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Analysis in Social Networks with Usage of Modified Raiffa Solution for Cooperative Games

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Abstract We present our approach based on Nash bargaining problem for n-player definition as a set B settled pairs (S, d). The elements B of are called instance (examples) of the problem B, elements S are called variants or vector of utility, point d is called the point of disagreement, or status quo. From the point of view that we develop it is interesting Raiffa's solution that was proposed in the early 1950's. Raiffa (1957) suggested dynamic procedures for the cooperative bargaining in which the set S of possible alternatives is kept unchanged while the disagreement point d gradually changes. He considers two variants of such process – a discrete one and the continuous one. Discrete Raiffa's solution is the limit of so called dictated revenues. Diskin, A., Koppel, M., Samet D. (2011) have provided an axiomatization of a family of generalized Raiffa's discrete solutions. The solution concept which is composed of two solution functions. One solution function specifies an interim agreement and the other specifies the terminal agreement. The solution that we suggest and that we called von Neumann-Morgenstern modified discrete Raiffa's solution for $n = 3$. Our approach modifies Raiffa solution as a value of compensation, that implies from affinity of the player Xr to the player Xs based on the assumption that they will be in the same $(n - 1)$ members of coalition.

Comparing the results of the original game with the game extended of player affinities brings valuable results if analysing various types of social networks. Particularly when examining relations based on investing in social status and when analysing the structures based on mutual covering of violations of the generally accepted principles.

Keywords: Nash bargaining problem for n-player, Raiffa solution, three-person game, social network, coalition affinity, social networks based on mutual covering violate the generally accepted principles, von Neumann-Morgernstern stable set

1. Introduction

The term social network and related term social capital became standard sociology terms in the 70s of the 20th Century. Various mathematical tools are searched for to shape them appropriately; it is mainly the graph theory that is applied. The definitions of the social network differ in details; they agree that it refers to a certain type of affinity-relationships based on various likings. There is no doubt that the existence of a social network has an impact on the formation of coalitions and on the fact whether those integrated in social networks or just exposed to social networks are going to cooperate or not.

How to assess the impact of social networks in various conditions? Which social networks are going to have the biggest impact on the behaviour of players and will their role be dominant in comparison with other social networks based on various affinity types? These questions may be answered through the apparatus of games theory. Our report is going to show one of the options of application. We will apply the theory of coalition games for three players with the possibility of extending the respective apparatus to coalition games for more players. In this context, the interpretation of Neumann-Morgenstern stable set and Nash (S, d) bargaining solution, specifically the established NM-modified sequential Raiffa solution (RmNM) is going to play a significant role.

In the end, we will mention a few options of results utilization during the examination of two significant social phenomena, in particular the positional investment and structures based on mutual covering of violations of the generally accepted principles.

2. Social network impact on the formation of coalitions, defining the phenomenon of affinity.

The definition of the term social network can be found in the works of numerous sociologists, e.g. P. Bourdieu (1985), R. Putnam (1995). A certain differentiation of basic sociological approaches toward the term social network was attempted by Matěju,Vitásková (2006): *"Irrespective of disciplinary focus, however, the formulation of a consistent theory of social capital continues to be complicated by the existence of two different yet equally useful and theoretically rewarding conceptual and methodological approaches:*

a) Social capital defined as mostly an 'attribute of an individual', as a person's potential to activate and effectively mobilise a network of 'social connections' based on 'mutual recognition' of proximity (in a social space) and maintained by symbolic and material 'exchanges' (Bourdieu, 1985). In this context, social capital has the properties of a 'private good', which individuals accumulate and use to achieve their own goals and personal advancements.

b) Social capital defined as mostly an 'attribute of a society', as a quality of networks and relationships enabling individuals to 'co-operate' and act collectively (Putnam, 1995). Within this framework, social capital is based on a high degree of interpersonal 'trust', as well as on the 'trustworthiness' of the public and political institutions that establish and uphold the 'rule of law', making all kinds of exchanges transparent and safe. For these reasons, social capital has the properties of a 'public good', facilitating the achievement of higher levels of efficiency and productivity; hence this form of social capital is often associated with economic growth." (just as Bourdieu (1985), Putnam (1995)).

The above-mentioned definition of social networks and social capital, a follow-up differentiation of two approaches towards those related issues is regarded as a most suitable answer to the question as to how to approach the analysis of affinity relationships that represent the basis of social networks. First, we will use the thought process utilized by J. Neumann and O. Morgenstern in the Theory of Games and Economic Behaviour (Neumann and Morgenstern, 1953), called a heuristic approach. Lets recall some practical chapters.

The basic case comes in §21: *The Simple Majority Game of Three Persons.* The following are the most important passages and paragraphs from which they were taken: *"Each player, by a personal move, chooses the number of one of the two other players. Each one makes his choice uninformed about the choices of the two other players. ... If two players have chosen each other's numbers we say that they form a couple. Clearly there will be precisely one couple, or none at all. If there is precisely one couple, then the two players who belong to it get one-half unit each, while the third (excluded) player correspondingly loses one unit. If there is no couple, then no one gets anything... Since each player makes his personal move in ignorance of those of the others, no collaboration of the players can be established during the course of the play."* (Neumann and Morgenstern, 1953, pp. 222–223). As thoroughly described, the game may end up either in two players receiving each and the third player -1, or in each player obtaining 0. This is one of the simplest three-person games, yet it can be extended into a more complex one. In §21.3., the authors stress that *"the game is wholly symmetric with respect to the three players"* (Neumann and Morgenstern, 1953, p. 224). This statement will prove very important. The authors are rather specific in claiming that any potential agreement among the players will always be reached outside the basic game (i.e. it would be an outcome of another game). As a follow-up, the authors take the first step and extend the basic (elementary) model of the simple majority game of three persons (§22.1.2.): *"...let us now consider a game in which each coalition offers the same total return, but where the rules of the game provide for a different distribution. For the sake of simplicity, let this be the case only in the coalition of players 1 and 2, where player 1, say, is favored by an amount ε...If the couple 1,2 forms, then player 1 gets the amount $\frac{1}{2}+\varepsilon$, player 2 gets the amount $\frac{1}{2}-\varepsilon$,and player 3 loses one unit. If any other couple forms (i.e. 1,3 or 2,3) then the two players which belong to it get one-half unit each while the third (excluded) player loses one unit. – What will happen in this game? – ...Prima facie it may seem that player 1 has an advantage, since at least in his couple with player 2 he gets more ε than in the original, simple majority game. – However, this advantage is quite illusory. If player 1 would really insist on getting the ε in the couple with player 2, then this would have the following consequence: The couple 1,3 would never form, because the couple 1, 2 is more desirable from 1's point of view; the couple 1, 2 would never form, because the couple 2,3 is more desirable from 2's point of view; but the couple 2,3 is entirely unobstructed, since it can be brought about by a coalition of 2,3 who then need pay no attention to 1 and his special desires. Thus the couple 2,3 and no other will form; and player 1 will not get $\frac{1}{2}+\varepsilon$ nor even one-half unit, but he will certainly be the excluded player and lose one unit. – So any attempt of player 1 to keep his privileged position in the couple 1,2 is bound to lead to disaster for him. The best he can do is to take steps which make the couple 1,2 just as attractive for 2 as the competing couple 2,3. That is to say, he acts wisely if, in case of the formation of a couple with 2, he returns the extra ε to his partner."* (Neumann and Morgenstern, 1953, p. 226). This point cannot be overstressed. It explains in depth why the players in the winning coalition have to share their payoff equally. If one of them wanted more, he would find himself outside the coalition and end up losing, rather than profiting (in a zero-sum game).

We will further extend the model described in the book and address a case of different amounts that can be gained at the expense of the third player if the two remaining players form a coalition. The problem is described in paragraph 22.2., entitled *Coalition of Different Strength* and can be briefly summarised as follows: Let's assume there are amounts a, b, c ($a =$ what players 2 and 3 may get from player 1, etc.). If player 1 wanted payoff x, then players 2 and 3, after subtracting payoff x, must be left with more than or as much as players 2 and 3 would obtain from player 1 if 2 and 3 cooperated, i.e. $(c-x)+(b-x) \geq a$. This means $x \leq (-a+b+c)/2$. Thus player 1 may count upon obtaining the maximum payoff $\alpha = (-a+b+c)/2$, and likewise players B and C may expect obtaining payoffs $\beta = (a-b+c)/2$ or $\gamma = (a+b-c)/2$ (Neumann and Morgenstern, 1953, p. 228).

Let us add the following to this brief summary: In every winning coalition, each player allied with either of the other players gets the same payoff. Or, to put it in the language of economic theory, the opportunity costs of forming any possible coalition are equal, based on the player's potential payoff in another coalition. In the next section we will look at how the initial very simple theoretic model of three-player games and their analysis can be further extended by analysis of more complex games containing other, important and real life elements.

If each of the players requires in each of a two-member coalitions, in which he can become a member, the payoff that he would have with the other of the players, the formation of each of the coalitions (unless there are no other external influences on the system) is likely to happen anyway. In this context, the situation is symmetrical. The average expected payoff of each of the players will be 2/3 of his payoff in the winning coalition. There is the same probability that three different coalitions may be formed and each player will be a member in two of them.

Let's assume now that there is a certain positive affinity between two players. The simplest case is that one of the players takes a liking to the other player. If the players decide to form two-member coalitions, the player who takes a liking to the other player will prefer a coalition with him, which results in this coalition. The liking of one player to the other brought a certain asymmetry in the game, which predetermines a formation of a respective coalition.

Now, we are facing one of the key moments of our approach. If the player, who would end up outside the coalition under these conditions or who would lose a 2/3 chance to become a member of the winning coalition, gave up part of his payoff, he could renew his (two-thirds) chance to participate in the winning coalition. Two conditions have to be met:

- He has to be informed, i. e. he needs to know that there is a certain liking between the players and to be aware of the level of appreciation by the relevant player.

- He must have a chance to give up part of his payoff.

As a result, we can now define what we regard as the affinity of one player towards the other. Affinity of one player towards the other is the gain for this player resulting from the formation of the coalition with the other player, whereas this gain can be expressed with the same values that are used to express the payoffs in the original game. If both players can benefit from the coalition, we can talk about reciprocal affinity that, however, may have different size in case of each of the players. Positive affinity can also be called a liking of one player towards the other; negative affinity as antipathy of one player towards the other. We see the original game as a game in which we do not consider the influence of external factors in the form of affinities among the players. .

Affinity, or liking among players may also be described differently in terms of the games theory. E.g. K. Binmore (1998) defines the liking of two players as follows:

"*Adam symphathiezes with Eva when he identifies with her aims that her welfare appearsas an argument in his utility funktcion.*" These issues are further analysed in section 2.5.2 Kinship; based on the evolution games Kant's and Hamilton's approach to ethics are compared.

Let's go back to our approach. What we presented about the relationship of two players in our concept is the liking in a common (intuitive) sense of this word and therefore there is no use in going round this term. One player benefits from the coalition with the other player. (In our approach we assume that this additional gain also originates in a game which is related to the original game, however, this is not substantial at the moment.)

In order to avoid the confusion with Binmore's concept, we shall use the term one-sided coalition liking of two players or coalition affinity; reciprocal liking of two players would then consist of two one-sided likings.

The situation in the simplest case then reads as follows: As one of the players (e.g. B) additionally benefits from the coalition with another player (e.g. A), B player receives additional payoff in the original game (without taking the effect of liking into consideration), which is why he will give preference to this coalition. In order to prevent the predetermining of the coalition (A, B) through the liking of B player towards A player, C player (or other players) has to counterbalance the influence of this liking, at the exact amount of the additional gain from the liking.

We have taken the first step towards utilizing a few tools of the games theory for modelling certain aspects of social networks.

3. Elementary Case and Basis of Mathematical Apparatus of Affinity Modelling

First, let's focus on the fact how the influence of coalition affinities can be expressed through a suitable mathematical apparatus. To keep things simple, we will analyse the case of three players again. May x, y, z be the salaries of relevant players and S the set of salaries which follows the formula $S(x, y, z) \leq 0$.

The points of discrete Neumann-Morgenstern internally and externally stable set are calculated from the following system of equations:

$$S(0, y, z) = 0$$

$$S(x, 0, z) = 0$$

$$S(x, y, 0) = 0$$

Let's mark the solution d_{x0}, d_{y0}, d_{z0}.

If A and B players form a coalition, their payoffs are d_{x0}, d_{y0} and the salary of the third player equals 0. Similarly, in case of other coalitions the average expected payoff of the players is $2/3d_{x0}$, $2/3d_{y0}$, $2/3d_{z0}$.

We assume that two players in each two-member coalition can fully discriminate the third player, whereas the lowest payoff they can give him equals zero. At the same time:

- we should expect collective rationality, i.e. we are interested only in the Pareto optimal points of S set marked with respective inequality.

- We will not take special cases of S set into consideration, where the task has no solution or has more solutions.

- For the time being we are not considering the fact that in case of full discrimination the lowest payoff of the discriminated player may differ from zero.

(These assumptions are not essential, however, quite important in terms of simplicity of another definition.)

The total payoff x^*, y^*, z^*, of each of the players in the coalition with the player towards whom he takes the affinity/liking equals his payoff in the basic game plus his payoff corresponding to the gain resulting from the formation of the coalition which will be marked as s_{xy}, s_{xz}, s_{yx}. s_{yz}, s_{zx}, s_{zy}. s_{xy} is a one-sided liking of player x towards player y, sxz one-sided liking of player y towards player x etc.

It follows that:

$$x^* = x + s_{xy} + s_{xz}$$

$$y^* = y + s_{yx} + s_{yz}$$

$$z^* = z + s_{zx} + s_{zy}$$

The sign may be positive (positive affinity, i.e. liking), or negative (negative affinity, i.e. antipathy).

As we have already mentioned... if there is a one-sided or reciprocal positive coalition affinity between two players and no affinity between neither of these players with the third player, it is predetermined that a coalition will be formed between these two players. It does not have to be the case if the third player counterbalances this positive affinity with his lower payoff. Let us assume that all players are perfectly informed about all affinities among the players. Then the original system of equations is modified to the system:

$$S(0, y^*, z^*) = s_{yz} + s_{zy}$$

$$S(x^*, 0, z^*) = s_{xz} + s_{zx}$$

$$S(x^*, y^*, 0) = s_{xy} + s_{yx}$$

The solution represents salaries $d^*_{x0}, d^*_{y0}, d^*_{z0}$ of respective players and the average expected payoff of the players is $2/3d^*_{x0}, 2/3d^*_{y0}, 2/3d^*_{z0}$.

To make it easier to understand, we will present a very simple case. There are players A, B, C. If a two-member coalition is formed:

- A and B split 5,

- A and C split 4,

- B and C split 3.

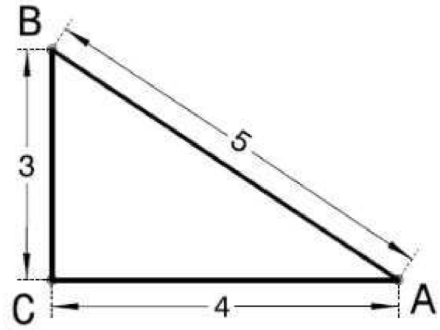

Fig. 1.

(Source: Černík - Valenčík)

The original solution (discovering a discrete NM-set), presenting conditions for the formation of a two-member coalition, can be calculated as follows:

$$\begin{array}{lll} x+y=5 & \text{thereafter} & x=3 \\ x+z=4 & & y=2 \\ y+z=3 & & z=1 \end{array}$$

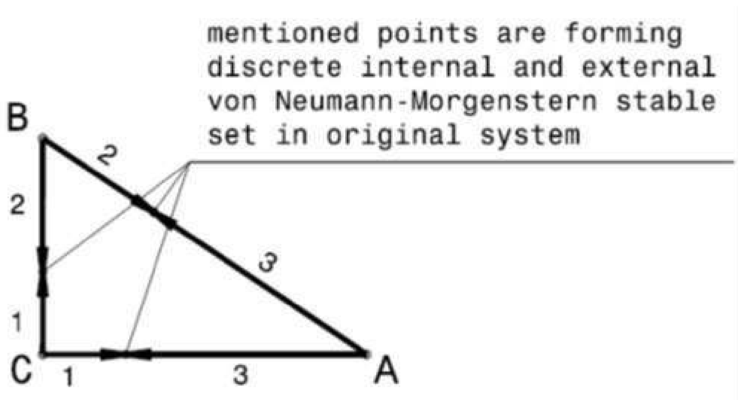

Fig. 2.

(Source: Černík - Valenčík)

The determined points form an internally and externally stable set (NM-set), the only discrete and final NM-set.

Let us enter the one-sided positive coalition affinity of B player towards A player (B player takes a liking to A player), equalling 1. The formation of the coalition with A player (without the participation of C player) has the same value 1 in the payoff in the original game for B player. It follows that:

$$y^* = y + syx$$

where

$$s = 1$$

The original system of equations in the presented simple case is modified as follows:

$$\begin{array}{rlll} x + y^* = 6 & \text{thereafter} & x = 3,5 & \\ x + z = 4 & & y^* = 2,5 & \text{then} \quad y = 1,5 \\ y^* + z = 3 & & z = 0,5 & \end{array}$$

As a result of positive affinity towards A, both C and B lose 0.5 in the payoff in the original game.

We have thus created a most simple tool allowing affecting some important aspects of social networks that we are going to work with.

Let us note the following points:

If one of the players (e.g. the first one) takes positive affinity towards the other player, then in the original game:

- the payoff of the first player decreases proportionately to the size of this affinity.

- the payoff of the second player increases proportionately to the size of this affinity.

- the payoff of the third player decreases proportionately to the size of this affinity.

(In the original game, the player who is likeable to the other player gains the most, without having to take any liking to that player or to another)

If players A and C were not informed about the one-sided positive coalition affinity of B player towards A player, a coalition between A and B would be formed, where A player would have the original payoff equalling 3 and C player a payoff equalling 0. As a result, we can appreciate information about coalition affinities in the games of this type.

4. Cooperative solution in games with coalition affinities

A cooperative game in our case offers the players a chance to upgrade the payoff. First, we are going to take into account the original game. If S set is compact and convex, players can substantially improve their positions in the original game compared with the average expected payoff, as indicated in the following figure 3.

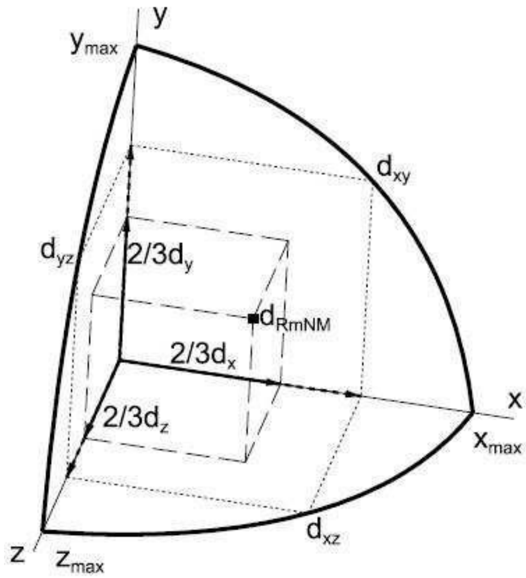

Fig. 3.

(Source: Černík - Valenčík)

The point with coordinates $2/3d_{x0}$, $2/3d_{y0}$, $2/3d_{z0}$ can be interpreted in the original game as the point of non-agreement. In that case we deal with the standard Nash's (S, d) bargaining problem. There is the S set corresponding to all possible payoffs of players, point of non-agreement d (that can be interpreted as the payoff of players in case they fail to come to terms) and we search for a payoff distribution that the players assent to. There are also other possible interpretations of this task.

J. Nash suggested one of possible solutions that bore his name, i.e. Nash solution (S, d) to the bargaining problem. He succeeded in proving that if five intuitively acceptable axioms are met, then this solution is unequivocally determined by these axioms and is the only one. At that time H. Raiffa also suggested another solution (S, d) to the problem which was based on the idea of progressive steps intuitively corresponding to gradual improvement of the initial payoff distribution (i.e. the salaries in point d). This solution was axiomatized early 80s. Meanwhile, other axiomatized solutions (S, d) to the problem emerged, e.g. by Kalai-Smorodinsky (1975), Kalai egalitarian solution, Shapley-Shubik etc. The axiomatic system corresponding to some of these solutions also turns out to be intuitively acceptable (if not the "only possible"), although the outcome is different.

This fact (i.e. that there are more intuitively acceptable axiomatizations corresponding to various solutions) can be interpreted differently. For instance, if we deal with a specific task, first we have to realize in which of the "possible worlds" it is dealt with. Which of the possible axiom systems is the "correct one" for the relevant case.

In our case we need such a solution that is sensitive to the existence of what we called coalition affinities among the players. Our simple case demonstrates that if there are affinities among the players, NM discrete set is changed as well as the factor that we called the average expected payoff of players in case that these players will strive to create a two-member coalition in a game with three players.

The most suitable approach seems to be the modification of sequential Raiffa solution. The set of generalized Raiffa's solution is certain kind of step-by-step negotiation solution $\{(f^p, g^p)\}$ $0 < p \leq 1$, where are f^p a g^p defined as: $f^p(S, d) = d+p/n(U(S, d)-d)$, $g^p(S, d) = d^\infty(S, d)$, where $d^\infty(S, d)$ is the limit of progression $\{d^k(S, d)\}$ of points constructed by induction follows: $d^0(S, d) = d$, $d^{k+1}(S, d) = f^p(S, d^k)$, where $U_i(S, d) = max\{x_i : x\}$.

Our approach will modify Raiffa solution as follows: $p = n - 1/n$ pro $n = 3$; instead of $U_i(S, d) = max\{x_i : x\}$ we shall consider $NM_i(S, d) = 2/3d_i$, where d_i is the solution of the system of equations:

$$S(d_{xi-1}, y, z) = 0$$
$$S(x, d_{yi-1}, z) = 0$$
$$S(x, y, d_{zi-1}) = 0$$

In other words – sequential Raiffa solution is modified in such a way that we do not deduce the solution from maximum payoffs of the respective players, but from the average expected payoff determined by discrete NM set, Valenčík, Černík (2014).

If there are coalition affinities among the players, the relevant points will be determined by the following system of equations:

$$S(d^*_{xi-1}, y^*, z^*) = s_{yz} + s_{zy}$$
$$S(x^*, d^*_{yi-1}, z^*) = s_{xz} + s_{zx}$$
$$S(x^*, y^*, d^*_{zi-1}) = s_{xy} + s_{yx}$$

A few notes about the proposed model of expressing coalition affinities among the players:

1. Similarly, it would be possible to modify the Shapley-Shubik solution.
2. The model allows distinguishing situations where it is worth for the players to create a three-member coalition without consideration for the impact of coalition affinities and when these affinities will play a certain role.
3. The model shows the changing NM-modified Raiffa solution (RmNM) in case that at least one player considers the role of coalition affinities more significant than the payoff increase by creating the three-member coalition.
4. The solution of a similar task for more than three players is not easy. There are several approaches to achieve it. However, we failed to create a satisfactory model. In our opinion, it is important to connect the creation of the respective model with the interpretation of its utilization. In the relevant case, it is likely to answer the questions related to the development of the system owned by more players and where more coalitions based on coalition affinities compete. This task needs to be regarded as dynamic.

5. Some applications of existing results

We have demonstrated the manifestations of coalition affinity in a situation where there is a group of players who split what they have obtained. The winning coalition can obtain more at the expense of the others. To make it easier to understand, we have chosen a case of three players. If there is some positive coalition affinity between the two of them, we can assume that it will result in the coalition of these two players. If this affinity did not exist, then – which can be proved – it would be better for each of the players to agree they create a big coalition (of all three players) than to risk that each of the players is most likely to become the one being discriminated by the other two. As a result, we can appreciate the influence of coalition affinity. If the player, who would be discriminated as a consequence of the existing positive (one-sided or two-sided) affinity between the other two players, knows how big this affinity is and if he can counterbalance this affinity with a concession (request for a lower payoff), it may restore the situation where all three two-member coalitions are equally probable and where it is worth to create a big coalition. It is logical to call the size of this compensation the power of affinity. Coalition affinity then appears as a certain additional payoff to the one that the player in the respective coalition would receive.

Two types of affinities turned out to be most significant:

- Those resulting from breaching generally accepted principles (ethical, legal etc., e.g. in a game of the Tragedy of the Commons type). Specifically affinities where one player knows that the other one breached generally accepted principles. The first player could discredit the other one, he blackmails him but he also covers for him and (in other contextual games) favours him. This affinity is quite significant as it represents a substantial difference in payoffs for the blackmailed player, and it cannot be counterbalanced – because the third player (in a model with more players) does not know about it.

- Those related to positional investment. If the player did not want to be outside the coalition that is going to split what was obtained, he would have to give up part of his payoff. As part of a repeated game, this would result in even stronger predetermining a coalition based on positional investment.

In social reality, these two types of affinities tend to interconnect. The competition in the area of positional investment makes those, who want to obtain necessary means, breach generally accepted principles. For a certain amount of time, this is connected with a risk of disclosure and subsequent punishment. However, sooner or later, it occurs that instead of being punished the player is blackmailed, but at the same time also favoured and covered for. It results in the interconnection of the positional investment with structures based on mutual covering of violations of the generally accepted principles.

The interconnection of the positional investment with growing based on mutual covering of violations of the generally accepted principles is fatal for the social organism and it may also affect institutions that should protect the society from the breach of generally accepted principles.

Every historical period presents different defence elements working against the concept of degeneration. They include:

- Tendency to cooperative behaviour (observance of principles as well as willingness to sacrifice something to the general benefit or even existential willingness to sacrifice oneself) as it was encoded in our mind through evolution games that were played since the birth of human civilization. (The source of morality is searched for e.g. by. K. Binmore (1998)).

- Particularities of certain superior institutions that are sufficiently superior through the process of formation and its historical continuity and permanently restore the resistance of other institutions. Those that are to prevent the breach of generally accepted principles.

- Competition among states. The state, the institutions of which are defeated, withdraws. It either falls apart, perishes, is absorbed by other states or there is more or less radical restoration of its institutions on a completely different basis.

6. Conclusions and discussions

We have presented one of the options to utilize selected tools from the area of games theory to analyse social networks which are the subject of sociology. We focused on three levels of abstraction:

- Mathematical model.
- Concept based on the model interpretation.
- Conceptual description of reality.

In the area of mathematical model we have defined the term "coalition affinity" and suggested a way of its quantification for the case of three players. We have presented one of the possibilities to consider the impact of coalition affinities (which is determined by the role of social networks) when searching for the solution to Nash bargaining (S, d) problem for three players. We thus created a concept that allowed demonstrating the role of social networks that are created (from the point of view of the Tragedy of the Commons type) as a consequence of the breach of generally accepted principles. Specifically then based on the affinities consisting in reciprocal cover, blackmailing and favouring within institutional structures that create relevant game context. Such a concept is of great importance for the understanding of some significant current phenomena such as corruption and especially corruption resistance, malfunction of institutions as a result of existing clientelistic networks etc. As it offers the interpretation and application of generally theoretical models, it stimulates the development of the games theory in respective areas. In our case, following research methods are considered relevant:

- Generalization of the proposed approach from three to more players (possibly with the utilization of dynamic models).
- Formulating general criteria which allow distinguishing Nash solutions to bargaining (S, d) problem that are sensitive to coalition affinities from those that are not.
- Focusing on the attributes of S set, also from the point of view of specific cases.

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