Equilibrium in Secure Strategies in the Bertrand-Edgeworth Duopoly Model*

Mikhail Iskakov[1] **and Alexey Iskakov**[2]

[1] *V.A.Trapeznikov Institute of Control Sciences, Ul.Profsoyuznaya 65, Moscow 117997, Russia*
E-mail: `mih_iskakov@mail.ru`

[2] *V.A.Trapeznikov Institute of Control Sciences, Ul.Profsoyuznaya 65, Moscow 117997, Russia*
E-mail: `isk_alex@mail.ru, iskakov@ipu.ru`

Abstract We analyze the Bertrand-Edgeworth duopoly model using a solution concept of Equilibrium in Secure Strategies (EinSS), which provides a model of cautious behavior in non-cooperative games. It is suitable for studying games, in which threats of other players are an important factor in the decision-making. We show that in some cases when Nash-Cournot equilibrium does not exist in the price duopoly of Bertrand-Edgeworth there is an EinSS with equilibrium prices lower than the monopoly price. The corresponding difference in price can be interpreted as an additional cost to maintain security when duopolists behave cautiously and secure themselves against mutual threats of undercutting. We formulate and prove a criterion for the EinSS existence.

Keywords: Bertrand-Edgeworth Duopoly, Equilibrium in Secure Strategies, Capacity Constraints.

1. Introduction

It is well known that the model of Bertrand-Edgeworth may not posses a Nash equilibrium (see e.g. d'Aspremont and Gabszewicz (1985)). Let the receipt function $pD(p)$ be strictly concave and reach its maximum at monopoly price p_M. When $D(p_M) \geqslant S_1 + S_2$ there is a Nash price equilibrium (p^*, p^*) in the Bertrand-Edgeworth duopoly model such that $D(p^*) = S_1 + S_2$. When $D(p_M) < S_1 + S_2$ there is no (pure strategy) Nash equilibrium in this game. There were several attempts to restore the concept of equilibrium in this model. For example d'Aspremont and Gabszewicz (1985) proposed the concept of quasi-monopoly, which restores the existence of pseudo equilibrium when one capacity is quite small compared to the other. Dasgupta and Maskin (1986) and Dixon (1984) demonstrated the existence of mixed-strategy equilibrium. However it proved not to be easy to characterize what the equilibrium actually looks like. Allen and Hellwig (1986a) were able to show that in a large market with many firms, the average price set would tend to the competitive price.

In this paper we analyze the Bertrand-Edgeworth duopoly model using a solution concept of Equilibrium in Secure Strategies (EinSS), which provides a model of cautious behavior in non-cooperative games (M.Iskakov 2005, 2008). It is suitable for studying games, in which threats of other players are an important factor in

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the decision-making (M.Iskakov and A.Iskakov 2012b). This concept proved to be effective and allowed to find equilibrium positions in some well-known economic games that fail to have Nash-Cournot equilibrium. In particular this approach has been successfully applied for the classic Hotelling's model with the linear transport costs (Hotelling 1929). There is no price Nash-Cournot equilibrium in this game when duopolists choose locations too close to each other (d'Aspremont et al., 1979). However, there is a unique EinSS price solution for all location pairs under the assumption that duopolists secure themselves against being driven out of the market by undercutting (M.Iskakov and A.Iskakov 2012a). Equilibria in secure strategies have been also successfully characterized for the contest described by Tullock (1969, 1980) as a rent-seeking contest. It is well known that a Nash-Cournot equilibrium does not exist in a two-player contest when the contest success function parameter is greater than two. However an EinSS always exists in the Tullock contest (M.Iskakov et al., 2013). Moreover, when the success function parameter is greater than two, this equilibrium is unique up to a permutation of players, and has a lower rent dissipation than in a mixed-strategy Nash equilibrium.

Our aim is to analyze the original Bertrand-Edgeworth duopoly model with capacity constraints, which may not possess a Nash-Cournot equilibrium. We show that in some cases when Nash-Cournot equilibrium does not exist there is an EinSS with equilibrium prices lower than the monopoly price. The corresponding difference in price can be interpreted as an additional cost to maintain security when duopolists are cautious and avoid mutual threats. We formulate and prove a criterion for the EinSS existence.

The remaining paper is organized as follows. In Section 2 we remind the Bertrand-Edgeworth model. In Section 3 the solution concept that we are going to use for analyzing the price duopoly game is presented. In Section 4 the equilibria in secure strategies are characterized for the linear demand function. In Section 5 the obtained results are generalized for the strictly concave receipt functions. Finally we provide an interpretation and summarize our results in the Conclusion.

2. Bertrand-Edgeworth duopoly model

In this section we consider a model of price setting duopolists with capacity constraints originated in papers of Bertrand (1883) and Edgeworth (1925). We consider the market for some homogeneous product with a continuous strictly decreasing consumer's demand function $D(p)$. There are two firms in the industry $i = 1, 2$, each with a limited amount of productive capacity S_i such that $D(0) \geqslant S_1 + S_2$. As in Edgeworth's work we assume these limits as physical capacity constraints, which are the same at all prices. Firms choose prices p_i and play non-cooperatively. The firm quoting the lower price serves the entire market up to its capacity and the residual demand is met by the other firm.

All consumers are identical and choose the lower available price on a first-come-first-serve basis. Following Shubik (1959) and Beckmann (1965) we assume in our analysis that the residual demand to the firm quoting the higher price is a proportion of total demand at that price. If duopolists set the same prices firms share the market in proportion to their capacities. Formally we define the payoff functions of players to be:

$$u_1(p_1,p_2) = \begin{cases} p_1 \min\{S_1, D(p_1)\}, & p_1 < p_2 \\ p_1 \min\{S_1, \frac{S_1}{S_1+S_2} D(p_1)\}, & p_1 = p_2 \\ p_1 \min\{S_1, \frac{D(p_1)}{D(p_2)} \max\{0, D(p_2) - S_2\}\}, & p_1 > p_2 \end{cases}$$
$$u_2(p_1,p_2) = \begin{cases} p_2 \min\{S_2, D(p_2)\}, & p_2 < p_1 \\ p_2 \min\{S_2, \frac{S_2}{S_1+S_2} D(p_2)\}, & p_2 = p_1 \\ p_2 \min\{S_2, \frac{D(p_2)}{D(p_1)} \max\{0, D(p_1) - S_1\}\}, & p_2 > p_1 \end{cases} \quad (1)$$

In the particular case of a linear demand function $D(p) = 1 - p$, which we consider below, the payoff functions can be written as:

$$u_1(p_1,p_2) = \begin{cases} p_1 \min\{S_1, 1-p_1\}, & p_1 < p_2 \\ p_1 \min\{S_1, \frac{S_1}{S_1+S_2} (1-p_1)\}, & p_1 = p_2 \\ p_1 \min\{S_1, \frac{(1-p_1)}{(1-p_2)} \max\{0, 1-p_2 - S_2\}\}, & p_1 > p_2 \end{cases}$$
$$u_2(p_1,p_2) = \begin{cases} p_2 \min\{S_2, 1-p_2\}, & p_2 < p_1 \\ p_2 \min\{S_2, \frac{S_2}{S_1+S_2} (1-p_2)\}, & p_2 = p_1 \\ p_2 \min\{S_2, \frac{(1-p_2)}{(1-p_1)} \max\{0, 1-p_1 - S_1\}\}, & p_2 > p_1 \end{cases} \quad (2)$$

The study of this case will allow us relatively easy to prove the main results in Section 4. However, these results will be generalized to arbitrary strictly concave receipt functions in Section 5.

3. Equilibrium in Secure Strategies

We now proceed to define the solution concept that we are going to use to analyze the Bertrand-Edgeworth duopoly model (1). Below we provide definitions of Equilibrium in Secure Strategies from (M.Iskakov and A.Iskakov 2012b). Consider n-person non-cooperative game in the normal form $G = (i \in N, s_i \in S_i, u_i \in R)$. The concept of equilibria is based on the notion of threat and on the notion of secure strategy.

Definition 1. A threat of player j to player i at strategy profile s is a pair of strategy profiles $\{s, (s'_j, s_{-j})\}$ such that $u_j(s'_j, s_{-j}) > u_j(s)$ and $u_i(s'_j, s_{-j}) < u_i(s)$. The strategy profile s is said **to pose a threat** from player j to player i.

Definition 2. A strategy s_i of player i is **a secure strategy** for player i at given strategies s_{-i} of all other players if profile s poses no threats to player i. A strategy profile s is **a secure profile** if all strategies are secure.

In other words a threat means that it is profitable for one player to worsen the situation of another. A secure profile is one where no one gains from worsening the situation of other players.

Definition 3. A secure deviation of player i with respect to s is a strategy s'_i such that $u_i(s'_i, s_{-i}) > u_i(s)$ and $u_i(s'_i, s'_j, s_{-ij}) \geq u_i(s)$ for any threat $\{(s'_i, s_{-i}), (s'_i, s'_j, s_{-ij})\}$ of player $j \neq i$ to player i.

There are two conditions in the definition. In the first place a secure deviation increases the profit of the player. In the second place his gain at a secure deviation covers losses which could appear from retaliatory threats of other players. It is important to note that secure deviation does not necessarily mean deviation into secure profile. After the deviation the profile (s'_i, s_{-i}) can pose threats to player i. However these threats can not make his or her profit less than in the initial profile s. We assume that the player with incentive to maximize his or her profit securely will look for secure deviations.

Definition 4. A secure strategy profile is **an Equilibrium in Secure Strategies (EinSS)** if no player has a secure deviation.

There are two conditions in the definition of EiSS. There are no threats in the profile and there are no profitable secure deviations. The second condition implicitly implies maximization over the set of secure strategies.

Any Nash-Cournot equilibrium poses no threats to players so it is a secure profile. And no player in Nash-Cournot equilibrium can improve his or her profit using whatever deviation. Both conditions of the EinSS are fulfilled. Therefore we obtain

Proposition 1. *Any Nash-Cournot equilibrium is an Equilibrium in Secure Strategies.*

The Nash equilibrium is the profile, in which the strategy of each player is the best response to strategies of other players. In a similar way, the strategy of each player in the EinSS turns out to be the best secure response.

Definition 5. A secure strategy s_i of player i is **a best secure response** to strategies s_{-i} of all other players if player i has no more profitable secure strategy at s_{-i}. A profile s^* is **the Best Secure Response profile** (BSR-profile) if strategies of all players are best secure responses.

The EinSS is a secure profile by definition. And it must be the best secure response for each player since otherwise there is a player who can increase the payoff by secure deviation. Therefore we get:

Proposition 2. *Any Equilibrium in Secure Strategies is a BSR-profile.*

This property provides a practical method for finding EinSS in three steps: (i) to analyze threats and determine conditions for secure profile, (ii) to find all BSR-profiles as a solution of the corresponding maximization problem in the set of secure profiles, and (iii) to verify the definition of EinSS for the found BSR-profiles.

4. Solution in secure prices for the linear demand function

In this Section we illustrate how to find an EinSS in the simplest case of the linear demand function $D(p) = 1 - p$. First of all, let us analyze the threats between players and define secure profiles in the Bertrand-Edgeworth duopoly model.

Proposition 3. *The profile (p_1, p_2) is a secure profile in the duopoly price game of Bertrand-Edgeworth with the linear demand function $D(p) = 1 - p$ and payoff functions (2) if and only if it lies in the set $M = \{(p_1, p_2) : 0 < p_i \leqslant p^*,\ i = 1, 2\}$, where $p^* = 1 - S_1 - S_2$.*

Proof. **(1)**. Consider the case $p^* < p_1 < p_2$. If $1 - p_1 > S_1$ player 1 always threatens player 2 by slight increasing his price p_1. If $1 - p_1 \leqslant S_1$ then player 2 can decrease his price till $\tilde{p}_2 < p_1$. In this case, according to (2): $\tilde{u}_1(p_1, \tilde{p}_2) = p_1(1 - p_1)\frac{\max\{0, 1-p_2-S_2\}}{1-p_2} < p_1(1-p_1) = u_1(p_1, p_2)$ and $\tilde{u}_2(p_1, \tilde{p}_2) = \tilde{p}_2 \min\{S_2, 1-p_2\} > 0 = u_2(p_1, p_2)$, and therefore there is always a threat of player 2 to player 1. Symmetrically, if $p^* < p_2 < p_1$ either player 2 threatens player 1 or vice versa. If $p^* < p_2 = p_1$ there is always a bilateral threat of undercutting by slight decreasing of price.

(2). If $p_1 \leqslant p^* < p_2$ player 1 always threatens player 2 by increasing his price till $p^* + 0 < p_2$ which exceeds p^* by an arbitrarily small amount. Indeed, in this case $1 - p_1 > 1 - (p^* + 0) \geqslant S_1$ and according to (2) $u_1(p_1, p_2) = p_1 S_1 < (p^* + 0)S_1 = u_1(p^* + 0, p_2)$. On the other hand, $u_2(p^* + 0, p_2) = p_2 \frac{1-p_2}{S_1+S_2} S_2 < p_2 S_2$ and $u_2(p^* + 0, p_2) = p_2(1-p_2)\left(1 - \frac{S_1}{1-(p^*+0)}\right) < p_2(1-p_2)\left(1 - \frac{S_1}{1-p_1}\right) \Rightarrow u_2(p^* + 0, p_2) < u_2(p_1, p_2)$. Symmetrically, if $p_2 \leqslant p^* < p_1$ player 2 always threatens player 1.

(3). From the above it follows that all secure profiles must lie in the set $M = \{(p_1, p_2) : 0 < p_i \leqslant p^*,\ i = 1, 2\}$. From the other hand if $p_1 \leqslant p^*$: $u_1(p_1, p_2) = S_1 p_1$ linearly increases in p_1 and does not depend on p_2. Hence there are no threats for player 1. Symmetrically, if $p_2 \leqslant p^*$ there are no threats for player 2. Therefore (p_1, p_2) is a secure profile in the game (2) if and only if it lies in the set $M = \{(p_1, p_2) : 0 < p_i \leqslant p^*,\ i = 1, 2\}$. □

Let us now find all Best Secure Response profiles in the set M of secure profiles.

Proposition 4. *In the duopoly price game of Bertrand-Edgeworth with the linear demand function $D(p) = 1 - p$ and payoff functions (2) there is a unique Best Secure Response profile (p^*, p^*), where $p^* = 1 - S_1 - S_2$.*

Proof. According to Definition 5 and Proposition 3 any Best Secure Response profile must lie in the set $M = \{(p_1, p_2) : 0 < p_i \leqslant p^*,\ i = 1, 2\}$ of secure profiles found in Proposition 3. According to (2) the payoff functions $u_1 = S_1 p_1$ and $u_2 = S_2 p_2$ increase in the set M linearly in p_1 and in p_2 respectively. Therefore there can be only one BSR-profile (p^*, p^*) in the set M (otherwise at least one player can securely slightly increase his price and get a more profitable secure strategy in M). Let us now prove that it is indeed a BSR-profile. Any decrease in price in the profile (p^*, p^*) is not profitable for players. On the other hand, as shown in paragraph 2 of the proof of Proposition 3 any increase in price in the profile (p^*, p^*) is not secure for players. Therefore no player has a more profitable secure strategy in (p^*, p^*) and therefore this profile by definition is a Best Secure Response profile in the game. □

According to Proposition 2 there can not be other EinSS in the Bertrand-Edgeworth duopoly game except the BSR-profile (p^*, p^*) found in Proposition 4. Below we formulate and prove a necessary and sufficient condition for the BSR-profile (p^*, p^*) to be an Equilibrium in Secure Strategies.

Proposition 5. *In the game of Bertrand-Edgeworth with the linear demand function $D(p) = 1 - p$ and payoff functions (2) there is an Equilibrium in Secure Prices (p^*, p^*) where $p^* = 1 - S_1 - S_2$ if and only if*

$$\frac{1 - S_1}{2} \leqslant p^* \quad \text{and} \quad \frac{1 - S_2}{2} \leqslant p^* \tag{3}$$

If the equilibrium price is not less than the monopoly price $p^* \geqslant p_M = \frac{1}{2}$ it is a Nash equilibrium. There are no other EinSS in the game.

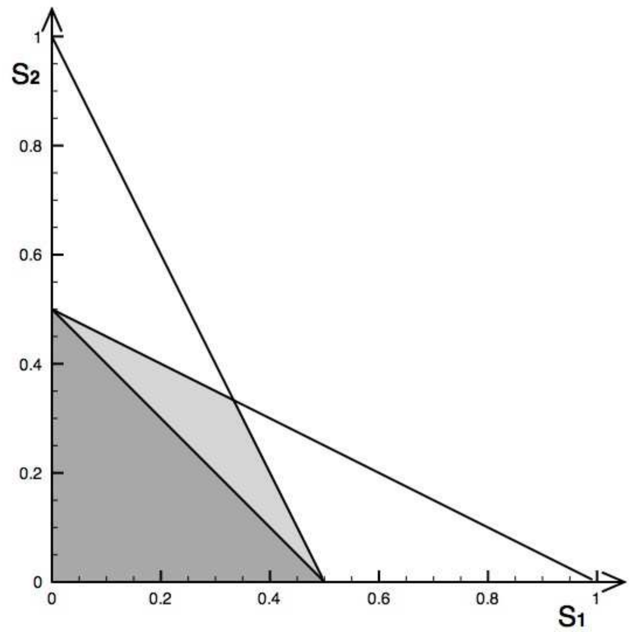

Fig. 1: Equilibria in secure prices in the Bertrand-Edgeworth duopoly model with $D(p) = 1 - p$ in the space of capacity parameters (S_1, S_2). Dark gray area: EinSS which coincide with Nash equilibria. Light gray area: EinSS which are not Nash equilibria. White area: there are neither Nash equilibria nor EinSS.

Equilibria in secure prices in the space of capacity parameters (S_1, S_2) are shown in Fig.1. The profile (p^*, p^*) is a Nash equilibrium if $S_1 + S_2 \leqslant \frac{1}{2}$ (dark gray area). Under the weaker conditions (3) it is an EinSS. The area of EinSS which are not Nash equilibria are shaded by light gray in Fig.1. If conditions (3) do not hold this profile is no longer an EinSS and corresponds to an unstable BSR-profile. The found solution can be compared with the price which would maximize the joint profits in the industry $p_M = \max\{1 - S_1 - S_2, \frac{1}{2}\}$. If Nash equilibrium exists (i.e. if $S_1 + S_2 \leqslant \frac{1}{2}$) then both equilibrium prices coincide. However if EinSS exists and Nash equilibrium does not exist (i.e. if $S_1 + S_2 > \frac{1}{2}$ and (3) holds) both EinSS prices $p^* = 1 - S_1 - S_2$ are lower than the monopoly price $p_M = \frac{1}{2}$. One can interpret the price difference $S_1 + S_2 - \frac{1}{2}$ as an additional cost to maintain security in the situation when players take into account mutual threats of undercutting and behave cautiously.

Proof. **(1) Necessity**. Let us consider profile (p^*, p^*) and prove the conditions (3). Suppose for example that $p^* < \hat{p}(S_2) \equiv \frac{1-S_2}{2}$. Then player 1 can deviate $p_1^* \to \hat{p}$. His payoff will increase since $p^* < \hat{p} \leqslant p_M = \frac{1}{2}$ and $u_1(p_1, p_2)$ is strictly increasing in p_1 if $p_1 \leqslant p_M = \frac{1}{2}$ according to (2). Any retaliatory threat of player 2 according to (2) can not make the payoff of player 1 less than $\min_{p_2} u_1(\hat{p}, p_2) = \min_{p_2 < \hat{p}} u_1(\hat{p}, p_2) = u_1(\hat{p}, p_2)|_{p_2 = \hat{p} - 0} = \hat{p} \min\{S_1, 1 - \hat{p} - S_2\}$. The payoff of player 1 in the initial profile does not exceed this value. Indeed $p(1-p-S_2)$ is strictly increasing at $p < \hat{p} = \frac{1-S_2}{2}$ and we have $u_1(p^*, p^*) \leqslant p^*(1 - p^* - S_2) < \hat{p}(1 - \hat{p} - S_2)$ and $u_1(p^*, p^*) \leqslant p^* S_1 < \hat{p} S_1$. Therefore the deviation of player 1 into $\hat{p}(S_2)$ is always a secure deviation according to Definition 3. Hence profile (p^*, p^*) is not an EinSS. Symmetrically if $p^* < \hat{p}(S_1) \equiv \frac{1-S_1}{2}$ then player 2 can make a secure deviation into $\hat{p}(S_1)$ and profile (p^*, p^*) is not an EinSS either. The necessity of (3) is proven.

(2) Sufficiency. Let us now assume that (3) holds (i.e. $\hat{p}(S_1) \leqslant p^*$ and $\hat{p}(S_2) \leqslant p^*$). Consider an arbitrary deviation $p^* \to p_1$ of player 1. If $p_1 < p^*$ it can not be a profitable deviation for player 1. Therefore $p_1 > p^*$. Player 1 increases the payoff if and only if $u_1(p^*, p^*) = p^* S_1 = p^* \frac{1-p^*-S_2}{1-p^*}(1-p^*) < u_1(p_1, p^*) = p_1 \frac{1-p^*-S_2}{1-p^*}(1-p_1)$, i.e. there must be $p^*(1-p^*) < p_1(1-p_1)$. Then there is retaliatory threat of player 2 to deviate from profile (p_1, p^*) into profile arbitrarily close to $(p_1, p_1 - 0)$. From $p^* S_2 < p_1 S_2$ and $p^*(1 - p^*) < p_1(1 - p_1)$ it follows that player 2 increases the payoff at this deviation. The payoff of player 1 in this profile is arbitrarily close to $u_1(p_1, p_1 - 0) = p_1 \min\{S_1, 1 - p_1 - S_2\}|_{p^* < p_1} = p_1(1 - p_1 - S_2)$. Since $p(1 - p - S_2)$ is strictly decreasing at $p \geqslant \hat{p}(S_2) = \frac{1-S_2}{2}$ and $p_1 > p^* \geqslant \hat{p}(S_2) = \frac{1-S_2}{2}$ then $u_1(p^*, p^*) = p^*(1 - p^* - S_2) > p_1(1 - p_1 - S_2) = u_1(p_1, p_1 - 0)$. Therefore the deviation of player 1 into profile (p_1, p^*) is not a secure deviation. Symmetrically an arbitrary deviation of player 2 is not a secure deviation either. No player can make secure deviation in the profile (p^*, p^*). By definition it is an EinSS. The sufficiency of (3) is proven.

(3) Nash equilibrium condition. One can easily check that $p_M \leqslant p^*$ $(S_1 + S_2 \leqslant \frac{1}{2})$ is the maximum condition of functions $u_1(p_1) = u_1(p_1, p^*)$ and $u_2(p_2) = u_2(p^*, p_2)$ in the points $p_1 = p^*$ and $p_2 = p^*$ respectively. In other words it is a condition of Nash equilibrium for the profile (p^*, p^*).

(4) Uniqueness of EinSS. It follows from Proposition 2 and the uniqueness of BSR-profile proven in Proposition 4. □

5. Solution in secure prices for the strictly concave receipt function

Obtained in the previous section results can be generalized to the more general case of the strictly concave receipt function $pD(p)$. Although the proofs become slightly more involved they follow the similar logic.

Proposition 6. *Let the receipt function $pD(p)$ be strictly concave and reach its maximum at p_M. Then in the game of Bertrand-Edgeworth with payoff functions (1) there is an EinSS (p^*, p^*) where $D(p^*) = S_1 + S_2$ if and only if*

$$\begin{cases} \arg\max\limits_{p>0}\{p(D(p) - S_1)\} \leqslant p^* \\ \arg\max\limits_{p>0}\{p(D(p) - S_2)\} \leqslant p^* \end{cases} \tag{4}$$

If $p^* \geqslant p_M$ it is a Nash equilibrium. There are no other EinSS in the game.

Remark. Since the receipt function $pD(p)$ is strictly concave then the function $p(D(p) - S)$ at a given S is also strictly concave in p and reaches the unique maximum at $p > 0$. Therefore $\arg\max\limits_{p>0}\{p(D(p) - S)\}$ can be considered as a function of S. The proof is given in (M.Iskakov, A.Iskakov, 2012b).

The condition (4) can be easily formulated in differential form.

Proposition 7. *If function $pD(p)$ is differentiable the condition (4) is equivalent to*

$$\frac{d}{dp}\Big(pD(p)\Big)\Big|_{p=p^*} \leqslant \min\{S_1, S_2\} \tag{4'}$$

Proof. One can easily check that $\hat{p} = \arg\max_{p>0}\{p(D(p)-S)\} \;<=>\; \frac{d}{dp}\Big(pD(p)\Big)\Big|_{p=\hat{p}} = S$. Besides $\frac{d}{dp}\Big(pD(p)\Big)$ is strictly decreasing. Therefore $\hat{p} \leqslant p^* \;<=>\; \frac{d}{dp}\Big(pD(p)\Big)\Big|_{p=p^*} \leqslant \frac{d}{dp}\Big(pD(p)\Big)\Big|_{p=\hat{p}} = S$. Hence the equivalence of (4) and (4'). □

The obtained results are generally similar to the results obtained for the linear demand function. In the EinSS all firms set equal prices such that market demand equals total supply. If the equilibrium price exceeds or equals the monopoly price this solution coincides with the well-known Nash equilibrium solution. However, in the EinSS which is not Nash equilibrium the prices are lower than the monopoly price. The corresponding difference in price can be interpreted as an additional cost to maintain security in the situation when players behave cautiously and secure themselves against mutual threats of undercutting.

6. Conclusion

In this paper we considered the Bertrand-Edgeworth duopoly model, with capacity constraints which may not possess a Nash-Cournot equilibrium. Whilst the existence of mixed-strategy equilibrium was demonstrated by Dixon (1984), it has not proven easy to characterize what the equilibrium actually looks like. It has been argued that non-pure strategies are not plausible in the context of the Bertrand-Edgeworth model. Therefore several alternative approaches have been proposed as a response to the non-existence of pure-strategy equilibrium. Allen and Hellwig (1986b) proposed a modification of the game, in which firms choose the quantity they are willing to sell up to at each price. Dastigar (1995) proposed that firms have to meet all demand at the price they set. Benassy (1989) introduced in the Bertrand-Edgeworth model the product differentiation. Dixon (1993) explored the Bertrand-Edgeworth model with "integer pricing" when firms cannot undercut each other by an arbitrarily small amount. All these approaches in some sense or another change the setting of the original game and introduce specific *ad hoc* modification to the Bertrand-Edgeworth model.

As an alternative approach to analyze the Bertrand-Edgeworth duopoly model we propose in this paper the concept of Equilibrium in Secure Strategies (EinSS). We present a new intuitive formulation of EinSS concept from (M.Iskakov and A.Iskakov, 2012b). An existence condition of the EinSS in the Bertrand-Edgeworth price duopoly is formulated and proved, which allowed us to extend the set of Nash-Cournot price equilibria in the game. If the equilibrium price exceeds or equals the monopoly price this solution coincides with the Nash-Cournot equilibrium solution. However, in the EinSS, which is not Nash-Cournot equilibrium the prices are lower than the monopoly price. The corresponding difference in price can be interpreted as an additional cost to maintain security when duopolists secure themselves against the threats of being undercut.

Although the proposed approach does not completely solve the problem of the non-existence of Nash-Cournot equilibria in the Bertrand-Edgeworth model, it nevertheless provides some advantages. In contrast to the above mentioned ad hoc equilibrium concepts developed in the context of the Bertrand-Edgeworth model, the EinSS is a general equilibrium concept that proved to be effective and allowed to find equilibrium positions in several well-known economic games that fail to have Nash-Cournot equilibrium (M.Iskakov and A.Iskakov, 2012b). On the other hand unlike equilibria in mixed strategies it offers a solution in an explicit form and can be easily interpreted in terms of cautious behavior.

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