Multi-period Cooperative Vehicle Routing Games

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Abstract In the paper we treat the problem of minimizing and sharing joint transportation cost in multi-agent vehicle routing problem (VRP) on large-scale networks. A new approach for calculation subadditive characteristic function in multi-period TU-cooperative vehicle routing game (CVRG) has been developed. The main result of this paper is the method of constructing the characteristic function of cooperative routing game of freight carriers, which guarantees its subadditive property. A new algorithm is proposed for solving this problem, which is called *direct coalition induction algorithm (DCIA)*. Cost sharing method proposed in the paper allows to obtain sharing distribution procedure which provides strong dynamic stability of cooperative agreement based on the concept of Sub-Core and time consistency of any cost allocation from Sub-Core in multi-period CVRG.

Keywords: VRP, vehicle routing problem, vehicle routing games, heuristics, multi-period cooperative games, dynamic stability, time consistency.

1. Introduction

When we study collaboration in cargo transportation and routing we have to address the following questions partly discussed in (Agarwal et al., 2009):

- How does one evaluate the maximum potential benefit from collaboration of carriers forming coalitions? However, to obtain such a benefit value is not easy because the underlying computational problem is NP hard.
- How should a membership mechanism be formed to be stable during sufficiently long period of time, and what are the desired properties that such a mechanism should possess? For logistics applications, this involves issues related to the design of the service network and utilization of assets, such as the allocation of ship capacity among collaborating carriers, assignment and scheduling vehicles on routes.
- How should the benefits achieved by collaborating be allocated among the members in a fair way? In the cargo transportation routing setting we investigate what does a fair allocation mean and how such an allocation may be achieved in the context of day-to-day operations to be time consistent during transportation process?
- How to overcome these disadvantages?

Dynamic cooperative game theory can provide us with models of coordination carrier's actions in order to reduce transportation costs. Cooperation issues in vehicle routing models are still an insufficiently studied problem. Possible applications of the cooperative game theory for such problems are demonstrated in the papers

(Ergun et al., 2007; Krajewska et al., 2008). The most important object under investigation of the cooperative game theory is the characteristic function of the game which reflects assessment of guaranteed values of total costs of participants united in a coalition. If one constructs a mathematical model of cooperation in practical tasks, it is important to select the method of such function building. Computational difficulties of finding the values of the characteristic function in a cooperative vehicle routing game (CVRG) are caused by the large size of the problem, which makes it unacceptable to use exact methods for solving wide class of routing problems with a comparatively small number of customers to be served (Baldacci et al., 2012; Kallehauge, 2008). At the same time, using of heuristic algorithms in the general case does not allow to guarantee fulfillment of the subadditive property of the characteristic functions, which has crucial importance for achievement of cooperative agreements and total cost reduction. Considering dynamic cooperation models, it is expedient to use imputation distribution procedures (IDP) which were first proposed by L.A.Petrosyan, as well as cooperation stability principles formulated by L.A.Petrosyan and N.A.Zenkevich (2009).

In our paper we propose mathematical setting of the freight carriers cooperation problem, a new approach to building the characteristic function of the multi-period CVRG and algorithm for constructing cost sharing scheme providing strong dynamic stability of the Sub-Core to meet condition of time consistency (dynamic stability) of cooperative agreements.

2. General Problem Statement

In this paper it is presumed that in the transportation service market there are several agents (companies) engaged in cargo transportation on a network. Each agent has a great number of customers located in nodes of network and its own resources, such as a depot and a non-empty fleet of vehicles. These companies consider various options of cooperation to reduce transportation costs. Each coalition meets the demand of customers for transportation services of all companies involved in cooperation using consolidated resources. Thus, within cooperative service, customers can be redistributed between participants in each coalition. In its turn, customers exchange between agents within a coalition would extend the set of feasible routes of consolidated fleet and provide additional possibility to improve current solution in comparison to non-cooperative case. On the other hand, when agents cooperate, the total number of customers that has to be dispatched at once to vehicles substantially increases along with the computational complexity of finding routes minimizing the total transportation costs of the coalition. Therefore, in operative decision-making environment there is a lack of time for quick assignment of customers to optimal routes, since this problem belongs to the class of NP-hard problems.

To find a good solution for vehicle routing problem with several depots the adaptation of well-known metaheuristic algorithm proposed by Ropke and Pisinger (2006) may be used for each coalition. Once the routes with minimum transportation costs for each possible coalition are found, the characteristic function value of the cooperative routing game can be calculated. To ensure that the agents have the motivation to form a coalition, the characteristic function has to satisfy subadditivity condition. In general, heuristic algorithms that find minimum of transportation costs of a coalition do not guarantee this property. Therefore, a special metaheuristic algorithms providing subadditive property of the characteristic function has to be proposed for VRP.

The solution of VRP is a set of vehicle routes, such that all customers are visited exactly once, each route starts and ends in a depot, the length of each route is limited to predetermined value. Additionally, in the vehicle routing problems with time windows each customer has specified service time and must be visited within the specified time interval.

Generally, the objective of such problems is to minimize the total length of routes. In real-life cases the number of used vehicles has more significant impact on the total transportation costs, because the cost of using additional vehicle appears to be much higher than benefit from shorter routes.

3. Mathematical Model of Static CVRG

Let N be a set of companies engaged in transportation service in the same transport network. Each company $i \in N$ provides transportation service to the given set of customers A_i. Each customer is served by only one company. Companies are considering possibilities of cooperation to reduce total transportation costs. Let $S \subseteq N$ be a proper coalition of companies (players or agents in the static CVRG with transferable utilities) to be formed. The total cost of the coalition S consists of two parts: costs of used vehicles and direct transportation costs. In this paper two assumptions are made concerning costs:

– direct transportation cost is linear function of the total length of routes;
– fleet of vehicles of coalition S includes homogenous vehicles of all companies from this coalition, and each vehicle has fixed utilization price. It is also assumed that each coalition has unlimited number of identical vehicles and pays only for those that are used in transportation service.

Thus, the cost function may be represented as follows:

$$cost(S, p_S) = a_S \cdot NT(S, p_S) + b_S \cdot TTC(S, p_S),$$

where

$p_S \in P_S$ — feasible routing plan for the vehicles of the coalition S, P_S — the finite set of feasible routing plans of the collation S;

a_S — the cost of one vehicle utilization for the coalition S;

$NT(S, p_S)$ — number of vehicles used by the coalition S at the particular routing plan p_S;

b_S — cost of one unit of the length for the coalition S;

$TTC(S, p_S)$ — the total length of routes of the coalition S at the particular routing plan p_S.

For the sake of simplicity, it is assumed that each company has only one depot. It is also assumed that companies may redistribute transportation costs among the collaborators using some cost sharing procedure.

In order to design subadditive characteristic function of CVRG consider for coalition $S \subseteq N$ the costs minimization problem over the set of feasible vehicle routes

$$min_{p_S \in P_S} cost(S, p_S) \tag{1}$$

Suppose the exact minimum value of the problem (1) is equal to $c^{opt}(S)$. In the case of using heuristic algorithm for solving this problem the obtained value of the minimum $c^h(S)$ will be not less than $c^{opt}(S)$, that is

$$c^{opt}(S) \leq c^h(S) \tag{2}$$

For two disjoint coalitions $S \subseteq N$ and $T \subseteq N$, for any pair of feasible routing plans $p_S \in P_S, p_T \in P_T$, the routing plan (p_S, p_T) consisting of the union of routing plans p_S and p_T is feasible in the routing problem for the joint coalition of carriers $S \cup T$, that is $(p_S, p_T) \in P_{S \cup T}$, moreover $P_S \cup P_T \subseteq P_{S \cup T}$ and $A_S \cup A_T = A_{S \cup T}$, then it is clear that the following inequality holds

$$c^{opt}(S \cup T) \leq c^{opt}(S) + c^{opt}(T)$$

Taking into account the inequality (2) we have

$$c^{opt}(S \cup T) \leq c^h(S) + c^h(T) \tag{3}$$

Last inequality can be rewritten for the arbitrary coalition $L \subseteq S$ and the corresponding values $c^h(L)$ and $c^h(S/L)$

$$c^{opt}(S) \leq c^h(S/L) + c^h(L) \tag{4}$$

We define the value of characteristic function $c(S)$ in cooperative CVRG in the following way

$$c(S) = min\{min_{L \subset S}\{c(S/L) + c(L)\}, c^h(S)\} \tag{5}$$

One can notice that if we start calculation of the characteristic function $c(S)$ with one-element coalitions and then gradually increase the size of coalitions by 1 until we obtain the value for the grand coalition N, the characteristic function designed by using (5) would fulfill the condition of the subadditive, i.e.

$$c(S \cup T) \leq c(S) + c(T),\ S \subseteq N,\ T \subseteq N,\ S \cap T = \emptyset \tag{6}$$

We call this algorithm for constructing characteristic function of TU-cooperative CVRG in the form (5) the direct coalition induction algorithm (DCIA). Thus, the following theorem holds.

Theorem 1. *The characteristic function* $c(S)$ *defined by* (5) *of static TU-cooperative VRG and calculated using direct coalition induction algorithm satisfies subadditivity condition* (6).

4. Example of Cooperative Routing

To illustrate the algorithm implementation we consider one artificial problem of cooperation with four transport companies $D1$, $D2$, $D3$, $D4$ having demand for cargo transportation from 54, 49, 44, 53 customers. The example has been generated using one benchmark (R2_2_1) proposed by Gehring and Homberger to compare heuristic algorithms that solve vehicle routing problems with time windows.

Thus, in case of full cooperation, the transportation companies together have to service 200 customers. Clients of each company are distributed evenly throughout the nodes of network where the servicing is provided. And they have wide time service windows (the time interval during which servicing is possible) which allows to use less vehicles, but at the same time increases the computational complexity of the problem. Use of a little number of vehicles is also facilitated by big carrying capacity thereof as compared to the customers' demand. The total costs will be calculated assuming that the value of use of one vehicle is 5000 condition monetary units, and the value of one unit of the route length is 5 conditional monetary units. The algorithm proposed by Ropke and Pisinger (2006) was used for solving corresponding routing problems. As a basic problem, this algorithm considers the more general problem, the particular case of which is the problem in question. In order to find efficient routes several basic heuristics were united in one algorithm with the help of the simulated annealing. One part of these heuristics removes several customers from the solution, and the other inserts them into the solution again. The adaptive mechanism tracks the performance of basic heuristics and chooses at each step of iterations two certain heuristics using obtained statistics of their previous effectiveness. Such mechanism is based on the special genetic algorithm. To diversify search process and enhance algorithm robustness the noise value is added to the value of objective function. Table 1 shows the solution of respective costs minimization problems for each coalition and values of the game characteristic function calculated using the direct coalition induction algorithm.

Table 1: Solutions of costs minimization problems and values of characteristic function

Coalition	Vehilces	Length	Characteristi function value
(D1)	3	2043,4	25217,05
(D2)	3	2013,3	25066,46
(D3)	2	1852,8	19263,80
(D4)	2	2245,9	21229,39
(D1, D2)	3	3879,6	34398,07
(D1, D3)	4	2750,9	33754,39
(D1, D4)	3	3573,3	32866,71
(D2, D3)	3	2949,2	29745,90
(D2, D4)	3	3130,7	30653,36
(D3, D4)	4	2502,3	32511,41
(D1, D2, D3)	4	4127,5	40637,45
(D1, D2, D4)	4	4166,2	40830,99
(D1, D3, D4)	5	3569,8	42848,97
(D2, D3, D4)	4	3570,8	37853,79
(D1, D2, D3, D4)	5	4575,6	47878,11

As one can see in Table 1, the sum of minimum costs of companies, if there is no cooperation, is equal to 90776.70. The minimum total costs in the case of cooperation are equal to 47878.11. Thus, the savings from cooperation in this example are about 47 per cent.

To share the total costs among players Shapley value is used. As Table 2 shows, considerable reduction of costs of the companies can be achieved in comparison to their minimum costs prior to cooperation.

It should be noted that reduction of costs of each of the companies under cooperation after redistribution of the total costs using the Shapley value was 43 to 54 percent.

Table 2: Solution for maximum coalition and cost sharing using Shapley value

Coalition	Shapley value	Minimum costs without cooperation	Cost reduction coefficient
(D1)	14382,5	25217,0	0,43
(D2)	11630,3	25066,5	0,54
(D3)	10571,1	19263,8	0,45
(D4)	11294,1	21229,4	0,47

5. Dynamic Model of CVRG

Suppose CVRG has duration from 0 to T. Let the interval $[0,T]$ be divided by periods $t_0, t_1, \ldots, t_m$. That is $[0,T] = (t_0, t_1, \ldots, t_m)$. Cost functions of players for the period $[0,T]$ and set of feasible routing plans (strategies) are determined in the same way like in section 3. It is assumed that for CVRG starting from origin t_0 the characteristic function $c(S,0)$ is calculated by the direct coalition induction algorithm.

For further calculations the following notation will be used:

$p_N^h(0)$ — is the optimal routing plan of grand coalition N in original game, which calculated by direct coalition induction algorithm and minimizes the total costs of the coalition for the periods $t_0, t_1, \ldots, t_m$;

$p_S^h(0)$ — is the optimal routing plan of coalition S in origin game, which is calculated by direct coalition induction algorithm and minimizes the total costs of the coalition for the periods $t_0, t_1, \ldots, t_m$, $S \subset N$;

$p_S^h(t_k)$ — profile of the optimal routing plan of the coalition S in period t_k, $S \subseteq N$, $k = 1, 2, \ldots, m$;

$p_N^h(0) = (p_N^h(t_0), \ldots, p_N^h(t_m))$ — vector of profiles of optimal routing plan;

$p_{N,i}^h(t_k)$ — optimal routing plan for vehicles of company $i \in N$ in period t_k as part of optimal plan $p_N^h(t_k)$;

$c(S, k, p_N^h(t_0), \ldots, p_N^h(t_{k-1}))$ — value of minimal total costs of coalition $S \subseteq N$ after implementation the optimal routing plan calculated by direct coalition induction algorithm.

One of the important issues of successful implementation of the routing plan $p_N^h(0) = (p_N^h(t_0), \ldots, p_N^h(t_m))$ during all periods of the game is optimality of each restriction of the original optimal plan $p_N^h(0)$ on the set of periods $t_k, t_{k+1}, \ldots, t_m$ for $k = 1, 2, \ldots, m$. We denote this restriction of the plan $p_N^h(0)$ by $p_N^h(k)) = (p_N^h(t_k), \ldots, p_N^h(t_m))$. When restriction of original optimal plan $p_N^h(0)$ appears to be not optimal for at least one period t_k, $k = 1, 2, \ldots, m$, we call this plan time inconsistent. Notice that unlike Bellman optimality principle it might be happened in routing optimization because of using heuristics instead of exact methods. To make an attempt to overcome time inconsistency of the plan formed by heuristic algorithm we propose to realize along originally calculated plan $p_N^h(0) = (p_N^h(t_0), \dots, p_N^h(t_m))$ the following iterative coalition induction algorithm (ICIA).

Iterative coalition induction algorithm.

Step 1. Put $k = 0$.

Step 2. Assume that plan $p_N^h(k)$ has been implemented within the period t_k. We exclude nodes visited in the period t_k from the set of customerâĂŹs nodes and consider current CVRG(t_{k+1}) under new conditions for the set of customers to be served in periods $t_{k+1}, \dots, t_(m)$ and new depots location taking into account current positions of vehicles at the end of routs executed in period t_k. If heuristic algorithm proposes new routing plan $(\overline{p_N(k+1)}) = (\overline{p_N(t_{k+1})}, \dots, \overline{p_N(t_m)})$ in current CVRG(t_k) which gives less total costs for the grand coalition N for periods $t_{k+1}, \dots, t_m$ than the plan $p_N^h(k+1) = (p_N^h(t_{k+1}), \dots, p_N^h(t_m))$ we make the following substitution to improve the plan considered for implementation before period t_{k+1}:

$$p_N^h(0) = \begin{cases} (p_N^h(t_0), \dots, p_N^h(t_k)) - \text{within the periods } t_0, t_1, \dots, t_k \\ (\overline{p_N(t_{k+1})}, \dots, \overline{p_N(t_m)} - \text{within the periods } t_{k+1}, \dots, t_m \end{cases} \tag{7}$$

And move to step 2 putting $k = k + 1$ and, if $k < m$. If plan $(\overline{(p_N(k+1))})$ in current CVRG(t_k) proposed by heuristic algorithm coincides with $p_N^h(k+1)$ or gives bigger value of total costs for grand coalition put $k = k + 1$, we do not make substitution (7) and move to step 2, if $k < m$. In any case, if $k = m$ go to step 3.

Step 3. Stop the procedure and use for implementation plan
$p_N^h(0) = (p_N^h(t_0), \dots, p_N^h(t_m))$ which has been gotten on the last iteration.

Let $p_N^h(0) = (p_N^h(t_0), \dots, p_N^h(t_m))$ be the optimal routing plan obtained by adjustment of the initial optimal plan with the help of the ICIA. For each period $t_1, \dots, t_m$ along optimal routing plan $p_N^h(0) = (p_N^h(t_0), \dots, p_N^h(t_m))$ we can calculate values of characteristic function $c(S, k, p_N^h(t_0), \dots, p_N^h(t_{k-1}))$ for current CVRG(t_k) using DCIA. Characteristic function for CVRG(t_0) is $c(S, 0)$. We can represent value of the characteristic function for the grand coalition in CVRG(t_0)

$$c(N, 0) = \sum_{i=1}^{n} \sum_{k=0}^{m} cost(i, p_{N,i}^h(t_k)) = c(N, p_N^h(0))$$

When all players form grand coalition N, for optimal routing plan $p_N^h(0)$ the set of imputations in the cooperative game $c(S, 0, p_N^h(0)) = c(S, 0)$ will be determined as follows

$$I(0, p_N^h(0)) = \{\alpha = (\alpha_1, \alpha_2, \dots, \alpha_n) \colon \ \alpha_i \le c(\{i\}, 0),\, i = 1, \dots, n,\, \sum_{i=1}^{n} \alpha_i = c(N, 0)\}$$

In this paper the Sub-Core was used as a solution of the cooperative game (Zakharov and Kwon, 1999; Zakharov and Dementieva, 2004).

Definition 1. Sub-Core of the cooperative game $c(S, 0, p_N^h(0))$ is called the set

$$SC(c(S, 0, p_N^h(0))) = \bigcup_{c^0(0) \in C_0(0)} SC(c(S, 0, p_N^h(0)), c^0(0)) \tag{8}$$

where

$$SC(c(S,0,p_N^h(0)),c^0(0)) = \\ = \Big\{\alpha = c^0 - \lambda\left(\sum_{i=1}^{n} c_i^0(0) - C(N,0,p_N^h(0))\right), \\ \lambda = (\lambda_1,\lambda_2,\ldots,\lambda_n)\colon \sum_{i=1}^{n}\lambda_i = 1,\ \lambda_i \geq 0,\ i = 1,2,\ldots,n\Big\}$$

and C_0 is the set of solutions of the following maximization problem

$$\max \sum_{i=1}^{n} c_i$$

provided that

$$\sum_{i\in S} c_i \leq c(S,0,p_N^h(0)),\ S\subset N$$

We shall call the set $C_0(0)$ as the basis of Sub-Core, any vector $c^0(0) = (c_1^0(0), c_2^0(0),\ldots,c_n^0(0)) \in C_0(0)$ — as the basis imputation of the cooperative game $c(S,0,p_N^h(0))$.

By the structure the Sub-Core is not empty if and only if the Core of cooperative game with the characteristic function $c(S,0,p_N^h(0))$ is not empty, and necessary and sufficient condition for the Sub-Core (and hence the Core) to be not empty is fulfillment the following inequality

$$\sum_{i\in N} c_i^0(0) \geq c(N,0,p_N^h) \tag{9}$$

Sub-Core in current CVRG(t_k) is determined by the same way. Presume that the Sub-Core $SC(c(S,k,p_N^h(t_0),\ldots,p_N^h(t_{k-1})),c^0(k))$ is not empty for $k = 1,\ldots,m$. Let $\alpha^k = (\alpha_1^k,\alpha_2^k,\ldots,\alpha_n^k) \in SC(c(S,k,p_N^h(t_0),\ldots,p_N^h(t_{k-1}))), k = 0,1,\ldots,m$, be vectors of cost sharing in the current games $c(S,k,p_N^h(t_0),\ldots,p_N^h(t_{k-1}))$. In this case costs of any coalition determined in the current game in accordance with the vector α^k will not exceed the values of the characteristic function for this coalition for any value $k = 0,1,\ldots,m$. Thus, there is no coalition interested in leaving the agreement at any stage of the game, which means strong dynamic stability of the Sub-Core. By analogy with the imputation distribution procedures (IDP) discussed e.g. in paper (Petrosyan and Zenkevich, 2009), the cost sharing procedure (CSP) $\beta_k = (\beta_1^k,\beta_2^k,\ldots,\beta_n^k)$ can be considered in the multistage cooperative game, where

$$\beta_i^k = \alpha_i^k - \alpha_i^{k+1}, k = 0,1,\ldots,m-1, i\in N \tag{10}$$

The crucial property of such procedure is fulfillment for any player i at any stage of the game of the condition

$$\sum_{j=k}^{m} \beta_i^j = \alpha_i^k, k = 0,1,\ldots,m,$$

that we call *condition of individual costs balance* of the player $i \in N$.

According to the definition of the Sub-Core, the following equation is valid for the vectors $\alpha^k = (\alpha_1^k, \alpha_2^k, \dots, \alpha_n^k)$ of cost sharing in the current games

$$\sum_{i=1}^{n} \alpha_i^k = c(N, k, p_N^h(0)), k = 0, 1, \dots, m$$

and taking into account (10), the following equation can be obtained

$$\sum_{i=1}^{n} \beta_i^k = c(N, k, p_N^h(t_0), \dots, p_N^h(t_{k-1})) - c(N, k+1, p_N^h(t_0), \dots, p_N^h(t_k)),$$
$$k = 0, 1, \dots, m-1$$

This condition will be called as *condition of collective balance of coalition costs* in the multistage cooperative game.

Presume that the numerical value β_i^k determines the size of payoff of the player i within the period t_k to a Costs Clearing Center (CCC), which accumulates funds for covering the costs of all players in the process of implementation of the routing plan $p_N^h(0)$ selected for implementation by coalition N. Then the economic meaning of the condition of individual costs balance will be, that the sum of payoffs of any player to Costs Clearing Center during the entire game will be equal to the size of costs, which player have to pay in accordance with the selected optimal distribution $\alpha^0 = (\alpha_1^0, \alpha_2^0, \dots, \alpha_n^0)$. And collective balance of coalitional costs will provide the possibility of covering the costs of participants of the coalition N within the same period, when these costs are made.

6. Example of Multi-Period CVRP

As illustration of the dynamic case, consider the static problem described earlier, but assume now that the entire servicing time is divided into 3 equal periods. All vehicles that are maintained in previous period by the grand coalition, begin their movement in current period from the last serviced customer's node. Each company participating in one or another coalition may use additional vehicles which begin their movement from the depot belonging to the company. The same heuristic algorithm as in the static case (Ropke and Pisinger, 2006) is used for finding efficient routes for CVRG in each period. To calculate characteristic function values given in Table 3 we apply algorithms DCIA and ICIA.

Using the obtained values of the characteristic function find the basis of Sub-Core for each period. In this case all three maximization tasks have the unique solution, and thus the set C_0 for each period contains only of one element.

It should be noted that Sub-Core will not be empty within each period due to fulfillment of the condition (9). In order to find certain imputation belonging to Sub-Core within each period, the value of each component of the vector λ was set to 0.25. After that, values of vectors β_k using the obtained sharing vectors were calculated. The calculation results are given in Table 5.

Negativity of payment values means that a company does not make payment to CCC within the respective period, but receives in this period compensation from CCC. Analyzing the data of Table 5, it is become clear that conditions of individual costs balance and collective costs balance in the three-period CVRG have been fulfilled.

Table 3: Values of characteristic function for three periods

Coalition	The characteristic function		
	$c(S,0)$	$c(S,1)$	$c(S,2)$
(D1)	25217,05	22268,11	13367,79
(D2)	25066,46	15418,88	12544,99
(D3)	19263,80	20099,82	12327,52
(D4)	21229,39	22199,77	12707,63
(D1, D2)	34398,07	29613,92	19046,92
(D1, D3)	33754,39	33180,69	24390,32
(D1, D4)	32866,71	34810,43	24110,87
(D2, D3)	29745,90	32592,22	19223,78
(D2, D4)	30653,36	33699,61	19324,51
(D3, D4)	32511,41	37683,95	23736,35
(D1, D2, D3)	40637,45	40119,31	24834,71
(D1, D2, D4)	40830,99	36572,44	20308,93
(D1, D3, D4)	42848,97	37133,72	29854,66
(D2, D3, D4)	37853,79	44562,17	24723,40
(D1, D2, D3, D4)	47878,11	38552,26	30364,98

Table 4: Basis of Sub-Core for three periods

	Sub-Core basis		
	Period 1	Period 2	Period 3
Company 1	16203	8720	9725
Company 2	11208	15419	2782
Company 3	13226	15980	12328
Company 4	13420	12433	7802
All companies	54057	52552	32637

Table 5: Values of imputations and vectors β_k

	Period 1		Period 2		Period 3	
	α_0	β_0	α_1	β_1	α_2	β_2
Company 1	14658	9438	5220	-3937	9157	9157
Company 2	9663	-2256	11919	9705	2214	2214
Company 3	11681	-799	12480	720	11760	11760
Company 4	11875	2942	8933	1699	7234	7234

7. Conclusions

The main result of this paper is the method of constructing the characteristic function of cooperative routing game of freight carriers, which guarantees its subadditive property. A new algorithm is proposed for solving this problem, which is called direct coalition induction algorithm (DCIA). To upgrade optimal routing plan and values of characteristic function of grand coalition we develop iterative coalition induction algorithm (ICIA) for dynamic CVRP. Both algorithms were built on the basis of the combination of various heuristic algorithms which are appropriate for solving large-scale VRP. For implementation of algorithms a special software has been developed and used for solving sample examples.

Proposed cost sharing method allow to obtain sharing distribution procedure which provide strong dynamic stability of cooperative agreement based on this Sub-Core optimality principle and time consistency of the Sub-Core in multi-period CVRG.

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