h(x(t)) - u(t)x(t), \quad x(0) = x_0 \tag{1}$$

Here $x(t)$ - a biomass of the harvested population (for example, fish); $u(t)$ - a catch share (in the moment t); h - a function of local homogeneous population dynamics (Malthus, Verhulst-Pearl, Ricker and other types); x_0 - an initial value of the biomass.

For description of the economic corruption we suppose that the catch share is constant: $u(t) \equiv u$. Then the total fishing income is equal to aux where a is a price of the unit of biomass (the dependence on time is omitted for simplicity). Suppose that a share r of that variable (where r can be treated as a tax rate) goes to the principal (the state), and the share $1-r$ goes to the agent (a fisher). In turn, the principal gives a share p from his part as a salary (or bonus) to the supervisor (an official of the fishing control agency), and the share $1-p$ keeps to himself. At last, the agent assigns a share b of his income to the bribe for the supervisor, retaining the share $1-b$. Thus, the final income values are equal to $ar(1-p)ux$ for the principal, $a(pr+b(1-r))ux$ for the supervisor, and $a(1-b)(1-r)ux$ for the agent. Using these balance considerations, the agent's payoff functional is given in the form

$$J_A = au\int_0^\infty e^{-\alpha t}(1-b(t))(1-r(t))x(t)dt \to max \tag{2}$$

In the case of economic corruption the tax rate is a function of the bribe. From the point of view of the agent this function is considered as given. For example, the linear capture function has the form

$$r(b(t)) = r_0 - r_0 b(t) \tag{3}$$

Here in the left side a real tax rate is situated, the first term in the right side r_0 is its official (normative) value, and the second term means its weakening in exchange to the bribe. In particular, $r(0) = r_0,\ r(1) = 0$.

Substituting (3) in (2) we get

$$J_A = au \int_0^{\infty} e^{-\alpha t} \left(1 - r_0 - (1 - 2r_0)b(t) - r_0 b^2(t)\right) x(t)dt \to max \tag{4}$$

where α is a discount factor. Adding the restrictions on control values

$$0 \leq b(t) \leq 1 \tag{5}$$

we get an optimal control problem (1), (4)-(5). Certainly, other functions of economic corruption can be used instead of (3).

From the point of view of the supervisor the function $\tilde{r}(t) = r(b(t))$ is sought as a solution of the game Γ_{2t} betweenthe supervisor and the agent (Gorelik et al., 1991). The supervisor's payoff functional can be written in the form

$$J_S = \int_0^{\infty} e^{-\alpha t}((pr(t) + b(t)(1 - r(t)))aux(t) - K\mu(r_0 - r(t)))dt \to max \tag{6}$$

with restrictions on controls

$$0 \leq r(t) \leq r_0 \tag{7}$$

To avoid the consideration of projection the function $r(t)$ is treated as a result of the map $r(b(t))$. The second term in the integrand function in (6) represents a penalty charged to the supervisor in the case of detection of the tax arrears, where K is the penalty factor, μ - probability of detection. In general, the relations (1)-(2), (5)-(7) define a hierarchical differential two-person game of the class Γ_{2t}.

At last, the principal's payoff functional can be written as

$$J_P = \int_0^{\infty} e^{-\alpha t}((1 - p(t))aux(t)r(t) - M(r_0 - r(t)) - \frac{m}{r_0 - z(t)})dt \to max \tag{8}$$

It is supposed that in general the principal can use two methods of control: impulsion or compulsion with the control variables respectively

$$0 \leq p(t) \leq 1 \tag{9}$$

$$0 \leq z(t) \leq r_0 \tag{10}$$

In the second case the restrictions (7) take the form

$$z(t) \leq r(t) \leq r_0 \tag{11}$$

i.e. the principal can restrict from below the weakening of the tax rate for a bribe. The first term in (8) represents the principal's income, the second one means a penalty charged on him in the case of tax arrears (M is a penalty factor), and the third one are the principal's expenditures for control on the supervisor's actions (m is the respective factor).

In general, the relations (1)-(2), (5)-(10) (where instead of (7) may be (11) in the case of compulsion) define a differential hierarchical three-person game. Its reglament depends on the methods of control used by the principal. If compulsion is used then the game between the principal and the supervisor has a form Γ_{1t}, if impulsion then Γ_{1t} or Γ_{2t} depending on that whether the function $p(t)$ depends only on time or also on the supervisor's control. The game between the supervisor and the agent always has a form Γ_{2t} (Gorelik et al., 1991) because the presence of a feedback on bribe is the principal moment in the description of corruption.

Thus, on the example of bioresource exploitation a series of dynamic models of economic corruption in the form "optimal control model - dynamic hierarchical two-person game - dynamic hierarchical three-person game" is defined.

Let's define a similar series for the case of administrative corruption. Now the catch share is a function of time, and the agent chooses two control functions, namely catch share and bribe. If the linear function of administrative corruption is given as

$$s(b(t)) = s_0 + (1 - s_0)b(t) \tag{12}$$

where $s(b(t))$ is a real value of the catch quota, s_0 is an official value of the quota, $(1 - s_0)b(t)$ is its weakening in exchange for the bribe then the agent's optimal control problem can be written in the form

$$J_A = a \int_0^\infty e^{-\alpha t}(1 - b(t))u(t)x(t)dt \to max \tag{13}$$

$$0 \leq u(t) \leq s_0 + (1 - s_0)b(t) \tag{14}$$

$$0 \leq b(t) \leq 1 \tag{15}$$

where the equation of dynamics of the controlled system with initial conditions has also the form (1). The payoff functional of the supervisor has the form

$$J_S = \int_0^\infty e^{-\alpha t}(ab(t)u(t)x(t) - K\mu(s(t) - s_0))dt \to max \tag{16}$$

with restrictions on controls

$$s_0 \leq s(t) \leq 1 \tag{17}$$

Similar to (6), the second term in the integrand in (16) is a penalty charged on the supervisor in the case of detection of excess of the official catch quota. The relations (1), (13)-(17) define a differential hierarchical two-person game of the class Γ_{2t}.

At last, the principal's payoff functional has the following form (similar to (8) but without consideration of the principal's personal interests)

$$J_P = \int_0^\infty e^{-\alpha t}(M(s(t) - s_0) + m\frac{1 - q(t)}{q(t) - s_0})dt \to min \tag{18}$$

with restrictions on controls

$$s_0 \leq q(t) \leq 1 \tag{19}$$

If (17) is replaced by the restrictions

$$s_0 \leq s(t) \leq q(t) \tag{20}$$

which determine the compulsion by principal then the relations (1), (16), (18)-(20) define a differential hierarchical three-person game. The reglament of the game between the principal and the supervisor is Γ_{1t} , and between the supervisor and the agent Γ_{2t}. In the rest of the paper some dynamic models of economic corruption are investigated.

3. Models of optimal exploitation of bioresources considering the economic corruption

Let's consider a model of economic corruption in the form

$$J = \int_0^T e^{-\alpha t}(1 - r(b(t)) - b(t)) f(u(t), x(t)) dt \to max$$

$$0 \leq u(t) \leq 1 \ ; \quad 0 \leq b(t) \leq 1$$

$$\dot{x} = (1 - u(t)) h(x(t)), \quad x(0) = x_0$$

where the variables have the same sense as earlier.

The functional describes profit of a fishing enterprise which may be a bribe-giver. From the profit the enterprise pays taxes and (perhaps) gives a bribe to an official of the fishing control agency. The real tax rate is a decreasing function of the bribe. If the bribe is equal to zero then the real tax rate coincides with the normative (established by the law) one. Only the case of capture is considered.

The function of economic corruption $r(b(t))$ is taken in the exponential form

$$r(b(t)) = r_0 e^{-kb(t)}$$

where the variables have the same sense as in (3); k is a bribe sensitivity. Suppose also that

$$f(u(t), x(t)) = \sqrt{u(t)x(t)}$$

Without loss of generality the price of fish biomass unit is supposed to be equal to one. Assume also for simplicity that $\alpha = 0$ (there is no discounting). The model has two control functions: the biomass of caught fish (in shares) and the bribe (also in shares). The Verhulst-Pearl model represents a natural dynamics of the fish population:

$$h(x(t)) = ax(t)(K - x(t))$$

where a is a natural increase factor, K is an environmental capacity. Thus, the following model is considered:

$$J = \int_0^T (1 - r_0 e^{-kb(t)} - b(t)) \sqrt{u(t)x(t)} dt \to max$$

$$0 \leq u(t) \leq 1 \ ; \quad 0 \leq b(t) \leq 1; \ \ k > 0; \ \ t \in [0, T]$$

$$\dot{x} = (1 - u(t)) ax(t)(K - x(t)), \quad x(0) = x_0$$

The optimal control problem is solved by the Pontryagin maximum principle (Grass et al., 2008). The Hamilton function has the form (the argument t is omitted):

$$H(x,u,b,\psi) = (1 - r_0 e^{-kb} - b)\sqrt{ux} + \psi(1-u)ax(K-x)$$

The conditions of maximum principle are supposed to be satisfied. Consider the first one. The derivative of H with respect to b is

$$\frac{\partial H}{\partial b} = \sqrt{ux}(r_0 k e^{-kb} - 1)$$

Equating the right side of the relation to zero and solving it with respect to b we get

$$b^* = \frac{ln(r_0 k)}{k} \tag{21}$$

Now calculate the derivative of H with respect to u:

$$\frac{\partial H}{\partial u} = \frac{(1 - r_0 e^{-kb} - b)\sqrt{x}}{2\sqrt{u}} - \psi a x(K-x)$$

Equating the derivative to zero and substituting (21) in the received expression we get:

$$u^* = \frac{(k - 1 - ln(r_0 k))^2}{4k^2\psi^2 a^2 x(K-x)^2} \tag{22}$$

Calculate the second derivatives of the Hamilton function with respect to the control variables considering (21):

$$\frac{\partial^2 H(x,u^*,b^*,\psi)}{\partial u^2} = \frac{2k^2\psi^3 a^3 x^2 (K-x)^3}{(k-1-ln(r_0 k))^2} \tag{23}$$

$$\frac{\partial^2 H(x,u^*,b^*,\psi)}{\partial b^2} = \frac{k-1-ln(r_0 k)}{2\psi a(K-x)} \tag{24}$$

Calculate also the mixed derivatives; due to (21) and (22) we get:

$$\frac{\partial^2 H}{\partial u \partial b} = \frac{\sqrt{x}(r_0 k/(r_0 k) - 1)}{2\sqrt{u}} \equiv 0 \tag{25}$$

The Hesse matrix has the form

$$\begin{pmatrix} -\frac{k-1-ln(r_0 k)}{2\psi a(K-x)} & 0 \\ 0 & -\frac{2k^2\psi^3 a^3 x^2 (K-x)^3}{(k-1-ln(r_0 k))^2} \end{pmatrix}$$

and it is negative definite. Thus, the sufficient condition of the maximum of Hamilton function with respect to the control variables is satisfied.

Let's analyze the expression (21). Consider the limit case $b^* = 0$ (the bribe is not profitable for the bribe-giver). It is possible when $k = 1/r_0$. In the model is supposed that the value r_0 (tax rate) is determined by the law, and the value k is determined by a fishing control agency official (bribe-taker). Given r_0 the bribe-giver chooses k such that the optimal for him value b^* takes the maximal value. It is found that the optimal for the bribe-taker value of k is defined by the formula

$k = e/r_0$. For example, if $r_0 = 0.2$ then $k \approx 13,59$. The substitution of k to (21) gives $b^* = r_0/e$. As r_0 is always positive then b^* is never equal to zero but b^* diminishes if r_0 diminishes. So, in the frame of the model the corruption cannot be eradicated completely but it can be restricted by decreasing of the tax rate.

Let's compose the expression for the conjugate variable $\psi(t)$:

$$\frac{d\psi(t)}{dt} = -\frac{(k-1-ln(r_0k))^2}{4k^2\psi a(K-x)^2} - \psi(t)a(K-2x(t)) \tag{26}$$

and define a boundary condition for (26):

$$\psi(T) = 0$$

Let's substitute (22) into the equation for the phase coordinate and transform it:

$$\dot{x}(t) = ax(t)(K-x(t)) - \frac{(k-1-ln(r_0k))^2}{4k^2\psi^2 a(K-x(t))} \tag{27}$$

The equations (26) and (27) form the system

$$\begin{cases} \dot{x}(t) = ax(t)(K-x(t)) - \frac{(k-1-ln(r_0k))^2}{4k^2\psi^2 a(K-x)}; \\ \dot{\psi} = -\frac{(k-1-ln(r_0k))^2}{4k^2\psi a(K-x)^2} - \psi(t)a(K-2x(t)) \end{cases} \tag{28}$$

with boundary conditions $\begin{cases} x(0) = x_0; \\ \psi(T) = 0 \end{cases}$

Let's solve the system by the explicit-implicit Euler method. To receive a numerical result let's give some specific values to the model parameters. Assume that $T = 3$; $r_0 = 0,2$; $x_{-3} = 8000$; $x_{-2} = 8800$; $x_{-1} = 9600$; $a = 1,14\ 10^{-5}$; $K = 104800$; $x_0 = 10385,45$; $k = 13,59$; $b = 0,07$. A common sorting method can be used for the determination of the initial value of ψ. It is appropriate because the explicit-implicit Euler method in solving (28) is stable and the value of grid function tends to zero when the number of steps grows. In this problem $\psi(0) = 0,0141$.

The function of natural dynamics of the population has the form

$$\tilde{x}(t) = \frac{x_0 K}{x_0 + (K-x_0)e^{-aKt}}$$

To receive the function $x(t)$ as an analytical expression a regression analysis is required because the values in the grid nodes are approximate. By means of the least squares method it is found that the function $x(t)$ is well approximated by the third degree polynomial

$$x(t) = -7961,28t^3 + 26486,57t^2 - 7845,41t + 13484,84$$

with determination factor $R^2 = 0,97$.

The equation (26) can be solved analytically, namely

$$\psi(t) = \sqrt{C_\psi e^{-2a(K-2x(t))t} - \frac{(k-1-\ln r_0 k)^2}{4k^2a^2(K-x(t))^2(K-2x(t))}},$$

where

$$C_\psi = \frac{(k-1-ln(r_0k))^2 e^{2a(K-2x(T))T}}{4k^2a^2(K-x(t))^2(K-2x(t))}$$

The fishing enterprise profit for three years is equal to 135 conditional units.

4. Game theoretic models of economic corruption in the water quality control systems

Let's investigate fight with corruption in a dynamic three-level water resource quality control system which includes the following control levels: top (federal agency or principal - he), middle (regional agency official or supervisor - she) and bottom (enterprise or agent - he), and the controlled dynamic system (water stream or CDS).

The agent tends to maximize his production profit. In the process of production some pollutants are thrown to the CDS. It is assumed that the supervisor can change in a range the normative pollution fee tending to maximize her income. The principal must ensure a stable state of the CDS (the stability is treated in the sense of Lagrange). The principal determines which part of the fees received from the agent goes to the supervisor and which one he keeps to himself respectively. The interests of principal and supervisor are different, and the supervisor may be interested to receive bribes from the agent and to decrease the pollution fee in exchange. For the supervisor bribes are considered as a factor, together with income from the fees, in the general balance of her interests. The principal should create such conditions in which to ensure the stable state of the CDS will be profitable for the supervisor even when the corruption exists.

The principal controls the supervisor by charging penalties for the bribes. The penalty size depends on the probability of detection of the corruption and on the deviation of the real pollution fee from the normative one. The principal's control costs are considered in the model. We speak about an economic corruption because impulsion is used as the method of control (Ougolnitsky, 2011).

A case of one type of pollutants (for example, nitric) and one agent is analyzed. It is supposed that the system is in the stable state if quality standards for river water

$$0 \le B(t) \le B_{max}; \quad 0 \le t \le \Delta \tag{29}$$

and sewage

$$\frac{W(t)(1-P(t))}{Q^0(t)} \le Q_{max}; \quad 0 \le t \le \Delta \tag{30}$$

are satisfied, where t - time; $B(t)$ - concentration of the pollutant in the river water in the moment t ; $Q^0(t)$ - amount of sewage; $W(t)$ - number of pollutant thrown to the river before refinement; $P(t)$ - share of the pollutant eliminated from the sewage due to the refinement; Δ - length of time period; values $B_{max},\ Q_{max}$ are given.

For description of the pollution dynamics an ordinary differential equation in the form

$$\frac{dB}{dt} = F(B(t), P(t), t) \tag{31}$$

is used where $F(B(t), P(t), t)$ is a given function.

Besides ensuring the stability of the system the principal tends to maximize his personal payoff functional

$$J_0(K(t), H(t), T(t), P(t), b(t)) = \int_0^{\Delta} (-C_{\Phi}(y(t)) + y(t)F(T(t))H(t)+ \tag{32}$$

$$+y(t)K(t)\mu(T_0 - T(t))b(t) - \frac{MK(t)}{T_0 - T(t)})dt \to max; \; y(t) = (1 - P(t))W(t),$$

where $y(t)$ is an amount of the pollutant thrown by the agent in the river after refinement; $C_{\Phi}(y(t))$ - the principal's water quality improvement cost function ($y(t)$); $F(T(t))$ - cost of the unit of thrown pollutant; $T(t) = T(b(t))$ - a real per unit pollution fee depending on the bribe ; T_0 - the normative per unit pollution fee; $H(t)$ - a share of the fee that goes to the supervisor; μ - a given probability of the bribery detection ($0 \leq \mu \leq 1$); $K = K(t)$- a penalty function; $M = const$ - a factor of the principal's bribery control cost.

The term $C_{\Phi}(y)$ in (32) represents the principal's water refinement cost; $y(t)H(t)F(T(t))$ - an amount of the pollution fee; $y(t)K(t)\mu(T_0 - T(t))b(t)$ - an amount of penalty charged on the supervisor in the case of corruption; $MK(t)/(T_0 - T(t))$ - the principal's control cost. A maximum of the functional (32) is sought with respect to two functions $K(t)$ and $H(t)$.

The supervisor's payoff functional has the form

$$J_1(K(t), H(t), T(t), P(t), b(t)) = \int_0^{\Delta} (-C_0(y(t)) + y(t)F(T(t))(1 - H(t)) - \quad (33)$$

$$y(t)K(t)\mu(T_0 - T(t))b(t) + b(t)y(t))dt \to max$$

In (33) the term $C_0(y)$ represents the supervisor's water quality improvement cost function; $y(t)(1-H(t))F(T(t))$ - a pollution fee paid by the agent to the supervisor; $y(t)K(t)\mu(T_0 - T(t))b(t)$ - the supervisor's penalty in the case of bribery detection; $y(t) = b(t)$ - the bribe received by the supervisor if the amount of pollutant is equal to $y(t)$.

The agent's objective is to maximize his profit in the presence of corruption:

$$J_2(T(t), P(t), b(t)) = \int_0^{\Delta} (zR(\Phi(t)) - C_P(P(t))W(t) - y(t)F(T(t) - \quad (34)$$

$$b(t)y(t))dt \to max$$

Here $C_P(P)$ - the agent's per unit cost of sewage refinement; $\Phi(t)$ - production funds; $R(\Phi(t))$ - the agent's production function; $z = const$ - price of the production unit. A maximum of (34) is sought with respect to two functions: $P(t)$ and $b(t)$.

The term $zR(\Phi)$ represents the agent's profit from sale of $R(\Phi)$ - units of production; $y(t)F(T(t))$ - the pollution fee; $C_P(P(t))W(t)$ - the agent's cost of sewage refinement; $y(t)b(t)$ - the bribe value.

The problems (32) - (34) are solved with the following restrictions on the control values:

- principal

$$0 \leq H(t) \leq 1; \quad 0 \leq K(t) \leq 1; \quad 0 \leq t \leq \Delta; \quad (35)$$

- agent

$$0 \leq P(t) \leq 1 - \varepsilon; \quad 0 \leq b(t) \leq b_{max}; \quad 0 \leq t \leq \Delta; \quad (36)$$

- supervisor

$$0 \leq T(t) \leq T_0; \quad 0 \leq t \leq \Delta, \quad (37)$$

(37) where ε is determined by the agent's technological capacity; $b_{max} = const$ is the maximal feasible bribe value per unit of the pollutant.

The dynamics of production funds of the agent is described by an ordinary differential equation in the form

$$\frac{d\Phi}{dt} = -\lambda\Phi + Y; \quad \Phi(0) = \Phi_0 \ , \tag{38}$$

(38) where λ is a depreciation factor; Y - constant production investments; a constant Φ_0 is given.

Suppose that the production functions are given in the form

$$W(t) = \beta R(\Phi(t)); \quad R(\Phi(t)) = \gamma\Phi^{0.5}(t); \quad \gamma, \beta = const \tag{39}$$

The algorithm of solution of the problem (29) - (39) consists in the following steps:
1) a Germeyer game of the type Γ_{2t} (Gorelik et al., 1991) for the supervisor and the agent is considered. The value L_{2t} of guaranteed payoff of the agent if he doesn't collaborate with the supervisor is determined:

$$L_{2t} = \sup_{P,b} \ \inf_{T} \ J_2(T(t), P(t), b(t))$$

The supervisor's strategy which minimizes the agent's payoff functional is called her punishment strategy and denoted $T^k(t)$;
2) the optimal control problem (33), (36), (37) with an additional condition

$$L_{2t} < J_2(T(t), P(t), b(t)) \tag{40}$$

is solved. A maximum of (33) is sought with respect to three functions: $T(t)$, $P(t)$ and $b(t)$. The optimal strategies depend on the principal's strategies and have the form

$$P^S(t) = P^S(K(t), H(t), t); \quad b^S(t) = b^S(K(t), H(t), t); \tag{41}$$
$$T^S(t) = T^S(K(t), H(t), t);$$

where'$T^S(t)$ is the supervisor's reward strategy. Thus

$$T^*(K(t), H(t), t) =$$

$$\begin{cases} T^K(t) \ \ if \ \exists t_0 : \ 0 \le t_0 \le \Delta; \ P(t_0) \ne P^S(K(t_0), H(t_0), t_0) \\ \qquad or \ b(t_0) \ne b^S(K(t_0), H(t_0), t_0); \\ T^S(K(t), H(t), t) \ \ if \ \ \forall t : \ 0 \le t \le \Delta; \ P(t) = P^S(K(t), H(t), t) \\ \qquad and \ b(t) = b^S(K(t), H(t), t) \end{cases}$$

Due to the condition (40) the reward strategy is the most profitable one for the agent;
3) the functions (41) are substituted in (30) - (32). The optimal control problem (32) with additional constraints (including the phase ones) (29) - (31), (35), (38), (39) is solved. The functions which solve the problem are denoted $K^*(t)$ and $H^*(t)$.
4) the Stackelberg equilibrium in the Germeyer games Γ_{2t} (between the supervisor and the agent) and Γ_{1t} (between the principal and the supervisor) has the form

$$\{K^*(t), \ H^*(t), \ T^S(K^*(t), \ H^*(t), t), \ P^S(K^*(t), \ H^*(t), t), \ b^S(K^*(t), \ H^*(t), t)\}$$

In the general case the optimal control problems defined on the steps 2 and 3 of the algorithm are solved numerically after their digitization in time by the direct ordered sorting method.

5. Conclusion

The general principles of building the dynamic models of the fight with corruption are proposed. The "genetic series" of dynamic models of economic and administrative corruption in the form "optimal control problem - dynamic hierarchical two-person game - dynamic hierarchical three-person game" are built. Some dynamic models of economic corruption with application to the exloitation of bioresources and water quality control problems are investigated and solved. In the latter case a Stackelberg equilibrium in the Germeyer games Γ_{1t} and Γ_{2t} is used.

It is supposed in future to analyze in more details the models of economic corruption for different classes of bribery functions as well as investigate the dynamic models of administrative corruption.

References

Basu, K., Bhattacharya, S., Mishra, A. (1992). *Notes on bribery and the control of corruption*. Journal of Public Economics, **48**, 349–359.

Olsen, T. E., Torsvik, G. (1998). *Collusion and Renegotiations in Hierarchies: A Case of Beneficial Corruption*. International Economic Review, **39(2)**, 143–157.

Yang, D. (2005). *Corruption by monopoly: Bribery in Chinese enterprise licensing as a repeated bargaining game*. China Economic Review, **16**, 171–188.

Lambert-Mogiliansky, A., Majumdar, M., Radner, R. (2007). *Strategic analysis of petty corruption: Entrepreneurs and bureaucrats*. Journal of Development Economics, **83**, 351–367.

Bhattacharya, S., Hodler, R. (2010). *Natural resources, bureaucracy and corruption*. European Economic Review, **54**, 608–621.

Balafoutas, L. (2011). *Public beliefs and corruption in a repeated psychological game*. Journal of Economic Behavior and Organization, **78**, 51–59.

Blackburn, K., Forgues-Puccio, G. F. (2010). *Financial liberalization, bureaucratic corruption and economic development* Journal of International Money and Finance, **29**, 1321–1339.

Cerqueti, R., Coppier, R. (2011). *Economic growth, corruption and tax evasion*. Economic Modeling, **28**, 489–500.

Antonenko, A., Ougolnitsky, G., Usov, A. (2012). *Static models of corruption in hierarchical control systems*. In: Collected papers presented on the Fifth International Conference Game Theory and Management (Petrosyan, L., Zenkevich, N., eds) Vol. V., pp. 20–32. Graduate School of Management.: SPbU.

Ougolnitsky, G. (2011). *Sustainable Management*. Nova Science Publishers, N.Y.

Gorelik, V. A., Gorelov, M. A., Kononenko, A. F. (1991). *Analysis of Conflict Situations in Control Systems*. CC. (in Russian).

Grass, D., Caulkins, J. P., Feichtinger, G. et al. (2008). *Optimal Control of Nonlinear Processes. With Applications in Drugs, Corruption, and Terror.* Springer-Verlag: Berlin Heidelberg.