Simulations of Evolutionary Models of a Stock Market*

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Abstract The main idea of this work is to present some simulation in evolutionary models of agents' interaction on the stock market. We consider game-theoretical model of agent's interaction, which evolving during long-time period. We consider three possible situations on the market, which are characterized by different types of agents' behavior.

Keyword: Evolutionary game, ESS strategy, stock market, replicative dynamic, imitation models, imitation dynamics.

1. Introduction

In this work, we construct and analyze evolutionary models of agents' behavior on the stock market in various situations. Consider stock market with large but finite group of agents. We suppose that each agent has own portfolio of different companies and can interact with randomly matched opponent. Various situations on the stock market are characterized by the actions that agents perform with their blocks of shares. Agents can hold their own blocks of shares, invest or sell the blocks of shares. Assume that in each model agents use only one pair of declared behavior, we will consider two variants of pairs: "invest" - "hold" and "invest" - "sell". Each model of agents' interaction is defined as basic symmetric two players game with corresponding payoff matrix. Describe these three situations particularly.

The first situation describes case in which one type of agent's behavior is hold the block of shares and receive fixed guaranteed profit from it. The second type of behavior is to invest the block of shares to get control of the company, but the main suggestion is that each agent can not invest control of the company independently; he can invest the control only in cooperation with the other agent, who has block of shares of the same company. Acquisition the control of the target company can bring some additional possibilities to the agents, for example they can influence on the company or receive extra profit.

The second situation on the stock market describes interaction between agents, which desire "invest" and "sell" the block of shares. Structure of agents' interaction is more complicated and will be described in details further.

In the third situation we suppose that agents coordinate their behaviors but they can get different payoffs. The one behavioral type is "invest" block of shares and to get the control of the company or large block of shares. The second behavior

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type is "hold" their block of shares and get small, but guaranteed profit from the own holding of shares.

These three base situations are extended by the addition of new behavior type. The additional behavior type is to detect purposes of the opponent and replay to the opponent's strategy rationally. In other words, in the first situation, in case that one agent meets his opponent, who wants to hold his block of shares, then he holds too, but if the agent meets the opponent, who wants to invest the control of the company, then agents cooperate and invest the control. In the second situation, if agent's opponent wants to invest controlling blocks of shares, then rational agent sells their shares, if his opponent prefers to sell his blocks of shares, then rational agent invests the shares. In the third situation rational agents behave symmetrically to their opponents. During the meeting of the rational agents in all modelled situations both players play Nash equilibrium strategies.

Assume that, in each period, an agent plays one basic model against all other agents and he chooses a best response to the distributions of actions of the other agents. The following assumptions are made in order to description the bounded rationality aspect of the agents (Sandholm, 2009; Subramanian, 2008):

- the number of agents on the stock market is large;
- inertia, that agent agent cannot detect the smallest shifts immediately and consider possibilities to switching strategies occasionally;
- myopia, that agents choices on current behavior and payoffs do not attempt include beliefs about the future course of play into their decisions. Each agent takes into account only the current strategy distribution;
- market agents have limited information about opponents' behavior, because the number of agents in interaction is large, exact information about their aggregate behavior typically is difficult to obtain.

This work pursues some purposes: considering some basic models of interaction between stock market agents, invasion new behavioral type to the models and analysis the agent's behavior on the stock market during the long-run period, constructing imitative evolutionary dynamics for all models.

2. Basic models

In this section we present three situations on the stock market and describe agents' behavior and payoffs more detailed. Consider stock markets with h types of shares and population (group) of agents on the market. Assume that shares have different investment attractiveness.

Let agent i has own portfolio of h types block of shares $\Pi_i = (\pi_1, \dots, \pi_h)$, Denote as $Pr(\Pi_i) = \sum\limits_{j=1}^{h} n_j A_{\pi_j}$ – portfolio profit of i-th agent. Where n_j is number of shares type π_j, A_{π_j} – cost of shares type π_j. Let $P_m in$ in minimal value of ivestment rate. Describe symmetric games between two randomly matched individuals. All agents have own portfolios and they have a choice receive guaranteed profit from own portfolio allocations or invest money into large block of shares In our game we have two strategies "invest" and "hold". Strategy Invest forces individuals to buy block of shares with high attractiveness.

Agents' interaction can be defined by one of the following basic models:

Situation A:

Each agent has two strategies "Invest" and "Hold". If agent "Hold" then he has guaranteed profit $Pr(\Pi_i)$

If agent "Invest" he has profit $Pr(\Pi_i) + B_I$, where $B_I = \frac{V_i - C_i}{C_i}$ is investment interest. V_I is received value of investment, C_i is cost of an investment.

Suppose that C_i is high and agent can invest only in cooperation with his opponent, he can not investment alone.

Describe agents interaction. Suppose that during single trade session different blocks of shares can be traded. When agents interact both agent have choice invest their money into large block of shares with high investment attractiveness or receive guaranteed profit from own portfolio $Pr(\Pi_i)$. If agent decides to invest into into large block of shares with high investment attractiveness he can receive additional profit B_I. If one agent wants to invest alone then he incurs investment cost $Pr(\Pi_i) - C_I$ but no profit in the same time his opponent has $Pr(\Pi_i)$.

In situation **A** market agents have two types of behavior. The first type is to hold the block of share and receive guaranteed profit from it. The second type is to invest the large block of share to get the control of target company. But agents have to cooperate with another agent if they prefer to receive the control, because each agent can not invest the control of company separately.

Matrix below illustrates symmetric game between the agents:

	H	I
H	$(Pr(\Pi_i), Pr(\Pi_i))$	$(Pr(\Pi_i) + \delta, C_i)$
I	$(C_I, Pr(\Pi_i) + \delta)$	$(Pr(\Pi_i) + B_I, Pr(\Pi_i) + B_I)$.

Define as $I \geq 0$ the income, that agent can get during the interaction.Denote players strategies as H and B, strategy B forces agents to invest and strategy H forces agents to hold his blocks of shares. We can give following interpretation the agents payoffs, if both agents hold their blocks of shares, then their get little payoff, which is equal to I. If one agent wants to invest the control, and other doesn't, then the first agent gets 0, because he expends money, but doesn't have the control, and the second player gets $3I/2$, because he has profit from his block of shares. If both players want to invest the control and cooperate, then they invest it and get payoff $2I$. Situation (B, B) is more risky, agents should to cooperate and to take into account own purposes and purposes of their opponents.

Obviously the basic game has three equilibriums (H, H), (B, B), $(2/3, 1/3)$. We verify that strategies H an B are evolutionary stable, in the sequel denote as Δ^{ESS} the set of evolutionary stable strategies. Evolutionary stability of some strategy x means that this strategy gives better payoff against any other strategy y and gives the best payoff against every alternative best reply y.

Situation B:

In this simple situation we consider one case in which agents enter into competition for the large block of shares with high investment attractiveness.

Each agent has two strategies "Invest" and "sell" and aspire to invest money into large block of shares with high investment attractiveness. If agent invest then his payoff is $Pr(\Pi_i) + B_I$. If he sells then he gets $Pr(\Pi_i)$.

Describe interaction: In this situation we suppose that during single trade session agents can invest into large block of shares with high investment attractiveness or sell own blocks of shares. If two agents with strategy "Invest" meet each other then they starts competition for the large block of shares with high investment attractiveness but only one agent can receive it. In this competition agents receive payoff $1/2(B - \overline{C_I})$, $\overline{C}_I$ is competition costs.

Consider another situation on the stock market and describe interaction between investers and sellers. Hence we will have another behavioral types for the agents. One part of agents wants to sell the block of share and thus their corresponding strategy is S. The other part of agents wants to invest the block of shares, then the strategy B corresponds to this behavior. As a result we get following situations:

- if two agents with "invest" type of behavior meet, then they start to fight for the blocks of shares and each agent can get the block of shares or lose it with probability $1/2$;
- if two agent with behavior "sell" meet, then each agents sells their assets and both agents receive guaranteed profit from the selling;
- if one agent invests the shares and the other sell, then seller has guaranteed profit and the invester has some profit from the shares and additional possibilities from large block of shares.

Payoff matrix is presented below:

	I	S
I	$((B_I - C_I)/2, (B_I - C_I)/2)$	(B_I, P_sell)
S	(P_sell, B_I)	$(Pr(\Pi_i)/2, Pr(\Pi_i)/2),$

here I is agent's income and C is agent's costs, $I < C$ both variables are nonnegative. There are three equilibriums $(S, B), (B, S), (\widetilde{x}, \widetilde{x})$, $\widetilde{x} = (I/C, 1 - I/C)$. Verification of evolutionary stability shows that in this game there is the unique evolutionary stable strategy $\widetilde{x}$, $x \in \Delta^{ESS}$.

Situation C:

In situation **C** assume that on the stock market agents provide symmetrical behavior to the opponents. Both agents can hold their blocks of shares or invest the large blocks of shares. However they cannot invest or hold assets separately and have to coordinate their actions with the other agents. Strategy S corresponds to behavior "hold", and strategy B corresponds behavior "invest". If both agents hold their shares then they get guaranteed profit but if they invest the large block of shares or the control of the target company then they receive additional possibilities (i.e. agents can influence on the companies decisions). Following matrix illustrate agents' payoffs:

	H	B
H	$((I_1, I_1)$	(0,0)
B	(0,0)	$(I_2,\ I_2).$

where $I_1 > I_2,\ I_1, I_2 \geq 0,\ I_j,\ j = 1, 2$ are agents' incames. There are three equilibriums in the game $(H, H), (B, B)$, $(\widetilde{x}, \widetilde{x})$ $\widetilde{x} = \dfrac{I_2}{I_1 + I_2}$. Two pure strategies B and H are evolutionary stable, $H, B \in \Delta^{ESS}$.

3. Extended games

Consider an extension of basic models. Suppose the small share of agents, which can recognize opponents' behavior during their meeting, invades on the stock market. This type of agents we will call rational agents or rationalist. We add new strategy E to each basic game, which describes new type of behavior of the market agents. Rationality of the player means that agents, which use strategy E can recognize actions of his opponents and adjust their behaviors in compliance with actions of the opponents. In each basic game we suppose that rational agents use their best responses as strategies and if rational agent meets another rational agent, then both play Nash equilibrium strategies. For each situation on the stock market construct new payoff matrices.

Situation **A**:

In basic game we have three equilibriums, hence extended game will have three variants, which describe agents preferences. We present payoff matrix which depends on the equilibriums profiles:

	H	B	E
H	(I, I)	$(3I/2, 0)$	(I, I)
B	$(0, 3I/2)$	$(2I, 2I)$	$(2I, 2I)$
E	(I, I)	$(2I, 2I)$	$(u(\widetilde{x}, \widetilde{x}), u(\widetilde{x}, \widetilde{x}))$

where values $(u(\widetilde{x}, \widetilde{x}), u(\widetilde{x}, \widetilde{x}))$ are players payoffs in the Nash equilibrium situation. In basic game we have three different equilibriums hence we get three variations of the extended game.

Situation **B**:

In situation **B** after invasion of rational agents,we get following payoff matrix. Basic game in situation **B** has three equilibriums hence extended game will have three modifications:

	B	S	E
B	$((I-C)/2, (I-C)/2)$	$(I, 1)$	$(1, 1)$
S	(1,I)	$(I/2, I/2)$	(I, I)
E	$(1, 1)$	(I, I)	$(u(\widetilde{x}, \widetilde{x}), u(\widetilde{x}, \widetilde{x}))$

where as in previous case values $(u(\widetilde{x}, \widetilde{x}), u(\widetilde{x}, \widetilde{x}))$ is agents payoff on the equilibrium strategies. Strategy E is the strategy of rational agents, which can identify behavioral type of their opponents.

Situation **C**:

In situation **C** we also have three modifications of payoff matrix, depending on the various agent's payoffs in Nash equilibriums profiles.

	B	H	E
B	(I_1, I_1)	$(0, 0)$	(I_1, I_1)
H	$(0, 0)$	(I_2, I_2)	(I_2, I_2)
E	(I_1, I_1)	(I_2, I_2)	$(u(\widetilde{x}, \widetilde{x}), u(\widetilde{x}, \widetilde{x}))$

where $u(\widetilde{x}, \widetilde{x})$ is equal to agent's payoff on corresponding equilibrium strategies.

4. Replication by Imitation

For all situations **A, B, C** we analyze, which behavioral type will prevail on the market during long-rum period with suggestion that in each situation at initial time moment small share of rational players are invaded. For all these models we will consider selection dynamics arising from adaptation by imitation. In all situations we suppose that all agents in the large group are infinitely live and interact forever. This assumption can be interpreted in following way, if one agent physically exits from the market, then he is replaced by another one. Each agent has some pure strategy for some time interval and then reviews his strategy and sometimes changes the strategy.

There are two basic elements in this model (Weibull, 1995). The first element is time rate at which agents in the group review their strategy choice. The second element is choice probability at which agents change their strategies. Both elements depend on the current group state and on the performance of the agent's pure strategy.

Let $K = \{H, B, S, E\}$ is the set of agents pure strategies. In each of the previously described situations agents match at random in total group and each agent use one of pure strategy from the set K. Player with pure strategy i will be called as i-strategist.

Denote as $r_i(x)$ an average review rate of the agent, who uses pure strategy i in the group state $x = (x_H, x_B, x_E)$. Variable $p^i_j(x)$ is probability at which i-strategist switches to some pure strategy j, $i, j \in K$. Here $p_i(x) = (p^1_i(x), p^2_i(x), p^3_i(x)), i = H, B, E, S$ is the resulting probability distribution over the set of pure strategies and depends on the population state. Value $p^i_i(x)$ is probability that a reviewing i-strategist does not change his strategy.

Consider imitation process generally in finite large group of agents. Suppose that each reviewing agent samples another agent at random from the group with equal probability for all agents and observes with some noise the average payoff to his own and to the sampled agent's payoff. If payoff of the samples agents is better then his own he can switch to the sampled agent's strategy.

In general case the imitation dynamics is described by the formula:

$$\dot{x}_i = \sum_{j \in K} x_j r_j(x) p^i_j(x) - r_i(x) x_i, \; i \in K. \tag{1}$$

In this paper we use special variation of the imitation dynamics of successful agents.

5. Imitation of Pairwise Comparison

Suppose that each agent samples another stock agent from the total group with equal probability for all agents and observes the average payoff to his own and the sampled agent's strategy. When both players show their strategies then player who uses strategy i gets payoff $u(e^i, x) + \varepsilon$ and player, who uses strategy j gets $u(e^j, x) + \varepsilon'$, where $\varepsilon, \varepsilon'$ is random variables with continuously probability distribution function ϕ. The random variables ε and ε' can be interpreted as individual preference differences between agents in the market. Each agent can get various preferences, i.e. agent, which receive the large block of shares or the control of target company, can be more satisfied, because he can influence to the company or have additional profit. Other agents, which hold own assets and receive only fixed payoff can be less satisfied of their profit. Use as distribution function $\phi(z) = exp(\alpha z)$, $\alpha \in R$.

Players compare their payoffs: if the payoff of the sampled agent is better than of the reviewing agent, he switches to the strategy of the sampled agent. In other words, if this inequality $u(e^j, x) + \varepsilon' > u(e^i, x) + \varepsilon$ is justify for player with pure strategy i then he switches to the strategy j.

For the general case the following formula describes the imitation dynamics of pairwise comparison:

$$\dot{x}_i = \left[\sum_{j \in K} x_j (\phi[u(e^i - e^j, x)] - \phi[u(e^j - e^i, x)]) \right] x_i, \; i \in K. \tag{2}$$

To simplify calculations use certain numerical values for the models parameters and construct dynamics for each extended game.

Situation A:

Using following values for incomes, which are $I = 2$, $\alpha = 1$ and values of parameter u are $u = 4, \; 2, \; 8/3$ then we get three different systems of differential equations, corresponding to various cases of the extended games:

$$\dot{x}_H = (x_B(e^{(4x_H + x_B - 2)} - e^{(-4x_H - x_B + 2)}) + \\ x_E(e^{(-x_B + (2-u)x_E)} - e^{(x_B + (-2+u)x_E)}))x_H;$$

$$\dot{x}_B = (x_H(e^{(-4x_H - x_B + 2)} - e^{(4x_H + x_B - 2)}) + \\ x_E(e^{(-2x_H + (4-u)x_E)} - e^{(2x_H + (-4+u)x_E)}))x_B;$$

$$\dot{x}_E = (x_H(e^{(x_B + (-2+u)x_E)} - e^{(-x_B + (2-u)x_E)}) + \\ x_B(e^{(2x_H + (-4+u)x_E)} - e^{(-2x_H + (4-u)x_E)}))x_E.$$

Situation B:

Let values for income and costs are: $I = 2$, $C = 4$, $\alpha = 1$ then we get three systems of differential equations with values of parameter u: $u = 4, \; 3, \; 1$ in various cases:

$$\dot{x}_H = (x_B(e^{(x_H + 5x_S - 3)} - e^{(-x_H - 5x_S + 3)}) + \\ x_E(e^{(-2x_H + (1-u)x_E)} - e^{(2x_H + (-1+u)(1 - x_H - x_S))}))x_H;$$

$$\dot{x}_S = (x_H(e^{(-x_H - 5x_S + 3)} - e^{(x_H + 5x_S - 3)}) + \\ x_E(e^{(-2x_B + (4-u)x_E)} - e^{(2x_S + (-4+u)x_E)}))x_B;$$

$$\dot{x}_E = (x_H(e^{(2x_H + (-1+u)x_E)} - e^{(-2x_H + (1-u)x_E)}) + \\ x_S(e^{(2x_S + (-4+u)x_E}) - e^{(-2x_S + (4-u)x_E)}))x_E.$$

Situation C:

Let incomes are $I_1 = 2$, $I_2 = 1$, $\alpha = 1$, then systems of differential equations that define pairwise comparison dynamics are following:

$$\dot{x}_B = (x_B(e^{(x_H - 2x_B + 1)} - e^{(-x_H + 2x_B - 1)}) +$$
$$x_E(e^{(-x_B + (2-u)x_E)} - e^{(x_B + (-2+u)x_E)}))x_B;$$
$$\dot{x}_H = (x_H(e^{(-x_H + 2x_B - 1)} - e^{(}x_H - 2x_B + 1)) +$$
$$x_E(e^{(-2x_H + (1-u)x_E)} - e^{(2x_H + (-1+u)x_E)}))x_H;$$
$$\dot{x}_E = (x_H(e^{(x_B + (-2+u)x_E)} - e^{(-x_B + (2-u)x_E)}) +$$
$$x_B(e^{(2x_H + (-1+u)x_E)} - e^{(-2x_H + (1-u)x_E)}))x_E;$$

Values of parameter u: $u = 2,\ 1,\ 4/3$.

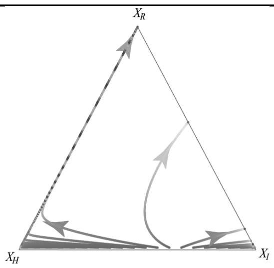
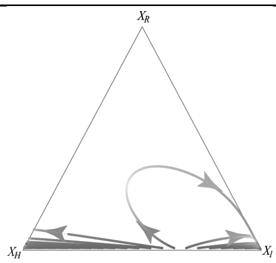
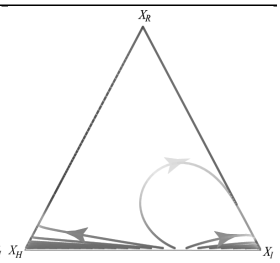
For each system we get numerical solution using next initial states: $x_E = 0.1,\ x_H = 0.01, 0.02, \ldots, 0.98,\ \ x_B = 0.98, 0.97, \ldots,\ 0.01$ and $x_E = 0.1,\ x_B = 0.01, 0.02, \ldots, 0.98,\ \ x_S = 0.98, 0.97, \ldots,\ 0.01$ solution trajectories are presented in Table 1, where rows represent different situations on the stock market and columns correspond to various modifications of extended games.

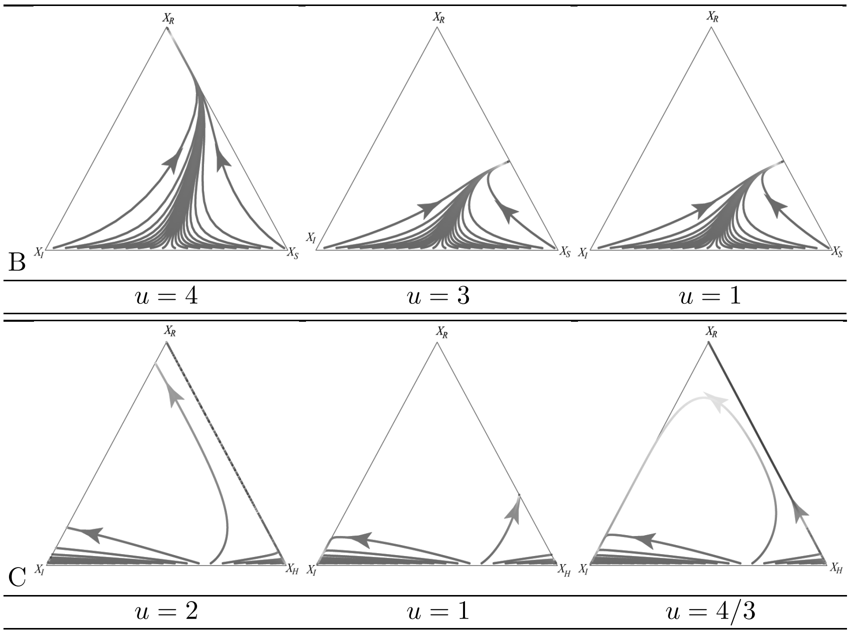
We get that in situation **A** for all cases of extended games behavioral type "invest" and behavioral type of rationalist are preferable and that strategies will survive in the long-run period, however in case 2 we can see that behavior "to hold" also can be preserved.

In situation **B** in case 1 only rational agents prevail on the market, in case 2 and 3 mixture of agents, who sell their blocks of shares and rationalists will survive and situation $(x_B, x_S, x_E) = (0, 1/3, 2/3)$ will be the rest point of the system.

In situation **C** we get different variants of prevailed behaviors. In case 1 solutions trajectories aspire to states x_B and x_E and on the stock market "investors" and "rationalists" will be survived in long-run period. In case 2 behaviors "invest" and partly "hold" will be conserved and and in case 3 only state x_B is stable.

Table 1: Imitation dynamics of pairwise comparison.

Case 1	Case 2	Case 3
A 		
$u = 4$	$u = 2$	$u = 8/3$



6. Imitation of successful agents

Suppose that the choice probabilities $p_i^j(x)$ are proportional to popularity of j's strategy x_j, and the proportionality factor is described by the currently payoff to strategy j. It is thus as if agent observes other agents choices with some small noise and would imitate another agent from the population with a higher probability for relatively more successful agents. Denote the weight factor that a reviewing agent with strategy i attaches to pure strategy j as $\omega[u(e^i, x), x] > 0$, where ω is a continuous function. Then

$$p_i^j = \frac{\omega[u(e^j, x), x]x_j}{\sum\limits_{p\in K} \omega[u(e^j, x), x]x_p}$$

Selection dynamics for that model is described by following equations:

$$\dot{x_i} = \left(\sum_{j\in K} \frac{\omega[u(e^j, x), x]x_j}{\sum\limits_{p\in K} \omega[u(e^j, x), x]x_p} - 1 \right) x_i. \qquad (3)$$

As in earlier case we have some additional assumptions for choice probability such as these is not that a reviewing agent necessarily knows the current average payoff to all pure strategies. It is sufficient that some agent have some possibly noisy empirical information about payoff to some pure strategies in current use and on average more agents prefer to imitate an agent with higher payoff than one with lower average payoff.

In this paper accept as weight function $\omega = exp(\alpha^{-1}u(e^i, x))$, where α is small noise of observation and get following expression (Sandholm, 2008, 2010):

$$\dot{x_i} = \frac{x_i exp(\alpha^{-1} u(e^i, x))}{\sum\limits_{k \in K} x_k exp(\alpha^{-1} u(e^k, x))}, i, k \in K. \quad (4)$$

To simplify calculations, as in previous section, we use certain numerical values for the models' parameters and construct dynamics for each extended game.

Situation A:

Let agents' income is $I = 2$ then we get three different systems of differential equations, corresponding to various cases of the extended games:

$$\dot{x}_H = \frac{x_H e^{(\alpha^{-1}(x_B+2))}}{x_H e^{(\alpha^{-1}(x_B+2))} + x_B e^{(\alpha^{-1}(4-4x_H))} + x_E e^{(\alpha^{-1}(2x_H+4x_B+x_E u))}};$$
$$\dot{x}_B = \frac{x_B e^{(\alpha^{-1}(4-4x_H))}}{x_H e^{(\alpha^{-1}(x_B+2))} + x_B e^{(\alpha^{-1}(4-4x_H))} + x_E e^{(\alpha^{-1}(2x_H+4x_B+x_E u))}};$$
$$\dot{x}_E = \frac{x_E e^{(\alpha^{-1}(2x_H+4x_B+x_E u))}}{x_H e^{(\alpha^{-1}(x_B+2))} + x_B e^{(\alpha^{-1}(4-4x_H))} + x_E e^{(\alpha^{-1}(2x_H+4x_B+x_E u))}};$$

Values of parameter u: $u = 4,\ 2,\ 8/3$.

Situation B:

Let agents' income is $I = 2$ and costs are $C = 4$ then we get three systems of differential equations describe imitation dynamics of successful agents with values of parameter u: $u = 4,\ 2,\ 1$.

$$\dot{x}_H = \frac{x_H e^{(\alpha^{-1}(-2x_H+3x_S+1))}}{x_H e^{(\alpha^{-1}(-2x_H+3x_S+1))} + x_S e^{(\alpha^{-1}(-3x_H-2x_S+4))} + x_E e^{(\alpha^{-1}(x_H+4x_S+x_E u))}};$$

$$\dot{x}_S = \frac{x_S e^{(\alpha^{-1}(-3x_H-2x_S+4))}}{x_H e^{(\alpha^{-1}(-2x_H+3x_B+1))} + x_B e^{(\alpha^{-1}(-3x_H-2x_S+4))} + x_E e^{(\alpha^{-1}(x_H+4x_S+x_E u))}};$$

$$\dot{x}_E = \frac{x_E e^{(\alpha^{-1}(x_H+4x_S+x_E u))}}{x_H e^{(\alpha^{-1}(-2x_H+3x_S+1))} + x_S e^{(\alpha^{-1}(-3x_H-2x_S+4))} + x_E e^{(\alpha^{-1}(x_H+4x_S+x_E u))}};$$

Situation C:

Let incomes are $I_1 = 2$ and $I_2 = 1$ then for different values of parameter u: $u = 2,\ 1,\ 4/3$. we have following systems of differential equations:

$$\dot{x}_H = \frac{x_H e^{(\alpha^{-1}(2-2x_B))}}{x_H e^{(\alpha^{-1}(2-2x_B))} + x_B e^{(\alpha^{-1}(1-x_H))} + x_E e^{(\alpha^{-1}(2x_H+x_B+x_E u))}};$$

$$\dot{x}_B = \frac{x_B e^{(\alpha^{-1}(1-x_H))}}{x_H e^{(\alpha^{-1}(2-2x_B))} + x_B e^{(\alpha^{-1}1-x_H)} + x_E e^{(\alpha^{-1}(2x_H+x_B+x_E u))}};$$

$$\dot{x}_E = \frac{x_E e^{(\alpha^{-1}(2x_H+x_B+x_E u))}}{x_H e^{(\alpha^{-1}(2-2x_B))} + x_B e^{(\alpha^{-1}(1-x_H))} + x_E e^{(\alpha^{-1}(2x_H+x_B+x_E u))}};$$

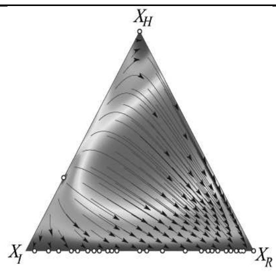
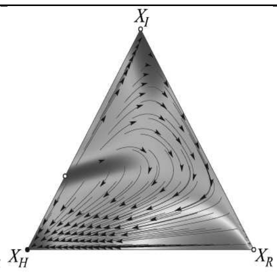
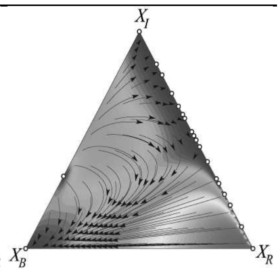
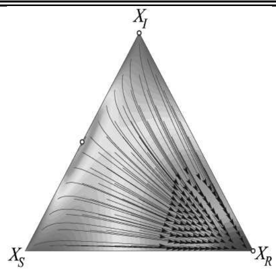
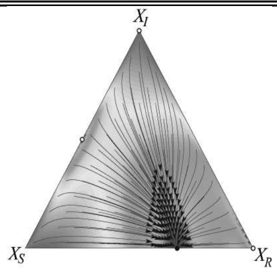
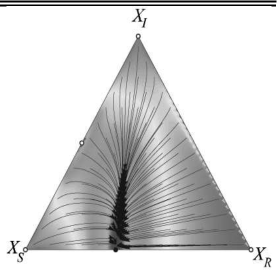
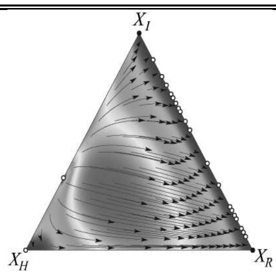
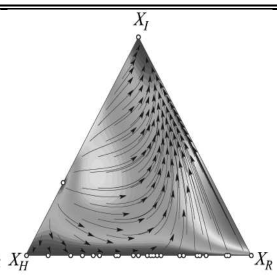
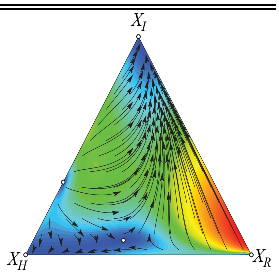
Table 2 contains pictures with solution trajectories for each extended game, as in Table 1 rows represent different situations on the stock market and columns correspond to various modifications of extended games.

We get that in situation **A** in case 1 all trajectories converge to stationary state x_E and in cases 2 and 3 the state x_H is stable and according behavioral type "hold" the blocks of shares will prevail on the market.

In situation **B**, in case 1 stable point is $(x_B, x_S, x_E) = (0, 0, 1)$, in case 2 system has only one stable rest point $(x_B, x_S, x_E) = (0, 1/3, 2/3)$, in case 3 stable point is $(x_B, x_S, x_E) = (0, 0.6, 0.4)$, hence we can say that during long-run period behaviors of rationalists and "sellers" will survive.

In situation **C** in case 1 solutions trajectories converge to border between x_E and x_B and in case 2 and 3 the stable point is $(x_B, x_S, x_E) = (1, 0, 0)$.

Table 2: Imitation dynamics of successful agents.

Case 1	Case 2	Case 3
A 		
$u = 4$	$u = 2$	$u = 8/3$
B 		
$u = 4$	$u = 3$	$u = 1$
Case 1	Case 2	Case 3
C 		
$u = 2$	$u = 1$	$u = 4/3$

7. Conclusion

This paper's main contribution is in using general results from evolutionary game theory to simulation of agents' interaction on the stock market and analysis the behavior stability over time. Applying numerical simulation, we get that in some situation behavior of agents with bounded rationality, which can not recognize the actions of the opponent, will survive in long-run period. And the behavior of rational players will preserve in some other situations. In future research we are planning to use other probability distributions for agent's revision profiles.

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