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Auctions of Homogeneous Goods: Game-Theoretic Analysis*

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Abstract This paper discusses results on Nash equilibrium and its refinements for several variants of the architecture for oligopolistic markets of homogeneous goods. For different kinds of one-stage and two-stage auctions, we compare the game-theoretic solution with the competitive equilibrium and estimate the loss of social welfare due to producers market power. We conclude on the optimal architecture of the market.

Keywords: Game theory; auctions; market power; social welfare.

1. Introduction

Markets of homogeneous goods play an important role for many economies including Russian one. A typical structure of such markets is oligopoly, and they are organized as auctions of some types. Consumers often have no market power and do not play an active role in these auctions. Their behavior corresponds to a known demand function. So an important problem for such markets is limitation of large producers market power. Splitting of the market into small companies is a bad way to deal with the problem because of the scale effect and the reliability requirements.

Another way is to design such market mechanism that its equilibrium state is sufficiently close to the Walrasian equilibrium the optimal state of the market according to the Welfare theorem (Debreu, 1954). The literature on the markets of homogeneous goods (Amir, 1996, Amir and Lambson, 2000, Ausubel and Cramton, 1999, Allen and Helwig, 1986, Vives, 1986, Vasin et al., 2003 and many others) models different mechanisms as strategic games where producers are the players, and examines Nash equilibrium or its refinement as behavior model. Other desirable properties of the mechanism are: existence of Nash equilibrium in dominating strategies; the strategy of each agent can be determined proceeding from his private information.

The present paper surveys results of game-theoretic analysis of economic mechanisms related to markets of homogeneous goods. The three following sections discuss different variants of the uniform price auction. In such action, a producer's bid determines the supplied volume depending on the price, the market price corresponds to the intersection of the total supply function with the demand function, and all agents buy and sell at this price. Section 2 considers the Cournot auction where each seller proposes a fixed amount of the good. Section 3 studies a model with bids corresponding to non-decreasing step functions (a typical design of the auction in practice). Section 4 examines the auction with continious bid functions and uncertain demand and discusses the concept of supply function equilibrium proposed by Klemperer and Meyer (1989). In Section 5 we discuss Vickrey auction that obtains the desirable properties. Section 6 considers another variant implemented in practice: pay as bid auction. Section 7 finishes with discussion of forward market as an instrument for the market power reduction and final conclusions from our study.

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2. Cournot oligopoly

Many papers (Novchek, 1985, Kukushkin, 1994, Amir, 1996, Amir and Lambson, 2000 and so on) study the Cournot auction where each seller proposes a fixed amount of the good. Vasin et al. (2007) prove existence of the unique Nash equilibrium under non-decreasing demand elasticity and marginal costs. They show that the relative deviation of the Cournot price from the Walrasian price is less or equal to the share of the largest company in the total production volume, divided by the demand elasticity. This estimate coincides with the Lerner index for the company and is precise if its marginal costs are equal for the Walrasian and the Cournot equilibria. Newbery (2009) considers these results in context of the data for Europian electricity markets. The share of the largest company typically exceeds 0.25, while the demand elasticity is less than 0.2. Thus, the data obviously contradicts to the Cournot model. Newbery calls it as Lerner Paradox and discusses different explanations considered below.

The formal model and the results are as follows. Consider a market for a homogenous good with a finite set A of producers. Each producer a is characterized by his cost function $C^a(v)$ with non-decreasing marginal costs for $v \in [0, V^a]$ where V^a is his production capacity. The precise form of C^a is his private information. Consumers behavior is characterized by a demand function $D(p)$, which is continuously differentiable, decreasing in p, tends to 0 as p tends to infinity, and is known to all agents.

The combination $(\widetilde{v}^a, a \in A)$ of production volumes is a *Walrasian equilibrium* (WE) and $\widetilde{p}$ is a *Walrasian price* of the market if, for any a,

$$\widetilde{v}^a \in S^a(\widetilde{p}) \stackrel{def}{=} \arg\max_{v^a}(v^a\widetilde{p} - C^a(v^a)), \sum_a \widetilde{v}^a = D(\widetilde{p})$$

Consider Cournot competition in this market. Then a strategy for each producer a is his production volume $v^a \in [0, V^a]$. Producers set these values simultaneously. Let $\overrightarrow{v} = (v^a, a \in A)$ denote a strategy profile. The market price $p(\overrightarrow{v})$ equalizes the demand with the actual supply: $p(\overrightarrow{v}) = D^{-1}(\sum_{a\in A} v^a)$. The payoff function of producer a determines his profit $f^a(\overrightarrow{v}) = v^a p(\overrightarrow{v}) - C^a(v^a)$ Thus, the interaction in Cournot model corresponds to the normal form game $\Gamma_C = \langle A, [0, V^a], f^a(\rightharpoonup v), \overrightarrow{v} \in \otimes_{a\in A}[0, V^a], a \in A\rangle$ where $[0, V^a]$ is the set of strategies for $a \in A$. The profile $\overrightarrow{v}^*$ of production volumes is a *Cournot equilibrium* (CE) if it is a NE in the game Γ_C. Let $(v^{a_*}, a \in A)$ denote the equilibrium production volumes and $p^* = D^{-1}(\sum_{a\in A} v^{a_*})$

Proposition 1. *(Vasin et al., 2007) Let the demand function $D(p)$ and the demand elasticity $e(p) \stackrel{def}{=} p\dfrac{|D'(p)|}{D(p)}$ meet one of the following conditions:*

a) $D(p) > 0$ and $e(p) \uparrow p$ for $p \in (\widetilde{p}, M)$, $D(p) = 0$ for $p \geqslant M$

b) $D(p) > 0$ and $e(p) \uparrow p$ for $p \geqslant \widetilde{p}, lim_{p\longrightarrow\infty} = L > 1/n$ where n is the total number of producers in the market. Then there exists a unique Nash equilibrium in the game Γ_C.

The F.O.C. for a Nash equilibrium is

$$v^{a^*} \in (p^* - C^{a'}(v^{a^*}))|D'(p^*)| \; for \; any \; a \; s.t. \; C^{a'}(0) < p^* \qquad (1)$$

$$v^{a^*} = 0 \; if \; C^{a'}(0) \geqslant p^* \tag{2}$$

where $C^{a'}(v) = [C^{a'}_{-}(v), C^{a'}_{+}(v)]$ at break points of the marginal cost function, $C^{a'}_{+}(V^a) = \infty$.

The combination $(p^*, v^{a^*}, a \in A)$ is called *a local Cournot equilibrium* if it meets the necessary conditions (1),(2). Define *Cournot supply function* $S^a_C(p)$ of a producer a as a solution of the system (1),(2). This function determines the optimal production volume of producer a if p is a Cournot equilibrium price. The Cournot price p^* is determined by the equation $\sum_a S^a_C(p^*) = D(p^*)$. Note that in general player a cannot determine his equilibrium volume v^{a^*} proceeding from his private information.

The next proposition evaluates the deviation of the Cournot outcome from the Walrasian equilibrium proceeding from the demand elasticity and the maximal share of one firm in the total production at the Cournot equilibrium.

Proposition 2. *For every firm a, its Lerner index at the Cournot equilibrium meets equation*

$$L^a_c \overset{def}{=} \frac{p^* - c^a(v^{a^*})}{p^*} = \frac{s^a(p^*)}{e(p^*)}$$

where $s^a(p^*) = \dfrac{v^{a^*}}{D(p^*)}$ *is the share of producer a in the total production volume,* $e(p)$ *is the demand elasticity. Moreover,*

$$\frac{p^* - \widetilde{p}}{\widetilde{p}} \leqslant \max_a \frac{S^a(p^*)}{e(p^*)}$$

The latter condition holds as the equality for a symmetric oligopoly with a fixed marginal cost $c = p$, and also for a large firm with a fixed marginal cost interacting with the competitive environment characterized by a smaller marginal cost and a limited total capacity (see Fig. 1).

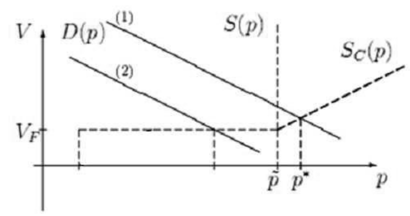

Fig. 1: Deviation of the Cournot price p^* from the Walrasian price $\widetilde{p}$ for a large firm in the competitive environment

3. Uniform price auction

Vasin et al. (2007) study a uniform price auction, where a strategy of each producer is a non-decreasing step function that determines the actual supply of the good depending on the price. Such auctions are typical for wholesale electricity markets, in particular, for day-ahead markets where the basic production and consumption components are determined. The real auctions differ in the rules for acceptable bids. Russian DAM accepts bids with at most 3 steps, a different bid for every hour of the next day, while the market of England and Welsh permitted up to 48 steps, but a unique bid for the whole day.

The mentioned study shows that, for any Nash equilibrium of the corresponding game, the market price lies between the competitive equilibrium price and the Cournot price, and vice versa, each price in this range corresponds to a Nash equilibrium. However, only the Nash equilibrium corresponding to the Cournot outcome is stable with respect to the dynamics of adaptive strategies.

Note that Moreno and Ubeda (2001)) obtain similar results for the two-step model where at the first step producers set capacities , and at the second step they compete by setting reserve prices. Kreps and Scheinkman (1983) show that the SPE outcome of the two-stage model ″first quantities, then price″ also corresponds to the Cournot equilibrium. Thus, for all these mechanisms the expected deviation of the market price from the Walrasian price is the same as for the Cournot model, and they are not responsible for solution of the Lerner paradox.

The formal model of the uniform price auction and the results are as follows. Every producer $a \in A$ simultaneously sends to the auctioneer his reported supply (r-supply) function $R^a(p)$ that determines the amount of the good this producer is ready to sell at price $p, p \geqslant 0$. Acceptable bid $R^a(p)$ is a non-decreasing step function with a limited number of steps. So this is not a usual function but a point-set mapping: at any jump point its value is a stretch, and it obtains the same properties as a Walrasian supply function.

A profile of r-supply functions determines the total r-supply $R(p) = \sum_a R^a(p)$ and the cut-off price $\widetilde{c}(R^a, a \in A)$ that meets condition $D(\widetilde{c}) \in R(\widetilde{c})$. Proceeding from the properties of the demand function, the cut-off price is uniquely determined for any non-zero r-supply, as well as the Walrasian price in the market model. In order to define payoff functions, consider two cases. Let $R^+(p) \stackrel{def}{=} \max R(p), R^-(p) \stackrel{def}{=} \min R(p)$. If $R^+(\widetilde{c}) = D(\widetilde{c})$ then each producer sells the reported volume $R^{a+}(\widetilde{c})$ at the cut-off price. Otherwise, first each producer sells $R^{a-}(\widetilde{c})$, and then the residual demand $D(\widetilde{c}) - R^-(\widetilde{c})$ is distributed among producers with $R^{a+}(\widetilde{c}) > R^{a-}(\widetilde{c})$ according to some rationing rule (typically the proportional rule).

Under a given rationing rule, the profit of producer $b \in A$ is determined as follows:

$$f^b(R^a(.), a \in A) = c(R^a, a \in A)v^b(R^a, a \in A) - C^b(v^b(R^a, a \in A)),$$

where $v^b(R^a, a \in A) \in [R^{b-}(\widetilde{c}), R^{b+}(\widetilde{c})]$ is the final demand for his production. Thus, we have defined the normal form game $\varGamma_S$ that corresponds to the auction.

Note that there might be three possible types of Nash equilibria for $\varGamma_S$: a) those Nash equilibria for which $R^+(\widetilde{c}) = D(\widetilde{c})$ (Nash equilibria without rationing), b) those for which $D(\widetilde{c}) \in (R^-(\widetilde{c}), R^+(\widetilde{c}))$ (Nash equilibria with rationing), c) those for which $D(\widetilde{c}) = R^-(\widetilde{c}) < R^+(\widetilde{c})$ (Nash equilibria with a barrier, see Figure 2).

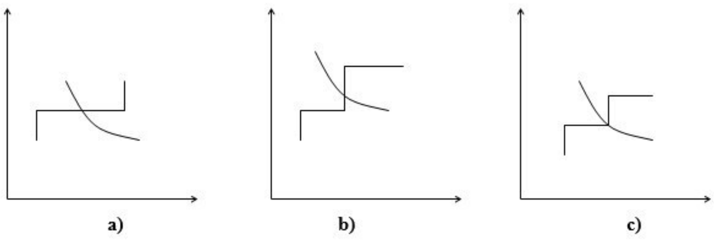

Fig. 2: Three types of Nash equilibria in the supply function auction.

Proposition 3. *(Vasin et al., 2007)*

a) *For every Nash equilibrium without rationing, the production volumes correspond to the local Cournot equilibrium. Vice versa, if $(v^a, a \in A)$ is a Cournot equilibrium, then the corresponding Nash equilibrium exists in Γ_S.*

b) *If $(R^a, a \in A)$ is a Nash equilibrium such that $D(\widetilde{c}) \in (R^-(\widetilde{c}), R^+(\widetilde{c}))$, then there exists at most one producer $b \in A$ such that $R^{b-}\widetilde{c}) < S^{b-}(\widetilde{c})$ (so $v^a \in S^a(\widetilde{c})$ for any $a \neq b$); the cut-off price lies in the interval $[\widetilde{p}, p^*]$.*

c) *For any Nash equilibrium of the type c), the cut-off price lies in the interval $[\widetilde{p}, p^*]$. Vise versa, for any $p \in [\widetilde{p}, p^*]$ there exists a Nash equilibrium $(R^a, a \in A)$ such that $\widetilde{c}(R^a, a \in A) = p$.*

Let us note that every Nash equilibrium of the types b and c is unstable and cannot occur as an outcome of the auction until some players act for the interests of some external regulator. Indeed, in any such equilibrium the excessive supply at price $\widetilde{c}$ creates a barrier that makes it unprofitable for any player to increase the cut-off price by reducing his supply level in the neighborhood of the price $\widetilde{c}$. However, keeping this barrier is unprofitable. Reduction of $R^a(\widetilde{c})$ to v^a for ever $a \in A$ does not change the profits of the players if other strategies are fixed. Moreover, as soon as the barrier is sufficiently small, some player finds it profitable to reduce his supply function and thus increase the profits of the other players. Thus, the expected outcome of the auction corresponds to the Cournot equilibrium, and the estimates from Proposition 2 also hold in this case.

Models by Baldick et al. (2000), Vasin and Daylova (2012), Klemperer and Meyer (1989) describe the uniform price supply function auction with continuous bids as a game in normal form and characterize the Nash equilibria of the auction. Klemperer and Meyer study the competition model with arbitrary bid functions, including non-monotonic. For a given demand function they find a lot of Nash equilibria corresponding to all prices grater than the Walrasian price. Green and Newbery (1992) consider a symmetric duopoly with linear marginal cost, demand and bid functions. They obtain the formula for calculation of the unique Nash equilibrium. Baldick et al. (2000) generalize the results to the asymmetric linear oligopoly. Abolmasov and Kolodin (2002) and Dyakova (2003) apply this approach to study electricity markets in two Russian regions. They use the affine approximations of the true supply functions and obtain in the simulations a significant reduction of the "market power" in the supply function auction compared with the Cournot auction.

Can a model of SFE with linear supply functions and marginal costs adequately describe and explain the Lerner paradox? Note that the assumption of affine structure of the supply function does not correspond to the actual cost structure of energy companies, nor the practice of the auction. In a typical DAM every producer may submit a bid corresponding to a non-decreasing piece-wise step function. In a first approximation the real structure of the variable costs of many power companies also corresponds to such function. Usually, such a company owns several power generators with limited capacities, each of them is characterized by approximately constant marginal cost, but the cost is specific for each generator. Under these conditions, the equilibrium bid is a nonlinear function of the price. This is confirmed by the results obtained in the other direction of research initiated in the same paper, Klemperer and Meyer (1989).

4. Supply function auction with uncertain demand

An important feature of electricity markets is uncertainty of demand, which is due to random changes of the environment and also to variations of the demand during the time for which the bids are submitted. In this context, Klemperer and Meyer (1989) proposed a promising auction model and theoretical results. They assumed a bid to be a monotone smooth function and the demand function to depend on a random parameter. Thus, the cut-off price that equalizes the total supply and demand is random. A bid profile is called a supply function equilibrium (SFE) if, for any parameter value, the bid of each firm maximizes its profit under fixed bids of other producers. For a symmetric oligopoly, the authors derive a differential equation for an equilibrium bid and describe the set of the SFE.

The SFE price is always lower than the Cournot oligopoly price. In some cases, the price reduction is significant (Green, 1999, Newbery, 1998). On this ground, some researchers claim that the supply function auction is an efficient mechanism for reduction of the "market power" of producers.

However, the computation of the SFE bids is a rather sophisticated mathematical problem. Even in a simple case with fixed marginal costs and a limited capacity, the equilibrium bid is a combination of a linear and a logarithmic functions. Moreover, calculation of the equilibrium bid requires full information on the demand function and the cost functions of all competitors, which in practice is lacking. Thus, SFE does not obtain those desirable properties we mentioned in the Introduction. Why should one expect that the actual behavior at the auction corresponds to this concept?

A similar question for Nash equilibria of normal form games is considered in the framework of adaptive and learning mechanisms investigation (see Milgrom and Roberts, 1990, Vasin, 2005). These studies show that for some classes of games rather simple mechanisms provide convergence of strategy profiles to stable NE. Models of adaptive dynamics require neither full information nor high rationality of the players. It suffices to be able to calculate and compare an agents profits under the current and alternative strategies. If the adaptive process converges to the Nash equilibrium, we can expect the appropriate behavior in the real life. But many NE are not stable in this sense.

Rudkevich (1993) examined best response dynamics for a symmetric oligopoly with linear demand function and linear marginal cost function and proved that the dynamics converges to the SFE with a geometric rate. Vasin and Dolmatova (2012) studied a symmetric oligopoly with fixed marginal cost and limited capacities. They showed that the best response does not exist at some stage under general conditions. Moreover, even if there exists a sequence of best responses in a certain range of the random parameter values, it is typically cyclic, so the convergence to the SFE does not hold.

4.1. Formal model of the auction and SFE concept.

Consider a market with a set $N = \{1, 2, ..., n\}$ of players (producers). For player i, $C_i(q)$ is the cost functions $C(q)$ depending on generation q, such that: $C_i'(q) > 0, C_i''(q) \geqslant 0$ for any $q \geqslant 0$. Demand is given by a function $D(p,t)$ of price $p \geqslant 0$, which also depends on a random factor t. The probability density of the factor is strictly positive for the set $[\underline{t}, \overline{t}]$. For any p, t, the demand function satisfies: $D_p < 0$, $D_{pp} \leqslant 0$, $D_{pt} = 0$, $D_t > 0$. A strategy of a player i is a non-decreasing supply function $S^i(p)$ that maps produced volume to the market price p. Making their choices $S^i(p)$, the players do not know the value of the random factor t. When t is known, a strategy profile $\overrightarrow{S} = (S^1(p), S^2(p), .., S^n(p))$ induces the price $p(\overrightarrow{S}, t)$ that balances the aggregate supply and demand: $D(p(t)) = \sum\limits_{i=1}^{n} S^i(p(t))$. Each producer i intends to maximize his pay-off function:

$$\pi_i(\overrightarrow{S}, t) = p(\overrightarrow{S}, t) S^i(p(\overrightarrow{S}, t)) - C(S^i(p(\overrightarrow{S}, t))), \ i \in N.$$

Strategy profile $\overrightarrow{S}^* = (S^{*i}, i \in N)$ is called an *SFE* if, for any $t \geqslant 0$ and $i \in N$, $S^{*i} \in \arg\max_{S_i}(\pi_i(S^i, S^{*-i}, t))$.

Proposition 4. *(see Proposition 1.2 in Klemperer and Meyer, 1989)*

For a symmetric oligopoly with the cost function $C_i(q) = C(q), i \in N$, *if* $sup_t D(0,t) = \infty$, *then* $\overrightarrow{S}^* = (S^{*1}, S^{*2}, ..., S^{*n})$ *is SFE if and only if* $S^i(p) \equiv S(p)$ $\forall i \in N$, *and* $S(p)$ *monotonously increases in* p *and meets equation:*

$$S'(p) = \frac{1}{n-1}\left[\frac{S(p)}{p - C'(S(p))} + D_p(p)\right]. \tag{3}$$

4.2. The model with linear marginal cost function.

Consider a symmetric oligopoly with n producers, where the cost function of each producer is $C(q) = (c_0 + 0,5c_1 q)q$, $c_0 > 0, c_1 > 0$, and the demand function is $D(p,t) = \overline{D}(t) - dp$, where $d > 0$ and $\overline{D}(t)$ is a maximal demand value depending on a random parameter t. According to Proposition 4, an equilibrium supply function for this case should meet the differential equation:

$$S'(p) = \frac{1}{n-1}\left(\frac{S(p)}{p - C'(S(p))} + D_p(p)\right).$$

There exists the infinite set of nonlinear solutions (see Rudkevich, 1993). However, the condition $\sup_t \overline{D}(t) = \infty$ results in a unique and linear SFE:

$$S^*(p) = (p - c_0)\frac{n - 2 - c_1 d + \sqrt{(n-2)^2 + 2nc_1 d + c_1^2 d^2}}{2c_1(n-1)}. \tag{4}$$

Consider the strong best response dynamics (SBRD) for the repeated auction with identical players. For any bid $S^1(p)$ bid $S^2(p)$ is called a strong best responce, if for each $t \in [\underline{t}, \overline{t}]$ solution $p(t)$ to $S^1(p) + S^2(p) = D(p,t)$ provides the maximum profit:

$$p(t) \longrightarrow \max_p \{(D(p,t) - S_1(p))p - C(D(p(t) - S_1(p)))\}.$$

At every time $\tau = 1, 2, ...,$ each firm sets a bid $S(p,\tau)$ that is a strong best response to its competitors' bids $S(p, \tau-1)$ at the previous time. We do not assign players in the considered dynamics, since we assume $S(p,0) = 0$, and so the best responses of all the players are the same. The sequince $S(p,\tau), \tau = 1, 2, ...,$ forms SBDR, if for any $t \in [\underline{t}, \overline{t}]$ $S(p,\tau)$ is a SBR for $S(p, \tau-1)$.

Below we recall the result by Rudkevich (1993) on the existance and convergence of the SBDR for the market under consideration.

Proposition 5. *Let* $\{\overline{D}(t), t \in T\} \supseteq [dc_0, \infty)$.

Then, the bid $S^i(p) = (p - c_0)\frac{d+(n-1)k}{1+c_1(d+(n-1)k)}$ *is the unique strong best response to the competitors' bids* $S^j = k(p - c_0)$, $j \in N\backslash\{i\}$.

The best response at time τ *is* $S(p,\tau) = k_\tau(p - c_0)$, *where* $k_\tau = \frac{d+(n-1)k_{\tau-1}}{1+c_1(d+(n-1)k_{\tau-1})}$. *The unique fixed point* $k^* = \frac{n-2-c_1 d+\sqrt{(n-2)^2+2nc_1 d+c_1^2 d^2}}{2c_1(n-1)}$ *for this dynamics corresponds to the SFE of the static auction model.*

4.3. Oligopoly with fixed marginal costs and capacity constraints.

Consider a market with n symmetric producers, each characterized by the capacity constraint Q and the linear cost function $C(q) = cq$, where $q \leq Q$ and $c > 0$. The equilibrium supply function bid is constructed as a continous monotone function satisfying (3).

For $n > 2$ the solution $S_n(p, A_n) = A_n(p - c)^{1/(n-1)} - \frac{d(p-c)}{n-2}$ depends on the integration constant A_n. The function reaches its maximum $q(A_n)$ under $p = p(A_n) \stackrel{def}{=} \frac{A_n(n-2)}{d(n-1)}^{\frac{n-1}{n-2}}$ at the point of intersection of its graph with the Cournot supply shedule. The inverce function is $A(q) = d \cdot (\frac{q}{d})^{\frac{n-2}{n-1}} \cdot \frac{n-1}{n-2}$.

Proposition 6. *(Vasin and Dolmatova, 2012)If* $D^* \geqslant (n+1)Q$, *then there exist a unique SFE in the oligopoly model. The equilibrium bid is*

$$S_n^*(p) = \begin{cases} S_n(p, A_n(Q)) & \textit{for } c \leq p \leq p(A_n(Q)), \\ Q & \textit{for } p \geq p(A_n(Q)). \end{cases}$$

Consider the BRD for the oligopoly where firms have limited capacities and their marginal costs are zero $c = 0$. For a linear demand $D(p) = \max\{0, \overline{D} - dp\}$ under fixed $\overline{D}$, we study the best response dynamics depending on the ratio between $\overline{D}$ and Q. At every stage $\tau = 1, 2, \ldots$, we search for the BR in the set of supply functions $S(p,\tau) = \min\{k(\tau)p, Q\}$. So the problem is to find the optimal slope $k(\tau)$. Our aim is to determine ranges of parameter $\dfrac{\overline{D}}{Q}$ with the same best response dynamics.

Proposition 7. *(Vasin and Dolmatova, 2012)*

The BRD for the model of n-firm oligopoly with $n \geqslant 3$ *depends on the ratio between parameters* $\overline{D}$ *and* Q *as follows:*

For $\overline{D} > (n+1)Q$, $S(p,\tau) = \min\{Q, kdp\}, \forall k \in [1,\infty) \forall \tau \geqslant 1$. *The BR is rather uncertain, but the outcome always corresponds to the Cournot equilibrium coinciding with the Walrasian equilibrium in this case.*

For $(n-1)Q < \overline{D} < (n+1)Q$, *the best response at the time* $\tau = 1, 2, \ldots, T(\overline{D}, Q)$ *is* $S(p,\tau) = \min\{Q, k(\tau)p\}$, *then the best response dynamics loops. The cycle length* $T(\overline{D}, Q)$ *is the smallest integer* T *meeting inequality* $\sum_{s=0}^{T}(n-1)^s > \dfrac{4Q(\overline{D}-Q)}{(\overline{D}-(n-1)Q)^2}$;

For $\overline{D} < (n-1)Q$, $S(p,\tau) = \min\{Q, k(\tau)p\}, \forall \tau \geqslant 1$, *where* $k(\tau) = d\sum_{s=0}^{\tau-1}(n-1)^s$. *For* $\tau \to \infty$ *the BRD converges to the Walrasian supply function.*

Figure 3 shows how the lenght of the BDR cycle varies depending on the ratio between $\overline{D}$ and the capacity constraint Q.

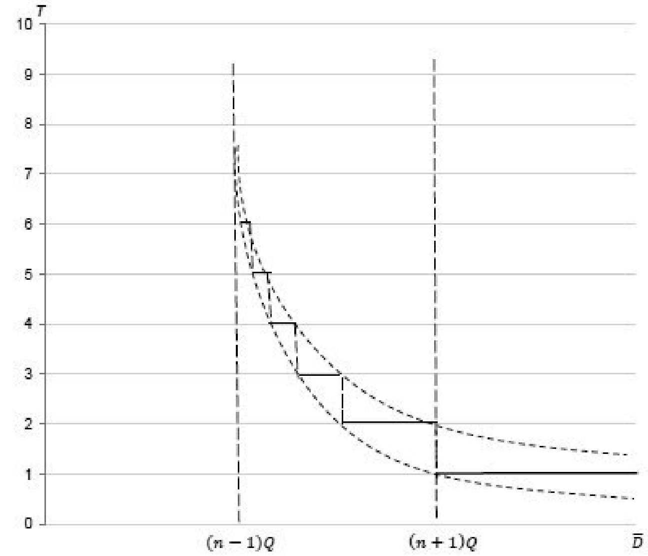

Fig. 3: The BDR cycle lenght T depending on the ratio between the maximal demand $\overline{D}$ and capacity Q.

Our study together with the previous research shows ambiguous results on the justification for the SFE concept. For market models with linear demand and marginal cost functions, the adaptive dynamics rapidly converges to the unique SFE existing under large demand shocks. A similar result holds if each generator either works all the time at its maximum capacity level, or never reaches this level, and its marginal cost is a linear function.

However, in general the real costs of generating companies do not meet these assumptions. Typically such a company owns several generators with limited capacities (and in practice the constraints are often binding in spike periods). The price of fuel per 1MW is usually the main component of the marginal cost. Sometimes this cost is approximately constant, for other generators it is necessary to use a more complicated relation. But in any case the SFE bid is not an affine function since the capacity constraint is binding. While the literature focuses on the SFE specified in proposition 6 as an instrument for the price forecast and evaluation of the market power in the supply function auction, our study shows that there is no ground to expect such kind of behavior in general.

5. Vickrey auction

An alternative possibility of SF auction organization, considered in several studies (Ausubel and Cramton, 1999, Wolfram, 1999, Vasin et al., 2007), is to use Vickrey auction. At this auction the cut-off price and production volumes are determined in the same way as in the uniform price auction. However, each producer is paid her reservation price for her goods. The marginal price is the minimum of the marginal cost of the same output for other producers and the marginal reservation price of this output for consumers. The marginal cost is calculated on the basis of the reported supply functions, but in this case reporting actual costs and production capacities is a weakly dominant strategy. In the absence of information on production costs, the guaranteed total welfare reaches its maximum at the corresponding Nash equilibrium, and each producer makes a profit equal to the increment of the total welfare of all participants in the auction as a result of his participation in the auction.

Our calculations for the Central Economic Region of Russia show that the Vickrey auction price for consumers exceeds the Walrasian price at 50% (to compare with 250-400% for the Cournot auction price). However, such increase seems to be also rather large. Besides, there exist reasonable arguments implying that the participants of Vickrey auction typically do not reveal their actual costs, that is, the specified equilibrium in dominant strategies is not realized (see Rotkkopf et al., 1990). The main argument is that reporting actual costs gives an advantage to the auctioneer (and also to other economic partners) in further interactions with this producer.

The situation is different if marginal costs and maximal capacity of each generator are common knowledge, and uncertainty pertains to a decrease of the capacities due to breakdowns and repairs. In this case the current state of working capacities is weakly correlated with the future state, and the specified argument against revealing the actual costs loses its validity. Moreover, available information may be used for redistribution of the total income in favor of consumers.

5.1. Formal model and results.

The set of strategies $\{R^a(p), p \geqslant 0\}$ of each articipant, the rule that determines for the cut-off price $\widetilde{c}(R^a(p), a \in A)$ and production volumes are the same as discussed for the uniform price auction. Producers a payment is calculated as follows.

The marginal reservation price $r^a(v^a)$ for an additional volume dv under production volume v^a is a cut-off price under the given bids $R^b(p), b \neq a$ and $P^a(p) \equiv v^a$:

$$D(r^a) \in R^{A \setminus a} + v^a \tag{5}$$

The total payment to producer a is equal to

$$I^a(\overrightarrow{R}) = \int_0^{\overline{v}^a} r^a(v)dv \tag{6}$$

Thus, we defined the game Γ_v corresponding to Vickrey auction. Bellow $S^a(p)$ denote Walrasian supply sunction.

Proposition 8. *(Vasin et al., 2007)*
$R^a(p) \equiv S^a(p)$ *is a weakly dominant strategy in* Γ_v. *Production volumes at the Nash equilibrium* $(S^a, a \in A)$ *are the same as at the competitive equilibrium, the profit of player a is* $W(A) - W(A \setminus a)$ *where* $W(K)$ *is the total welfare of the market with the set of producers* $K \subseteq A$.

Proposition 9. *The rule (5, 6) determines the minimal payment functions* $C^a(v), a \in A$ *under which, for any cost function, the optimal production volume of each firm is equal to* $s^a(p)$.

It is possible to distribute the total payment among the consumers in different ways. Consider the following variant taking into account reservation prices of consumers and at the same time minimizing the maximal price that they pay for the good. Consumer b buys the good at the maximal price p_v until it exceeds his marginal reserve price. The rest of the amount $\widehat{v}^b$ he buys out at his reservation prices. Thus, the total cost of the good for consumer b is

$$C^b(p_v) = p_v D^b(p_v) + \int_{D^b(p_v)}^{D^b \widetilde{p}} (D^b)^{-1}(v) dv$$

The price p_v equalizes the total cost of the good for consumers and the total payment to producers: $\sum_{b \in B} C^b(p_v) = \sum_{a \in A} I^a(\overrightarrow{R})$. Since each cost function monotonously increases in p_v, the unique solution of the latter equation may be obtained by a standard computational method.

5.2. Vickrey Auction with Incomplete Information on Cost Functions.

Consider now the case where the marginal costs c_i^a and the maximal capacities v_{iM}^a for each participant a and for each generator i are common knowledge. Then the optimal auction procedure differs from the previous case in two aspects. The auctioneer restricts the set of possible bids in accordance with the obtained information, accepting from player a only bids corresponding to the specified values of c_i^a and some $v_i^a \leqslant v_{iM}^a, i = 1, \dots m$. Besides, he takes into account this information in the computation of the prices used to determine the auction outcome. As in the previous variants, the production volumes are determined by the accepted bids as $v^a = R^a(\widetilde{c}(R^b, b \in A)), a \in A$. The payment to firm a for the good is calculated according to (6) on the basis of the reservation prices but these prices are reduced in comparison with (5) taking account the given information. Let us describe the algorithm to calculate the minimal reservation price $\overline{r}^a(v)$. This function is determined by the reservation price $r^a v$ for the standard Vickrey auction and the marginal cost function $c_M^a(v)$ corresponding to $(c_i^a, v_{iM}^a, i = 1, \dots m(a))$.

Stage 1.

Let us find $i_1 = \max\{i | c_i^a \leqslant r^a(0) = \widetilde{c}(R^b, b \in A \setminus a)\}$. Let $\overline{V}_1 = \sum_{i \leqslant i_1} V_{iM}^a$, $\overline{r}^a(v) = c_M^a(\overline{V}_1 - v)$, until inequality $c_M^a - (\overline{V}_1 - v) > r_+^a(v)$ does not hold or $v = \overline{V}_1$. In the first case let us define $\overline{v}_1$ as a minimal volume for which the specified inequality holds.

Stage *l*.

For a given value $\overline{v}_{l-1}$ let $i_l = \max\{i|c_i^a \leqslant r_+^a(v_{l-1})\}$, $\overline{V}_l = \sum_{i\leqslant i_l} V_{iM}^a$, $\overline{r}^a(\overline{v}_{l-1} + \Delta v) = c_M^a(\overline{V}_l - \Delta v)$, until $c_M^a - (\overline{V}_l - \Delta v) > r_+^a(\overline{v}_{l-1} + \Delta v)$ does not hold or $\Delta v = \overline{V}_l$. In the latter case the algorithm finishes its work. In the former case let us define $\overline{v}l$ as the minimal value of $\overline{v}_{l-1} + \Delta v$ such that the specified inequality holds, and go to stage $l+1$.

The proposed algorithm calculates the maximal marginal cost allowing the firm a to produce the volume dv under the available information on her costs and the evidence of selling this volume in the auction under given bids of other players.

Proposition 10. *Let the payment to each firm for the supplied volume be calculated according to (6) but substituting of $\overline{r}^a(v)$ for reservation price $r^a(v)$. Then for any $V_i^a \leqslant V_{iM}^a$ and any player a the strategy $R^a = S^a$ is weakly dominant, and the maximum of the total welfare is reached at the corresponding Nash equilibrium. The reservation price $\overline{r}^a(v)$ is minimal among reserve prices providing the specified property.*

5.3. Empirical study.

In this section we compute the NE prices of the standard supply function auction, the standard and the modified Vickrey auctions for two variants of the electricity market in the Central economic region of Russia. The paper by Dyakova (2003) based on the data from the RAO UES provides the following values of the marginal costs and the production capacities of the generating companies in this region. (See table 5.3..)

Table 1.

Generator	*Marginal cost (rub/mqth)*	*Capacity (bln kwth per year)*	*Generator*	*Marginal cost (rub/mqth)*	*Capacity (bln kwth per year)*
Mosenergo			GC2		
G1	0	5	1	95	2.5
G2	75	10	2	110	2.5
G3	80	10	3	120	4
G4	85	25	4	128	13
G5	90	10	5	135	6
G6	100	5	6	145	2
G7	165	10	7	162	15
Rosenergoatom					
	12.5	125.4			
GC1			GC3		
1	0	16	1	0	3.5
2	60	2	2	100	2.5
3	112	3	3	120	21
4	125	2	4	150	3.5
5	150	16	5	170	4.5
6	200	2	6	200	4.5
7	255	2	7	215	3
8	340	10			

We consider several demand function $D(p) = N - \gamma p$ corresponding to the average price and consumtion volume in 2000:

γ	0.1	0.2	0.4	0.6
N	279.9	316.1	388.4	460.7

For each slope ratio γ, we find the Cournot price, standart and modified Vickrey prices, end we evaluate the deviations of the NE prices from the Walrasian prices for two variants of the market structure:

a) 5 independents companies (Mosenergo, Rosenergoatom, GC1, GC2, GC3).
b) 3 independents companies (Mosenergo, Rosenergoatom and UGC including all the rest generators).

Table 2: Walrasian ($\widetilde{p}$), Cournot (p^*), Vickrey (p_V) and modified Vicrey ($\overline{p}_V$) prices for electricity market in the Central economic region od Russia. The cases with 5 and 3 generating companies.

γ	$\widetilde{p}$	$p_5^*/\widetilde{p}$	$p_3^*/\widetilde{p}$	$p_{V5}^*/\widetilde{p}$	$p_{V3}^*/\widetilde{p}$	$\overline{p}_{V5}^*/\widetilde{p}$	$\overline{p}_{V3}^*/\widetilde{p}$
0.1	135	4.24	5.65	1.59	2.19	0.51	0.62
0.2	150	2.45	3.10	1.49	1.92	0.44	0.57
0.4	172.5	1.56	1.87	1.49	1.76	0.42	0.49
0.6	219.67	1.15	1.34	1.30	1.46	0.33	0.38

6. Pay-as-bid auction

Another possible form of the organized market is a pay-as-bid auction. Sales volumes are defined in the same way as for a uniform price auction, but the payment is made to each participant according to the prices specified in her bid. This form was used for the electricity market in England and Wales, as well as in Russia in the capacity market. As a trivial argument in its favor, we note that, for fixed bids, the sales price for consumers is less than under the uniform price auction. However, this form has serious drawbacks. Rational behavior of participants is significantly different from the above options. Even under conditions of perfect competition, submission of a bid corresponding to real costs is unreasonable. The optimal strategy for a producer is to calculate the competitive equilibrium price and to offer at this price the supply function value. Given the incompleteness of the information, it is practically impossible. In the case of imperfect competition, the Nash equilibrium in the corresponding game typically does not exist, because the auction is similar to the Bertrand-Edgeworth model of price competition (see Wolfram, 1999, Vasin et al., 2003).

Proposition 11. *For the game corresponding to the pay-as-bid auction, the Nash equilibrium exists if and only if the Cournot price coincides with the Walrasian price.*

The latter condition typically does not hold. This situation is pushing sellers to conclude cartel agreements as a means to ensure the stable operation of the market. This, of course, increases their bargaining power. Therefore, in our opinion, everyone should agree with C. Wolfram who does not recommend this type of an auction.

7. Forward market

The final part of our survey is devoted to the impact of the forward market on the market power of large companies. In the literature, the forward market is studied as one of the most important mechanisms to reduce the market power. One of the first papers investigating its impact on the level of competition in oligopolistic markets is Allaz and Vila (1992). They consider a symmetric duopoly. Two producers compete in N rounds of forward sales, and then in the spot market. The prices at all stages are equal proceeding from no arbitration condition. The results show that the introduction of forward markets increases competition among producers, as well as social welfare. As the number of steps N tends to infinity, the SPE outcome tends to the competitive equilibrium. Hughes and Kao (1997) show that this result can be achieved only under assumption of firms' forward positions being perfectly observed, and that in case of firms' positions not being transparent the Cournot outcome arises.

Mahenc and Salanie (2004) show that under Bertand-Edgeworth competition at the spot market, a possibility of forward cotracts may increase the market power and reduce the social welfare. They establish that at the equilibrium each producer buys forwards on its own production in order to increase the spot price.

Bushnell (2005) considered a two-stage Cournot auction with a constant marginal cost, and showed that the ability to make forward contracts reduces the bargaining power of producers as well as an increase in their number in the market from n to n^2.

Note the following problems related to the latter study. First, the actual price trends in the electricity markets are not consistent with the hypothesis of equality of prices in the spot and forward markets. Usually the price in the spot market is slightly lower, but sometimes there are jumps in which the spot price significantly exceeds the price in the forward market. The second problem relates to the assumption of the priority of consumers with high reserve prices when buying goods in the forward market. It is hard to imagine the possibility of such a distribution of consumers without special rationing which does not exist in real markets.

Vasin et al. (2009) and Vasin and Daylova (2012) consider a two-stage model with a random market price in the spot market. We take into account the presence of risk-neutral arbitrageurs, the competition between them leads to equality of the forward price to the spot price expectation. Consumers operate under conditions of perfect competition and are free to choose between the spot and forward markets. Our model describes a strategic interaction between producers, consumers and arbitrageurs. We find the optimal strategies of rational consumers, depending on the reserve price and the parameter characterizing risk aversion. We examine properties of the subgame perfect equilibrium (SPE) for the model under the assumption that the proportion of risk-preferring consumers with high reserve prices is constant.

In our model at the equilibrium the producers employ correlated mixed strategies, and the corresponding outcome is random: the expected (rather than actual) spot market price coincides with the price in the forward market. Consumers with low reserve prices buy goods at the spot market if the price is lower than their reserve price, otherwise they refuse the purchase. The risk-preferring consumers with high reserve prices always buy goods at the spot market. Risk-averse consumers buy in the forward market if their reserve price is higher than the forward price and the risk aversion parameter is above a certain threshold.

Fluctuations of the spot price are usually explained by the existence of random external factors. Our model shows that external factors are not necessarily the main reason. In the game describing the spot market there are two local equilibria. The first (with the low price) corresponds to the steep slope of the residual demand ($p < pf$, "bear market"). The second (with the high price) corresponds to the small slope of the residual demand ($p > pf$, "bull market"). In the subgame perfect equilibrium, the "bear market" with the low spot prices realizes often, the "bull market" with the high prices is seldom.

Finally, we estimate the reduction in the market power of producers owing to forward contracting.

7.1. The model of interaction on a two-stage market.

Consider a symmetrical oligopoly with constant marginal costs c. Let n producers present on the market. Each consumer b wants to purchase a unit of the product and is characterized by the reserve price r_b. The demand function $D(p)$ is defined by the distribution density $\rho(r)$ of the consumers: $D(p) = \int_{\rho}^{r_{\max}} \rho(r)dr$.

Risk-neutral arbitrageurs either sell contracts on the forward market and then purchase the product on the spot market, or perform the inverse operation in the conditions of perfect competition.

Trading runs in two stages, on the forward market, firms supply their sales volumes $q_a^f, a \in \overline{1,n}$. Denote by q^f the total volume supplied by all producers on the forward market, by q_{arb} - the volume supplied by arbitrageurs on the forward market, $q_{arb} > 0$. In the case when arbitrageurs first purchase contracts on the forward market and then sell the product on the spot market, the quantity q_{arb} shows the volume purchased by them on the forward market, $q_{arb} < 0$.

Each consumer b who decides to participate in trading on the forward market specifies the reserve price r_b in her ask and purchases the product if the market price does not exceed the reserve price. Denote by $D^f(p)$ the demand function of consumers on the forward market. The price on the forward market p^f follows from the condition $D^f(p^f) = q_t^f \stackrel{def}{=} q^f + q_{arb}$.

The spot market operates as the Cournot auction with the residual demand function $D^s(p)$ of consumers: $D^s(p) = D(p) - q_t^f$ for $p < p^f$, and $D^s(p) = D(p) - D^f(p)$ for $p > p_f$. Producers supply the sales volumes $q_a^s, a \in \overline{1,n}$ on the spot market. The spot price p^s balancing the demand and supply meets the condition $D^s(p^s) + q_{arb} = \sum_{a=1}^n q_a^s$.

Random events occur between trading on the forward market and trading on the spot market. Let a random factor possess values $i \in \overline{1,k}$ with probabilities $w_i > 0$, $\sum_{i=1}^n w_i = 1$. The strategy of a firm is defined by the set $(q_a^f, q_a^s(i); i \in \overline{1,k})$ that specifies the volumes of supplies on the forward and spot markets. The price on the spot market represents a random variable and can be found from the expression $D^s(p_i + q_{arb} = \sum_{a=1}^n q_a^s(i), i \in \overline{1,k})$. Let $p_{\min} = p_1 < p_2 < \ldots < p_k = p_{\max}$.

Since the condition of perfect competition holds true for arbitrageurs and consumers, their strategies meet the principle of economic equilibrium (Makarov, 1973).

The equilibrium condition for arbitrageurs implies that the price on the forward market equals the mathematical expectation of the price on the spot market: $p^f = \mathsf{E}(p^s)$.

Find the optimal strategies of consumers taking into account their attitude towards risk. For consumer $b \in B$, the attitude is described by the parameter $\lambda_b \in [\lambda_{\min}, \lambda_{\max}]$, $\lambda_{\min} < 0 < \lambda_{\max}$. The positive domain of λ_b reflects risk-averse consumers. Next, the zero value of λ_b corresponds to risk-neutral consumers. And finally, the negative domain of λ_b characterizes risk-seeking consumers. The utility function of consumer b depends on the price difference Δ_b and the parameter λ_b: $U_b(p) = U(\Delta_b, \lambda_b)$ where $\Delta_b = r_b - p$. If $\Delta < 0$, we obtain $U(\Delta, \lambda) = 0$ for any λ, since purchasing fails. Under $\Delta \geqslant 0$, the function $U(\Delta, \lambda)$ is concave in Δ for risk-averse consumers, linear in Δ for risk-neutral consumers and convex in Δ for risk-seeking consumers.

Theorem 1. *The equilibrium behavior of consumers is defined depending on the reserve prices and their attitude towards risk as follows:*

1) *Customers with reserve prices from the interval $r_b < p^f$, as well as risk-neutral and riskseeking consumers with reserve prices from the interval $p^f < r_b < p_{\max}$ purchase on the spot market if $p^s < r_b$.*

2.1) *Let $U'_\Delta(\Delta, \lambda)$ be concave in Δ and consumers with $\lambda = \lambda_{\max}$ choose trading on the forward market. Then for risk-averse consumers with $p^f < r_b < p_{\max}$ there exists a threshold $\lambda(r)$ such that consumers with $\lambda_b > \lambda(r)$ prefer product purchase on the forward market, whereas consumers with $\lambda_b < \lambda(r)$ prefer product purchase on the spot market.*

2.2) *Suppose that the price on the spot market possesses only two values, the condition*

$$(\ln U(\Delta, \lambda))''_{\lambda\Delta} < 0 \tag{7}$$

takes place and $U(\Delta, \lambda_{\max}) \equiv U_{\max}$ for $\Delta > 0$.

Then the optimal behavior of risk-averse consumers with $p^f < r_b < p_{\max}$ is defined by point 2.1 and the function $\lambda(r)$ decreases monotonically from $\lambda_{\max}$ to 0.

3) For risk-seeking consumers with $r_b > p_{\max}$, the optimal behavior is product purchase on the spot market for any realized price. For risk-averse consumers with $r_b > p_{\max}$, the optimal behavior is product purchase on the forward market.

Note 1. For the utility function $U(\Delta, \lambda) = (\frac{\Delta}{\Delta_{\max}})^{1-\frac{\lambda}{2m}}$, $U'_\Delta(\Delta, \lambda)$ is concave in Δ for $\lambda_{\max} \leqslant m$. The condition $U(\Delta, \lambda_{\max}) = U_{\max}$ is valid under $\lambda_{\max} = 2m$, $0 < \Delta \leqslant \Delta_{\max}$, $U_{\max} = 1$. The condition (7) holds true for $\lambda < 2m$, $0 < \Delta \leqslant \Delta_{\max}$.

Theorem 2. *The residual demand function of the consumers in the equilibrium has the form*

$$D^s(p) = \begin{cases} D(p) - q_t^f & \text{for } p < p^f, \\ \int_p^{r_{\max}} \int_{\lambda_{\min}}^{\lambda(r)} \rho(r, \lambda) d\lambda dr & \text{for } p^f < p < p_{\max}, \\ \alpha(p) D(p) & \text{for } p > p_{\max}. \end{cases}$$

For SPE evaluation, we first find the equilibrium strategies of producers at the second stage under fixed q_a^f using the residual demand function.

Theorem 3. *If the model admits a subgame perfect equilibrium and the residual demand function is smooth under $p = p_i, i \in \overline{1,k}$, then the prices $p_1, \ldots, p_k$ meet the conditions:*

$$(p_1 - c)|D'(p_1)| = \frac{D(p_1) - q_t^f + q_{arb}}{n},$$

$$(p_i - c)|D^{s\prime}(p_i)| = \frac{D^s(p_i) + q_{arb}}{n}, i = 2, \ldots, k-1,$$

where

$$D^s(p) = \int_{\rho}^{r_{\max}} \int_{\lambda_{\min}}^{\lambda(r)} \rho(r,\lambda) d\lambda dr,$$

$$(p_k - c)|(\alpha(p_k)D(p_k))'| = \frac{\alpha(p_k D(p_k)) + q_{arb}}{n}. \tag{8}$$

Note 2. If the function $D^s(p)$ is concave under $p > p_f$ then $k = 2$, the value $p_2 > p^f$ meets conditions (8).

Next, consider the SPE when $D(p) = \max\{d(r_{\max} - p), 0\}$, the share of risk-seeking consumers is constant: $\alpha(p) = \alpha$ and the random factor possesses only two values. Consequently, the spot market has two possible realizations of the price, p_1 with a probability w and $p_2 > p_1$ with the probability $1 - w$, $p^f = wp_1 + (1-w)p_2$. Denote by q_a^{si} the volume sold by producer a on the spot market under the realized price $p_i, i = 1, 2$.

Theorem 4. *The SPE prices p_1, p_2, p^f and the volumes q_a^{s1}, q_a^{s2} meet the equations:*

$$\begin{gathered} p_1 = p^* - \frac{q^f}{d(n+1)}, p_2 = p^* - \frac{q_{arb}}{\alpha d(n+1)}, p^f = p^* - \frac{wq^f}{d(n+1)} + \frac{(1-w)q_{arb}}{\alpha d(n+1)}, \\ q_a^{s1} = d\left(\Delta^* - \frac{q^f}{d(n+1)}\right), q_a^{s2} = \alpha d\left(\Delta^* + \frac{q_{arb}}{\alpha d(n+1)}\right). \end{gathered} \tag{9}$$

Here p^* is the Nash equilibrium pricec in the classical Coutnot oligopoly model for this market, $\Delta^* = p^* - c$.

Let us find the equilibrium strategies at the first stage. Consider the demand-supply balance equation in the price p^f:

$$q^f + q_{arb} = D^f(p^* - \frac{wq^f}{d(n+1)} + \frac{(1-w)q_{arb}}{\alpha d(n+1)}). \tag{10}$$

Since the left-hand (right-hand) side of equation (10) increases (decreases, respectively) in q_{arb}, the value $q_{arb}(\overrightarrow{q^f})$ is uniquely defined by (10).

The profit of producer j on the forward market is calculated by the formula

$$\pi_j^f = q_j^f(p^f - c) = q_j^f\left(\Delta^* - \frac{wq^f}{d(n+1)} + \frac{(1-w)q_{arb}(\overrightarrow{q^f})}{\alpha d(n+1)}\right).$$

And the mathematical expectation of the total profit π_j of producer j is described by

$$\begin{gathered} \pi_j(\overrightarrow{q^f}) = q_j^f\left(\Delta^* - \frac{wq^f}{d(n+1)} + \frac{(1-w)q_{arb}(\overrightarrow{q^f})}{\alpha d(n+1)}\right) \\ +wd\left(\Delta^* - \frac{q^f}{d(n+1)}\right)^2 + (1-w)\alpha d\left(\Delta^2 + \frac{q_{arb}(\overrightarrow{q^f})}{\alpha d(n+1)}\right)^2. \end{gathered} \tag{11}$$

Theorem 5. *If the model admits a subgame perfect equilibrium with* $q_j^f > 0, j = \overline{1,n}$*, then the equilibrium volumes* q_j^f *result from the system* $\frac{\partial \pi_j(\overrightarrow{q^f})}{\partial q_j^f} = 0, j = \overline{1,n}$*, where* $\pi_j(\overrightarrow{q^f})$ *and* $q_{arb}(\overrightarrow{q^f})$ *are specified by (10) and (11).*

However, the stated necessary condition is not sufficient in the general case.

For the low-price local equilibrium on the spot market, a deviating producer can increase the equilibrium price by reducing the volume of supply. A possible beneficial deviation is when the new price corresponds to the gentle slope domain of the residual demand function.

The produser gains the profit $\pi_1 = q_a^{s1}(p_1 - c) = q_{new} = \alpha d(p^{new} - c)$, where the new price p^{new} follows from the equality $(n-1)q_a^{s1} + q^{new} = \alpha(\overline{D} - dp^{new}) + q_{arb}$. And the equilibrium profit is $\widetilde{\pi}_2 = q^{new}(p^{new} - c) = \alpha d(p^{new} - c)^2$. The equilibrium is stable if $\pi_1 \geqslant \widehat{\pi_2} \Longleftrightarrow d(p_1 - c)^2 \geqslant \alpha d(p^{new} - c)^2 \Longleftrightarrow$

$$\left(n - 1 + 2\sqrt{\alpha}\right) \frac{p_1(w,n,\alpha) - c}{\Delta^*} \geqslant (n+1)\alpha + \frac{q_{arb}(w,n,\alpha)}{d\Delta^*}. \tag{12}$$

For the high-price local equilibrium on the spot market, a producer may benefit by a unilaterial deviation from the evaluated strategy so that the new price corresponds to the steep slope domain on the residual demand function. In the local equilibrium, the producer obtains the profit $\pi_2 = q_a^{s2}(p_2 - c) = \alpha d(p_2 - c)^2$. The optimal volume of supply under the deviation makes up $\overline{q^{new}} = d(\overline{p^{new}} - c)$, where the new price $\overline{p^{new}}$ follows from the equality $(n-1)q_a^{s2} + \overline{q^{new}} = \overline{D} - d\overline{q^{new}} - q^f$. And the resulting profit is $\widetilde{\pi_1} = \overline{q^{new}}(\overline{p^{new}} - c) = d(\overline{p^{new} - c})^2$. The equilibrium is stable if $\pi_2 \geqslant \widehat{\pi_1} \Longleftrightarrow \alpha d(p_2 - c)^2 \geqslant d(\overline{p^{new}} - c)^2 \Longleftrightarrow$

$$\frac{p_2(w,n,\alpha) - c}{\Delta^*}\left(2\sqrt{\alpha} + (n-1)\alpha\right) \geqslant n + 1 - \frac{nq_a^f(w,n,\alpha)}{d\Delta^*}. \tag{13}$$

Under $q_{arb} > 0$, the total residual demand function equals q_{arb} for sufficiently large p. Therefore, for stability it is necessary that a separate player appears unable to reduce the supply of products below this level (by decreasing its volume of production). In other words, the following condition must hold satisfied: $q_{arb} \leqslant (n-1)q^{s2} \Longleftrightarrow$

$$\frac{q_{arb}(w,n,\alpha)}{d} \leqslant (n-1)(p_2(w,n,\alpha) - c)\alpha. \tag{14}$$

The range $w_1(n,\alpha), w_2(n,\alpha)$ of the parameter w, where local equilibria represent an SPE, is defined by (12) - (14) (see Table 3). The value $w_1(n,\alpha)$ results from the expression

$$\frac{p_2(w,n,\alpha) - c}{\Delta^*}\left(2\sqrt{\alpha} + (n-1)\alpha\right) = n + 1 - \frac{nq_a^f(w,n,\alpha)}{d\Delta^*}.$$

The value $w_2(n,\alpha) = \min w_2^1, w_2^2$, where w_2^1 is determined from the equation

$$\left(n - 1 + 2\sqrt{\alpha}\right) \frac{p_1(w,n,\alpha) - c}{\Delta^*} = (n+1)\alpha + \frac{q_{arb}(w,n,\alpha)}{d\Delta^*},$$

and w_2^2 is specified by the equality

$$\frac{q_{arb}(w,n,\alpha)}{d} = (n-1)(p_2(w,n,\alpha)-c)\alpha.$$

Table 3: The admissible range of the parameter w

	$\alpha = 0.1$		$\alpha = 0.5$		$\alpha = 0.9$	
	w_1	w_2	w_1	w_2	w_1	w_2
$n = 2$	-	-	0.4741	0.7185	-	-
$n = 3$	**0.6560**	**0.8105**	0.4328	0.7574	0.3481	0.6812
$n = 4$	**0.6386**	**0.9171**	0.4113	0.7759	0.3270	0.7018
$n = 5$	**0.6275**	**0.9227**	0.3981	0.7868	0.3144	0.7141
$n = 6$	**0.6198**	**0.9263**	0.3892	0.7939	0.3059	0.7223
$n = 7$	0.6141	**0.9287**	0.3828	0.7990	0.2998	0.7281
$n = 8$	0.6098	**0.9306**	0.3779	0.8028	0.2953	0.7325
$n = 9$	0.6064	**0.9319**	0.3741	0.8057	0.2917	0.7359
$n = 10$	0.6036	0.9330	0.0311	0.8080	0.2889	0.7386

Table 4 characterizes the reduction in the market power of producers due to introduction of the forward market. It contains the ratio $\frac{p^f(w,n,\alpha)-c}{\Delta^*}$, hereinafter referred to as the reduction coefficient of the market power. This coefficient is obtained using (9) and the values $w = w_1$, $w = w_2$ from Table 3.

Table 4: The reduction coefficient of the market power

	$\alpha = 0.1$		$\alpha = 0.5$		$\alpha = 0.9$		Bushnell
	w_1	w_2	w_1	w_2	w_1	w_2	
$n = 2$	-	-	0.7019	0.6629	-	-	0.6
$n = 3$	**0.7551**	**0.6617**	0.5129	0.4545	0.4169	0.4084	0.4
$n = 4$	**0.6509**	**0.4252**	0.3991	0.3383	0.3093	0.3009	0.29
$n = 5$	**0.5728**	**0.3390**	0.3249	0.2671	0.2441	0.2364	0.23
$n = 6$	**0.5117**	**0.2805**	0.2734	0.2197	0.2009	0.1939	0.18
$n = 7$	0.4625	**0.2387**	0.2356	0.1862	0.1703	0.1640	0.16
$n = 8$	0.4220	**0.2074**	0.2069	0.1614	0.1477	0.1420	0.13
$n = 9$	0.3880	**0.1832**	0.1843	0.1423	0.1303	0.1251	0.12
$n = 10$	0.3591	0.1639	0.1661	0.1272	0.1165	0.1117	0.1

The feasibility of forward contracting appreciably decreases the market power of producers. Here possible cases are $q_{arb} > 0$ (boldfaced in the tables) and $q_{arb} < 0$. Clearly, the results demonstrate that, as the probability of low-price outcome on the spot market goes up, the market power of producers is reduced. In addition, the growing share of risk-seeking consumers also reduces the market power of producers. Table 5 shows the ratio of sales volumes on the forward and spot markets. This ratio has been calculated using (9) and the values $w = w_1$, $w = w_2$ from Table 3. Obviously, producers sell the prevailing volume of products on the forward market.

Table 5: The ratio of sales volumes on the forward and spot market

	$\alpha = 0.1$		$\alpha = 0.5$		$\alpha = 0.9$	
	w_1	w_2	w_1	w_2	w_1	w_2
$n = 2$	-	-	1.4384	1.2828	-	-
$n = 3$	**3.3408**	**2.5790**	2.4759	2.2941	2.0708	2.0516
$n = 4$	**4.4422**	**3.5697**	3.4944	3.3072	3.0731	3.0552
$n = 5$	**5.5089**	**4.5668**	4.5053	4.3179	4.0744	4.0580
$n = 6$	**6.5562**	**5.5660**	5.5123	5.3262	5.0753	5.0602
$n = 7$	7.5916	**6.5661**	6.5173	6.3328	6.0760	6.0619
$n = 8$	8.6186	**7.5664**	7.5210	7.3381	7.0764	7.0633
$n = 9$	9.6400	**8.5669**	8.5238	8.3425	8.0768	8.0644
$n = 10$	10.6578	9.5675	9.5259	9.3461	9.0771	9.0654

8. Conclusions and Discussion

The performed analysis of the ratio of the equilibrium prices and sales volumes for the two-stage market and the classical Cournot oligopoly testifies that introduction of the forward market appreciably restricts the market power of firms. Moreover, the growing share of risk-seeking customers also reduces the market power of producers.

Another mechanism with nice properies is Vickrey auction, especialy its modification with account of information on producer characteristics. The corresponding equilibrium in dominating strategies provides the maximal total welfare and substantial price reduction in fawour of consumers. The both mechanisms may be recommended for increasing the efficiency of markets for homogenious goods.

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