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Models of Concordance of Public and Private Interests in Control Systems

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Abstract The problem of resource allocation is considered by taking into account the allocation among public and private agent's interests.The problem of concordance of public and private interests (CPPI) in several forms is considered, the comparative characteristics of different approaches to the problem in terms of social welfare are found. The values of price of anarchy and social price of anarchy for different private and public interests functions are calculated. The problem of system compatibility is considered, preliminary results on system compatibility mechanisms of economic compulsion and impulsion and administrative compulsion in CPPI-models are obtained. In the most cases, system compatibility in CPPI-models means that all agents are either pure individualists or pure collectivists.

Keywords: system compatability, hierarchy, union in coalition, public purpose, private purpose, .

1. Introduction

The problem of resource allocation is considered by taking into account two aspects:
1) resource allocation among public and private agents' interests;
2) maximization of welfare by taking into account individual interests.
Germeier and Vatel (Germeyer, Vatel, 1975) considered a system in which there are N subjects with objective functions

$$f_i(x_i) \to \max,$$

where variable x_i is controlled by the i-th subject. In addition to these objective functions there is a public purpose which is described by a function $F(y_1, y_2, ..., y_n)$ which depends on actions of all players, where y_i is a resource which the i-th subject assigns for the public purpose. Let a_i be some aggregate resource, which is available to the i-th subject. So, each of the subjects must share his resource: a part of it y_i is assigned for the public purpose and the rest x_i for the private purpose, i.e.: $x_i + y_i = a_i$.
It was shown that if the objective functions of all agents have the form of convolution by minimum of the public and private purposes

$$G_i(x_i, y_i) = \min(f_i(x_i), F(y_1, y_2, ..., y_n)),$$

then in the game there is a Pareto-optimal Nash equilibrium (Germeyer, Vatel, 1975).
It is well known that the social welfare value in the case of egoistic behavior of

independent active agents is often less than the one when their cooperative actions are agreed. The quantitative aspect of this problem was introduced by Ch. Papadimitriou (Papadimitriou, 2001) and called "price of anarchy" which represents the ratio of the worst value of welfare function on the set of Nash-equilibria to its optimal value.
In the case when price of anarchy is very far from one, it is necessary to coordinate agents' actions to increase social welfare. This formulation of the problem is closed to the base problem of mechanism design (Nisan etc, 2007, Novikov, 2013). But in mechanism design a more restricted problem is considered: how to induce agents to express their true preferences, i.e. to build strategy-proof mechanisms.
The aim of our work is to investigate the price of anarchy in models of concordance of public and private interests (CPPI-models). These models are based on Germeier-Vatel idea but use the linear convolution instead of minimum:

$$G_i(x_i, y_i) = f_i(x_i) + s_i F(y_1, y_2, ..., y_n),$$

Each player gets some share s_i of social welfare. Also the concept of system compatibility is introduced which means that individually optimal controls form globally optimal social welfare. Administrative and economic mechanisms with or without feedback are proposed.
The rest of the paper is organized as follows. In the section 2 the considered model is described. In the section 3 different types of game theoretic formulations of the problem are considered. In the section 4 the definitions of price of anarchy and social price of anarchy are given and their values are calculated for some models. In the section 5 administrative and economical mechanisms of system compatibility are investigated. Section 6 concludes.

2. Model of concordance of public and private interests

We consider the following model of resource allocation:

$$g_i(u_1, u_2) = a_i(u_1, u_2) + s_i c(u_1, u_2), \quad (1)$$

$$0 \le u_i \le r_i, a_i \ge 0, s_i \ge 0, \sum_{j=1}^{n} s_j = \begin{cases} 1, \exists i : s_i > 0, \\ 0, \forall i s_i = 0, \end{cases} \quad (2)$$

$$i = 1, ..., n.$$

where r_i is the i-th player's resource, u_i is the amount of resource assigned for the public purpose by the i-th player, c is a public gain function, a_i is a private gain function of the i-th player, s_i is a share of the public gain obtained by the i-th player. The functions a_i and c must be continuous, differentiable and concave in both arguments.
Let us introduce three sets: $C = \{i \in N | u_i = r_i\}$- the set of collectivists; $I = \{i \in N | u_i = 0\}$ - the set of individualists; $C' = \{i \in N | u_i > 0\}$.

3. Different problem formulations

We investigate the model (1) - (2) as
- game in normal form of symmetric players;
- hierarchical game $\Gamma 1$ (Stackelberg game);
- hierarchical game $\Gamma 2$ (Germeier game);

- union of the players into coalition without hierarchical resource allocation;
- union of the players into coalition with hierarchical resource allocation.

At first, consider a game in normal form.There is a one-level control system consisting of several (for example two) players (Fig.1). Each of them has r_i units of resource, a part u_i of them the player assigns for the public purposes and the rest - $r_i - u_i$ - for the private purposes. The public gain is divided between two players completely.

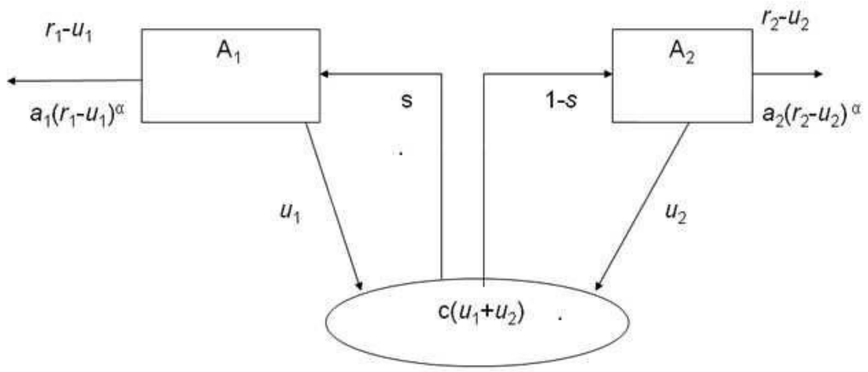

Fig. 1: A control system consisting of two symmetric agents

Example 1. Consider the case when the private gain functions are power with an exponent α, $0 < \alpha < 1$ and public gain function is linear

$$g_1(u_1, u_2) = a_1(r_1 - u_1)^\alpha + sc(u_1 + u_2) \to \max_{u_1}, \tag{3}$$

$$g_2(u_1, u_2) = a_2(r_2 - u_2)^\alpha + (1 - s)c(u_1 + u_2) \to \max_{u_2}, \tag{4}$$

$$0 \le u_i \le r_i. \tag{5}$$

in which the Nash-equilibrium is shown in Fig.2.

$$\overline{u} = \begin{cases} (0; 0), & a_1 > \frac{scr_1^{1-\alpha}}{\alpha}, a_2 > \frac{(1-s)cr_2^{1-\alpha}}{\alpha}, (I) \\ \left(0; r_2 - \sqrt[1-\alpha]{\frac{a_2\alpha}{(1-s)c}}\right), & a_1 > \frac{scr_1^{1-\alpha}}{\alpha}, a_2 < \frac{(1-s)cr_2^{1-\alpha}}{\alpha}, (II) \\ \left(r_1 - \sqrt[1-\alpha]{\frac{a_1\alpha}{sc}}; 0\right), & a_1 < \frac{scr_1^{1-\alpha}}{\alpha}, a_2 > \frac{(1-s)cr_2^{1-\alpha}}{\alpha}, (III) \\ \left(r_1 - \sqrt[1-\alpha]{\frac{a_1\alpha}{sc}}; r_2 - \sqrt[1-\alpha]{\frac{a_2\alpha}{(1-s)c}}\right), & a_1 < \frac{scr_1^{1-\alpha}}{\alpha}, a_2 < \frac{(1-s)cr_2^{1-\alpha}}{\alpha}, (IV) \end{cases}$$

As can be seen, each player has a critical value of private activity coefficient. If this coefficient is less than critical value it is profitable for a player to assign a part of his resource for the public purposes, otherwise it is profitable to assign all the resource for the private purposes.

The following conclusions may be made:

- In the considered case there are no situations when it is profitable for at least one player to assign all the resource for the public purposes.

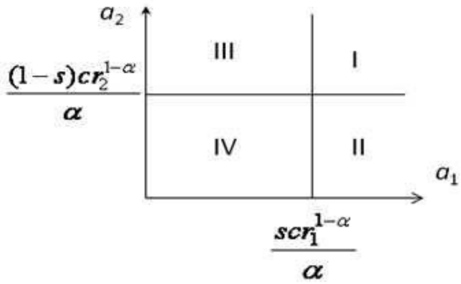

Fig. 2: Outcomes in the game in normal form of example 1

- For other types of production functions, situations may happen,when there is no profit to the players to assign all the resource for the private purposes (when both public and private activity functions are power with an exponent less than one) or there is a profit to players to assign all the resources only for the public purpose or only for the private purposes (when both production functions are linear or convex).
- The more gain from private activity holds, the more resource will be assigned for the public purposes.

Now let's consider a hierarchical setting. There is a two-level control system consisting of two elements (Fig. 3). The top level has some amount of resource r, a share of which u_1 it allocates to the bottom level for the public purpose, and the rest of the resource it assigns for its own private purpose. The bottom level in turn assigns a share u_2 of u_1 for the public purpose and the rest of them for its own private purpose. The public gain is completely divided between two players.

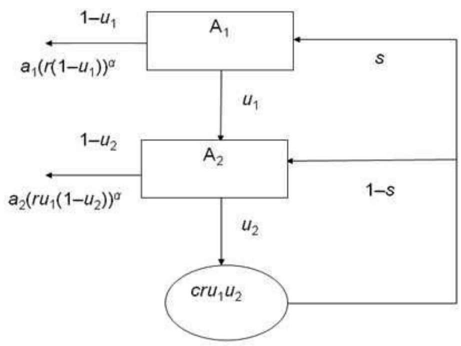

Fig. 3: Hierarchical control system of two agents

Example 2. Consider the model

$$g_1(u_1, u_2) = a_1(r(1-u_1))^\alpha + scru_1u_2 \to \max_{u_1}, \tag{6}$$

$$g_2(u_1, u_2) = a_2(ru_1(1-u_2))^\alpha + (1-s)cru_1u_2 \to \max_{u_2}, \tag{7}$$

$$0 \le u_i \le 1, i = 1, 2. \tag{8}$$

The Stackelbergequilibrium is shown in Fig. 4.

$$\overline{u} = \begin{cases} (0;0), & a_1 > \frac{scr^{1-\alpha}}{\alpha}, or \quad (I) \\ & a_2 > \frac{(1-s)cr^{1-\alpha}}{\alpha}\left(1 - \frac{1}{r}\sqrt[1-\alpha]{\frac{a_1\alpha}{sc}}\right)^{1-\alpha}, \\ \left(1 - \frac{1}{r}\sqrt[1-\alpha]{\frac{a_1\alpha}{sc}};\right. & a_1 < \frac{scr_1^{1-\alpha}}{\alpha}, \\ \left. 1 - \frac{1}{\left(1-\frac{1}{r}\sqrt[1-\alpha]{\frac{a_1\alpha}{sc}}\right)}\sqrt[1-\alpha]{\frac{a_2\alpha}{(1-s)c}}\right) & a_2 < \frac{(1-s)cr^{1-\alpha}}{\alpha}\left(1 - \frac{1}{r}\sqrt[1-\alpha]{\frac{a_1\alpha}{sc}}\right)^{1-\alpha}, (II) \end{cases}$$

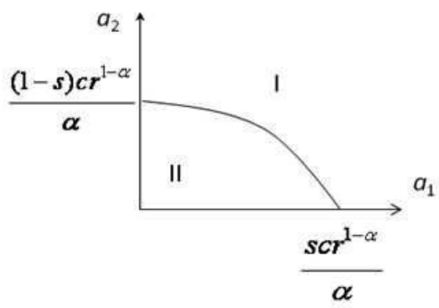

Fig. 4: Outcomes in the hierarchical game for example 2

There are two outcomes: 1)if the private activity coefficient of at least one of the players is big enough, it is profitable for both players to assign all the resource for the private purposes; and 2) if the private activity coefficients of both players are small enough, it is profitable for both players to assign some part of the resource for the public purposes.

The following conclusions may be made:

- In the given case it is unprofitable for the players to assign all the resource for the public purpose.
- For other types of production functions, situations may be where it is profitable to assign only part of the resource for the public purpose, and the rest for the private purpose (when both public and private activity functions are power with exponent less than one).
- In the case of hierarchy, if it is profitable for one of the players to assign all the resource for the private purpose, it is disadvantageous for the second player to spend even some part of resource on public purpose.

If we compare with the case of player equality then:

- In the case of hierarchy, there are no situations where it is profitable for one player to assign all the resource for the private purpose, and for the other player it is not

profitable.
- In the case of hierarchy, there are more situations where it is profitable for both players or at least for one of them to assign all the resource for the private purposes. We may make a conclusion that symmetry of players is more profitable for the public purpose than the hierarchy.
Now consider a case of players' union in coalition (Fig. 5).There is a coalition consisting of several (for example two) players. Each of them has r_i units of resource, a player assigns the part of them u_i for the public purpose and the rest $r_i - u_i$ for the private purpose.

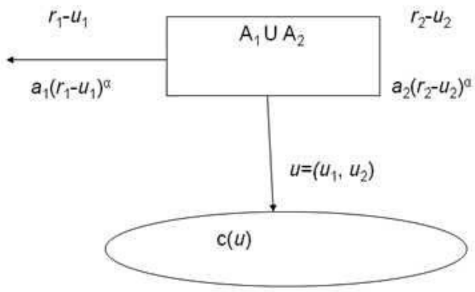

Fig. 5: The player union in coalition

Pareto-optimal set in the case of coalition in model (3) - (5)is

$$\overline{u} = \begin{cases} (0;0), & a_1 > \frac{cr_1^{1-\alpha}}{\alpha}, a_2 > \frac{cr_2^{1-\alpha}}{\alpha}, (I) \\ \left(0; r_2 - \sqrt[1-\alpha]{\frac{a_2\alpha}{c}}\right), & a_1 > \frac{cr_1^{1-\alpha}}{\alpha}, a_2 < \frac{cr_2^{1-\alpha}}{\alpha}, (II) \\ \left(r_1 - \sqrt[1-\alpha]{\frac{a_1\alpha}{c}}; 0\right), & a_1 < \frac{cr_1^{1-\alpha}}{\alpha}, a_2 > \frac{cr_2^{1-\alpha}}{\alpha}, (III) \\ \left(r_1 - \sqrt[1-\alpha]{\frac{a_1\alpha}{c}}; r_2 - \sqrt[1-\alpha]{\frac{a_2\alpha}{c}}\right), & a_1 < \frac{cr_1^{1-\alpha}}{\alpha}, a_2 < \frac{cr_2^{1-\alpha}}{\alpha}, (IV) \end{cases}$$

Therefore, critical value of private activity coefficient and the amount of the resources assigned for the public purpose are greater than if there is no coalition. Pareto-optimal set in the case of coalition in model (6) - (8) is

$$\overline{u} = \begin{cases} (0;0), & a_1 > \frac{cr^{1-\alpha}}{\alpha} \quad (I) \\ \left(1 - \frac{1}{r}\sqrt[1-\alpha]{\frac{a_1\alpha}{sc}}; 0\right), & a_1 < \frac{cr^{1-\alpha}}{\alpha} \quad (II) \\ & a_2 > \frac{cr^{1-\alpha}}{\alpha}\left(1 - \frac{1}{r}\sqrt[1-\alpha]{\frac{a_1\alpha}{c}}\right)^{1-\alpha}, \\ \left(1 - \frac{1}{r}\sqrt[1-\alpha]{\frac{a_1\alpha}{c}};\right. & a_1 < \frac{cr_1^{1-\alpha}}{\alpha}, \\ \left. 1 - \frac{1}{\left(1 - \frac{1}{r}\sqrt[1-\alpha]{\frac{a_1\alpha}{c}}\right)}\sqrt[1-\alpha]{\frac{a_2\alpha}{c}}\right) & a_2 < \frac{cr^{1-\alpha}}{\alpha}\left(1 - \frac{1}{r}\sqrt[1-\alpha]{\frac{a_1\alpha}{c}}\right)^{1-\alpha}, (III) \end{cases}$$

In terms of public gain the union into coalition with players equality is better than with hierarchy.

4. Price of anarchy and system compatibility

Now we introduce a social welfare function guiding by the idea of meta-game synthesis problem (Burkov, Opoitsev, 1974): the center seeks to determine a game between the elements of the system so that the maximum of his own payoff is reached at Nash equilibrium for the agents.

Definition 1. The welfare function is the sum of system elements payoff functions

$$g(u_1, ..., u_n) = \sum_{j=1}^{n} g_j(u_1, ..., u_n) = \sum_{j=1}^{n} p_j(r_j - u_j) + c(u_1, ..., u_n)$$

Definition 2. The model is system-compatible if one of the Nash equilibria $NE = \{u_{(1)}^{NE}, ..., u_{(n)}^{NE}\}$ coincides with the vector of player strategies $g_{max} = \max_{u \in U} g(u) = g(u^{\max})$, in which the social welfare function is maximal.

Definition 3. The ratio of the function at the worst Nash equilibrium $g_{min}^{NE} = \min\{g(u_{(1)}^{NE}), ..., g(u_{(n)}^{NE})\}$ and the maximum value is the price of anarchy,

$$PA = \frac{g_{\min}^{NE}}{g_{\max}}$$

which is not greater than one. As the price of anarchy is closer to one, the equilibrium is more effective and the necessity of model coordinating is lower.

Definition 4. Social price of anarchy is defined as

$$SPA = \frac{c_{\min}^{NE}}{c_{\max}}$$

where $c_{\min}^{NE} = \min\{c(u_{(1)}^{NE}), ..., g(u_{(n)}^{NE})\}$,$c_{\max} = \max_{u \in U} c(u) = c(r_1, ..., r_n)$.
The social price of anarchy in contrast to the price of anarchy characterizes the efficiency or inefficiency of equilibria in terms of public gain defined as the function c.

Example 3. We consider these values on the example of CPPI-models in the system which consists of n elements where the public gain is allocated among all players equally.

$$g_i(u) = \sqrt{r_i - u_i} + \frac{1}{n}\sum_{j=1}^{n} u_j, g(u) = \sum_{j=1}^{n} \sqrt{r_i - u_i} + \sum_{j=1}^{n} u_j, s_i = \frac{1}{n}, i = 1, ..., n.$$

In the table 1 the dominant and optimal in terms of social welfare public function strategies are given. As we can see, interests of the players and society are compatible

Table 1.

	u_i^D	$u_i^{\max}$
$r_i < \frac{1}{4}$	0	0
$\frac{1}{4} < r_i < \frac{n^2}{4}$	0	$r_i - \frac{1}{4}$
$r_i > \frac{n^2}{4}$	$r_i - \frac{n^2}{4}$	$r_i - \frac{1}{4}$

only in the first case, although the public gain is zero. The greater n is, the more probable the second case becomes which is the most unprofitable in terms of price of anarchy and social price of anarchy.

5. Administrative and economical compatible mechanisms

We assume that maximization of social welfare is a responsibility of a certain subject (called Leader) which may apply the next mechanisms (Table 2):
1) Administrative:Leading affects the set of admissible control of the player.
2) Economical: Leading affects the player gain.
Each of these methods may be applied with or without feedbacks: impulsion and compulsion, respectively.

Table 2.

Leader's controls:	Without feedback (Γ_1) - compulsion	With feedback (Γ_2) - impulsion
the set of admissible controls of the player (administrative) $U_i = U_i(q_i)$	Administrative compulsion $\widetilde{q_i} \le u_i \le \overline{q_i}, \widetilde{q_i}, \overline{q_i} = const$	Administrative impulsion $\widetilde{q_i}(u_i) \le u_i \le \overline{q_i}(u_i)$
player gain function (economical) $g_i = g_i(u_i, b_i)$	Economical compulsion $s_i = const$	Economical impulsion $s_i = s_i(u_i)$

Definition 5. Control mechanism is called compatible if the model is compatible as a result of players' optimal response on its implementation.

Example 4. Consider the case of economical compulsion in the model

$$g_i(u) = k_i\sqrt{r_i - u_i} + s_i K \sum_{j=1}^{n} u_j \to \max, 0 \le u_i \le r_i, i \in N;$$

$$g_0(u) = \sum_{j \in N} k_j\sqrt{r_j - u_j} + K \sum_{j \in N} u_j \to \max,$$

$$0 \le s_i \le 1, \sum_{j=1}^{n} s_j = \begin{cases} 1, \exists i : s_i > 0, \\ 0, otherwise. \end{cases}$$

The dominant for agents and globally optimal strategies are respectively

$$u_i^* = \begin{cases} r_i - \frac{k_i^2}{4K^2 s_i^2}, \ s_i \ge \frac{k_i}{2K\sqrt{r_i}}; \\ 0, \quad otherwise, \end{cases} u_i^{max} = \begin{cases} r_i - \frac{k_i^2}{4K^2}, \ k_i \le 2K\sqrt{r_i}; \\ 0, \quad otherwise. \end{cases}$$

The system compatibility is reached if $\forall i \in N \ k_i \ge 2K\sqrt{r_i}$. In this case all players are individualists $N = I$ and the maximum of social welfare is $g_0^{max} = g_0^I = \sum_{j \in N} k_j \sqrt{(r_j)}$.
If this condition is not satisfied we can pose the problem of system compatibility in a weaker form in which the economical compulsion mechanism of maximizing the price of anarchy is found. In this example we solve the problem

$$g_0(s) = \sum_{j \in I(s)} k_j\sqrt{r_j} + \sum_{j \in C'(s)} \left[K r_j + \frac{k_j}{2K s_j} - \frac{k_j^2}{4K s_j^2} \right] \to \max$$

by Lagrange-multipliers method.
Now we consider economical impulsion mechanism in CPPI-models. We can use two approaches to analyze it: empirical and theoretical ones. Within the empirical approach widespread practice methods of public gain allocation are investigated.
Let's consider the proportional allocation mechanism

$$s_i(u) = \begin{cases} \frac{u_i}{\sum_{j \in N} u_j}, & \exists m : u_m > 0, \\ 0, & otherwise. \end{cases}$$

in which a share of the public gain of the i-th player is proportional to its resource amount assigned for the public purpose. Here is the first order condition:

$$\sum_{j \neq i} u_j \left[\frac{\partial c(u)}{\partial u_i} \sum_{j \in Ni} u_j - c(u) \right] = 0, i \in N.$$

The expression in square brackets is equal to zero at the linear function c, therefore proportional mechanism is system compatible in CPPI-models with a linear public gain function and any private gain function.

Example 5. Consider the case of economical impulsion with proportional allocation mechanism if $g_i(u) = k_i \sqrt{r_i - u_i} + s_i K \sum_{j \in N} u_j$.

$$u_i^* = u_i^{max} = \begin{cases} r_i - \frac{k_i^2}{4K^2}, & k_i \leq 2K\sqrt{r_i}; \\ 0, & otherwise. \end{cases}$$

As we can see the dominant and optimal strategies coincide.
Theoretical approach is based on Germeier theorem.

Example 6. Apply Germeier theorem to the model

$$g_i(u) = k_i \cdot (r_i - u_i) + s_i K \cdot \sum_{j=1}^{n} u_j \to \max, 0 \leq u_i \leq r_i, i \in N;$$

$$g_0(u) = \sum_{j \in N} k_j \cdot (r_j - u_j) + K \cdot \sum_{j \in N} u_j \to \max, \sum_{j=1}^{n} s_j = \begin{cases} 1, \exists i : s_i > 0, \\ 0, otherwise. \end{cases}$$

Dominant strategy s_i^D is arbitrary between zero and one, because the social welfare function g_0 does not depend on s. The penalty strategy is $s_i^P \equiv 0$. L_i is the maximal player gain if the Leader chooses punishment strategy $L_i = k_i r_i$. This value is reached at $E_i = \{u_i = 0\}$. D is the player strategy set at which the player gain is greater than L_i:

$$D_i = \left\{ (s_i, u_i) : s_i > \frac{k_i u_i}{K \sum_{j \in N} u_j} \right\}.$$

K_2 is the maximal social welfare if the chosen strategy from E_i:

$$K_2 = g_0^I = \sum_{j \in N} k_j r_j.$$

K_1 is the maximal social welfare value if the strategy from D is chosen. To find K_1 it is necessary to solve optimization problem

$$g_0(u) = \sum_{j\in I} k_j \cdot (r_j - u_j) + K \cdot \sum_{j\in C} u_j \to \max$$

under constraints

$$\frac{k_i u_i}{K\sum_{j\in N} u_j} < s_i \leq 1, \sum_{j=1}^{n} s_j = \begin{cases} 1, \exists i : s_i > 0, \\ 0, \, otherwise \end{cases} 0 \leq u_i \leq r_i, i \in N.$$

It is proved that it is possible to choose s_i from the set D.In this case K_1 is greater than K_2.
From the first order condition

$$\frac{\partial g_0}{\partial u_i} = K - k_i \Rightarrow u_i^* = \begin{cases} r_i, \quad k_i \leq K \quad \text{(set C)} \\ 0, \, otherwise \; \text{(set I)}. \end{cases}$$

Therefore,

$$g_0(u^*) = \sum_{j\in I} k_j r_j + K \sum_{j\in C} r_j.$$

6. Conclusion

In the article the problem of system compatibility was analyzed. In the beginning of the article the problem of concordance of public and private interests in several forms was considered, the comparative characteristics of different approaches to the problem in terms of public revenue were found. Further, the values of price of anarchy and social price of anarchy were described and calculated for various private and public interest functions. Then the social welfare function was introduced, and the problem of system compatibility was considered. Some preliminary results about system compatibility of the CPPI-models were obtained. Thus, system compatibility in CPPI-models when $s_i = const$ is reachable only if all agents are pure individualists (all resources are assigned for private interests) or pure collectivists (all resources are assigned for public interest).
The research perspectives include:
- investigation of the system compatibility for more general classes of models;
- considering of corruption (Antonenko, Ugol'nitskii, Usov, 2013) (an additional feedback on bribe);
- analysis of dynamic settings, including phase constraints (requirements of sustainable development), investigation of the conditions of time consistence.

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