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Phenomenon of a "Snag" in Financial Markets and its Analysis via the Cooperative Game Theory

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Abstract The paper describes the development of financial markets and changes in the nature of economic growth using the theory of cooperative games. These issues have developed since the early 1950s under the influence of theoretical problems based on the game theory itself and interacting with real problems outside of the game theory (mostly from economics). It turned out that various applications and contexts correspond to numerous possible solutions of standard tasks, e.g. Nash (S, d) bargaining problem. Some of the significant solutions are responded to questions arising in the context of social welfare economic theory, respectively issues are related to the redistribution of wealth between different groups in population and the rationale of such reallocation. We show that under conditions of sufficiently effective financial markets the question of the relationship between efficiency and equality, which is typical of the theory of social welfare, may be replaced by the question of making full utilization of investment opportunities associated with the acquisition, preservation and application of human capital. We define "sufficient efficiency of financial markets" as ability to fully utilize investment opportunities related – to put it simply – to human development, regardless of its initial assets or income position. This is related to the fact that instead of different ways of reasoning for solution (S, d) of the problem we can take advantage of technical solution (based on the equality of marginal returns of investment opportunities, or rather based on sum payments maximization), e.g. the solution used in problem of optimal allocation of water (water allocation problem) (Brink, et al., 2011). The question of compensation payments in relation to solutions based on technical optimum has important interpretation. Sufficiently efficient functioning of financial markets (in the above mentioned sense) assumes also good functioning of such financial market instruments, e.g. human capital contracts associated with the use of transferred prices and mediated utilization of transferred prices. In case of full utilization of those tools, compensation would not be necessary. Above mentioned topics are part of wider research focused on changes in the nature of economic growth. This research is based on the hypothesis that the existing possibilities of economic growth have become exhausted and that it is necessary to transition towards the economy based on the dominant role of productive services, i.e. services which have immediate effect on the acquisition, preservation and utilization of human capital

(e.g. education, health care etc.)(Friedman, 1957). The development of financial markets in the above direction is prerequisite to economic growth.

Keywords: Nash bargaining problem, investment opportunities, human capital; financial markets; cooperative games; investment opportunities

1. Introduction

In our contribution, we are pointing out an interesting and from the practical point of view important area of interpretation of solving cooperative games (especially the Nash (S, d) problem) (Nash, 1950), namely from the perspective of relation between the level of reality (practical application), a definition of assumptions (in the language of micro-economy), a drawing up of a concept (by defining assumptions based on the cooperative game theory) and a setting of a corresponding axiomatic system. The application of the cooperative game theory apparatus for financial markets, specifically for the analysis of supply and demand of investment funding and investment opportunities is original and innovative. As part of the defined objective, we identify a general cause of a certain type of problems that arise in financial markets (we call it a "snag" in financial markets) and we point out a practical purpose of this phenomenon. The achieved results are well applicable at searching an answer to the question of what causes some of the phenomena that we are currently encountering in financial markets. At present, they are being applied in the financial markets research which is being carried out by the University of Finance and Administration based in Prague.

We will take the following steps:

1. We will point out the microeconomic and practical dimension of the problem and apply a numerical model to it.
2. We will identify the "snag" phenomenon.
3. We will describe it by means of the cooperative game theory and point out some interesting and from the practical point of view important characteristics of this phenomenon.
4. We will discuss our results with respect to some methodological issues of the cooperative game theory (relation between a theoretical solution and its practical purpose) and with respect to possible interpretations of various solutions of the Nash (S, d) problem and with respect to what we encounter in financial markets in the Czech Republic.
5. Lastly, we will outline a possibility of substantial expansion of the area of application of the processes which we deal with in our contribution. Since the single steps that we make require their permanent relation to a practical context, or more precisely identification of practical relevance of the rolling results, we will put emphasis on comprehensibility and a clear picture for a wide circle of experts, as it results from, inter alia, our experience of cooperation with specialists in financial markets.

2. Introduction to the problem – Microeconomic view of financial markets

We consider a simple model of the financial market which includes two entities, each of them having investment opportunities and investment funding. Combination of

a particular amount of investment funding and a particular investment opportunity results in a particular yield. We consider the current income that the business entities have at disposal to be investment funding. The future income that they will receive by combining investment opportunities and investment funding is considered to be their yield. We assume that both business entities will maximise their future yield and therefore they will utilise investment opportunities in the order of their rate of return, i.e. the function of the marginal rate of return on investment opportunities is a function non-increasing in its whole domain. Functions of the marginal rate of return on investment opportunities of both entities are continuous, when the minimum of one of the functions is smaller than the maximum of the second function and the maximum of the first function is greater than the minimum of the second function, see Fig. 1:

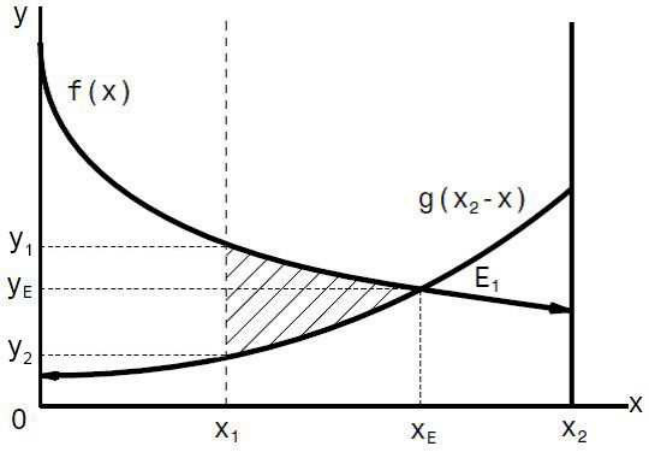

Fig. 1: Supply and demand of investment funds and investment opportunities

Here: $x_1,\ x_2 - x_1$ are the quantities of investment funding that business entities 1 and 2 have available, y is the future yield in marginal quantities, $f(x)$, $g(x)$, or $g(x_2 - x)$ non-deceasing continuous functions of the marginal yield on investment opportunities; $g(x)$ is modified for a better graphical illustration of the situation in question.

$E_1(x_E, y_E)$ is the point in which $f(x) = g(x) = f(x_2 - x) = g(x_2 - x)$; in this point all the investment opportunities of both entities are used depending on the rate of return. The hatched area shows a size of the maximum possible Pareto improvement as a result of the financial market effect if one of the entities gives up his less profitable investment opportunities and provides funding to the second entity.

The total yield of the first (analogically the second) business entity is as follows:

$$\int_0^{x_1} f\,(x)\,d\,(x) = \int_{x_1}^{x_2} f\,(x_2 - x)\,d(x),$$

resp.

$$\int_0^{x_1} g\,(x)\,d\,(x) = \int_{x_1}^{x_2} g\,(x_2 - x)\,d(x).$$

In the event that the cost of investment funding is determined by equality of marginal yields, i.e. by the fact that $f(x) = g(x)$, investment opportunities of both entities will be utilised depending on the rate of return on them. The compensation of the entity that provided his investment funding to implement an investment opportunity of the other entity will equal to $y_E(x_E - x_1)$.

Such process is satisfactory for the microeconomic approach. It has reached the Pareto equilibrium; both entities improved their positions compared to the previous ones; the origin and volume of interest have been clarified (as a compensation for use of investment funding of the other entity to implement their own investment opportunities). This solution seems to be problem-free.

$y\,(1) = \int_0^{x_1} f\,(x)\,d\,(x)$ is the function of the payoff of the entity 1,

$y(2) = \int_1^{x_2} g\,(x_2 - x)\,d\,(x)$ is the function of the payoff of the entity 2.

Distribution of the yields in Figure 1 can be mathematically described as follows:

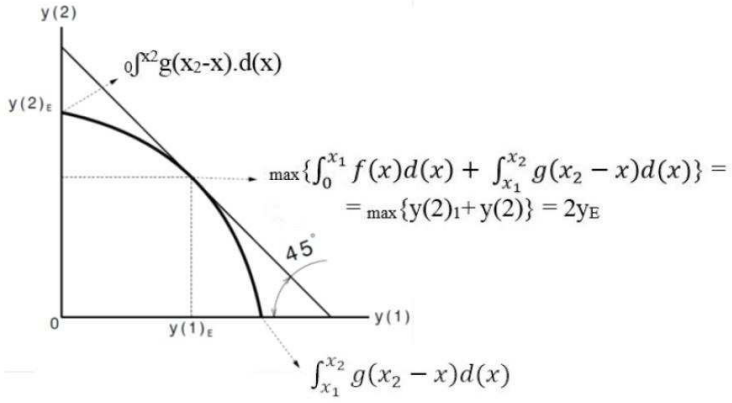

Fig. 2: Limit of achievable payoffs of both entities

where:

$\int_0^{x_1} f\,(x)\,d\,(x)$, $\int_{x_1}^{x_2} g\,(x_2 - x)\,d(x)$ are maximum payoffs of business entities,

$\int_0^{x_1} f\,(x)\,d\,(x) + \int_{x_1}^{x_2} g\,(x_2 - x)\,d(x)$ is a function delimiting a set of permissible payoffs,

$\max\{\int_0^{x_1} f\,(x)\,d\,(x) + \int_{x_1}^{x_2} g\,(x_2 - x)\,d(x)\}$ is the maximum total of business entities' payoffs.

The points inside the area of delimitation by the curve of the sum of the payoffs can be interpreted as points corresponding to business entities' payoffs in the situation when they did not use all of their investment funding.

As long as the financial market is in operation (investment funding of one business entity can be used for implementing investment opportunities of the other business entity), both entities can increase their payoffs if the cost of investment funding – marked with the letter y – is in the interval between $f(x_1) = y_1$ and $g(x_1) = y_2$, i.e. if it is true that $g(x_1) < y_i < f(x_1)$. At the cost of investment opportunities equal to y_E, all the investment opportunities prioritised by their return on them will be used.

2.1. Microeconomic model testing

Before we applied the apparatus of the cooperative game theory to the microeconomic issues of relation between supply and demand of investment funding and investment opportunities, we had created a simple numerical model to test various situations. To understand the achieved results better and to illustrate how financial markets work from the viewpoint considered by us, we will point out some findings from the numerical model testing.

$$y' = 2(5 - x),\ y = 10x - x_2,$$
$$x_1 = 2,\ y'_1 = 6,\ y_1 = 16,$$
$$x_2 = 5,\ y'_2 = 0,\ y_1 = 25,$$
$$y_1 = 2.6 + 2.4.$$

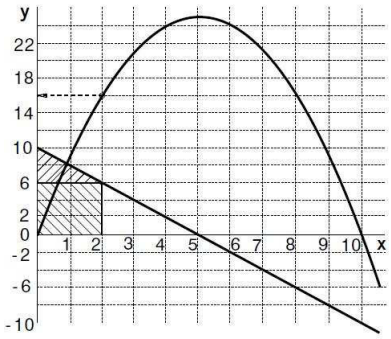

Fig. 3: Quadratic production function simulating a decrease of the marginal yield on investment opportunities

First part of the Figure 4 on the left side shows supply and demand of investment funding and investment opportunities of two entities in case of a quadratic production function of both entities, when one of its possible situations is presented in Figure 3. In first part of the right side of Figure 4. you can see the Pareto improvement in the graph, where axes ${}^1y$ and ${}^2y$ show yields of individual entities. Notice the typical "heart" shape which appears in most models and implies that the maximum yield of individual entities is usually not in the point of full utilization of investment opportunities. Because of significance of the above-mentioned, we present one more Figure (5,6) which shows the area of Pareto improvements in detail:

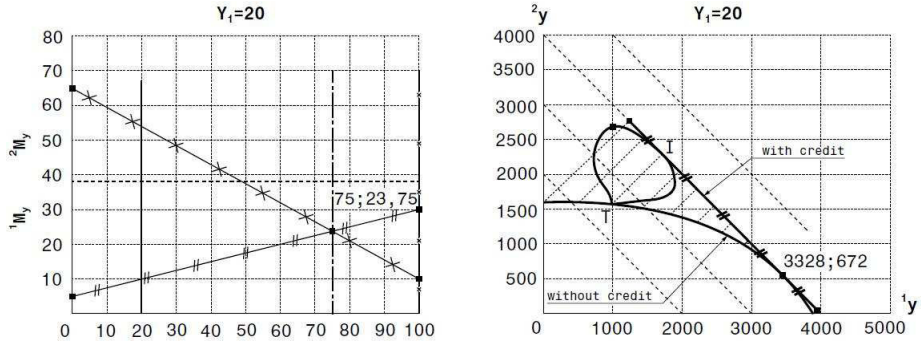

Fig. 4: Supply and demand of investment funding and investment opportunities of two entities in case of a quadratic production function

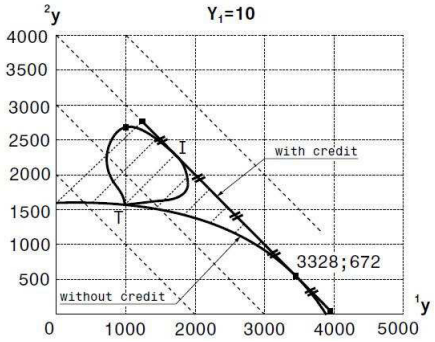

Fig. 5: Detail of Pareto improvement areas of financial market stakeholders

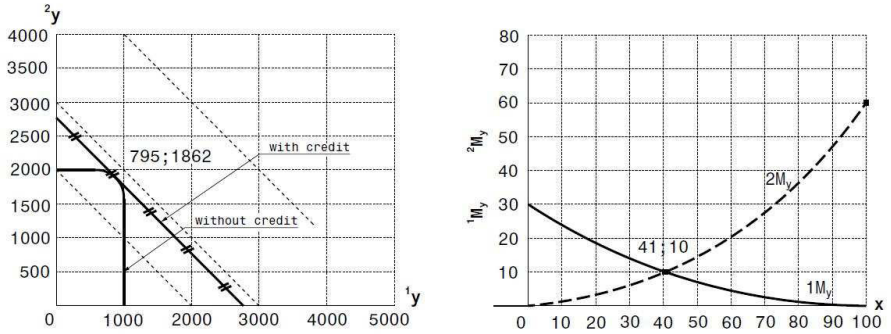

Fig. 6: Example of another marginal yield functions (quadratic function of decreasing marginal yields)

The concept we have used to illustrate the relation of supply and demand of investment funding and investment opportunities can have other interpretations as well. One of them is the water problem, for details see e.g. (Beal, 2013), (Brink,

2011), (Houba, 2013). From the practical point of view, we have two very different tasks, however, in terms of description by an abstract concept, the problems are almost identical. We will comment on this topic at the end of our contribution. To monitor multiple possible interpretations during the presentation (and moreover without knowledge of what the presentation will bring) would make comprehensibility of our steps significantly harder.

3. Problem of the yield distribution in the language of cooperative games, Nash (S, d) problem

As we approach the area of game theory, we will call business entities players. If in the point $(y(1)_E, y(2)_E)$ both players achieved the maximum payoff at the costs of investment funding being in the bounded interval $\langle y_1, y_2 \rangle$ Fig. 1. If in that point both players achieved the maximum payoff at the costs of investment funding being in the bounded interval, assumption of the individual rationality would suffice to regard the point $(y(1)_E, y(2)_E)$ as an intuitively acceptable solution of the respective cooperative task. However, this need not to be true, see Figure 7:

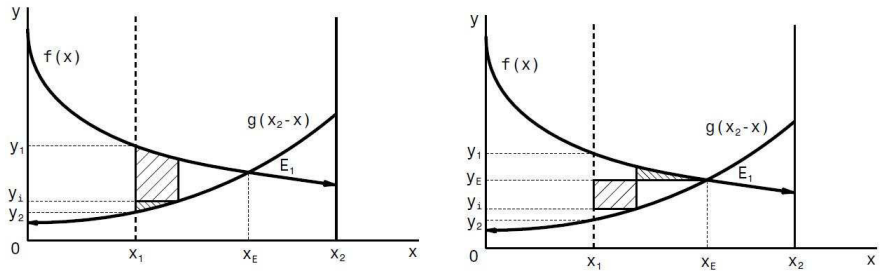

Fig. 7: Players' payoffs at changing costs of investment funding

The figure on the left side shows an increase of the first player's and second player's payoff at the cost of investment funding y_i. The next figure shows the change which would occur if the cost of investment funding changed from y_i to y_E. As seen in Figure 7, at the cost of investment funding y_E (i.e. at the cost which we consider equilibrium from the microeconomic point of view), the payoff of the first player will decrease as a result of the decline in the cost of investment funding (interest, compensation) more (see the hatched area 45° in the figure on the left side) than it increases as a result of the fact that more investment opportunities will be utilised (see the thickly hatched area 135° in the figure on the left side).

This is a very significant moment. It turns out that the individual rationality assumption need not suffice to find an unequivocal solution. Therefore, it is useful to convert the problem encountered by us to the form of the Nash (S, d) bargaining problem.

Now, let us look at the following Figure 8.

It corresponds to Figure 3-7; however, it shows the compensation area. We regard compensation as the part of the yield which the person who owns utilisable investment opportunities will transfer to the person who lent investment funding (i.e. in our case interest paid by the debtor to the creditor as well as the cost of

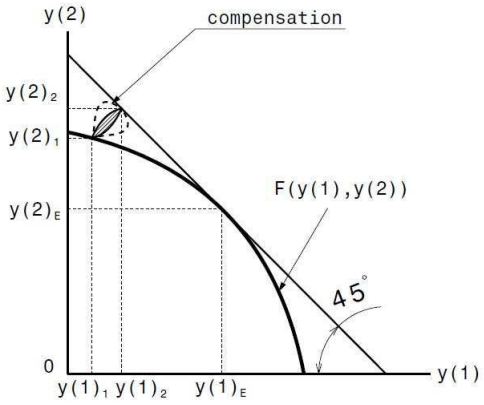

Fig. 8: Compensation area

investment funding). The size of compensation naturally depends on the interest rate, i.e. on the cost of investment funding.

The particular task can be also viewed as the Nash (S, d) bargaining problem. In the following figure we will illustrate the problem which occurs at compensations. For this purpose, we will look at the compensation area in more detail. Figure 9 is an enlarged section of Figure 8.

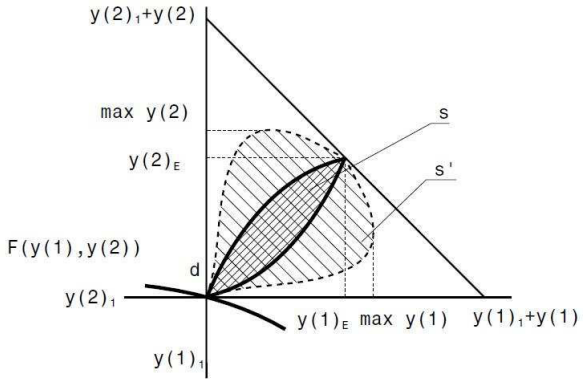

Fig. 9: Compensation area as the Nash (S, d) problem

S is a set of payoff distribution possibilities in the situation when $\max y(1)$ and $\max y(2)$ are smaller than $y(1)_E$ and $y(1)_E$. S' is a set of payoff distribution possibilities in the situation when $\max y(1)$ and $\max y(2)$ are greater than $y(1)_E$ and $y(1)_E$. Pareto improvement functions are in Graph 5. The solid-line curve represents the situation when $\max y(1)$ and $\max y(2)$ are smaller than $y(1)_E$ and $y(1)_E$; the broken-line curve the situation when $\max y(1)$ and $\max y(2)$ are greater than $y(1)_E$ and $y(1)_E$. In the first situation (provided that the cost of investment funding is constant and use is made of all the investment opportunities the yield of which is

greater than this cost), the solution of the cooperative task in question is univocally determined by the individual rationality assumption.

In the second example, the individual rationality assumption does not suffice. A "**snag**" (term introduced by us) occurs at **utilisation of investment opportunities.** It is possible to apply a number of potential approaches to a solution of the cooperative game in question. Let us view the particular problem (the cooperative task in question) in more detail.

Each of the players may claim his maximum payoff: $\max y(1)$, or as the case may be $\max y(2)$. The bargaining will then take place in the bounded interval between $y1_{max}$ and $y2_{max}$ (which is the cost of investment funding at which one or the other of the players will achieve the maximum payoff), see Figure 10.

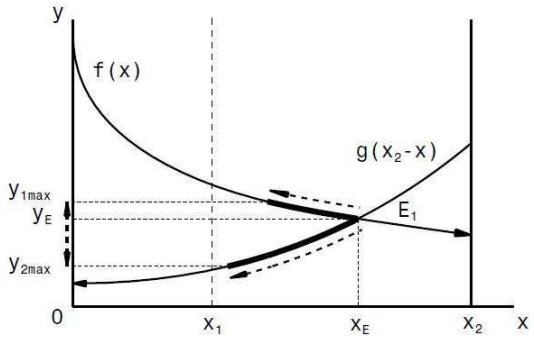

Fig. 10: Possiblity for bargaining

The same can be illustrated in Figure 11.

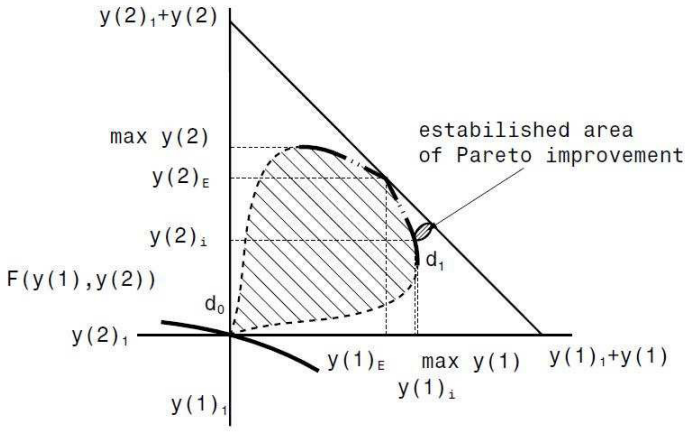

Fig. 11: Area of Pareto improvement

Only the points on thick solid lines in both pictures correspond to the requirements (axioms) of achievability and collective rationality.

Each cost of investment funding which the players opt for or more precisely agree on corresponds to a particular distribution of yields between them. However, only if the cost equals to y_E, use will be made of all the investment opportunities prioritised by their rate of return.

It means that if the players opt for any solution of the cooperative game which results in a different cost of investment funding than y_E, they have a chance to act in such a way that they will improve their position compared to this solution of the cooperative game, see Figure 10.

If they opt for the solution that results in the cost y_i, then the hatched 45° area of Pareto improvements is offered. We can show this hatched 45° area in the Figure 11 as well.

If we want to meet the requirement of collective rationality, i.e. to exploit all the Pareto improvements, to use the language of microeconomy, then in the situation when the (S, d) solution of the Nash bargaining problem does not result in the cost when the sum of the payoffs is maximised, another sequential improvement is always possible at a different cost of investment funding. In the limit situation, sequential solutions constructed this way and derived from any type of the cooperative solution of the Nash bargaining problem will reach the line of the maximum sum. All the investment opportunities, regardless to which of the entities (players) they belong, are utilised depending on their rate of return. Nevertheless, the distribution of players' payoffs need not (and usually will not) correspond to the point of the maximum sum in the primary task, see Figure 12.

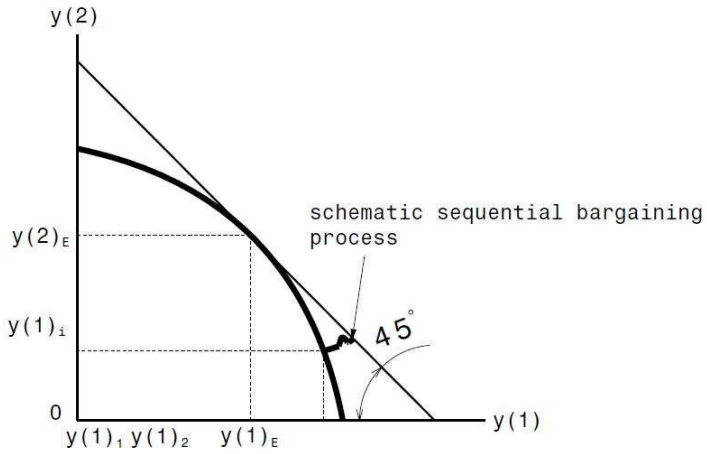

Fig. 12: Sequential bargaining process

4. Results and Discussion

The task that we are dealing with has a number of interpretations. With regard to what we know especially two of them come into focus:

- Financial markets, i.e. supply and demand of investment funding and investment opportunities.

- Water problem – supply of water and a possibility of its use in order to realise a yield in a various way.

Generally, this task can be understood as an application of a production factor in different ways if we have two owners who have different possibilities of applying the respective production factor in order to achieve some yield.

We assume that each of the owners of the resource is in question and he uses his own opportunities to use this resource according to the rate of return on the opportunities that he owns. The question is whether or under what conditions opportunities prioritised by their rate of return are utilised regardless of who is the owner of such opportunities for application of the production factor (and who is the owner of the production factor). From this point of view, let us repeat our initial assumption – business entities (players) use their investment funding to implement investment opportunities prioritised by their rate of return. Let us proceed from this assumption to an analysis of assumptions under which investment opportunities prioritised by their rate of return are implemented regardless which of the business entities (players) owns them. And it turns out that it is a very difficult topic.

4.1. Recapitulation of the findings and a discussion about the theoretical and methodological aspect of the topic in question

If the cost of the production factor may change in the sense that it is paid (as a compensation) by the person who uses the resource for his own opportunities, at costs at various stages of bargaining and rolling agreements (realised acts of exchange), then all the investment opportunities prioritised by their rate of return are used (regardless to whom they belong); the solution has a sequential character.

The requirement of a single cost for all resource units and simultaneous utilisation of all the opportunities of resource use prioritised by the rate of return (i.e. if we want both requirements to be applied simultaneously) is met by the only solution at which the sum of yields is maximised right in the first step. Any other solution conflicts with a simultaneous meeting of both requirements. If we do not insist on the requirement of a single cost, there exist an endless number of different sequential solutions at which the sum of yields is maximised by gradual compensations.

On the basis of a given task, a great number of currently known solutions can be interpreted – dictator's, egalitarian, Kalai-Smorodinski's, Raiffa's solution, etc. At a certain shape of the function delimiting yields in the point of the maximum sum (unless the function is smooth), some of the solutions merge into one (those of Nash (1950), Kalai-Smorodinski (1977, 1975), Raiffa (1953)).

As the first step for evaluating the presented result theoretically, it is possible to compare the currently known solutions of the Nash (S, d) bargaining problem with the possibility of interpreting them from the perspective of their relevance (information values) for the problem of the "snag" in financial markets:

- Which of the known solutions have a real interpretation?
- For which do we lack an interpretation?
- Are there any of the solutions of the Nash (S, d) bargaining problem which, for principal reasons, cannot have any interpretation in a given area and why?

One of the extremely interesting applications is a survey of the financial market development which can be mentioned based on the above-mentioned facts. It is as follows:

– Imperfections of the financial market can be understood as impossibility of using all the resources (investment funding) according to their rate of return.

– One reason may be that problems can not be solved on the basis of maximization the amount and at the same time, only acceptable solutions are those which set a single cost.

– These solutions can be described empirically, notionally, conceptually and presumably axiomatically as well. It means that each imperfection agrees with a certain type of a cooperative game solution which can be expressed at a various level of abstraction.

The cooperative game theory then gives a highly effective instrument for describing financial markets and their development:

– Identification of standard situations.

– Identification of standard transitions between different stages and ways of the financial market development.

– Definition of ways of improving financial markets (or, on the contrary, their degeneration), etc.

It would be interesting to see whether there are other tasks which would similarly induces a wide spectrum of problems associated with a search and comparison of various cooperative games in terms of relation among reality, concept, model and different levels of abstraction, or transition up to its axiomatic expression. At this moment, we consider the area that we have touched upon particularly interesting from the above-mentioned point of view.

From theoretical, methodological and philosophical perspectives, these are interesting issues of relation of our real world on one hand and possible worlds on the other hand that are partly a result of our abstractions and partly corresponding to possible changes of our real world.

4.2. Outline of one of the possible applications of achieved results to examine current problems in financial markets in the Czech Republic

One of the conclusions arising from the attained findings can be briefly formulated as follows: If entities in the market of supply and demand of investment funding and investment opportunities are in an asymmetric position, when the stronger position belongs to the entity offering investment funding, this stronger entity will offer investment funding for differentiated costs in such a way to increase his yield as much as possible to the detriment of the other entity. After he uses investment opportunities of the other entity at the cost which is the most advantageous for the entity offering investment opportunities, (only then) he will offer his additional investment opportunities at a lower cost. This process can repeat.

Now, let us look at one of the topical or rather acute problems of the Czech financial market (to discuss the question of whether it shows the financial market "snag" problem by us revealed and analysed, or not). We will show that the concept of analysing the snag in financial markets can be used at analysing real situations which occur at present. Real situations always comprise multiple, various influences. A good model facilitates their precise identification. Let us try to do so in the context of the following situations that occurred in the Czech financial market.

Continuation in interventions of the Czech National Bank as the most visible part of its monetary policy under the current macroeconomic conditions results in accomplishing new "records" in some key monetary indicators. In recent days, the

volume of ready money in circulation has exceeded CZK 500,000 million and we may discuss or rather speculate about reasons for its permanent growth without regard to the real economic growth.

On 13 November 2015, the total value of all banknotes and coins which are currently in circulation ("money in circulation") exceeded the amount of CZK 500,000 million for the very first time. This amount represents more than 2,000 million banknotes and coins in circulation. In the long term, the most numerous representations among Czech money have been had by the one-thousand-koruna banknote and the one-koruna coin, as shown in the statistics of the Czech National Bank.

A new record was also shown in CNB deposits of domestic commercial banks which recently for the very first time exceeded CZK one trillion, i.e. one million million. The fact that this amount is twice as big as the total value of all the banknotes and coins which are currently in circulation is related to the effect of the money multiplier, or more precisely to multiplication of deposits.

In the past, a long-term massive surplus of liquidity in domestic banks was invested in Czech state bonds that account for a majority of the securities in their portfolio structure. A high demand for Czech state bonds resulted in a slump of their yield, across maturities. In shorter maturities (up to two years), yields of Czech state bonds gradually moved to red figures and recent state bond auctions show that a negative yield appears in longer maturities (three to five years) as well. However, lower yields of Czech state bonds, also when compared with German bonds that are generally considered the lowest-risk state bonds, do not mean that the Czech Republic is a more solvent debtor than Germany, but they reflect the fact that almost all foreign investors expect the Czech koruna appreciation after the CNB's interventions end.

The fact that the time of foreign currency interventions is quickly drawing to an end and at the same time some members of the Bank Board admit aloud that interventions may continue even after the originally announced half of the year 2016 gives the impression of the Bank Board of the Czech National Bank not having any exit strategy from the intervention regime drawn up, let alone possible exit scenarios. At the high quality of work of Czech National Bank' analysts, it is strange that the possibility of a jump appreciation of the Czech koruna after the end of interventions is considered highly unlikely, opposed to a majority of economic experts. The development of the Czech economy in relation to its main business partners has been showing a convergent character in the long term and an artificial prevention of fluency of this convergence (through foreign currency interventions) only increases the future pressure on koruna appreciation. The situation when at comparable work productivity and costs of living the labour force receives a wage of third or fourth the amount (when compared to developed European countries) is not sustainable in the long run. And jumps and shocks of all kind rank among the factors that are not beneficial to any normally functioning economy. Without comparing the Czech Republic and the Swiss Confederation, the jump appreciation of the Swiss franc after the end of interventions that we witnessed is at least food for thought (not only for our central bankers). A permanent inflow of euros from the EU funds which will be higher than the outflow still for few years results in additional demand for Czech koruna and a pressure on its appreciation. Moreover, the sale of Czech state bonds with a negative yield at primary auctions (organised by the Czech National

Bank) indicates that all the "big players" in financial markets already expect the appreciation of koruna.

The current structure of the CNB' liabilities in [million Kč]			
1.	Bank notes and coins in circulation	495 873	-829
2.	Liabilities to the IMF	49 655	918
3.	Liabilities abroad	21 792	-1 641
3.1	Loans from foreign banks	6 335	-3 000
3.2	Other liabilities abroad	15 457	1 359
4.	Liabilities to domestic banks	955 876	1 625
4.1	Loans received	395 300	81 100
4.2	Minimum reserve requirement	81 959	3 117
4.3	Other liabilities to banks	478 617	-82 592
5.	Liabilities to the state and other public institutions	1 913	107
6.	Other liabilities	12 174	147
7.	Reserves	282	0
8.	Registered capital and reserve funds	15 561	0
9.	Revaluation differences	11 670	0
10.	Profit or loss for the previous period	0	0
11.	Profit or loss for the accounting period	5 475	10 875
	LIABILITIES IN TOTAL	1 570 271	11 202

The current structure of the CNB' liabilities is illustrated in the table.

Now, let us view the topic in question from the perspective of two players of a cooperative game, when one of the players are commercial banks and the other player the other financial market stakeholders, including the Czech National Bank. It is rather simplified, especially with regard to the fact that:

– Commercial banks compete with each other.

– The other stakeholders in the financial market represent a substantially heterogeneous mix (starting from the already-mentioned Czech National Bank through the state bond market up to companies who are granted loans by commercial banks and last but not least households as the financial market stakeholders at whom consumer loans and mortgage loans in particular target).

With awareness of the reservations we have mentioned, it is however evident that the following behaviour prevails in commercial banks (that, as it turns out, act in a certain agreement):

– They first select such entities whom they can grant loans at a very high interest rate.

– They differentiate among them depending on how big the loans are, how high the related risk is, what the transaction costs are and how transparent the information about these stakeholders is.

– Subsequently, they invest a huge amount through deposits in the central bank at very low interest; (however, in the context of this particular investing as a cost of sacrificed opportunity in relation to the other investment possibilities) they strengthen their asymmetric or privileged position against the other owners of investment opportunities.

To what extent this phenomenon is related to our analysed phenomenon of the "snag" in financial markets is a subject of another analysis. It is necessary to take

into consideration various aspects that we mentioned in connection with a reduction of the task to two players of the cooperative game. A certain connection undoubtedly exists here and the instruments of the financial market "snag" analysis discussed by us (as well as those that we will acquire by further examination directing the discussion towards theoretical and methodological aspects of the respective problem) may produce immensely interesting findings.

5. Conclusions

Since the early 1950s the cooperative game theory has been developing under the influence of theoretical problems arising both in the field of the game theory itself and in interaction with real problems outside the game theory (mostly in the economic field). It turned out that different applications or contexts correspond to different potential solutions of standard tasks, e.g. Nash (S, d) bargaining problem. Also, a number of different axiomatic systems describing the (S, d) problem have been established. Some of the solutions responded to the issues arising in the then important economic theory of social welfare, or the issue related to redistribution of wealth among various groups of population and justification of such redistribution. To do so, some of the arbitrary solutions of the Nash (S, d) bargaining problem were mostly applied.

In our contribution, we have presented an interesting and significant area of interpretation of different solutions of the Nash (S, d) bargaining problem in connection with some other issues of the cooperative game theory and the instruments employed by this theory (e.g. in relation to the water problem solution) at analysing supply and demand of investment funding and investment opportunities. And this both at the general level and in specific conditions of the Czech Republic, where we interpreted this task as a cooperative game of two players in which one of the players is commercial banks and the other player is the other financial market stakeholders, including the Czech National Bank. The interpretation and the subsequent discussion about the theoretical and methodological aspect of the topic in question have produced many interesting findings concerning the relation among the level of reality (practical application), a definition of assumptions (in the language of microeconomy), a drawing up of a concept (by defining assumptions based on the cooperative game theory) and a setting of a corresponding axiomatic system. Equally, the application to the issues of the Czech financial market and the related discussion about achieved results shows that it is a useful and promising topic.

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