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On a Dynamic Traveling Salesman Problem *

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Abstract In this paper we consider a dynamic traveling salesman problem (DTSP) in which n objects (the salesman and m customers) move on a plane with constant velocities. Each customer aims to meet the salesman as soon as possible. In turn, the salesman aspires to meet all customers for the minimal time. We formalize this problem as non-zero sum game of pursuit and find its solution as a Nash equilibrium. Finally, we give some examples to illustrate the obtained results.

Keywords: dynamic traveling salesman problem, non-zero sum game, Nash equilibrium.

1. Introduction

We consider the classical traveling salesman problem (TSP). The idea of the TSP is to find a route of a given number of cities, visiting each city exactly once and returning to the starting city where the length of this tour is minimized. The first instance of the traveling salesman problem was from Euler in 1759 whose problem was to move a knight to every position on a chess board exactly once. The traveling salesman first gained fame in a book written by German salesman B.F. Voigt in 1832 (Michalewicz, 1994) on how to be a successful traveling salesman. He mentions the TSP, although not by that name, by suggesting that to cover as many locations as possible without visiting any location twice is the most important aspect of the scheduling of a tour. The origins of the TSP in mathematics are not really known - all we know for certain is that it happened around 1931 (Michalewicz, 1994).

Currently the only known method guaranteed to optimally solve the traveling salesman problem of any size, is by enumerating each possible route and searching for the tour with the shortest length. When n gets large, it becomes impossible to find the cost of every tour in polynomial time. Many different methods of optimization have been used to try to solve the TSP.

The traveling salesman problem has many different real world applications, making it a very popular problem to solve. For example, some instances of the vehicle routing problem can be modeled as a traveling salesman problem. Here the problem is to find which customers should be served by which vehicles and the minimum

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number of vehicles needed to serve each customer. There are different variations of this problem including finding the minimum time to serve all customers.

The TSP: given a list of cities and the distances between each pair of cities, what is the shortest possible route that visits each city exactly once and returns to the origin city?

For the classical traveling salesman problem there are the following difficulties:

- The rule that one first should go from the starting point to the closest point, then to the point closest to this, etc., in general does not yield the shortest route.
- It is an NP-hard problem in combinatorial optimization, important in operations research and theoretical computer science.
- Algorithms for finding exact solutions work reasonably fast only for small problem sizes.

In this paper we consider a dynamic traveling salesman problem (DTSP) allowing all considered objects (the salesman and customers) to move on a plane with constant velocities. We apply a game theoretical approach to solving the DTSP. In fact, we propose to use some methods of pursuit game theory (Isaaks, 1965) for this purpose (Petrosjan and Shirjaev, 1981; Petrosjan, 1983; Kleimenov, 1993; Tarashnina, 1998; Pankratova and Tarashnina, 2004; Pankratova, 2007). This means that each agent is considered as a player that has his own aim and his profit is described by a payoff function. The players may use admissible strategies and interact with each other. Here we find a solution of the DTSP as a Nash equilibrium in a non-zero sum game of pursuit. In other words, we define strategies of all players that provide the minimal length of the salesman route.

2. The game

We have m customers $C_1, \dots, C_m$ who are initially located in different cities and move on a plane with constant velocities, and a salesman S who wants to meet all of them. The players start their motion at the moment $t = 0$ at initial positions $z_1^0, \dots, z_m^0, z^0$. At each instant t they may choose directions of their motion. Let α be the velocity of salesman S, β_j be the velocity of customer C_j, $j = 1, \dots, m$, $\alpha < \beta_j$.

Suppose that the salesman never meets the same customer twice and does not return to the starting point (he she stays in the last meeting point). Thus, the salesman tries to find the shortest route that passes through the customers' current positions once and each customer also wants to meet the salesman as soon as possible.

In contrast to the classical problem, where customers are located at fixed points and may not move, here they move with constant velocities.

A strategy of salesmen S

$$u_S(t, z_1^t, \dots, z_m^t, z^t) = u_S.$$

The salesman uses piecewise open-loop strategies.

A strategy of customer C_j is a function of time, players' positions and a velocity-vector of the salesman at a current time moment, i.e.

$$u_{C_j}(t, z_1^t, \dots, z_m^t, z^t, \mathbf{u}_S^t) = u_{C_j},$$

where $z_1^t, \ldots, z_m^t, z^t$ are current positions of the players and $\mathbf{u}_S^t$ is a vector-velocity of S at time instant t. In this game we suppose that the customers use the parallel pursuit strategy (Π-strategy) (Petrosjan, 1965). Denote by $\mathcal{U}_S$ and $\mathcal{U}_{C_j}$ the sets of admissible strategies of the players, $j = 1, \ldots, m$.

The game is played as follows: at the initial moment of time the salesman informs customers $C_1, \ldots, C_m$ about a chosen direction of his motion. After that, S meets the customers on his route if they cross it. The game is finished when the salesman meets the last customer. S aspires to minimize the total meeting time, i.e. to meet all customers for the minimal time. At the same time each customer wants to minimize his own meeting time.

The payoff function of customer C_j is

$$K_{C_j}(z_1^0, \ldots, z_m^0, z^0, u_{C_1}, \ldots, u_{C_m}, u_S) = -T_j, \tag{1}$$

where T_j is a meeting time of S and customer C_j.

The payoff function of salesman S is

$$K_S(z_1^0, \ldots, z_m^0, z^0, u_{C_1}, \ldots, u_{C_m}, u_S) = -\max\{T_1, \ldots, T_m\}. \tag{2}$$

The objective of each player in the game is to maximize his own payoff function. So, we define this problem in a normal form

$$\Gamma(z_1^0, \ldots, z_m^0, z^0) = \langle N, \{\mathcal{U}_i\}_{i \in N}, \{K_i\}_{i \in N} \rangle, \tag{3}$$

where $N = \{C_1, \ldots, C_m, S\}$ is the set of players, $\mathcal{U}_i$ is the set of admissible strategies of player i, and K_i is a payoff function of player i defined by (1) and (2), $i \in N$. The constructed game depends on initial positions of the players. Let us fix players' initial positions and consider the game $\Gamma(z_1^0, \ldots, z_m^0, z^0)$.

3. Basic notions and definitions

Give some notions of pursuit game theory that help to find a solution of the DTSP.

Definition 1. The parallel pursuit strategy (Π-strategy) is a kind of motion of a customer C regard the motion of salesman S which provides a segment C^tS^t connecting current players' positions C^t and S^t at each time moment $t > 0$ to be parallel to the initial segment C^0S^0 and its length strictly decreases.

Since we suppose that all customers use the parallel pursuit strategy, the following definition of the Apollonius circle is needed.

Definition 2. The Apollonius circle $A(z_j^0, z^0)$ for initial positions $C_j^0 = z_j^0$ and $S^0 = z^0$ of customer C_j and salesman S, respectively, is the set of points M such that

$$\frac{|S^0M|}{\alpha} = \frac{|C_j^0M|}{\beta_j},$$

where $\beta_j > \alpha > 0$ (see Fig. 1).

First let us consider a three person game: with salesman S and two customers C_1, C_2. We have two intersection points of the Apollonius circles. The set of all intersection points of the Apollonius circles is denoted by Z. In this game $Z = \{z_{12}, z_{21}\}$

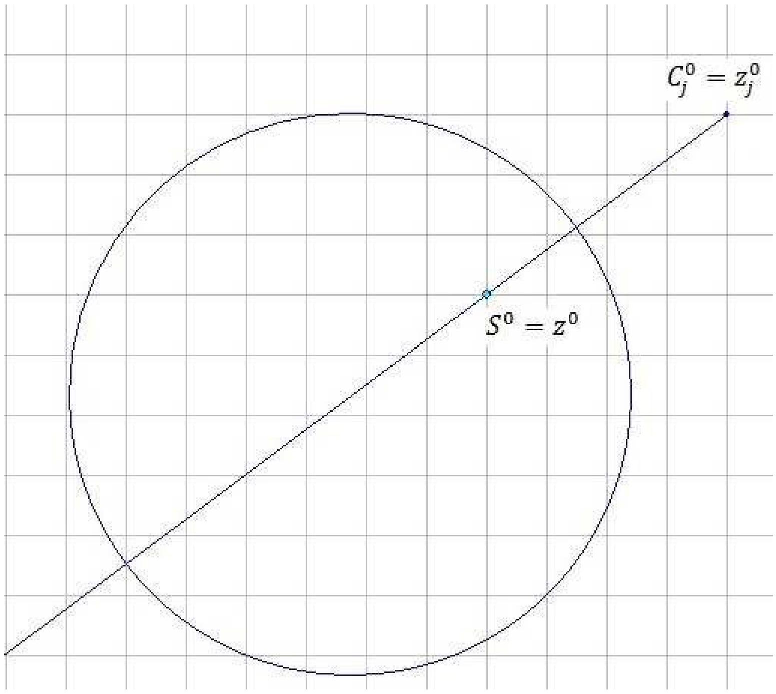

Fig. 1. The Apollonius circle for game $\Gamma(z_j^0, z^0)$

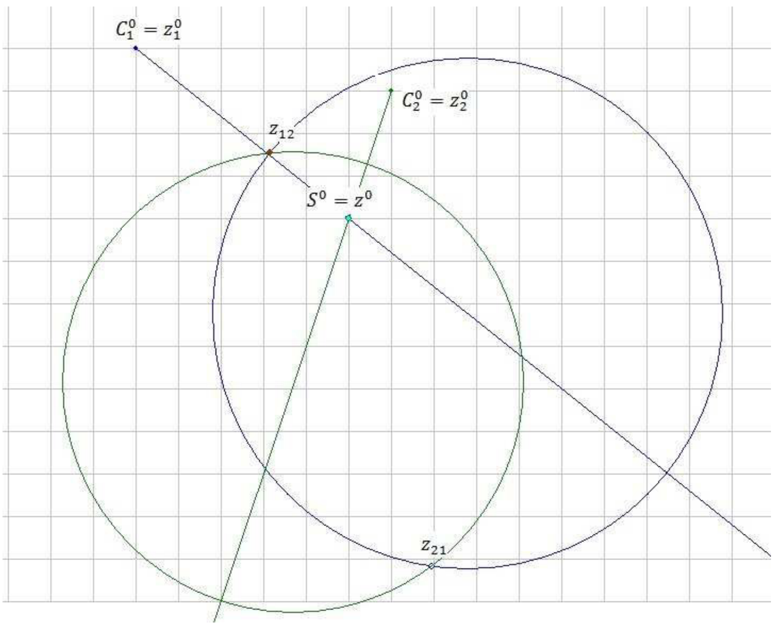

Fig. 2. The Apollonius circles for game $\Gamma(z_1^0, z_2^0, z^0)$

(Fig. 2). In Pankratova, Tarashnina, Kuzyutin, 2016 the analytical formulas for finding coordinates of the intersection points of the Apollonius circles are given.

In Fig. 3 there are the Apollonius circles for all pairs of S and C_j, $j = 1, \ldots, m$. Denote by A_j the Apollonius disk corresponding to the Apollonius circle $A(z_j^0, z^0)$.

It is known that if customer C_j uses the parallel pursuit strategy and salesman S uses any admissible strategy from $\mathcal{U}_S$, then all possible meeting points of the salesman and customer C_j cover the Apollonius disk (Petrosjan, 1983). In particular, if the salesman moves along a straight line, then a meeting point of S and C_j lies on the Apollonius circle.

The union of all Apollonius disks is denoted by A, i.e. $A = A_1 \cup \ldots \cup A_m$ and $\partial A = \partial(A_1 \cup \ldots \cup A_m)$ is a boundary of the set A.

In addition, we introduce a notion of the level of the boundary. The boundary $\partial A = \partial^1 A$ is called the boundary of the first level. If we remove the boundary of the first level, then the remaining Apollonius disks form a new boundary, we call it the boundary of the second-level and denote by $\partial^2 A$, etc.

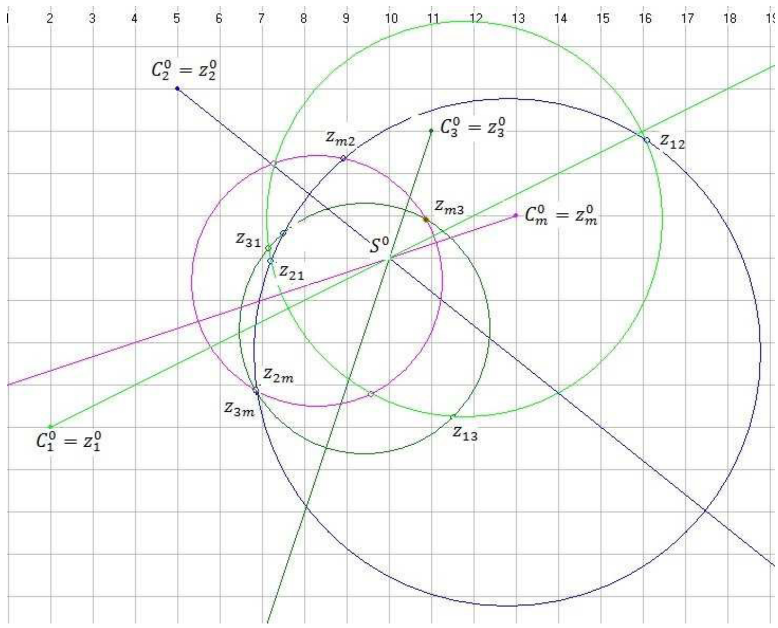

Fig. 3. The Apollonius circles for game $\Gamma(z_1^0, \ldots, z_m^0, z^0)$

4. Nash equilibria

Introduce the following types of behavior of salesman S.

Behavior u_S^1: Salesman S uses the type of behavior u_S^1, according to which he moves along a straight line towards customer C_j, that is, to the nearest point on the boundary of the union of all Apollonius disks A_j, $j = 1, \ldots, m$ (Fig. 4).

Behavior u_S^2: Salesman S uses the type of behavior u_S^2, according to which he moves along a straight line to the nearest intersection point of the Apollonius circles $A(z_j^0, z^0)$ and $A(z_k^0, z^0)$ $(j \neq k)$ that belongs to the boundary ∂A (Fig. 5).

Behavior u_S^3: Salesman S uses the type of behavior u_S^3, according to which he moves along a straight line to the nearest intersection point of the Apollonius

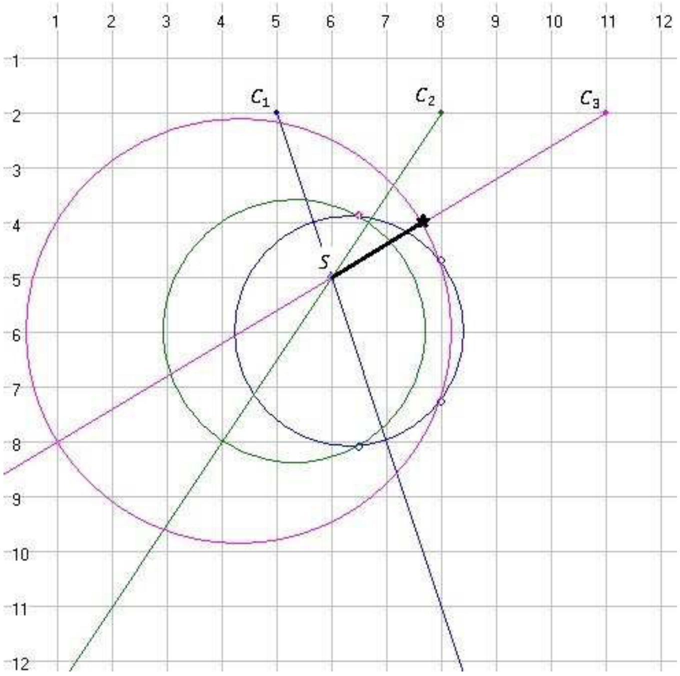

Fig. 4. Behavior u_S^1

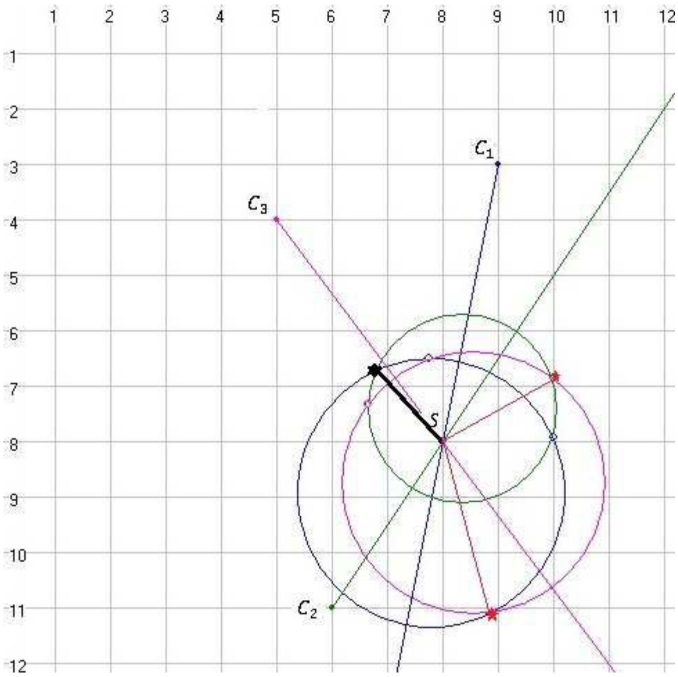

Fig. 5. Behavior u_S^2

circles $A(z_j^0, z^0)$ and $A(z_k^0, z^0)$ $(j \neq k)$ that belongs to the boundary $\partial^2 A$ (Fig. 6) and then changes his direction and moves along a straight line towards the last customer C_l, $l \neq j \neq k$.

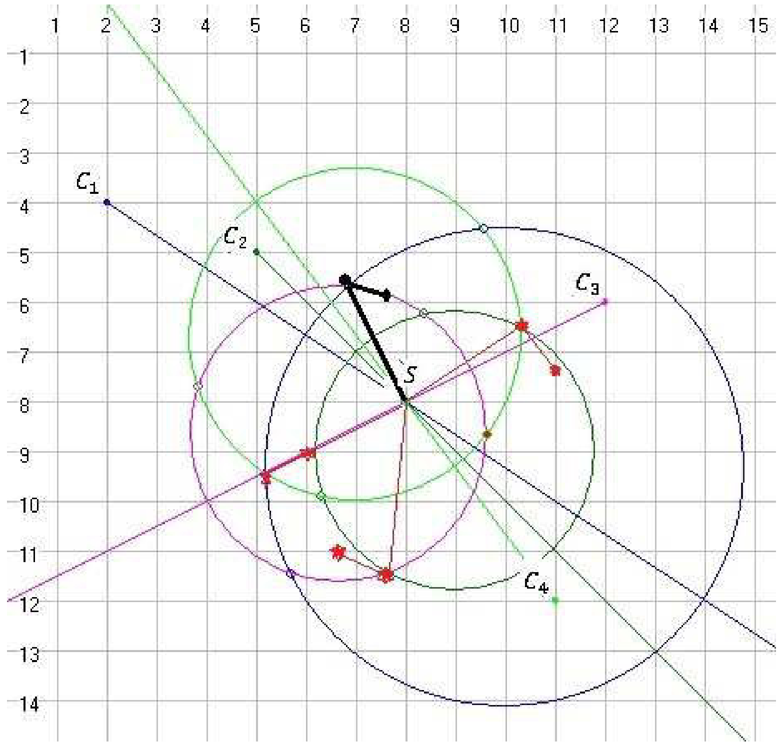

Fig. 6. Behavior u_S^3

Theorem 1. *In the dynamic traveling salesman problem $\Gamma(z_1, z_2, z_3, z_4, z)$ there exists a Nash equilibrium. It is constructed as follows:*

- *The salesman chooses strategy u_S^* that prescribes to him one type of behavior u_S^1, u_S^2 or u_S^3 and gives the minimal meeting time.*
- *The customers use Π-strategy.*

Remark 1. If there exists behavior u_S^1, then it provides the salesman the minimum meeting time and there is no sense to consider the other types of behavior: u_S^2 and u_S^3.

Example 1. Consider a game $\Gamma(z_1^0, \ldots, z_4^0, z^0)$ with salesman S and four customers C_1, C_2, C_3, C_4 with initial conditions:

$$S^0 = (8; 8),\ C_1^0 = (7; 1),\ C_2^0 = (3; 5),\ C_3^0 = (9; 2),\ C_4^0 = (12; 4)$$

and velocities

$$\alpha = 2,\ \beta_1 = 3,5,\ \beta_2 = 4,\ \beta_3 = 4,\ \beta_4 = 4,$$

respectively.

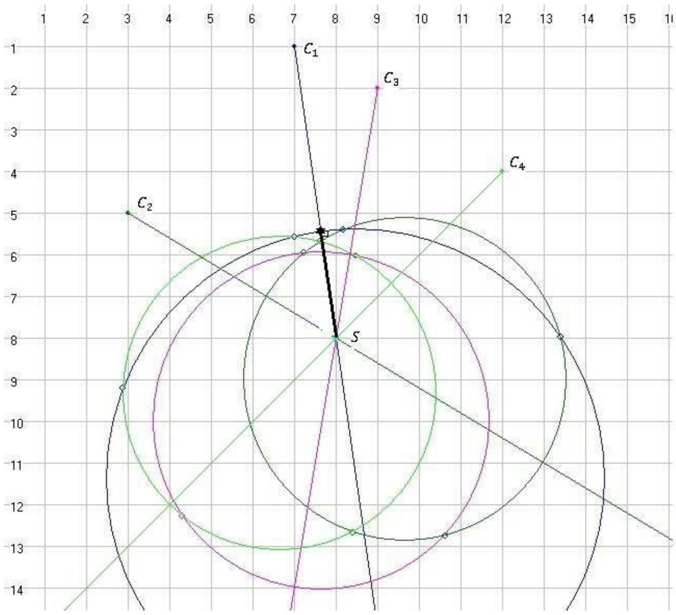

Fig. 7. Nash equilibrium in $\Gamma(z_1^0, z_2^0, z_3^0, z_4^0, z^0)$: behavior u_S^1

In this game there exists Nash equilibrium in which salesman S uses behavior u_S^1. In other words, S moves along a straight line towards customer C_1 to point (7,636; 5,454). The corresponding trajectory of his motion is shown in Fig. 7 (the thick line). The customers move to the following points using Π-strategy:

- C_1 moves to point (7,636; 5,454),
- C_2 moves to point (7,666; 5,666),
- C_3 moves to point (7,706; 5,945),
- C_4 moves to point (7,671; 5,698).

In fact, the salesman meets the customers in the order C_3, C_4, C_2, C_1. That is, customer C_1 meets with S last.

The players' payoffs in the Nash equilibrium are equal to:

$$K_{C_1}(z_1^0, z_2^0, z_3^4, z_4^0, z^0, u_{C_1}, u_{C_2}, u_{C_3}, u_{C_4}, u_S) = -1{,}285649,$$

$$K_{C_2}(z_1^0, z_2^0, z_3^4, z_4^0, z^0, u_{C_1}, u_{C_2}, u_{C_3}, u_{C_4}, u_S) = -1{,}178511,$$

$$K_{C_3}(z_1^0, z_2^0, z_3^4, z_4^0, z^0, u_{C_2}, u_{C_2}, u_{C_3}, u_{C_4}, u_S) = -1{,}037922,$$

$$K_{C_4}(z_1^0, z_2^0, z_3^4, z_4^0, z^0, u_{C_3}, u_{C_2}, u_{C_3}, u_{C_4}, u_S) = -1{,}162512,$$

$$K_S(z_1^0, z_2^0, z_3^4, z_4^0, z^0, u_{C_3}, u_{C_2}, u_{C_3}, u_{C_4}, u_S) = -1{,}285649.$$

Example 2. Consider game $\Gamma(z_1^0, \ldots, z_4^0, z^0)$ with salesman S and four customers C_1, C_2, C_3, C_4 with initial conditions:

$$S^0 = (7; 8),\ C_1^0 = (2; 11),\ C_2^0 = (3; 6),\ C_3^0 = (4; 14),\ C_4^0 = (5; 3)$$

and velocities

$$\alpha = 2,\ \beta_1 = 4,\ \beta_2 = 4,\ \beta_3 = 4,\ \beta_4 = 4,$$

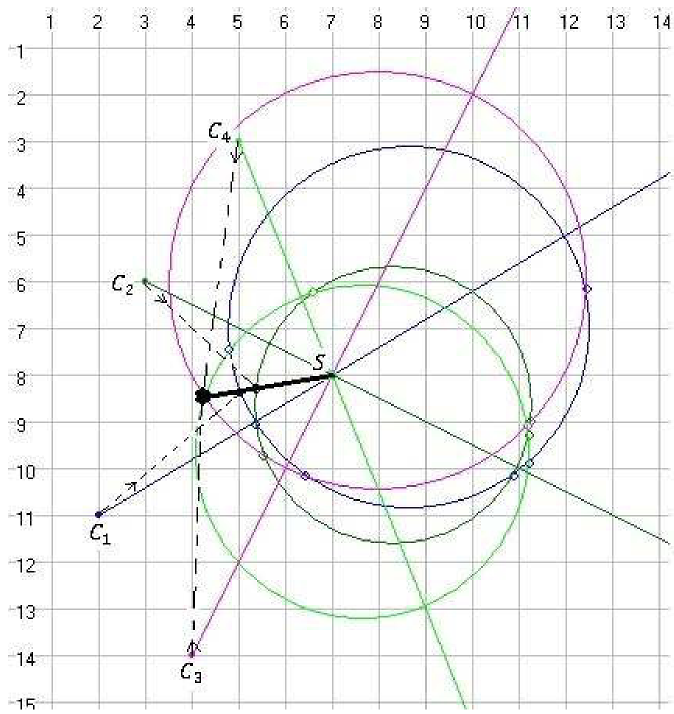

Fig. 8. Nash equilibrium in $\Gamma(z_1^0, z_2^0, z_3^0, z_4^0, z^0)$: behavior u_S^2

respectively.

In this game there exists a Nash equilibrium in which salesman S uses behavior u_S^2. In other words, S moves along a straight line to the nearest intersection point of the Apollonius circles $A(z_3^0, z^0)$ and $A(z_4^0, z^0)$ with coordinates $(4{,}278; 8{,}479)$.

The corresponding trajectory of the salesman's motion is shown in Fig. 8 (the thick line). By the bold dot we mark the end of the route, and the dashed lines correspond to trajectories of the customers' motions C_1, C_2, C_3 and C_4. The customers move to the following points using Π-strategy:

- C_1 moves to point $(5{,}02, 8{,}348)$,
- C_2 moves to point $(5{,}376; 8{,}286)$,
- C_3 and C_4 move to point $(4{,}278; 8{,}479)$.

In this case the salesman meets the customers in the order C_2, C_1, C_3&C_4. Note that two customers C_3 and C_4 meet the salesman simultaneously.

The players' payoffs in the Nash equilibrium are equal to:

$$K_{C_1}(z_1^0, z_2^0, z_3^4, z_4^0, z^0, u_{C_1}, u_{C_2}, u_{C_3}, u_{C_4}, u_S) = -1{,}004803,$$

$$K_{C_2}(z_1^0, z_2^0, z_3^4, z_4^0, z^0, u_{C_1}, u_{C_2}, u_{C_3}, u_{C_4}, u_S) = -0{,}824385,$$

$$K_{C_3}(z_1^0, z_2^0, z_3^4, z_4^0, z^0, u_{C_2}, u_{C_2}, u_{C_3}, u_{C_4}, u_S) = -1{,}381790,$$

$$K_{C_4}(z_1^0, z_2^0, z_3^4, z_4^0, z^0, u_{C_3}, u_{C_2}, u_{C_3}, u_{C_4}, u_S) = -1{,}381790,$$

$$K_S(z_1^0, z_2^0, z_3^4, z_4^0, z^0, u_{C_3}, u_{C_2}, u_{C_3}, u_{C_4}, u_S) = -1{,}381790.$$

Example 3. Consider a game with one salesman and four customers C_1, C_2, C_3, C_4 with initial conditions:

$$S^0 = (7;8),\ C_1^0 = (2;11),\ C_2^0 = (13;6),\ C_3^0 = (4;14),\ C_4^0 = (5;3)$$

and velocities

$$\alpha = 2,\ \beta_1 = 4,\ \beta_2 = 4,\ \beta_3 = 4,\ \beta_4 = 4,$$

respectively.

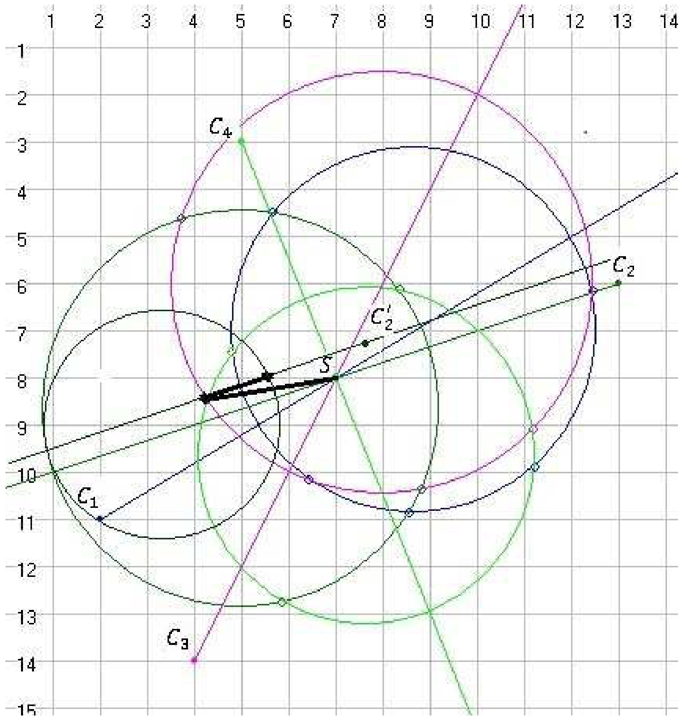

Fig. 9. Nash equilibrium in $\Gamma(z_1^0, z_2^0, z_3^0, z_4^0, z^0)$: behavior u_S^3

In this game there exists a Nash equilibrium in which salesman S uses behavior u_S^3. In other word, S moves along a straight line to the nearest intersection point of the Apollonius circles $A(z_3^0, z^0)$ and $A(z_4^0, z^0)$ with coordinates (4,278; 8,479), and then changes his direction and moves along a straight line towards last customer C_2 to point (5,399; 8,106).

The corresponding trajectory of the salesman's motion is shown in Fig. 9 (the thick line). In Fig. 9, besides the Apollonius circles at the initial time moment one can see the Apollonius circle of the last customer C_2 at the moment of meeting the salesman with customers C_3 and C_4. The position of customer C_2 at this moment is marked by point C_2'.

The customers move to the following points using Π-strategy:

- C_1 moves to point (4,830; 8,382),
- C_2 moves to point (5,399; 8,106),
- C_3 and C_4 move to point (4,278; 8,479).

In this case the salesman meets the customers in the order C_1, C_3&C_4, C_2. This means that at first the salesman meets only one customer C_1, then C_3 and C_4 at the same time, and the last he meets C_2.

The players' payoffs in the Nash equilibrium are equal to:

$$K_{C_1}(z_1^0, z_2^0, z_3^4, z_4^0, z^0, u_{C_1}, u_{C_2}, u_{C_3}, u_{C_4}, u_S) = -1{,}004803,$$
$$K_{C_2}(z_1^0, z_2^0, z_3^4, z_4^0, z^0, u_{C_1}, u_{C_2}, u_{C_3}, u_{C_4}, u_S) = -1{,}972784,$$
$$K_{C_3}(z_1^0, z_2^0, z_3^4, z_4^0, z^0, u_{C_2}, u_{C_2}, u_{C_3}, u_{C_4}, u_S) = -1{,}381790,$$
$$K_{C_4}(z_1^0, z_2^0, z_3^4, z_4^0, z^0, u_{C_3}, u_{C_2}, u_{C_3}, u_{C_4}, u_S) = -1{,}381790,$$
$$K_S(z_1^0, z_2^0, z_3^4, z_4^0, z^0, u_{C_3}, u_{C_2}, u_{C_3}, u_{C_4}, u_S) = -1{,}972784.$$

Example 4. Now we consider a case in which the customers are symmetrically positioned relatively to salesman S. In this game there exist several Nash equilibria. Consider a game with one salesman and four customers C_1, C_2, C_3, C_4 with initial conditions:

$$S^0 = (7;8),\ C_1^0 = (11;12),\ C_2^0 = (3;4),\ C_3^0 = (3;12),\ C_4^0 = (11;4)$$

and velocities

$$\alpha = 2,\ \beta_1 = 3{,}5,\ \beta_2 = 4,\ \beta_3 = 3{,}5,\ \beta_4 = 4,$$

respectively.

Here we have the symmetrically located Apollonius circles and, therefore, we get two Nash equilibria in this game. In both equilibrium situations salesman S uses behavior u_S^3:

1 Starting at the initial time moment salesman S moves along a straight line to the intersection point of the Apollonius circles $A(z_3^0, z^0)$ and $A(z_4^0, z^0)$ with coordinates $(9{,}116; 10{,}857)$, and then changes his direction and moves along a straight line towards customer C_2 to point $(8{,}625; 10{,}366)$. In this case the salesman meets the customers in the order C_1, C_3&C_4, C_2.

$$K_{C_1}(z_1^0, z_2^0, z_3^4, z_4^0, z^0, u_{C_1}, u_{C_2}, u_{C_3}, u_{C_4}, u_S) = -1{,}034963,$$
$$K_{C_2}(z_1^0, z_2^0, z_3^4, z_4^0, z^0, u_{C_1}, u_{C_2}, u_{C_3}, u_{C_4}, u_S) = -2{,}124748,$$
$$K_{C_3}(z_1^0, z_2^0, z_3^4, z_4^0, z^0, u_{C_2}, u_{C_2}, u_{C_3}, u_{C_4}, u_S) = -1{,}777778,$$
$$K_{C_4}(z_1^0, z_2^0, z_3^4, z_4^0, z^0, u_{C_3}, u_{C_2}, u_{C_3}, u_{C_4}, u_S) = -1{,}777778,$$
$$K_S(z_1^0, z_2^0, z_3^4, z_4^0, z^0, u_{C_3}, u_{C_2}, u_{C_3}, u_{C_4}, u_S) = -2{,}124748.$$

2 Starting at the initial time moment salesman S moves along a straight line to the intersection point of the Apollonius circles $A(z_1^0, z^0)$ and $A(z_2^0, z^0)$ with coordinates $(4{,}883; 10{,}857)$, and then and then changes his direction and moves along a straight line towards customer C_4 to point $(5{,}374; 10{,}366)$. In this case the salesman meets the customers in the order C_3, C_1&C_2, C_4.

$$K_{C_1}(z_1^0, z_2^0, z_3^4, z_4^0, z^0, u_{C_1}, u_{C_2}, u_{C_3}, u_{C_4}, u_S) = -1{,}777778,$$
$$K_{C_2}(z_1^0, z_2^0, z_3^4, z_4^0, z^0, u_{C_1}, u_{C_2}, u_{C_3}, u_{C_4}, u_S) = -1{,}777778,$$
$$K_{C_3}(z_1^0, z_2^0, z_3^4, z_4^0, z^0, u_{C_2}, u_{C_2}, u_{C_3}, u_{C_4}, u_S) = -1{,}034963,$$
$$K_{C_4}(z_1^0, z_2^0, z_3^4, z_4^0, z^0, u_{C_3}, u_{C_2}, u_{C_3}, u_{C_4}, u_S) = -2{,}124748$$
$$K_S(z_1^0, z_2^0, z_3^4, z_4^0, z^0, u_{C_3}, u_{C_2}, u_{C_3}, u_{C_4}, u_S) = -2{,}124748.$$

So, the route of the salesman can be finished at points (8,625; 10,366) or (5,374; 10,366). The corresponding trajectories of the salesman's motion is shown in Fig. 10 (the thick line).

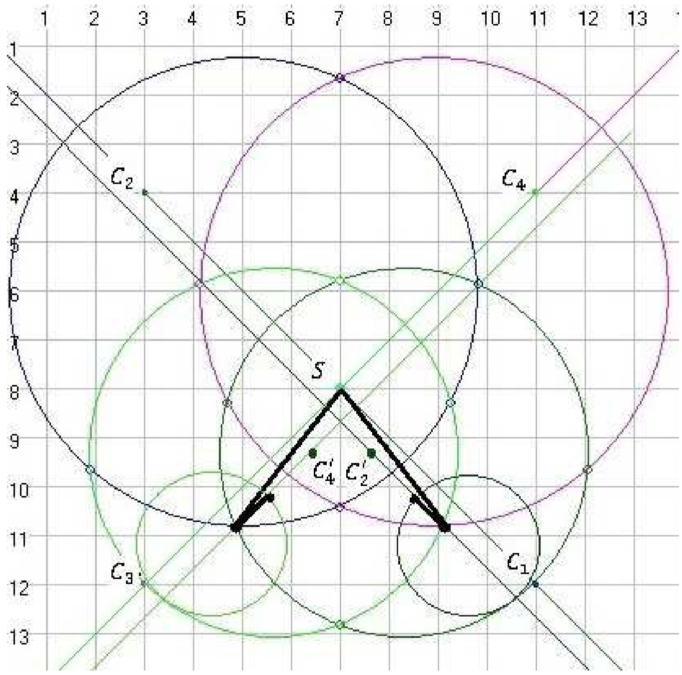

Fig. 10. Nash equilibria in $\Gamma(z_1^0, z_2^0, z_3^0, z_4^0, z^0)$: behavior u_S^3 in a symmetric case

5. Conclusion

In the considered dynamic traveling salesman problem we propose a new approach to finding a solution of this task. Applying methods and solution concepts of pursuit game theory we describe motion of the salesman and customers in a form of differential equations and assign them goals to meet the salesman as soon as possible. We find Nash equilibria and consider different examples which illustrate all possible cases of behavior. Further research could be deal with a cooptative version of this dynamic traveling salesman problem taking into account some companies which have many branches interacting with each other and the main office. In cooperative dynamic games the core is often considered as a main solution concept. However, it is important for a solution being time-consistent (Petrosjan, 1977). This property and also strong time-consistency of the core are investigated in (Tarashnina, 2002; Pankratova, 2010; Sedakov, 2015).

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