

Power in Game Theory

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Abstract The main aim of this paper is to discuss power in game theory, in order to model the asymmetries of forces among the players of the game. The starting point is that games are strategic interactions between rational individuals in a social environment, and the players do not have equal forces. Game theory has increased greatly in recent years, including several new branches. However, the concept of power in game theory has not been explored to any great extent. Indeed, power is a broad concept that has no clear definition. This paper formalizes a model, taking account the asymmetric forces between players. Examples are given, using some well-known games, to illustrate this relation involving power. As a result, this paper presents an approach in two still distant scientific fields: sociology and game theory.

1. Introduction

Some words or concepts do not have a precise meaning. Power is one of these words. It is said that the president of a nation has power, while civilians do not. On the other hand, in a democratic environment, the coalition of people is a source of power because it is precisely this coalition that will decide who will be the governor. The director of a company has power, while the workers do not, but one of these workers could be a union leader with as much power as the director. A teacher has power over his students because he can assign them a grade.

Everyone knows that in recent times Europe and the United States have been losing their economic power to China. In fact, like love, power is an abstract noun. It is impossible to see, touch or measure it, but it is possible to feel it. Nye (2011) states that it is very common to compare power with other things, such as energy (physics) or money (economics). Both these comparisons are mistaken. It is impossible to compare power with energy, as energy can be measured, as can money, which is liquid and interchangeable.

To understand the "nature" of this word, different theorists and sciences, including sociology, psychology, politics and economics, have developed different or complementary concepts of power and its relation to society. Power, in fact, is a broad concept, involved in different sciences and with a long historical approach. Hobbes, for instance, was one the first philosophers to analyze the meaning of power in the modern ages as far back as the 17th Century. These ideas were published in the *Leviathan*. To him, power is the ability to secure well-being or personal advantage in order to achieve some apparent good in the future.

After Hobbes, many concepts were developed for understanding power, but in all cases the meaning is the same: the asymmetry of forces. It is clear that power has a different view and differs in accordance with time and space. Power, in fact, is an important concept within society, and arises from the interaction between

people. In this sense, power could be thought of in terms of different views, including sociological, anthropological, internationally political and economic.

In this sense, the aim of this paper is to discuss power in game theory in order to model the asymmetries of forces between the players of the game. The starting point is that games are strategic interactions between rational individuals in a social environment and the players do not have equal forces. Game theory has increased greatly in recent years and several new branches have developed. However, the concept of power in game theory has not been explored to any great extent. Indeed, power is a broad concept with no clear definition. This paper formalizes a model, taking into account the asymmetric forces between players. In addition to this introduction, the paper discusses two specific topics: the concepts of power and a game model that includes power. These are followed by the conclusions.

2. The Concept of Power

Most people do not know the exact meaning of power, but they have an intuitive notion regarding it. This difference arises because power is a universal phenomenon, affecting people of different ages and in different places and times. Besides any kind of definition, people feel it. A little child can identify the power of its father or mother, an employee in an organization respects the power of his boss and, in normal circumstances, all the people obey the president of the nation. On the other hand, it is possible to identify its branches, such as political power, economic power and international power.

It is very common to hear a conversation about power at different levels. Daily newspapers publish news about power, who is involved and its consequences. But what is power? The answer to this question is not simple and covers a long process throughout the history of humanity. Some definitions of power are given below, but this list is not exhaustive. Power is normally associated with glory, but this association could distort the real meaning. Dahl (1957) p. 203 was led to think that "A has power over B to the extent that he can get B to do something that B would not otherwise do". This definition indicates that power exists over people. However, it is also possible to have power over animals and objects.

Weber (1968), for instance, provided one of the most important definitions of power: "the probability that one actor within a social relationship will be in a position to carry out his own will despite resistance". Foucault (1978) p. 92 said that "power must be understood in the first instance as the multiplicity of force relations immanent in the sphere in which they operate and which constitute their own organization". In this sense, Foucault advocates the existence of numerous powers, what he referred to as the micro-physics of power. Wartenberg (1990) distinguishes the exercise and possession of power. The exercise of power has attracted the attention of theorists due empiricist assumptions, but it is fundamental to have a strong concept of the possession of power. In this sense, the possession of power can be defined as "a social agent A possesses power over another agent B if A controls B's action-environment in a fundamental manner" Wartenberg (1990), p. 10. Action-environment means the structure, in a whole sense, within which as agent exists as a social actor. The action-environment supports, or provides conditions for, action-alternatives. It can be understood as "a course of action that is available to the agent in the situation" (p. 7). Power can be thought of as a capacity for control that one agent has over the action-environment of another agent. The main

important notion is that one agent has power over another agent without having exercised power in a practical sense. In this sense, if an agent has power, he could exercise it if he chose to, even in situations where he chooses not to, but he possesses that power nevertheless.

On the other hand, power has a practical meaning. Agents use power over other agents to obtain material or immaterial elements or to gain any kind of advantage. The use of power differs in intensity and type. The use is called the exercise of power, which can be defined as an "agent A exercises power over an agent B if A uses his control of B's action-environment to change it in some fundamental manner" Wartenberg (1990), p. 11. In this sense, power can be understood as the realization of a capacity. At this juncture, it is important draw a distinction between possessing and exercising power. As discussed above, possessing is capacity, and exercising is the realization of that capacity. Wartenberg (1990) emphasizes that power is used by restructuring the options of the other agent. This means that if agent A has power over agent B, the options of B are changed by the power of agent A. "It states that an agent who exercises power over another does so by changing the circumstances within which the other agent acts and makes choices" (p. 12).

The exercise of power can take four forms: force, coercion, influence and manipulation. It is important to say that manipulation is a kind of influence. The relationship of force occurs when "A's power over B is an instance of force if A physically keeps B from pursuing an action-alternative that B has reason to pursue or makes B's body behave in a way that B would avoid if possible" (Wartenberg (1990), p. 13). The second form of power is coercion, which is related to threats, and these threats must be recognized and accepted by the threatened agent. Coercion exists when agent A exercises "power over social agent B if (1) A has the ability to affect B in a significant way; (2) A threatens to do so unless B acts in a certain way; and (3) B accedes to A's threat and alters his course of action" (Wartenberg (1990), p. 15). Influence, the third form, can be thought of as when agent A "influences another agent B if A provides B with some putative information which results in B altering his assessment of his action-environment in a fundamental manner" (Wartenberg (1990), p. 21). The last form is manipulation, which can be understood as "agent A manipulates another agent B if A influences B for purposes or ends that he keeps concealed from B" " (Wartenberg (1990) p. 15). For the purpose of this paper, the understanding of (Galbraith (1983), p.2) has been adopted. Indeed, Galbraith uses the definition proposed by Weber: "power is the ability to impose one's will on the behavior of other persons". He goes on to divide it into three categories: a) condign, b) compensatory, and c) conditioned power.

The first category is based on brute force, meaning that the person who does not obey will be punished. In this situation, threat or intimidation is an essential action. Condign is a kind of power which is based on "the ability to impose an alternative to the preferences of the individual or group that is sufficiently unpleasant or painful so that these preferences are abandoned" (Galbraith (1983), p. 4). Compensatory power means that a quantity of resources is used to exchange what the owner wants. "Compensatory power, in contrast, wins submission by the offer of affirmative reward by the giving of something of value to the person so submitting" (p. 5.). Conditioned power refers to changing belief. Galbraith states that "persuasion, education, or the social commitment to what seems natural, proper, or right, causes the individual to submit to the will of another or of others" (p.6).

Galbraith also proposed three sources of power: personality, property or wealth and organization. Personality is "the quality of physique, mind, speech, moral certainty or other personal trait" (Galbraith (1983), p. 23). It is the ability to create or persuade. Belief, is the ability to change the mind or the way that people think. Personality is associated with conditioned power.

Property is associated with compensatory power, and means that agent one (the wealthy) can offer a quantity of money to another player, who accepts the conditions proposed by the first agent. In this sense, property and income "provide the wherewithal to purchase submission" (Galbraith (1983), p. 23). The third source of power is the organization, which is, in modern societies, the most important source of power. An organization is a number of persons or groups that have united for some purpose. This means that an organization concentrates both property and personality power. The State is a kind of organization that accesses condign power (punishment). In some circumstances, it is possible to connect the proscription of a company as a kind of punishment. (Nye (2006)) separated power into two categories: soft power and hard power. To him, conditioned power is hard, while compensatory power and condign power are soft. Figure 1, below, shows some specifications concerning these two kinds of power.

Type of Power	Behavior	Sources	Examples
Soft	Attract and co-opt	Inherent qualities Communications	Charisma Persuasion, example
Hard	Threaten and induce	Threats, intimidation. Payment rewards	Hire,fire,deote Promotions, compensation

Fig. 1: Soft and Hard Power.

Source: (Nye (2006))

(Balzer (1992)) has compared both game theory and power theory. Even power theory has no status of generally acknowledged theory like game theory, and the author has identified some particular converging aspects. First of all, the individuals (players) in both models can be the same, as well the alternatives for the individuals involved in the game. Nevertheless, the comparison described by Balzer requires two distinct events, before and after, and this is a situation that is only present in power theory. (Wiese (2009)) understands that compensatory power is a particular case from co-operative games, because it means transferable utility. However, condign power and conditioned power represent a non-transferable utility case, which plays an important role in social relations.

Formalizing the Idea of Power-asymmetric Games

In this section the idea of power-asymmetric games is formalized in a broad sense. It is important to emphasize that the power game does not invalidate the two main postulates of game theory: developing criteria for rational behavior and the assumption that players maximize their own utility functions. It should be emphasized that the only form of power is the exercise of power, not its mere possession. This means that an agent with power will always exercise it, as generates better results for him. It is also important to highlight that game theory usually encompasses different

games when the action-environment changes. For instance, when power is incorporated into a specific game, the use of power by an agent generates a new game, which transforms the options set of the player subjected by the power relation. However, instead of different games, the model proposed in this paper understands it as a single or unique game. This notion is important for capturing some elements of changing, and allows for comparison ex ante and ex post action-environment when the options are changed.

Definition 1. For an arbitrarily fixed $n \in \mathbb{N}$, an n-player power-asymmetric game Γ_n^P is given by the 5-uple:

$$\Gamma_n^P = \langle I_n, (S_i)_{i \in I_n}, (u_i)_{i \in I_n}, (p_i)_{i \in I_n}, \mathcal{P} \rangle,$$

where $I_n = 1, \dots, n$ is the set of players, S_i is the set of strategies of player $i \in I_n$, the function

$$u_i : \times_{(j \in I_n)} S_j \rightarrow \mathbb{R}$$

stands for the utility of player i , p_i is the power of player $i \in I_n$ and \mathcal{P} is the power map of the game.

The power map

$$P : \mathcal{R}^n \rightarrow 2^{\times_{j \in I_n} S_j}$$

is a set-valued function. For each point (p_1, \dots, p_n) it assigns a subset of $\times_{j \in I_n} S_j$ so that the strategies in $\mathcal{P}(p_1, \dots, p_n)$ are no longer available in the game. The power function represents the power of player i over player j . If the power function is valid, the strategic choices of player j are affected by the influence or force of player i , thus, the power function restricts the strategic space of player j . Being a rational player and having power over j , player i will influence the decisions of player j , rendering invalid one or more strategies of j that result in a lower payoff to i . In other words, player i constrains the space of strategy of player j to guarantee the maximization of his utility. To explain the meaning of the power game, some examples are presented below in which the main implications of the game when power relations are included are shown.

Example 1: (generic game). Assuming a static game of complete and perfect information, such as the normal-form representation below, where A and B are the players, a_i and b_i are the strategy sets of A and B , and the number are the players' utility.

		B		
		b_1	b_2	b_3
A	a_1	2.4	3.2	2.1
	a_2	2.3	4.1	3.0
	a_3	1.5	2.2	1.5

Fig. 2: Generic Game

Source: the author.

First of all, it is necessary to define the existence of the power function and who has the power, for instance by assuming that B has power over A . An inspection reveals that the matrix can show that when player A chooses a_2 , this results in a

lower payoff to player B . B , in turn, exerts his power and will preclude A choosing a_2 . By doing so, he avoids the possibility of having lower utilities. If player B did not have power over A , A would be able to freely choose all available strategies, the forces between them would be equal, and player B would have lower utilities.

Example 2 (Battle of the sexes - BoS). Ballet versus soccer. It is known that there are two Nash equilibriums. Instead of the regular formulation that the woman and man do not have power, it is supposed that the woman has a power p_w and the man has a power p_m . Furthermore, it is assumed that the game is endowed with a power function \mathcal{P} that excludes soccer from the man's possible strategies whenever $p_w > p_m$ and excludes ballet from the woman's possible strategies if $p_m > p_w$. The formalization of this game is:

$$\Gamma_{BoS}^P = \langle \{man = 1, woman = 2\}, (\{ballet, soccer\}, \{batter, soccer\}), (u_1, u_2), (p_1, p_2), \mathcal{P} \rangle$$

$$\prod_{i \in I_n} S_i = \{(B, B), (B, S), (S, B), (S, S)\} = \mathcal{S}$$

The set of parts of $\mathcal{S}, 2^{\mathcal{S}}$, is given by:

$$2^{\mathcal{S}} = \left\{ \begin{array}{l} \Phi, \mathcal{S}, \{(B, B)\}, \{(B, S)\}, \{(S, B)\}, \{(S, S)\}, \{(B, B), (B, S)\}, \\ \{(B, B), (S, B)\}, \{(B, B), (S, S)\}, \{(B, B), (B, S), (S, B)\}, \\ \{(B, B), (B, S), (S, S)\}, \{(B, B), (S, B), (S, S)\}, \{(B, S), (S, B), (S, S)\} \\ \{(B, S)\}, \{(S, B)\}, \{(B, S), (S, S)\}, \{(S, B)\}, \{(S, S)\} \end{array} \right\}$$

The power map $\mathcal{P} : \mathbb{R}^2 \rightarrow 2^{\mathcal{S}}$ is:

$$\mathcal{P}(x, y) = \begin{cases} \{(B, S), (S, S)\} & \text{if } x < y \\ \{(B, S), (B, B)\} & \text{if } x > y \\ \phi & \text{if } x = y \end{cases}$$

where $x = p_1 = \text{man power}$, and $y = p_2 = \text{woman power}$. Assuming that \mathcal{P} constrains the set of strategies, as shown below:

$$\prod_{i \in I_n} S_i|_{\mathcal{P}(p_1)} = \prod_{i \in I_n} S_i|_{\mathcal{P}(p_2)} = \prod_{i \in I_n} S_i \setminus \mathcal{P}(p)$$

In the game of the BoS, the available strategies are:

$$\prod_{i \in I_n} S_i|_{\mathcal{P}(x < y)} = \mathcal{S} \setminus \{(B, S), (S, S)\} = \{(S, B)\}, \{(B, B)\}$$

and

$$\prod_{i \in I_n} S_i|_{\mathcal{P}(x > y)} = \mathcal{S} \setminus \{(B, S), (B, B)\} = \{(S, B)\}, \{(S, S)\}$$

Example 3 (Prisoner's Dilemma). Supposing that criminal A is more powerful than B, A will use his power to influence the decisions (set of strategies) of agent B. For this particular situation, (Williamson (2010), p. 26) offered an explanation regarding the relation between the prisoners that implicitly uses the concept of power: "rather than assume that players are accepting of the coercive payoffs that are associated with the prisoners' dilemma - according to which each criminal is

induced to confess, whereas both would be better off if they could commit not to confess - Transaction Costs Economy assumes that the criminals (or their handlers, such as the mafia) can, upon looking ahead, take ex ante actions to alter the payoffs by introducing private ordering penalties to deter defections. This latter is a governance move, variants of which can be introduced into many other bad games". The ex ante actions are defined exactly by the power relations between the criminals, which could make use of condign, compensatory or conditioned power.

Assuming asymmetric forces between both, the criminal with more power imposes the strategy not to confess on the man with less power, he chooses the strategy not to confess, while the criminal with more power chooses the strategy to confess. In this situation, the criminal with more power goes free while the man with less power will be punished. Taking into account that power is present in the relationship, there is no more Nash equilibrium.

The examples above show that the powerful agent constrains the set of strategies of the other agent. In this sense, the other agent does not have all the options available to choose, only part of them. In this new condition, the equilibrium is not the same that would be achieved if the options were unconstrained.

Due to this, it is possible to think of a measure or index of power which the powerful player has over the other. In a two person game, and supposing that both have mixed strategies, the measure of power of the powerful agent (player i) can be described as:

$$\mathcal{IP}_i = \frac{u_i|P_{ij}}{u_i}$$

Where u_i is the expected utility for the player i and $u_j|P_{ij}$ is the expected utility for the player i when he uses his power (P_{ij}) to constrain the strategy set of player j . The power is the "force" that the powerful player uses to restrict the choices of the other player. In fact, by the definition, \mathcal{IP}_i lies in the interval $[1, \infty]$. If $\mathcal{IP}_i = 1$, it means that $u_i|P_{ij}$ is equal to u_i , and in this situation the function w_{ij} does not restrict the strategy set of player j . Thus, player i does not have power over player j . On the other hand, if $u_i|w_{ij}$ is greater than u_i , \mathcal{IP}_i will be greater than one. In this sense P_{ij} acts and restricts the strategy set of player j . When the power is acting, some of the options are not available in the strategy set, meaning that this player does not choose the option that he would if all the options were available.

Example 4. The matrix below shows the same game as example one, but here the strategies are mixed, with equal probabilities for both players and for all strategies.

Table 1: Example 4

		B		
		b_1	b_2	b_3
A	a_1	2.4	3.2	2.1
	a_2	2.3	4.1	3.0
	a_3	1.5	2.2	1.5

The expected utility for player B is 2.55. Assuming again that player B has power over player A , and that he uses this power, it is clear that the strategy a_2 is restricted by the power. In this way, player A cannot choose this strategy (a_2) because it is not available, and player B is free to choose any strategy in his set.

Afterwards, the new expected utility for player B is 3.16, and the measure of power is 1.23. This measure shows an increase in expected utility when the powerful player exercises his power.

3. Conclusions

Power is not a philosophical or remote subject, despite the fact that usually no one can see or measure it. The understanding of this subject has been developed, over many years, mainly by social scientists, including economists. Game theory is an applied branch of mathematics that analyses the rational interaction between agents, but the models constructed have not embodied the asymmetric forces between them.

In this way, it is possible to understand in a formal model regarding game theory how power can affect the relationship between players, assuming that one of the agents has more forces than others, and uses his forces to guarantee that his will is easily achieved. The model was constructed taking into account that power should be analyzed in the strategic interaction between individuals.

In games where there are one or more Nash equilibria, the power relationship between the players removes one or all of the equilibria due to the fact that one of the players is not doing what is best, and does exactly is imposed on him. Finally, this paper unites two different sciences that developed separately: sociology and game theory.

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