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Evolutionary Model of Tax Auditing

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Abstract Tax audit is repeated process which occurs annually. Therefore, we consider a dynamic model of tax auditing in this work.
It is supposed that each taxpayer can declare his income less or equal to his true level of income after every tax period. To simplify the model it is also supposed that taxpayers have only two levels of income — low and high. The tax authority assumed to use some statistical information about taxpayers' evasions from the previous tax periods and to choose the probability of tax auditing in the current period depends on this information.
Every taxpayer has one of three possible risk-statuses (risk averse, risk neutral and risk preferred) and change it depending on the external conditions. As well as tax audit is a regular action then it can be modeled as an evolutionary process, which take into account a revision of taxpayers' statuses. Several modifications of evolutionary dynamics were considered.

Keywords: tax auditing, tax evasions, risk status, evolutionary games, imitation dynamics.

1. Introduction

Creation of an effective tax system is the actual trend in the mathematical modeling of economical processes, and tax control is one of the most important components in this system.

The game-theoretical models of interaction between the tax authority and the finite number of taxpayers were studied earlier in such works as (Chander and Wilde, 1998), (Vasin and Morozov, 2005) and (Boure and Kumacheva, 2010). The model (Boure and Kumacheva, 2010) is based on a hierarchical game, where the high-level player is the tax authority and the low-level players are finite number of taxpayers. In this model the tax authority assumed to get some statistical information about taxpayers' evasions from the previous tax periods. Earlier the problem of tax auditing using statistical information about taxpayers was considered in (Kumacheva, 2012) (in (Macho-Stadler and Perez-Castrillo, 2002) such information was called "a signal"). In this contribution the probability of tax auditing in the current period is supposed to depend on the mentioned information. This static model, where taxpayers have only two levels of income (low and high), we consider as a basis to our study. In the paper, we extend the basic model and suppose that all considered taxpayers possess one of three statuses, they can be risk averse, risk neutral and risk preferred. These three statuses define the behavior of taxpayers, according to their intentions to evade the tax payment. For example, risk averse

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taxpayers prefer to avoid the punishment from the tax authority, therefore, they pay taxes. Risk preferred taxpayers choose risky behavior and try to evade the tax payment. Risk neutral taxpayers follow to flexible and adaptive behavior, they can behave as a risk preferred or risk averse taxpayers in different conditions. Hence, the entire population of taxpayers can be divided into three subgroups: risk averse, risk neutral and risk preferred, which correspond to taxpayers' statuses.

As well as tax audit occurs regularly, in different groups of taxpayers we present a dynamic model which combine the tools of evolutionary games with the hierarchical static game. Recent literature has demonstrated a certain interest in using game theoretical approach and dynamic modeling to describe the social dilemmas in particular the tax audit problem (Bloomquist, 2006), (Antunes et al, 2006), (Antocia et al, 2014).

In our study we suppose that every taxpayer, receiving a signal, assesses the probability of tax audit and reacts to this information subject to his status. We also introduce a rule, which describes the reaction of a taxpayer to the probability of tax audit and the adaptation to the current social environment. As we mention above, every taxpayer behaves one of the three possible manners, which lead to different bonuses and punishments. Then the taxpayer can shift his behavior and hence his status at the end of the tax period, if current status brings insignificant benefits and the probability of punishment from tax authority is high.

In our contribution we extend the static hierarchical model and formulate a dynamical model, in which individuals from each subgroup have a possibility to observe a situation (population state) after every tax period and at a given signal they can change their statuses.

The paper is organized as follows. In Section 2., we present the static hierarchical model and define the different groups of taxpayers, according to their statuses. Subsection 2.2. presents the behavior of tax authority. In Subsection 2.3., we establish the rule of taxpayers reaction on the behavior of tax authority. In section 3., we present the evolutionary model of annual process of tax audit. In section 4., we present numerical simulation to illustrate the results. The paper is concluded in Section 5.

2. Static model

A hierarchical model of tax control, in which the high-level player of the hierarchy is the tax authority and the low-level players are finite number of taxpayers, is in the basis of the static model. Every taxpayer has true income ξ and declares income η after each tax period, $\eta \leq \xi$.

2.1. Players' strategies and payoffs

To simplify the model let's suppose (as it was done in (Vasin and Morozov, 2005)), that the whole set of taxpayers is divided on two groups – with high and low levels of income (if it is necessary, the number of partitions can be increased, but it does not effect on the following arguments and conclusions). By the other words, taxpayers' incomes can take only two values $\xi \in \{L, H\}$, where L is the low level and H is the high level ($0 \leq L < H$). Declared income η also can take values from the mentioned binary set $\eta \in \{L, H\}$.

Thus, there are three possibilities for every taxpayer, which depend on the true and declared income:

1. $\eta(\xi) = L(L)$;
2. $\eta(\xi) = H(H)$;
3. $\eta(\xi) = L(H)$.

It is obvious that the taxpayers from the first and the second groups declare their income correspondingly to its true level and they do not evade. The third group is the group of tax evaders.

In every tax period the tax authority audits those taxpayers, who declared $\eta = L$, with the probability P_L. In this study we suppose that tax audit is absolutely effective, i. e. it reveals the existing evasion. The simplest (proportional) case of penalty is considered: when the tax evasion is revealed, the evader must pay $(\theta + \pi)(\xi - \eta)$, where θ and π are the tax and the penalty rates correspondingly. For the agents from the studied groups the payoffs are:

$$u\,(L(L)) = (1 - \theta) \cdot L; \tag{1}$$

$$u\,(H(H)) = (1 - \theta) \cdot H; \tag{2}$$

$$u\,(L(H)) = H - \theta L - P_L(\theta + \pi)(H - L). \tag{3}$$

The tax authority gets information about taxpayers from their tax declarations and audits those, who declared $\eta = L$. The fraction of audited taxpayers is P_L. It's obvious that the agents from the first group (who actually have true income $\xi = L$) and the evaders from the third group are in this fraction of audited taxpayers.

The whole set of the taxpayers is divided to the next groups: wealthy taxpayers, who pay taxes honestly ($\eta(\xi) = H(H)$), insolvent taxpayers ($\eta(\xi) = L(L)$) and wealthy evaders ($\eta(\xi) = L(H)$). The diagram, presented in the picture 1.a, illustrates this distribution.

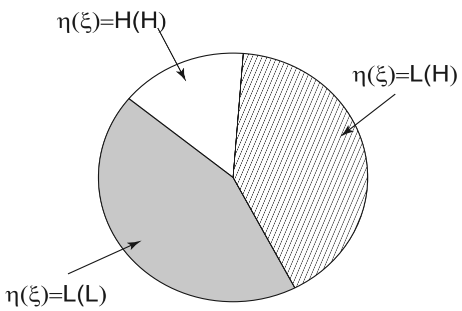


Fig. 1.a.

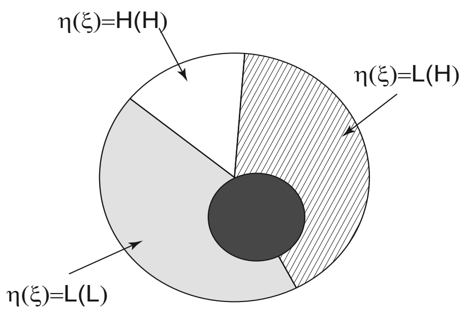


Fig. 1.b.

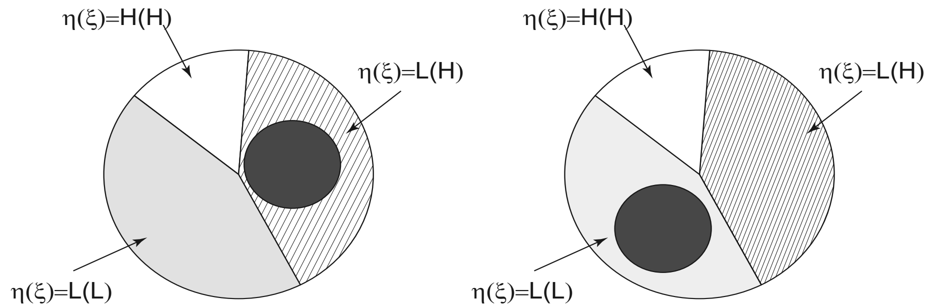


Fig. 1. Auditing of the different groups of the taxpayers

In the figure 1. cases: b, c, d, the little circle, inscribed in the diagram, corresponds to the fraction of audited taxpayers. Let's call it an "audited circle". Only those, who declared $\eta = L$, are in the interest for auditing. Therefore, the mentioned circle is inscribed into the sectors, which correspond to the situations $\eta(\xi) = L(H)$ and $\eta(\xi) = L(L)$.

In the figure 1.b the case, when either evaders or simply insolvent taxpayers are audited, is considered. In the figure 1.c the "audited circle" is contained in the area, which satisfies the condition $\eta(\xi) = L(H)$. This is the optimistic situation, when every audit reveals the existing tax evasion. On the other hand, the figure 1.d illustrates the pessimistic case, when none of the audits reveals the evasion, because only insolvent taxpayers, declared their true income $\eta(\xi) = L(L)$, are audited. Certainly, all presented diagrams illustrate only some boundary ideal cases. But the tax authority is obviously to make the real situation closer to the illustration in the figure 1.c.

Hence, the following arguments, related to the searching of possible tax evasions, apply to the third group of the agents, declared $\eta(\xi) = L(H)$. Thereby, precisely this group is expedient to be considered as the **studied population**, speaking in the terms of the evolutionary games.

It is supposed, that each considered taxpayer can possess one of the three possible risk-statuses: risk averse A, risk neutral N and risk preferred R. Choice of one of three statuses determines the future behavior of the taxpayer, and, moreover, his payoffs. Therefore, each agent belongs to one of three groups (A, N, R), which are the subsets of the set of taxpayers. The similar arguments are transferred from the whole set of taxpayers to the studied population.

2.2. The tax authority chooses its strategy

Let k be the number of current tax period. Let W be the random variable, which characterizes the taxpayers' propensity to tax evasion. Let's suppose, that W is beta-distributed with parameters α_{k-1}, β_{k-1} in the k-th tax period. Let's consider the term a "tax story", which is a characteristic of the taxpayers' behaviour in the previous periods. This method of estimation of the propensity to tax evasion was earlier considered in (Boure and Kumacheva, 2010).

For $k = 1$ let's suppose, that there is no information from the previous periods. When the a-priori information is absent, it is expedient to suppose W as a uniform-distributed (that is a particular case of the beta-distribution with parameters α_0 =

$= 1$, $\beta_0 = 1$). In this case the tax authority audits the taxpayer, declared $\eta = L$, with some fixed probability P_L^1, which can be defined as a quantile of the uniform distribution.

Let's denote as γ_{HL}^k the part of those taxpayers, who declared $\eta = L$ and was found evaded after auditing (it is the fraction of revealed evasions).

The tax authority assumed to have a criteria γ (let's call it "the level of tax crime") for choosing its further strategy. Then the tax story after the first tax period is a result of observation (audit), presented as a Bernoulli-distributed random quantity Y_1:

$$Y_1 = \begin{cases} 1, & \text{if } \ \gamma_{HL}^k \geq \gamma \\ 0, & \text{if } \ \gamma_{HL}^k < \gamma. \end{cases}$$

Let's apply the results of the theorem[1], consisting of the fact that the family of beta distributions is conjugate to the family of Bernoulli distribution. Using the mentioned feature, we obtain that the a posteriori distribution of W is beta-distributed with parameters $\alpha_1 = 2$, $\beta_1 = 1$ if the fraction of evasions γ_{HL}^k is equal or bigger then the critical value γ, and $\alpha_1 = 1$, $\beta_1 = 2$ if the fraction of evasions is less then the critical value.

For the next tax periods ($k > 1$) an observation Y_{k-1} of the previous tax period is considered as a tax story. This is caused by that an a posteriori distribution of the taxpayers' propensity to evasion in the $(k-1)$-th period becomes an a priori distribution for the k-th period.

It is reasonable to assume that in each tax period k the value of P_L^k (which is the probability of audit those who declared $\eta = L$) is chosen as a quantile of the a posteriori distribution of W (which is the distribution of tax evasions after the previous audit).

2.3. Impact of the tax authority's strategy on the behaviour of population

Risk neutral taxpayers' behaviour supposed to be absolutely rational: their tax evasion is impossible only if the risk of punishment is so high that the tax evader's profit is less or equal to his expected post-audit payments (in the case when his evasion is revealed):

$$P_L(\theta + \pi)(H - L) \geq \theta(H - L).$$

The critical value of audit probability P_L (due to the taxpayer's decision to evade or not) is

$$P_L^* = \frac{\theta}{\theta + \pi}. \tag{4}$$

Let's also assume the existence of "sensitivity thresholds": $\underline{P_L}$ is a sensitivity threshold for risk averse taxpayers and $\overline{P_L}$ is a sensitivity threshold for risk preferred taxpayers. Thus, the audit probability's segment of values $[0, 1]$ is divided to the subsets, which are presented in the table 1. It is reasonable to assume that

$$0 \leq \underline{P_L} < P_L^* < \overline{P_L} \leq 1.$$

[1] See (De Groot, 1974, p. 165)

Pure strategies for every risk status are the reactions on the tax authority's strategy P_L. They are presented in the next equations:

$$A: \ \eta_A = \begin{cases} H, \, P_L \geq \underline{P_L}; \\ L, \ P_L < \underline{P_L}. \end{cases} \tag{5}$$

$$N: \ \eta_N = \begin{cases} H, \, P_L \geq P_L^*; \\ L, \ P_L < P_L^*. \end{cases} \tag{6}$$

$$R: \ \eta_R = \begin{cases} H, \, P_L \geq \overline{P_L}; \\ L, \ P_L < \overline{P_L}. \end{cases} \tag{7}$$

where A, N, R are the pure strategies of the corresponding risk statuses. Hence, let's denote the players' payoffs, obtained on the pure strategies A, R, N, as u_A, u_R, u_N.

Table 1: Impact of the tax authority's strategy on the behavior of population

Values of P_L	Evaders	Declared $\eta = \xi$
$[0, \underline{P_L})$	everybody (A, R, N)	nobody
$[\underline{P_L}, P_L^*)$	R, N	A
$[P_L^*, \overline{P_L})$	R	A, N
$[\overline{P_L}, 1]$	nobody	everybody (A, R, N)

3. Stochastic evolutionary model

We model repeated process of tax audit, taking into consideration two main hypothesis of the evolutionary modeling: myopia and inertia. Here **myopia** means that any agent, who receives the "signal" can revise his current status and payoff opportunities, but in the same time he does not attempt to involve his beliefs about the future population's state. In addition, when population is large, then each taxpayer acts anonymously and independently. **Inertia** means that individual taxpayer does not reevaluate his status regularly, and receive this possibility only occasionally (Weibull, 1995), (Sandholm, 2010).

We assume that in well-mixed population of economic agents, taxpayers and tax auditors the series of random matches are occurred. Every random strategical interaction between agents is described by an instant hierarchical game presented in the section 2. Pure strategies for each players are defined by (5), (6), (7). We formulate the evolutionary game, which describes the changes of the number of taxpayers, who have different risk-statuses over the long run period.

The strategy of the tax authority is the selection of probability P_L in every tax period. The population state, which is formed by the fractions of risk-averse, risk-neutral and risk-preferred taxpayers, can be changed depending on the choice of strategy of tax authority, as far as the probability P_L determines the frequency of tax audits and then it indirectly indices the changes of taxpayers status. We say that the strategy of tax authority influence the behavior of taxpayers indirectly, because they usually do not have a reliable information about the time of tax audit and can only estimate the probability of this event. Taxpayers can analyze historical date about previous tax audits. In other words, they study the "tax story" and, having information about the principle of the construction of tax authority strategies, they react to the tax authority's choice. Here the reaction of taxpayer is the transition from one status to another, according to the rules (5), (6), (7).

Let say that every tax period each taxpayer receives a signal that he has the ability to observe the current economic situation or the population state and, probably, change his own status. This signal maybe interpreted as the "stochastic alarm" which is presented in (Sandholm, 2010). The individual time alarms are independent. For example, as a signal we can consider information about deadline of tax payments.

Since the tax authority chooses the taxpayer for auditing at random from different subgroups, then, as a result of the series of instant games, we obtain the results of the audit of the entire population of taxpayers. The outcome of the current tax audit updates the "tax story", and in the next period population agent will take into account new data.

Thus, we have the evolutionary process of the changes of the population state, which is caused by the adaptation of the taxpayers behavior to the updated economic environment. If a taxpayer switches on another status, then he transfers to the new subgroups, and thus the qualitative structure of the population is changed. This population process resembles an evolutionary game. Therefore, we can use the tools of evolutionary game theory, such as stochastic evolutionary dynamics, to describe the changes in the taxpayers behavior.

Below we present the general formalization of the evolutionary game and the stochastic procedure of adaptation of taxpayers behavior.

Let $G = \langle n, K, u \rangle$ be the population game, where $K = \{A, R, N\}$ is the set of pure strategies, n Is the total number of evaders in population and u is payoff function. According to general formulation, the size of population is large but the quantity of population agents is finite. So, as we mentioned above, there are three different subgroups of evaded taxpayers in the population: risk averse, risk neutral and risk preferred taxpayers. It is assumed that taxpayers in each subgroup follow one pure strategy A, R or N. Let $\nu_i(t),\ i = A, R, N$ be the number of evaders in each subgroup, then $n = \sum_i \nu_i(t),\ i = A, R, N$.

Variable $x_i = \dfrac{\nu_i(t)}{n}, i = A, R, N$ is the fraction of taxpayers in relevant subgroup. Due to the evolutionary modeling, we suppose that a taxpayer's status defines his behavior or strategy, hence, in the game G we have three strategies – A, N and R.

Vector $x = (x_A, x_R, x_N)$ determines the state of the population of evaded taxpayers, the $u(x) = (u(x_A), u(x_R), u(x_N))$ is the payoff vector of the entire population at the state x. Variables $u(x_A)$, $u(x_R)$, $u(x_N)$ are defined by (1), (2), (3).

The updating procedure, which is called revision protocol, combines the aggregate behavior and current payoffs of taxpayers as input and its output is the conditional switching rate from strategy from one behavior to another. We use this procedure to characterize how frequently taxpayer, having status i and reviewing switching statuses, switch to status j for given population state and current payoff vector.

Definition 1. Revision protocol is a map ρ: $R^n \times X \rightarrow R_+^{n\times n}$. The scalar $\rho_{ij}(u_i(x), x)$ is called the conditional switch rate from strategy $i \in K$ to strategy $j \in K$ given payoff vector $u(x)$ and population state x.

Revision protocol is a flexible tool, which allows to accommodate a wide range of decision rules. Here the evolutionary game describes a strategic environment and revision protocol defines the adaptation procedure. Together, revision protocol ρ and evolutionary game G, define a stochastic evolutionary process, which models adaptation of taxpayers status under the pressure of tax authority.

According to the rule, which is presented in (Sandholm, 2010) and (Weibull, 1995) revision protocol generates a stochastic evolutionary dynamics.

Definition 2. Let G be a population game, and let ρ be a revision protocol, then mean dynamics corresponding to G and ρ is:

$$\dot{x}_i = \sum_{j\in K} x_j \rho_{ji}(u_j(x), x) - x_i \sum_{j\in K} \rho_{ij}(u_i(x), x), \tag{8}$$

where the first term corresponds to the income flow to the subgroup i from any others subgroups, and the second term describes the outcome flow from i to j subgroup. Equation (8) describes the general mean dynamics, where revision protocol ρ_{ji} can be adapted to the particular imitation rule, which is appropriate to considering model.

In this paragraph we choose two variations of revision protocol, which lead to different rules of adaptation depending on taxpayers' preferences. Taxpayer, who receive the opportunity to change his status, following a revision protocol, chooses an opponent at random and switches from status i to status j according to the conditional rate ρ_{ij}. In other words the taxpayer can compare his behavior with the behavior of the random agent. If the exampled strategy gives better payoff then he changes his status (strategy).

Pairwise imitation revision protocol and pairwise comparison dynamics. Here we present the first imitation rule which can be used by taxpayers. Suppose that after receiving a revision opportunity a taxpayer chooses a random opponent to imitate. Revising taxpayer switches to example strategy only if opponent's payoff is higher than his own.

This imitation rule is called pairwise comparison revision protocol $\rho : R \to R_+$ and it is defined as follows:

$$\rho_{ij}(u(x), x) = x_j \phi(u_j(x) - u_i(x)), \tag{9}$$

where function $\phi : R \to R_+$ is

$$\phi = \begin{cases} 0, & \text{on } (-\infty, 0], \\ \text{is increasing}, & \text{on } [0, \infty)\,. \end{cases}$$

By using (9) we receive the imitation dynamics to describe the stochastic evolutionary process in population:

$$\begin{aligned} \dot{x}_i = \sum_{j\in K} x_j (\rho_{ji}(u(x), x) - x_i \sum_{j\in K} (\rho_{ij}(u(x), x) = \\ = x_i \sum_{j\in K} x_i (\phi(u_i(x) - u_j(x)) - \phi(u_j(x) - u_i(x))) = \\ = \sum_{j\in K} x_j [u_i(x) - u_j(x)]_+ - x_i \sum_{j\in K} [u_j(x) - u_i(x)]_+, \ i \in K. \end{aligned}$$

Imitation of success. Imitation dynamics of successful agents. Here when revising taxpayer receives an opportunity to change his strategy, then he imitates status (strategy) with probability proportional to weight function $\omega(u_j(x))$. If the agent does not imitate the opponent, he draws a new opponent at random and repeats the procedure.

This revision protocol is called imitation of success rule $\rho : R \to R_+$ and it is defined as follows:

$$\rho_{ij}(u(x), x) = \frac{x_j \omega(u_j(x))}{\sum\limits_{h \in K} x_k \omega(u_k(x))}, \quad \sum_{h \in K x_k} \omega(u_k(x)) > 0. \tag{10}$$

By using (10) we receive the imitation dynamics to describe the stochastic evolutionary process in population:

$$\dot{x_i} = \left(\sum_{j \in K} \frac{\omega[u(e^j, x), x] x_j}{\sum\limits_{p \in K} \omega[u(e^j, x), x] x_p} - 1 \right) x_i.$$

4. Numerical simulation

In our numerical simulation we use the distribution of the income among the population of Russian Federation in April of 2011[2].

Table 2: The distribution of income among taxpayers

1	less 4200
2	4200 – 10600
3	10600 – 20200
4	20200 – 30000
5	30000 – 40000
6	40000 – 50000
7	50000 – 75000
8	more 75000

In the studied model two levels of income (L and H) are considered. Let's suppose:

- for L (income level from 0 to 75000): income is uniform distributed;
- for H (income level more than 75000): income is Pareto-distributed.

The mathematical expectations of these distributions are considered as the values of L and H.

[2] See The web-site of the Russian Federation State Statistics Service, 2011

Let's recall that the density $f(x)$ and function $F(x)$ of the uniform distribution of the value X on the interval $(b-a, b+a)$ are defined by the next way[3]:

$$f(x) = \begin{cases} \frac{1}{2a}, & \text{if } \ |x-b| \leq a, \\ 0, & \text{if } \ |x-b| > a, \end{cases}$$

$$F(x) = \begin{cases} 0, & \text{if } \quad x < b-a, \\ \frac{1}{2a}(x-b+a), & \text{if } \ |x-b| \leq a, \\ 1, & \text{if } \quad x > b+a, \end{cases}$$

The mathematical expectation MX of the uniform distribution is $MX = \frac{a}{a-1} \cdot b$.

The Pareto distribution[4], which is often used in the modeling and prediction of an income, has the next density

$$f(x) = \begin{cases} \frac{ab^a}{x^{a+1}}, & \text{if } \ x \geq b, \\ 0, & \text{if } \ x < b, \end{cases}$$

function

$$F(x) = \begin{cases} 1 - \left(\frac{b}{x}\right)^a, & \text{if } \ x \geq b, \\ 0, & \text{if } \ x < b, \end{cases}$$

and the mathematical expectation $MX = \frac{a}{a-1} \cdot b$.

The scatter of income levels in the group with the highest income may be extremely wide. Therefore, as a value of parameter of the distribution we consider $a = 2$: higher or lower values significantly postpone or approximate average value to the lower limit of income.

After calculations, we use the following values of the income levels L and H:

Table 3: The values of the income levels L and H

L	less 75000	37500
H	more 75000	150000

As the values of the tax and penalty rates let's consider $\theta = 0.13$ (which corresponds to the income tax rate in the Russian Federation) and $\pi = 0.13$ correspondingly. The optimal value of the audit probability P_L^* is obviously obtained from (1) with a given ratio of the parameters θ and π. The values of the threshold probabilities $\underline{P_L}$ and $\overline{P_L}$ are obtained as a result of the assumption of very low fractions of risk-preferred and risk-averse. The value $\underline{P}_L = 0.1$ was obtained from the simulation where $\gamma_{HL}^k < \gamma$ in three passed audits, and the value $\overline{P}_L = 0.9$ was obtained from the simulation where $\gamma_{HL}^k \geq \gamma$ in three passed audits.

Using for the calculation the equations (1) − (3) and considered values of parameters, we obtain the values of payoffs functions, depending on the taxpayer's decision to declare or not to declare their incomes honestly. These values are $u(L(H)) \ = =$ 145.125 and $u(H(H)) = 139.50$ correspondingly.

[3] See Kendall and Stuart, 1966, p. 81
[4] See Kendall and Stuart, 1966, p. 82

We calculate the payoffs of taxpayers from each subgroup, taking into account that all pure strategies are the functions of probability P_L, and show the algorithm below:

- If $P_L \geq \underline{P_L}$ then the payoff on the strategy A is $u_A = u(H(H))$, else $u_A = u(L(H))$;

- If $P_L \geq P_L^*$ then the payoff on the strategy N, $u_N = u(H(H))$, else $u_N = u(L(H))$

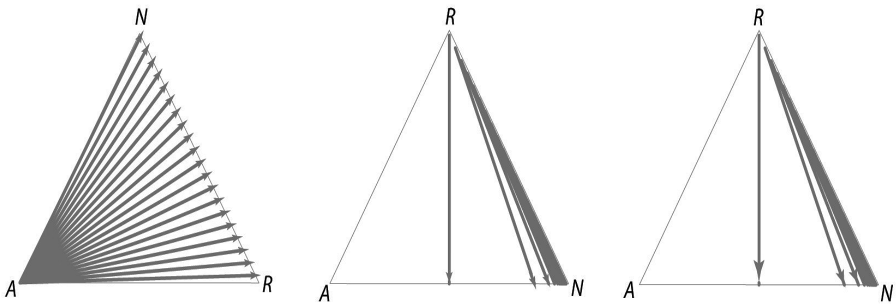
- If $P_L \geq \overline{P_L}$ then the payoff on the strategy R, $u_R = u(H(H))$, else $u_R = u(L(H))$.

The evolutionary process of the regular tax audit is modeled by using the imitation dynamics of pairwise comparison (10):

$$\begin{aligned}
\tfrac{d}{dt}x_A(t) &= [x_R(t)((u_R - u_A) - (u_A - u_R)) + x_N(t)((u_N - u_A) - (u_A - u_N))]x_A(t);\\
\tfrac{d}{dt}x_R(t) &= [x_A(t)((u_A - u_R) - (u_R - u_A)) + x_N(t)((u_N - u_R) - (u_R - u_N))]x_R(t);\\
\tfrac{d}{dt}x_N(t) &= [x_A(t)((u_A - u_N) - (u_N - u_A)) + x_R(t)((u_R - u_N) - (u_N - u_R))]x_N(t);
\end{aligned} \tag{11}$$

In figure 2, for parameters values $H = 150$, $L = 37.5$, we present the trajectories of the system (11) and show the stationary states.

If probability of tax audit is $P_L = 0.25$, then all trajectories converge to the bound RN, that means that all taxpayers are distributed between two statuses "Risk neutral" and "Risk preferred". If audit probability is $P_L = 0.6$ or $P_L = 0.75$, then all trajectories of the system converge to the bound AN, then another two statuses prevail in the population: "Risk neutral" and "Risk averse". However, in this case the distribution of taxpayers between statuses is not uniform and depending on the initial states. Taxpayers will prefer "Risk neutral" status.



$P_L = 0.25$ $P_L = 0.6$ $P_L = 0.75$

Fig. 2. Numerical simulations of the evolutionary process of tax audit

That is to say that two ideal cases were considered. In the first case the penalties assumed to be high enough in comparison with the tax rates. In the second case the certain model of the "ideal" community, in which there is no need in penalties, is considered.

As the results of this numerical simulation we obtained that the trajectories of the system converge to the bound NR in the case when the probability $P_L^* > 0.5$, and they converge to the bound AN (as it was shown in the second picture) in another case.

5. Conclusion

In this paper, we have studied an epidemic model of regular tax audit process, that takes into account the stochastic evolutionary dynamics. By using static hierarchical game between tax authority and taxpayers, we have introduced a new rule of the behaviour of taxpayers, which depends on the "tax story" and reformulated the strategy of tax authority. We use two stochastic dynamics which define the imitation process of changes the state of population of taxpayers and show how taxpayers change their statuses over the long-run period.

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