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Social Welfare under Oligopoly: Does the Strengthening of Competition in Production Increase Consumers' Well-Being?*

Mathieu Parenti[1], Alexander V. Sidorov[2,3] and Jacques-François Thisse[4]

[1] *European Centre for Advanced Research in Economics and Statistics (ECARES) , Av. F.D., Roosevelt, 39, 1050 Bruxelles, Belgium,*
E-mail: mathieu.parenti@ulb.ac.be
[2] *Sobolev Institute of Mathematics, 4 Acad. Koptyug avenue, 630090, Novosibirsk, Russia,*
E-mail: alex.v.sidorov@gmail.com
[3] *Novosibirsk State Univercity, 2 Pirogova Street, 630090 Novosibirsk, Russia*
[4] *CORE-Université Catholique de Louvain, 34 Voie du Roman Pays, 1348 Louvain-la-Neuve, Belgium,*
E-mail: jacques.thisse@uclouvain.be

Abstract The paper studies the detailed comparison of the Social welfare (indirect utility) under three types of imperfect competition in a general equilibrium model: quantity oligopoly (Cournot), price oligopoly (Bertrand) and monopolistic competition (Chamberlin). The folk wisdom implies that an increasing toughness of competition in sequence Cournot-Bertrand-Chamberlin results in increasing of consumers' welfare (indirect utility). We show that this is not true in general. This is accomplished in a simple general equilibrium model where consumers are endowed with separable preferences. We find the sufficient condition in terms of the representative consumer preference providing the "intuitive" behavior of the indirect utility and show that this condition satisfy the classes of utility functions, which are commonly used in examples (e.g., CES, CARA and HARA). Moreover, we provide a series of numerical examples (and analytically verifiable conditions as well), which illustrate that violation of this condition may results in "counter-intuitive" behavior of indirect utility, when the weakest level of competition (Cournot) provides the highest amount of the consumer's welfare.

Keywords: Cournot competition, Bertrand competition, free entry, Lerner index, indirect utility.

1. Introduction

In oligopolistic markets, price (Bertrand) and quantity (Cournot) competition deliver market solutions that typically differ, making it hard to formulate robust predictions. The purpose of this paper is to contribute to this debate by providing a comparison of these types of competition from the consumer's point of view. This is accomplished in an economy involving one sector and a population of consumers endowed with separable preferences and a finite number of labor units. Although we recognize that additive preferences are restrictive, they are widely used in the

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literature and suffice to shed new light on old questions. Note also that the budget constraint implies that firms do not behave like monopolists.

Our main findings are as follows. Using the concepts of relative love for variety, which measures the intensity of the preference for variety, and social mark-up, which measures the proportion of the utility gain from adding a variety, we show that ranking of consumers' well-being under these two types of imperfect competition is ambiguous and depends on behavior, to be more precise, on their derivatives, in the neighborhood of zero.

2. The model

The present paper deals with the model, which was introduced and studied in the paper (Parenti et al., 2017). There were proved the existence and uniqueness of oligopolistic equilibria, its comparative statics and limit behavior. The welfare was out of the scope of that paper. To save reader's time, we borrowed the model description, definitions and the key results without proofs, which can be found in cited paper.

2.1. Firms and consumers

There is one sector supplying a horizontally differentiated good and one production factor - labor. Consumption sector is continuum $[0, L]$ of identical consumers. Each consumer supplies one unit of labor and owns $1/L$ of firms' profits. The labor market is perfectly competitive and labor is chosen as the numéraire. The differentiated good is made available under the form of a finite number $n \geq 2$ varieties. Each variety is produced by a single firm and each firm produces a single variety. To operate every firm needs a fixed requirement $f > 0$ and a marginal requirement $c > 0$ of labor. Without loss of generality we can normalize $c = 1$. Since wage can be also normalized to 1, the cost of producing q_i units of variety $i = 1, ..., n$ is equal to $f + 1 \cdot q_i$.

Consumers share the same additive preferences given by

$$U(\mathbf{x}) = \sum_{i=1}^{n} u(x_i), \tag{1}$$

where u is thrice continuously differentiable, strictly increasing, strictly concave over $\mathbb{R}_+$, and such that $u(0) = 0$. The strict concavity of u implies that consumers have a *love for variety*: when a consumer is allowed to consume X units of the differentiated good, she strictly prefers the consumption profile $x_i = X/n$ to any other profile $\mathbf{x} = (x_1, ..., x_n)$ such that $\sum_i x_i = X$.

Following (Zhelobodko et al., 2012), we define the *relative love for variety* (RLV) as follows:

$$r_u(x) \equiv -\frac{xu''(x)}{u'(x)},$$

which is strictly positive for all $x > 0$. Very much like the Arrow-Pratt's relative risk-aversion, the RLV is a local measure of consumers' variety-seeking behavior. A higher value of the RLV means a stronger love for variety. On the contrary, $r_u(x) = 0$ means that the consumer perceives the varieties as perfect substitutes. Under the CES, we have $u(x) = x^\rho$ where ρ is a constant such that $0 < \rho < 1$, thus implying a constant RLV is constant and given by $1 - \rho$. Other examples include: (i) the

CARA utility $u(x) = 1 - \exp(-\alpha x)$ where $\alpha > 0$ is the absolute love for variety (Behrens and Murata, 2007), while the RLV is increasing and given by αx; and (ii) the quadratic utility $u(x) = \alpha x - \beta x^2/2$, with $\alpha, \beta > 0$; the RLV is increasing and given by $\beta x/(\alpha - \beta x)$.

The budget constraint is given by

$$\sum_{i=1}^{n} p_i x_i = y. \tag{2}$$

A consumer's income y is equal to her wage plus her share of total profits:

$$y = 1 + \frac{1}{L}\sum_{i=1}^{n} \Pi_i \geq 1, \tag{3}$$

where the profits earned by firm i is given by

$$\Pi_i = (p_i - 1)q_i - f, \tag{4}$$

p_i being the price set by firm i.

The first-order condition for utility maximization yields

$$u'(x_i) = \lambda p_i,$$

where λ is the Lagrange multiplier defined by

$$\lambda(\mathbf{x}, y) = \frac{\sum_{j=1}^{n} x_j u'(x_j)}{y} \geq 0. \tag{5}$$

A consumer's inverse demand for variety i is such that

$$p_i(x_i, \mathbf{x}_{-i}, y) = \frac{u'(x_i)}{\lambda}, \tag{6}$$

where $\mathbf{x}_{-i} = (x_1, ..., x_{i-1}, x_{i+1}, ...x_n)$.

2.2. Market equilibrium

The market equilibrium is defined by the following conditions.

(E.1) Each consumer maximizes her utility (1) subject to (2).

(E.2) Each firm i maximizes its profit (4) with respect to q_i (under Cournot competition) or p_i (under Bertrand competition).

(E.3) Product market clears:

$$Lx_i = q_i \qquad \text{for } i = 1, ..., n.$$

(E.4) Labor market clears:

$$nf + \sum_{i=1}^{n} q_i = L.$$

The last condition implies that

$$\bar{q} \equiv \frac{L}{n} - f \iff \bar{x} \equiv \frac{1}{n} - \frac{f}{L}$$

are the only candidate symmetric equilibrium output and consumption, which both decrease with n. Note than nf is the minimum labor requirement for n firms to operate. Therefore, n cannot exceed L/f, which implies $\bar{x} \geq 0$.

Cournot Using (5) and (6), we obtain firm i's inverse demand:

$$p_i(\mathbf{x}) = \frac{y^C u'(x_i)}{\sum_{j=1}^n x_j u'(x_j)}, \tag{7}$$

where y^C is a consumer's income under Cournot competition. Firm i's profit function is then given by

$$\Pi_i^C(\mathbf{x}) = [p_i(x_i, \mathbf{x}_{-i}) - 1]Lx_i - f = \left[\frac{y^C u'(x_i)}{\sum_{j=1}^n x_j u'(x_j)} - 1\right] Lx_i - f.$$

For any given $n \geq 2$, a *Cournot equilibrium* is a vector $\mathbf{x}^* = (x_1^*, \ldots, x_n^*)$ such that each strategy x_i^* is firm i's best reply to the strategies $\mathbf{x}_{-i}^*$ chosen by the other firms. This equilibrium is symmetric if $x_i^* = x^C$ for all $i = 1, ..., n$.

Bertrand Assume now that firms compete in prices. Let $\mathbf{p} = (p_1, ..., p_n)$ be a price vector. In this case, consumers' demand functions $x_i(\mathbf{p})$ are obtained by solving the system of equations (7) with $i = 1, ..., n$, where y^C is replaced with

$$y^B = 1 + \frac{1}{L}\sum_{i=1}^n \Pi_i^B(\mathbf{p})$$

that is, a consumer's income under Bertrand competition. Her the firm i's profits are given by

$$\Pi_i^B(\mathbf{p}) = (p_i - 1)Lx_i(\mathbf{p}) - f.$$

A Nash equilibrium $\mathbf{p}^* = (p_1^*, ..., p_n^*)$ of this game is called a *Bertrand equilibrium*. This equilibrium is symmetric if $p_i^* = p^B$ for all i.

Income Effect and the Income-taking Firms One major difficulty in general equilibrium with oligopolistic firms is the income effect. Ever since (Gabszewicz and Vial, 1972), it is well known that firms operating in an imperfectly competitive environment are able to manipulate individual incomes through the profits they redistribute to consumers. By changing consumers' incomes, firms affect their demand functions, whence their profits. Accounting for such feedback effects typically leads to the nonexistence of an equilibrium because the resulting profit functions are not quasi-concave (Roberts and Sonnenschein, 1977). This negative result probably explains why many economic models involving imperfectly competitive product markets rely on the CES model of monopolistic competition, where the existence of an equilibrium can be established under very mild conditions. In this paper, we assume that firms recognize that income is endogenous because they operate in a general equilibrium environment. However, firms treat income parametrically, which means that they behave like "income-takers". This approach is in the spirit of (Hart, 1985) for whom firms may take into account only some effects of their policy on the whole economy.[1] Even though our model does not capture all possible strategic aspects, it is a full-fledged general equilibrium model in which oligopolistic firms account for strategic interactions within their group, as well as for endogenous

[1] When product markets are imperfectly competitive, it is common to assume that firms do not manipulate wages, even though firms also have market power on the labor market. The paper (d'Aspremont et al., 1996) is a noticeable exception.

incomes through the distribution of profits. Speaking technically, firms are said to be income-takers when they are aware that the income is endogenous, but treat y parametrically:

$$\frac{\partial y}{\partial x_i} = 0 \qquad \left(\frac{\partial y}{\partial p_i} = 0\right) \qquad \text{for all } i.$$

Proposition 1. *Assume that a symmetric equilibrium exists under Cournot and Bertrand competition when the number of firms is equal to n¡L/f. If firms are income-takers, the equilibrium markups are given by*

$$m^C(n) = \frac{1}{n} + \frac{n-1}{n} r_u \left(\frac{1}{n} - \frac{f}{L}\right), \; m^B(n) = \frac{n r_u \left(\frac{1}{n} - \frac{f}{L}\right)}{n - 1 + r_u \left(\frac{1}{n} - \frac{f}{L}\right)}, \tag{8}$$

while $m^C(n) > m^B(n)$.

For the **Proof** see Proposition 1 in (Parenti et al., 2017).

2.3. Free Entry Condition

Let p is a price in symmetric equilibrium, no matter Cournot or Bertrand, then denote as

$$m \equiv \frac{p-1}{p} \in (0,1)$$

mark-up, i.e., relative difference between price and marginal cost. Taking into account that marginal cost coincides with the equilibrium prices under perfect competition, we obtain an another interpretation of mark-up as Lerner index of market power. Note that zero value of Lerner index characterizes perfect competition, while imperfect competition, e.g., Cournot or Bertrand oligopoly, is characterized by positive values of $m < 1$.

Note that in equilibrium, profits must be non-negative for firms to operate. The budget constraint can be rewritten as follows:

$$y = 1 - \frac{nf}{L} + \frac{1}{L}\sum_{j=1}^{n} \frac{p_j - 1}{p_j} p_j q_j,$$

which, after symmetrization, yields

$$y = 1 - \frac{nf}{L} + \frac{1}{L} m \cdot np \cdot q = 1 - \frac{nf}{L} + m \cdot y \iff y = \frac{1 - nf/L}{1 - m}. \tag{9}$$

Moreover, the strictly positive profit in industry is an incentive to enter for new firms. Thus, assuming that the enter is free we obtain one more condition of equilibrium

(E.5) For all firms $i = 1, \ldots, n$ profit $\Pi_i^C(\mathbf{x}) = 0$ (resp. $\Pi_i^B(\mathbf{p}) = 0$)

Applying (E.5) to (3) and (9) we obtain that zero-profit condition holds if and only if

$$\frac{1 - nf/L}{1 - m} = 1$$

or, equivalently

$$n = \frac{L}{f} m. \tag{10}$$

Therefore, the equilibrium number of firms increases with the market size and the degree of firms' market power, which is measured by the Lerner index.

The new problem we face now is that "zero-profit" number of firms is typically *non-integer*. It is not technical problem for symmetric equilibria, because in this case the upper limit of sum n turns into multiplier. As for substantial interpretation of "fractional" firms, see, for example, short discussion in Subsection 4.3 of (Parenti et al., 2017)

Note also that (10) implies

$$\bar{x} = \frac{f(1-m)}{Lm} > 0, \tag{11}$$

provided that m satisfies $0 < m < 1$.

Applying (10) and (11) to (8) we obtain that the equilibrium markups under free-entry must solve the following equations:

$$m^C = \frac{f}{Lm^C} + \left(1 - \frac{f}{Lm^C}\right) r_u \left(\frac{f}{L}\frac{1-m^C}{m^C}\right), \tag{12}$$

$$m^B = \frac{f}{L} + \left(1 - \frac{f}{L}\right) r_u \left(\frac{f}{L}\frac{1-m^B}{m^B}\right). \tag{13}$$

Under the CES, the right-hand side (13) is a constant K while the right-hand side of (12) is a decreasing function of m^C, which exceeds K over $[0,1]$. Therefore, it must be that $m^B < m^C$. It then follows from (10) and (11) that $n^C > n^B$ and $q^C < q^B$.

First, we determine sufficient conditions on preferences and market size for a free-entry equilibrium to exist and to be unique. Second, we show that the above inequalities hold for any utility u.

Since x can take on any positive value, for an equilibrium to exist under any collection of the parameter values, it must be that

$$r_u(x) < 1 \text{ for all } x \geq 0. \tag{14}$$

It is well known that a firm's profit function is strictly quasi-concave if the second-order condition for profit-maximization is satisfied at any solution to the first-order condition. The second-order condition always holds if

$$r_{u'}(x) = -\frac{xu'''(x)}{u''(x)} < 2. \tag{15}$$

This condition highlights the need to impose restrictions on the third derivative of the utility u to prove the existence and uniqueness of a Nash equilibrium.

Proposition 2. *Assume that (14) and (15) hold. If $f > 0$, then there exists a value $L_0 > 0$ such that, for every $L \geq L_0$, there exists a unique symmetric free-entry Cournot equilibrium and a unique symmetric free-entry Bertrand equilibrium. The equilibrium markups, outputs and numbers of firms satisfy*

$$m^C > m^B \qquad q^C < q^B \qquad n^C > n^B$$

and

$$\lim_{L\to\infty} m^C(L) = \lim_{L\to\infty} m^B(L) = r_u(0).$$

For the **Proof** see Proposition 2 in (Parenti et al., 2017).

3. Consumers' Welfare

Proposition 2 highlights the existence of a trade-off between *per variety* consumption and product diversity. To be precise, when free entry prevails, Cournot competition leads to a larger number of varieties $n^C > n^B$, and at the same time, consumption level per variety is lower, than for Bertrand competition $x^C < x^B$. Therefore, the comparison between $V^C = n^C \cdot u(x^C)$ and $V^B = n^B \cdot u(x^B)$ is *a priori* ambiguous.

In what follows we assume additionally that the elemental utility satisfies

$$\lim_{x \to \infty} u'(x) = 0,$$

which is not too restrictive and typically holds for basic examples of utility functions. To solve the Welfare problem we consider an imaginary Social Planner, who manipulates with masses of firms n trying to maximize consumers' utility

$$V(n) = n \cdot u(x)$$

subject to the labor market clearing condition

$$(f + L \cdot x)n = L.$$

Let $\varphi = f/L$, then the Social Planner's problem is equivalent to maximization of the following function

$$V(n) = n \cdot u\left(\frac{1}{n} - \varphi\right)$$

on the interval $n \in (0, \varphi^{-1})$. Note that

$$V(0) = V(\varphi^{-1}) = 0$$

$$V'(n) = u\left(\frac{1}{n} - \varphi\right) - \frac{1}{n} \cdot u'\left(\frac{1}{n} - \varphi\right)$$

$$V''(n) = \frac{1}{n^3} \cdot u''\left(\frac{1}{n} - \varphi\right) < 0$$

which implies that graph of $V(n)$ is bell-shaped and there exists unique social optimum $n^* \in (0, \varphi^{-1})$, and $V'(n) \le 0$ (resp. $V'(n) \ge 0$) for all $n \ge n^*$ (resp. $n \le n^*$.) This implies the following statements:

1 Let Bertrand equilibrium number of firms lies to the right of Social Optimum $n^B \ge n^*$, then $V^C < V^B$ holds

2 Let Cournot equilibrium number of firms lies to the left of Social Optimum $n^C \le n^*$, then $V^C > V^B$ holds

3 In the intermediate case $n^B < n^* < n^C$ the relation between V^B and V^C is ambiguous.

In what follows, the first case will be referred as pro-Bertrand case, the second one - as pro-Cournot case.

Now the problem formulated in the title of this paper may be represented in the following form. Let $\varphi = f/L$ be given. Due to Proposition 2, there exist unique Cournot and Bertrand equlibria for all sufficiently small φ. This means that these equilibria are parametrized by φ, i.e., functions $m^C(\varphi)$, $m^B(\varphi)$, $x^C(\varphi)$, $x^B(\varphi)$,

$n^C(\varphi)$, $n^B(\varphi)$ are well-defined for all sufficiently small $\varphi \in (0, \hat{\varphi})$. Moreover for all $\varphi > 0$ there exists the unique socially optimal number of firms $n^*(\varphi)$, while the corresponding consumption of representative consumer

$$x^*(\varphi) = \frac{1}{n^*(\varphi)} - \varphi.$$

Therefore, to obtain the pro-Bertrand case we have to prove that $n^*(\varphi) \leq n^B(\varphi)$ for all sufficiently small φ, while pro-Cournot case holds when $n^*(\varphi) \geq n^C(\varphi)$. In addition, it is (almost) obvious that $x^C(\varphi) \to 0$, $x^B(\varphi) \to 0$ vanish when $\varphi \to 0$ (for the rigorous proof see (Parenti et al., 2017)),thus "for sufficiently small φ" is actually equivalent to "for sufficiently small x."

Let's determine the following function

$$\mathit{\Delta}_u(x) \equiv [1 - \varepsilon_u(x)] - r_u(x) = \frac{u(x) - xu'(x)}{u(x)} + \frac{xu''(x)}{u'(x)} = \frac{u(x) - xp(x)}{u(x)} + \frac{xp'(x)}{p(x)}$$

Vives in (Vives, 2001) points out that $1 - \varepsilon_u(x)$ is the degree of preference for a single variety as it measures the proportion of the utility gain from adding a variety, holding quantity per firm fixed. The subtrahend term, $r_u(x)$, may be characterized as "relative love for variety" (RLV), see, (Zhelobodko et al., 2012). In (Dhingra and Morrow, 2014) these values are referred as *social mark-up* and *private mark-up*, respectively. See the cited paper for the more detailed discussions on these characteristics of the consumer's demand.

Lemma 1. Let $r_u(0) < 1$ holds, then

$$\mathit{\Delta}_u(0) \equiv \lim_{x\to 0} \mathit{\Delta}_u(x) = 0.$$

Proof. Note that the function $xu'(x)$ is strictly increasing and positive for all $x > 0$. Indeed, $u'(x) > 0$ and $(xu'(x))' = u'(x) + xu''(x) = u'(x)(1 - r_u(x)) > 0$, therefore there exists limit

$$\lambda = \lim_{x\to 0} x \cdot u'(x) \geq 0.$$

Assume that $\lambda > 0$. This is possible only if $u'(0) = +\infty$. Using the L'Hospital rule we obtain

$$\lambda = \lim_{x\to 0} x \cdot u'(x) = \lim_{x\to 0} \frac{x}{(u'(x))^{-1}} = \lim_{x\to 0} -\frac{(u'(x))^2}{u''(x)} = \lim_{x\to 0} \frac{xu'(x)}{-\frac{xu''(x)}{u'(x)}} = \frac{\lambda}{r_u(0)} > \lambda.$$

This contradiction implies that $\lambda = 0$. Q.E.D.

The CES case is characterized by identity $\mathit{\Delta}_u(x) = 0$ for all $x > 0$, in the other cases the sign and magnitude of $\mathit{\Delta}_u(x)$ may vary, as well as the directions of change for terms $1 - \varepsilon_u(x)$ and $r_u(x)$ may be arbitrary, see (Dhingra and Morrow, 2014). Let

$$\delta_u \equiv \lim_{x\to 0} \mathit{\Delta}'_u(x)$$

finite or infinite. Then the following theorem provides the sufficient conditions for pro-Bertrand and pro-Cournot cases, the obvious gap between (a) and (b) corresponds to the ambiguous case 3. above.

Theorem 1.

(a) Let $\delta_u < r_u(0)$*, then for all sufficiently small* $\varphi = f/L$ *an inequality* $V^B > V^C$ *holds.*

(b) Let $\delta_u > 1$*, then for all sufficiently small* $\varphi = f/L$ *an inequality* $V^C > V^B$ *holds.*

Proof. See Technical Appendix.

It is obvious that in CES case $u(x) = x^\rho$ we obtain immediately $\delta = 0 < r_u(0) = 1 - \rho$, thus CES is pro-Bertrand function. Considering the CARA $u(x) = 1 - e^{-\alpha x}$, $\alpha > 0$, HARA $u(x) = (x + \alpha)^\rho - \alpha^\rho$, $\alpha > 0$, and Quadratic $u(x) = \alpha x - x^2/2$, $\alpha > 0$, functions, we obtain $r_u(0) = 0$, while the direct calculations show that $\delta_{CARA} = -\alpha/2 < 0$, $\delta_{HARA} = -(1-\rho)/2\alpha < 0$ and $\delta_{Quad} = -1/2\alpha < 0$. This implies that these popular classes of utility functions also provide the pro-Bertrand case.

To illustrate the opposite, pro-Cournot case, consider the following function $u(x) = \alpha x^{\rho_1} + x^{\rho_2}$. Without loss of generality we may assume that $\rho_1 < \rho_2$, then

$$1 - \varepsilon_u(x) = \frac{\alpha(1-\rho_1) + (1-\rho_2)x^{\rho_2-\rho_1}}{\alpha + x^{\rho_2-\rho_1}}, \; r_u(x) = \frac{\alpha\rho_1(1-\rho_1) + \rho_2(1-\rho_2)x^{\rho_2-\rho_1}}{\alpha\rho_1 + \rho_2 x^{\rho_2-\rho_1}}.$$

Using the L'Hospital rule and the obvious calculations we obtain

$$\lim_{x\to 0} \Delta'_u = \lim_{x\to 0} \frac{1 - \varepsilon_u(x) - r_u(x)}{x} = \lim_{x\to 0} \frac{\alpha(\rho_2-\rho_1)^2 x^{-\rho_1-(1-\rho_2)}}{(\alpha + x^{\rho_2-\rho_1})(\alpha\rho_1 + \rho_2 x^{\rho_2-\rho_1})} = +\infty > 1.$$

Remark 1. It is obvious, that the difference of social and private mark-ups may be equivalently represented as elasticity of elasticity of utility

$$\Delta_u(x) = \frac{x\varepsilon'_u(x)}{\varepsilon_u(x)}.$$

Moreover, using L'Hospital rule we obtain that

$$\delta_u = \lim_{x\to 0} \Delta'_u(x) = \lim_{x\to 0} \frac{\Delta_u(x)}{x} = \lim_{x\to 0} \frac{\varepsilon'_u(x)}{\varepsilon_u(x)} = \frac{1}{\varepsilon_u(0)} \lim_{x\to 0} \varepsilon'_u(x),$$

where $\varepsilon_u(0) = 1 - r_u(0) > 0$ exists due to our assumptions. Therefore, the sufficient conditions for pro-Bertrand and pro-Cournot cases may be transforms as follows

$$\lim_{x\to 0} \varepsilon'_u(x) < r_u(0)\,(1 - r_u(0)) = (1 - \varepsilon_u(0))\,\varepsilon_u(0) \Rightarrow n^* < n^B < n^C \Rightarrow V^B > V^C$$

$$\lim_{x\to 0} \varepsilon'_u(x) > 1 - r_u(0) = \varepsilon_u(0) \Rightarrow n^B < n^C < n^* \Rightarrow V^C > V^B$$

Consider the class of additive utility functions satisfying

$$\varepsilon'_u(x) = -(1 - \varepsilon_u(x))' < 0,$$

i.e., social mark-up is strictly increasing function in some neighborhood of 0. This means that consumers have a higher preference for variety when they consume more per variety, see Subsection 2.1.1 in (Dhingra and Morrow, 2014). Due to (Spence, 1976) and (Vives, 2001; Chapter 6), such type of consumer's behavior is considered as "normal", or, "intuitive", though there are various types of utility functions, which generate "counter-intuitive" behavior. Note that the classes of utility CARA, HARA, Quadratic satisfy this condition of strictly increasing of the social mark-up, while in case of CES utility, social mark-up is constant.

Corollary 1. *Let social mark-up strictly increases at zero, then $V^B > V^C$.*

This easily follows from Remark 1 and an obvious fact that $r_u(0)(1 - r_u(0)) \geq 0$. This is not necessary condition, however, which may be shown by function $u(x) = \sqrt{x+1} - e^{-\sqrt{x}}$, because we obtain there $\varepsilon_u'(0) = 1/6 > 0$, while $(1 - \varepsilon_u(0))\,\varepsilon_u(0) = 1/4 > 1/6$ which implies $V^B > V^C$.

4. Conclusion

Additive preferences are widely used in theoretical and empirical applications of monopolistic competition. This is why we have chosen to compare the market outcomes under two different competitive regimes when consumers are endowed with such preferences. It is our belief, however, that most of our results hold true in the case of well-behaved symmetric preferences. Unlike most models of industrial organization which assume the existence of an outside good, we have used a limited labor constraint. This has allowed us to highlight the role of the marginal utility of income in firms' behavior. Another distinctive feature of our approach is that firms recognize that consumers' incomes are endogenous through the distribution of profits. The assumption of income-taking firms seems to be a reasonable alternative to the polar cases in which incomes are taken as exogenous, as in partial equilibrium analyses, or incomes are strategically manipulated by firms, which leads to intractable general equilibrium models. In brief, even though our setup is restrictive, it is sufficient to show that whether strengthening of imperfect competition will increase the social welfare depends on the nature of preferences.

Appendix

Proof of Theorem 1

Let

$$x = \frac{1}{n} - \varphi \iff n = \frac{1}{x + \varphi},$$

besides an equilibrium mark-up

$$m = \frac{f}{L} n = \varphi n$$

which implies

$$m = \frac{\varphi}{x + \varphi}.$$

Note that Bertrand equilibrium mark-up is determined by equation

$$m = \varphi + (1 - \varphi) r_u(x).$$

Substituting

$$m = \frac{\varphi}{x + \varphi}$$

we obtain

$$\varphi = \frac{1-x}{2} - \sqrt{\left(\frac{1-x}{2}\right)^2 - \frac{xr_u(x)}{1-r_u(x)}}$$

which implies that

$$n^B \geq n^* \iff u(x) \leq \left(\frac{1+x}{2} - \sqrt{\left(\frac{1+x}{2}\right)^2 - \frac{x}{1-r_u(x)}}\right) u'(x),$$

at $x = x^B$. The direct calculation shows that the last inequality is equivalent to

$$\Delta_u(x) \leq (1-r_u(x)) \left[1 - \frac{1+x}{2}\left(1 + \sqrt{1 - \frac{4x}{(1-r_u(x))(1+x)^2}}\right)\right]. \tag{16}$$

Taking into account the obvious inequality $\sqrt{1-z} \leq 1 - z/2$, we obtain that (16) will hold provided that

$$\Delta_u(x) \leq (1-r_u(x)) \left[1 - (1+x)\left(1 - \frac{x}{(1-r_u(x))(1+x)^2}\right)\right] = x\left[r_u(x) - \frac{x}{1+x}\right]. \tag{17}$$

holds. Let

$$F_u(x) = x\left[r_u(x) - \frac{x}{1+x}\right],$$

then $F_u(0) = 0 = \Delta_u(0)$, therefore (17) will hold in some neighborhood of 0 provided that

$$\Delta_u'(0) < F_u'(0) = r_u(0).$$

Similarly, Cournot mark-up satisfies

$$m = \frac{\varphi}{m} + \left(1 - \frac{\varphi}{m}\right) r_u(x).$$

Taking to account that

$$m = \frac{\varphi}{x+\varphi}$$

we obtain the following equation

$$(1-r_u(x))(x+\varphi)^2 - (1-r_u(x))(x+\varphi) + x = 0,$$

which implies

$$x + \varphi = \frac{1}{2}\left(1 - \sqrt{1 - \frac{4x}{1-r_u(x)}}\right)$$

and

$$n^C \leq n^* \iff u(x) \geq \frac{1}{2}\left(1 - \sqrt{1 - \frac{4x}{1-r_u(x)}}\right) u'(x)$$

at $x = x^C$. The direct calculation shows that the last inequality is equivalent to

$$\Delta_u(x) \geq \frac{1}{2}(1-r_u(x))\left[1 - \sqrt{1 - \frac{4x}{1-r_u(x)}}\right]. \tag{18}$$

Taking into account the inequality $\sqrt{1-z} \geq 1 - \alpha z/2$ holds for any $\alpha > 1$ and $x \in \left[0, 4(\alpha-1)/\alpha^2\right]$, we obtain that (18) will hold provided that

$$\varDelta_u(x) \geq \frac{1}{2}(1 - r_u(x))\left[1 - \left(1 - \frac{2\alpha x}{1 - r_u(x)}\right)\right] = \alpha x \tag{19}$$

for all sufficiently small x. Now assume that $\delta_u > 1$ and let $\alpha = \frac{1+\delta_u}{2}$, then $\alpha > 1$ and $\varDelta_u'(0) = \delta_u > \alpha$, which implies that (19) holds in some neighborhood of 0.

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