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Application of Game Theory in the Analysis of Economic and Political Interaction at the International Level

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Abstract The main objective of the work is to consider possible approaches to the application of the theory of cooperative games for the simulation of the relationship between the major centers of economic and political influence in the modern world.
The authors discuss three main interval repositioning of political forces in the world until 2008, from 2008 to 2014 and after 2014. Model presented in the building on political cooperation are intended to reflect the economic component of this interaction on the world market.
The results are subject to more detailed content analysis. Special attention is paid to the issues of construction of characteristic functions for cooperative games, the underlying model interaction of the centers of political forces. In particular, the technique of construction of characteristic function, involving the use of baskets, constructible on currencies of countries coming together in a coalition.

Keywords: cooperative games, cooperative models interaction of the international centers of power, stochastic cooperative games, imputations, Shapley value

1. Introduction

Recent years we have seen impressive shifts not only in economics sphere, but also in those spheres which have traditionally characterized by polysemantic definition politics.

Just two or three years, significantly transformed the system of international political and economic relations. Simple comparison of the problems which was most actual for period 2009-2010 with most actual today problems looks symptomatic.

Just years ago, the question: "Will there be a second wave of the crisis?" was the most discussed. The vast majority of experts responded in the affirmative. Opinions differed on the timing of the crisis only. This greatly reduces the value of these opinions. It is extremely difficult to verify the quality of the forecasts of perspectives of evolution of Russia and the global economic system which were based on threats, challenges and wave of the crisis.

It is hard to overestimate the complexity of the problems faced by the Russian economy in recent year.

However, we have to admit that the main reason for today's problems are economic sanctions, and not the second wave of crisis.

2. Introductory provisions

There is common set of issues about the relationship between economics and politics spheres.

On the one hand, it‘s generally accepted the basis foundation of political phenomena are economic processes. At the same time, there is no doubt that the political processes affect the economy macro-economic systems of individual countries and regions, and the global economic system as a whole.

There are many works devoted to explaining the objective economic reasons, political conflicts, crises and wars. There is a lot of papers considering the economic consequences of political decisions. There is also a lot of research devoted to the economic consequences of the economic policy decisions. However, significantly fewer studies in which these issues would be considered in the logic chain economy-politics-economics. See, for example, (Dergachov, 2011; Dergachov, 2005).

The researches aimed at finding methods and tools to describe and analyze the causal relationship between economic processes and political events are very actual and interesting.

Let's illustrate this issue by a concrete example. The current economic and political situation in Russia is characterized by complex set of problems, such as Crimea incorporation, cardinal shift in relations with Ukraine, course on economic cooperation with China and the BRICS countries, building new relationships with the countries of Western Europe and freezing cooperation with United States. There are two possible approaches to explain the sequence of these events. One of them based on the priority political factors, the other on the economics.

In the first case, the key causes are taken decision of the political leadership of the country, has decided to pursue a more independent course. This has led to conflicts with the leading powers of the modern world. They are trying to restore the status quo using economic sanctions. The sanctions have caused harm not only to Russia but also to Western countries, at the same time. However, they are guided, basic values are not ready to go to their abolition as long as Russia does not back down. With the weight of this argument is difficult to disagree. As historical experience shows, concessions a separate state on fundamental ideological issues can lead to their destruction.

The second case explanations the fundamental cause of the current events base on issue of balance of power in the world. There were very serious socio-economic and socio-political transformation in the first decade of the XXI-st century in Russia. No less important transformation took place with China, India and with other countries, which undoubtedly changed their relative weight in the international arena. This led to the emergence of new claims on their side. This version looks more realistic than ideological value-oriented version.

The both approaches have a common defect. Namely, they have speculative character. More preferable from a scientific standpoint is the conclusion, based on a model researches.

This paper focuses on the mathematical model allowing to represent and analyze the economic and political processes of the modern world in terms of the interaction of a coalition of world powers, international associations and countries. The key place occupied by cooperative effects in such cooperation and using methods of cooperative game theory.

3. The basic game-theoretic model of cooperative interaction centers of political influence (Base-3)

We will look at several versions for constructing cooperative game models assessment of political influence, next. Of course, we are talking about an extremely simplified conceptual models.

We restrict our discussion to the game with three participants (players) Russia (1st player), China (2nd player), West (3rd player). We agree to call this game briefly Base-3 with adding postfix a, b, c, etc.

Recall that the classical cooperative game with transferable utility is given where a pair (I, v):

- $I = \{1, ..., n\}$ - a set of players;
- v - the characteristic funtion ($2^I \longrightarrow R$), that each subset of players $S \subset I$ (a coalition of players) assigns the value of the usefulness of the coalition in the event of its formation ($v(\{S\})$).

The definition of "cooperative game solution" based on the concepts of "imputation" and "pre-imputation". Let define for cooperative game (I, v) as vector $x = (x_1, x_2, ..., x_n)$ satisfying the condition

- individual rationality

$$x_i \geq v(\{i\}), \forall i \in \{1..n\}; \tag{1}$$

- group rationality

$$\sum_{i=1}^{n} x_i = v(\{I\}). \tag{2}$$

In other words, imputation is a distribution of the full utility of the coalition of all players (the so-called grand or full coalition). Imputation gives each player the utility of not less than he could get individually (without entering into any coalition).

Widely known methods of the index influence calculation proposed in the works (Penros, 1946; Shapley, 1954; Banzhaf, 1965; Johnston, 1978; Deegan, 1978; Holler, 1983) and others.

There are many studies devoted to the application of the theory of cooperative games for analyzing the interaction of political forces within the elected bodies (parliaments, legislatures, etc). In particular, in this connection it is appropriate to recall such works as (Aleskerov et al., 2008), which analyzes the distribution of power between the fractions of the State Duma of the Russian Empire, (Sokolov, 2008), dedicated to the calculation of the indices of influence and consider them examples of the Council of Ministers of the European Union and modern Russia's State Duma

As the values of the characteristic function of the considered games Base-3 at baseline studies can be used conventional assessment of the impact on the situation in the world that has this or that party or coalition of parties. The level of influence is the variable taking values in the interval $[0, 1]$ (0 - lack of influence, 1 - the maximum impact). The usefulness of the countries and coalitions bases on the criterion of influence. We are developing the traditions already mentioned earlier political science applications cooperative games associated with the assessment of the impact of the parliamentary parties and coalitions on the basis of indices: Shapley-Shubik

(Shapley, Shubik, 1954), Banzhaf (Banzhaf, 1965), Penrose (Penrose, 1946), and others.

In the simplest case, a quantitative assessment of the influence of player (the coalition) may, for example, based on the share in the pool of all considering international problems those problems which can‘t be solved without agree of this player (coalition). This approach can be justified by those circumstances that in the sphere of interstate cooperation veto-players, usually defined enough simply.

Table 1. The characteristic function of the game Base-3-a for the period 1992 - 2008.

i	v(i)	S	v(S)
1	0	{1,2}	0
2	0	{1,3}	1
3	0.8	{2,3}	1
		{1,2,3}	1

Source: conditional data

Table 1 shows the characteristic function corresponding to the balance of power in the world for the period about 1992 - 2008 (game Base-3-a). The predominance of the third player (West) observed at this period. Players 1 and 2 are not able to create a counterbalance to him.

Fig. 1 demonstrates a geometric interpretation of the set of non-negative pre-imputation for game Base-3-a. In this case pre-imputations are vectors lying on the plane

$$x_1 + x_2 + x_3 = v(\{I\}) \tag{3}$$

in three-dimensional space. Traditionally for geometric illustrations are using flat picture, which represents the plane (3).

One of the main concepts of the solution of cooperative games is Core, or set of non-dominated imputations

$$C(v) = \{x \in R^n | (\forall S \subset I) \sum_{i \in S} x_i \geq v(\{S\}), \sum_{i \in I} x_i = v(\{I\})\}. \tag{4}$$

From (4) it follows that any imputation belonging to Core, allows any coalition no less than it could obtain without cooperation with other any players. As can be seen in Fig. 1 Core games Base-3-a (period 1992 - 2008) is a singleton

$$C(v) = (0, 0, 1), \tag{5}$$

which marks a maximum of third player (West) influence.

The method of constructing the characteristic function is controversial. At the same time the logic on which it is based looks quite plausible. In particular, highest level of influence reflects the dominance of the West at the stage that followed the collapse of the Soviet Union. Complete dominance of the West is due to the inability to create a winning coalition without him

$$v(\{1,3\}) = v(\{2,3\}) = 1$$

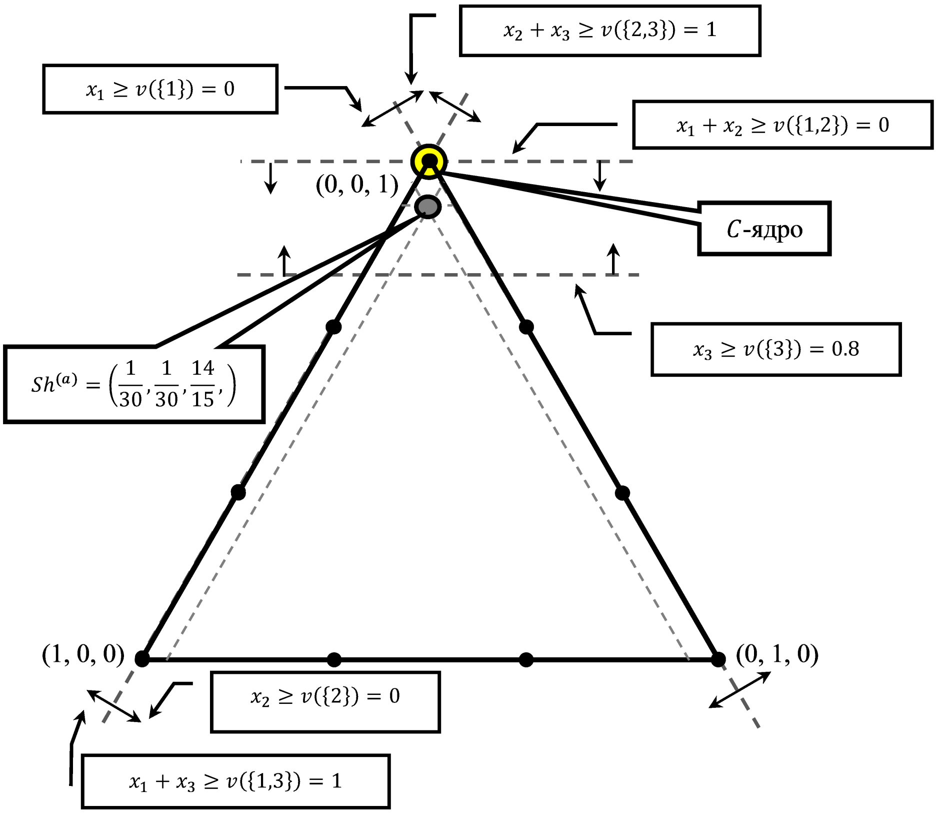

Fig. 1. Geometric illustration of the game Base-3-a, 1992-2008.

in that $v(\{1,2\}) = 0$.

This occurs despite the fact that the individual payoff of the West ($v(\{3\}) = 0.8$) less than 1.

Another possible distribution of influence between players in the game Base-3-a is based on the vector (the values) Shapley

$$Sh^{(a)} = (\tfrac{1}{30}, \tfrac{1}{30}, \tfrac{14}{15}).$$

Recall that the values of Shapley vector can be expressed as

$$Sh_i = \sum_{S: i \notin S} \frac{s!(n-s-1)!}{n!} (v(\{S \cup i\}) - v(\{S\})), \tag{6}$$

where n - the total number of players, s - the number of players in the coalition S. Shapley vector give for player $i \in I$ a value representing a weighted sum of the increments of utility coalition, which are caused by the addition to these coalitions player i.

Shapley value of the game Base-3-a does not belong to Core. This means that there are potential challenges to coalitions $\{1,2\}$, $\{1,3\}$. The Shapley values could be interpreted as possible marks of concessions from the players 3 in relation to the players 1 and 2 when it is interested in reaching a consensus (the creation of a full coalition)

One of the advantages of the game Base-3-a is that it allows one to clearly describe the changes in the system of interstate relations that are observed after the economic crisis of 2008. The highlighting of 2008 as the beginning of new stage is highly conditional. As an argument in favor of such a choice, we can take an involvement of Russia in the events in South Ossetia (August 2008), which is visibly different from the Russian foreign policy during the events in Kosovo in 1999, is limited only by diplomatic demarches.

The changes taking place in relations between the world centers of power in the years 2008 - 2014 are shown in Table 2 (game Base-3-b).

Table 2. The characteristic function of the game Base-3-b (Period 2008 - 2014).

i	v(i)	S	v(S)
1	0	{1,2}	0.2
2	0	{1,3}	1
3	0.8	{2,3}	1
		{1,2,3}	1

Source: conditional data

In the new conditions of the possibility of players 1 and 2 (Russia, China) set as

$$v(\{1,2\}) = 0.2$$

The changes mean that Russia and China can achieve predominant influence in those 20 procents of cases that are beyond the control of the player to the West, when he did not enter into any coalition

$$v(\{1,2\}) = 0.2 = 1 - v(\{3\}).$$

A geometric interpretation of the game Base-3-b is shown in Fig. 2.

The game Base-3-b has empty Core. This means that now none of the players can not reasonably undivided claim to influence, unlike the previous case (see Fig. 1). This fact can be interpreted as a mathematical proof of the thesis about the end of unipolar world in this model.

In a situation of Core emptiness can be considered an alternative solution concepts, for example Nucleolus. This concept is based on the solution providing a maximum of at least lexicographical excesses coalitions $S \neq \emptyset, I$. Strict definition of Nucleolus can be found in Schmeidler (1969) or other professional books on the cooperative games theory, for example Pechersky Yanovskaya (2004).

Remind also that the excess of coalition S, for imputation x is expressed as

$$e(S, x) = v(\{S\}) - x(\{S\}), \text{ where } x(\{S\}) = \textstyle\sum_{i \in S} x(i).$$

The excess $e(S, x)$ contrapose own capabilities of the coalition $v(\{S\})$ and payoff which it receives in accordance with the imputation Thus, the smaller the excess, the more favorable imputation for the coalition and vice versa.

In our example, the nucleolus takes on values

$$N^{(b)} = (\frac{0.2}{3}, \frac{0.2}{3}, 0.8 + \frac{0.2}{3}). \quad (7)$$

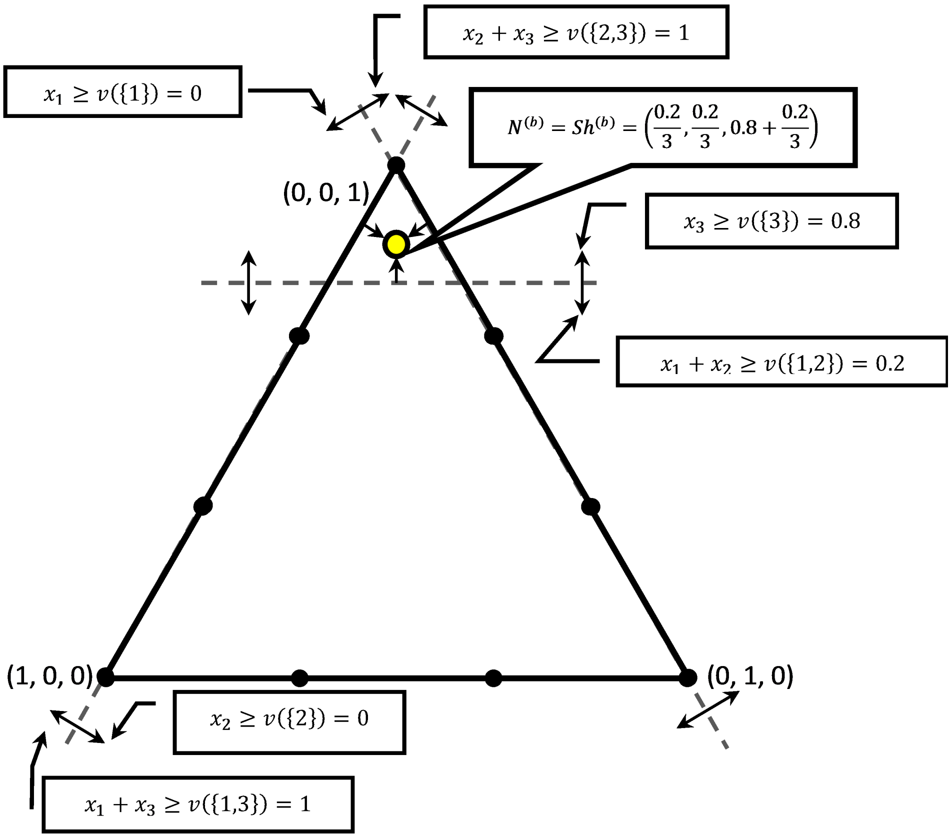

Fig. 2. Geometric illustration of the game Base-3-b, 2008 - 2014.

It should be emphasized that, Nucleolus values it is not the uncontested, for reasons of Core emptiness. For example, the player 3 can separately negotiate with the player 1 on the conditions

$$(\delta, 0, 1 - \delta) \text{ where } \tfrac{0.2}{3} < \delta < 0.2,$$

excluding player 2. Similarly, it may come with the player 1, player 2 had agreed

$$(0, \delta, 1 - \delta), \text{ where } \tfrac{0.2}{3} < \delta < 0.2.$$

At the same time, players 1 and 2, there is also the opportunity to challenge the nucleolus, get off at either the division, which provides them with a total of not less than limiting, thus, third player claims.

Shapley value $Sh^{(b)}$ in this game is the same as nucleolus.

The following qualitative changes associated with a further increase in the capacity of the coalition $\{1, 2\}$. We may waive the requirement of equality of the sum influence for possible configurations of coalitions of unit. In this case, we can get another version of the game Base-3-c, see Table 3.

Source: conditional data

A geometric interpretation of the game Base-3-c is shown in Fig. 3. It is also, like the preceding examples, is non-convex, i.e. the condition

Table 3. The characteristic function of the game Base-3-c (after 2014).

i	v(i)	S	v(S)
1	0	{1,2}	0.4
2	0	{1,3}	1
3	0.8	{2,3}	1
		{1,2,3}	1

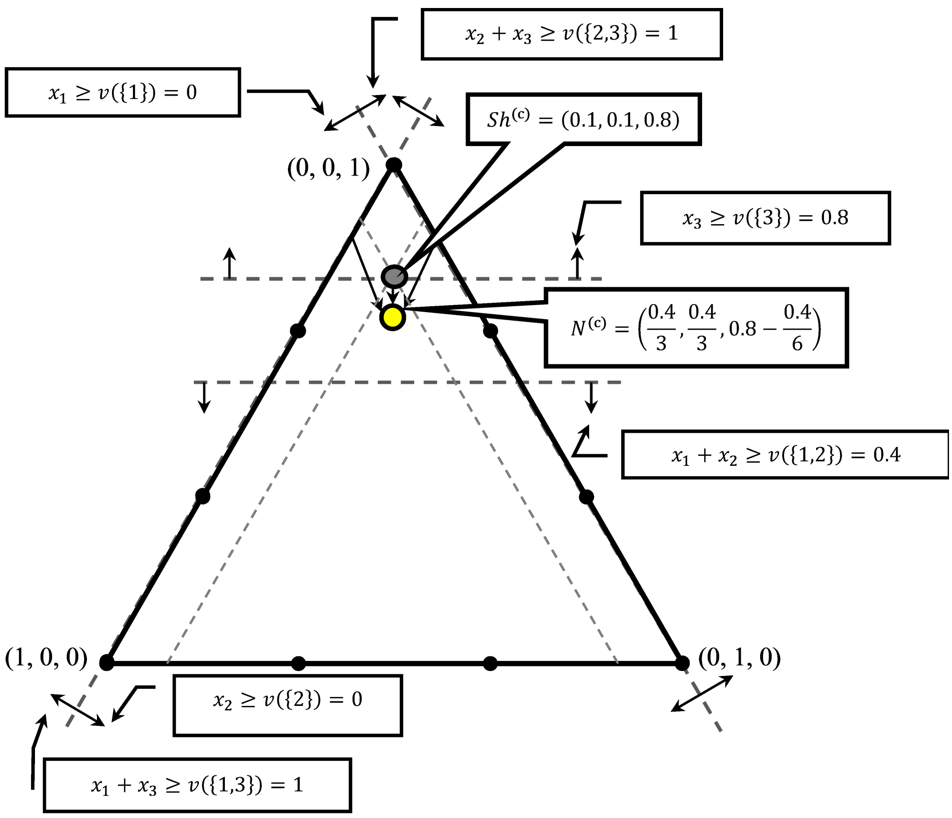

Fig. 3. Geometric illustration of the game Base-3-c, after 2014.

$$(\forall S, T) v(\{S \cup T\}) + v(\{S \cap T\}) \geq v(\{S\}) + v(\{T\}) \tag{8}$$

is not satisfied.

Moreover, unlike them, it is not a superadditive. Indeed, in the case of accession Player 3 to the coalition $\{1, 2\}$ we see that

$$v(\{1,2\}) + v(\{3\}) = 0.4 + 0.8 = 1.2 > 1 = v(\{1,2,3\}).$$

As can be seen, in this situation, we got an additional factor incompatibility coalition rationality conditions

$$v(\{1,2\}) \geq 0.4, v(\{3\}) \geq 0.8.$$

The nucleolus of the game is determined by the vector

$$N^{(c)} = (\frac{0.4}{3}, \frac{0.4}{3}, 0.6 + \frac{0.4}{3}) = (\frac{0.4}{3}, \frac{0.4}{3}, 0.8 - \frac{0.4}{6}). \qquad (9)$$

The principal difference between the solution (9) of the solution (7) is that it assumes the need for concessions from the players 3 (West) in order to form a full coalition. Indeed the proportion of the influence of the West in the nucleolus of the Base-3-c

$$N_3^{(c)} = 0.8 - \tfrac{0.4}{6} <= 0.8 = v(\{3\}),$$

which implies serious doubts about the desirability and possibility of a full coalition of all players (achieve a global consensus). We have to admit that this is not the positive conclusion drawn on the level of the model. But this conclusion does not contradict the reality that we have observed over the last year. Few of the experts and analysts agree with the thesis reducing the stability and security in the world for 2013 - 2014 years. Another feature of this game is a mismatch nucleolus and Shapley

$$Sh^{(c)} = (0.1, 0.1, 0.8).$$

These values $Sh^{(c)}$ are a direct consequence of non-superadditivity of the game. This means that the third player indifferent between joining or not joining the coalition $\{1, 2\}$. In in other words, this may mean the appearance of objective tendencies to dismissal third players from other game participants.

Widely known thesis about the importance of the transition from a unipolar to a multipolar world. Appearing at the beginning of the third millennium, it has taken a leading position in the contemporary political science discourse.

Let's try to analyze the possible relationship version of our model of players in a multipolar world. Table 4 (Base-3-d) describes a situation in which none of the players (Russia, China, the West) can not get influence in the world alone. At the same time, any coalition of the two parties receives the absolute influence (it can impose its rules to a third party). Of course, the absolute influence has full coalition.

Table 4. The characteristic feature of the game Base-3-d ("Multi-polar world")

i	v(i)	S	v(S)
1	0	{1,2}	1
2	0	{1,3}	1
3	0	{2,3}	1
		{1,2,3}	1

Source: conditional data

Geometric Vector game Base-3-d shown in Fig. 4.

As you can see the game Base-3-d has an empty Core. Crossing lines coalition rationality gives us three basic points

$$x^{1,2} = (\tfrac{1}{2}, \tfrac{1}{2}, 0),\ x^{1,3} = (\tfrac{1}{2}, 0, \tfrac{1}{2}),\ x^{2,3} = (0, \tfrac{1}{2}, \tfrac{1}{2}).$$

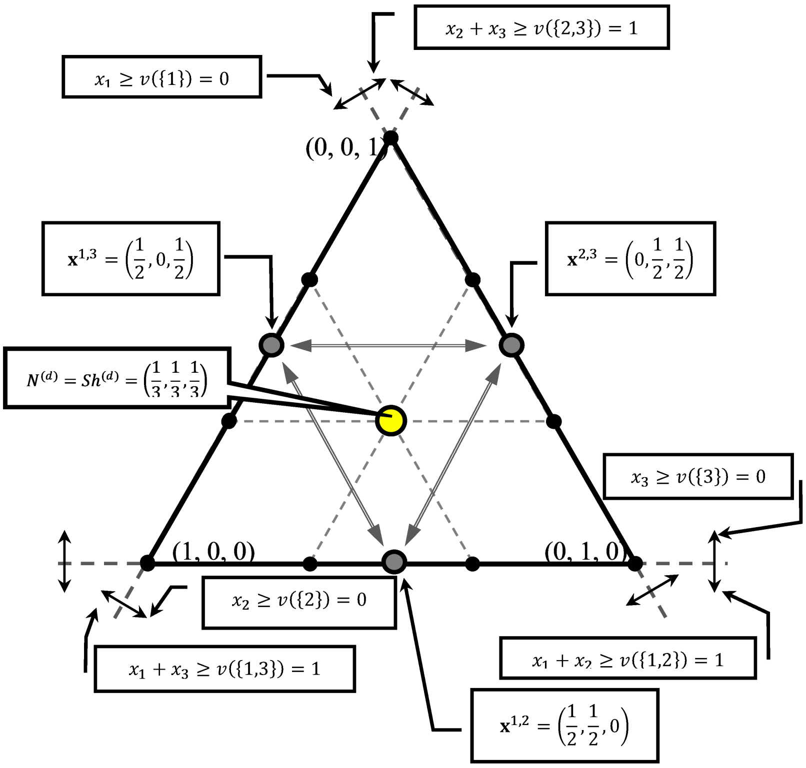

Fig. 4. Geometric illustration of the game Base-3-d

Each of them corresponds to the "agreement" between the two players, the effect of dividing equally, to the exclusion of a third party, do not receive anything. These distributions are characterized by instability evident since against them there are obvious threats. If any configuration, each of "united party" has a reason to suspect a partner that one can without loss of utility for himself agrees with "the odd man out."

Nucleolus Base-game 3-d by the symmetry of the capacity of members is a vector

$$N^{(d)} = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3}). \tag{10}$$

Note that in the Base-3-d coincides with the nucleolus of the Shapley value. However, (10) has all the disadvantages of "unstable, contested" decision from which players deviate profitable $x^{(1,2)}$, $x^{(1,3)}$ or $x^{(2,3)}$.

"Symmetry" instability of the solutions $x^{1,2}$, $x^{1,3}$, $x^{2,3}$ in the game Base-3-d due to the symmetry of the player to complete of the players capabilities. Credibility of such assumptions are raises serious doubts obviously. This disadvantage can be partly overcome due to the differentiation of individual utility player, see Table 5.

The characteristic function given by Table 5, based on estimates, according to which the supposed relative increase in economic and political potential of China in relation to the United States and its allies. We can agree with the proposed values, at least at the level of the preliminary analysis. It is not hard to guess that

Table 5. The characteristic feature of the game Base-3-e (Multi-polar world with asymmetry).

i	$v(i)$	S	$v(S)$
1	0	{1,2}	1
2	0.2	{1,3}	1
3	0.4	{2,3}	1
		{1,2,3}	1

the fundamental importance is not so much the absolute values as their ordering relative to each other.

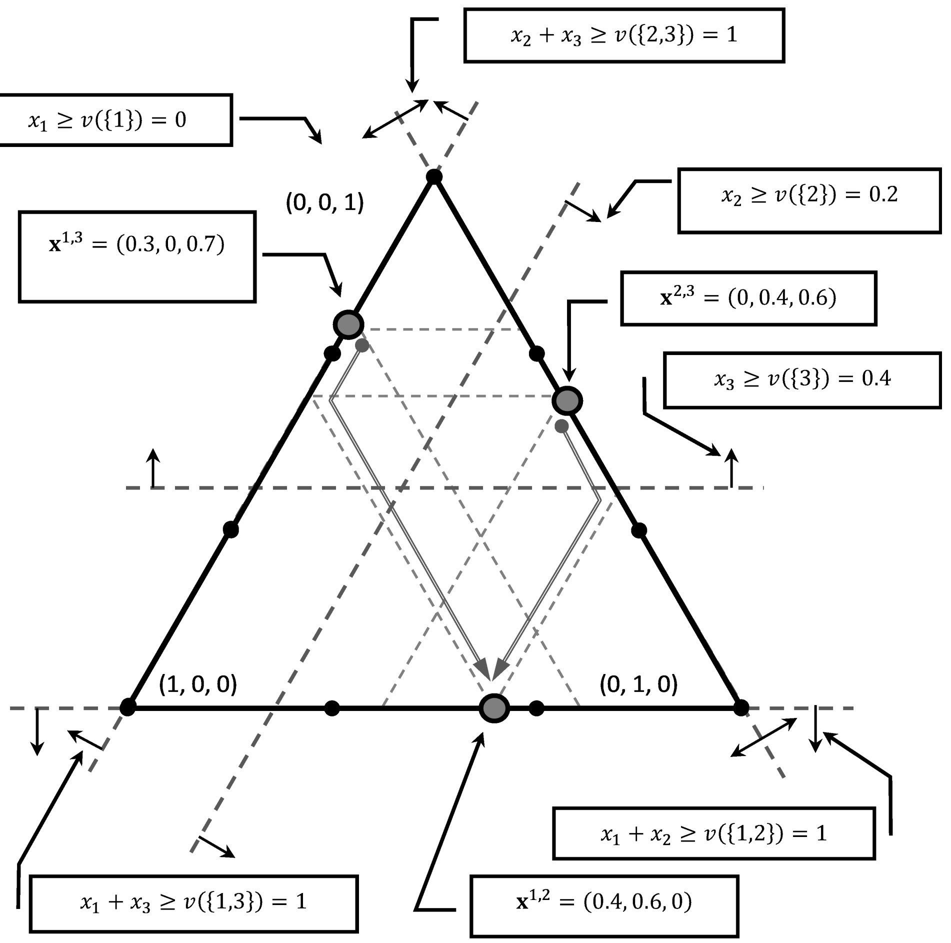

Fig. 5. Geometric illustration of the game Base-3-e

Fig. 5 (geometric illustration games Base-3-e) demonstrates the transformation of solutions $x^{1,2}, x^{1,3}, x^{2,3}$, assuming the achievement of agreement between the two players with the exception of the third (it's assumed an equal distribution of power between the contracting players). For example,

$x^{2,3} = (0, v(\{2\}) + \frac{v(\{I\})-v(\{2\})-v(\{3\})}{2}, v(\{3\}) + \frac{v(\{I\})-v(\{2\})-v(\{3\})}{2}) =$
$= (0, 0.2 + \frac{1-0.2-0.4}{2}, 0.4 + \frac{1-0.2-0.4}{2}) = (0, 0.4, 0.6).$

As you can see, in Base-3-e players 1 and 2 in the case of the realization of the distribution $x^{1,2} = (0.4, 0.6, 0)$ does not have incentives to abandon the alliance in favor of the unions with the third player in which it gets a smaller share.

The higher an individual utility of the third player and therefore a higher level of his initial claims are working against him. It makes less favorable separate agreement with him for other players.

Substantial differences between the games Base-3-d and Base-3-e can be demonstrated by comparing Nucleolus

$$N^{(e)} = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3}) \tag{11}$$

(remains the same as the Base-3-d, which is determined by the immutability capabilities paired coalitions) and the Shapley vector

$$Sh^{(e)} = (\frac{7}{30}, \frac{1}{3}, \frac{13}{30}). \tag{12}$$

Comparing (11) and (12), we find that these decisions are indifferent to second player (China). At the same time, (12) makes it possible to take into account the inequality of thr opportunities of players 1 and 3 (Russia and the West). If the players overcome the temptation twinning and try to reach a consensus in the game Base-3-e (to form a great coalition), the possible contours of such an agreement are determined by the following parameters:

- player 1 - from 23 to 33 procents of influence;
- player 2 - about 33 procents of influence;
- player 3 - from 33 to 43 procents of the influence.

4. Extended model of interaction centers of political influence (Base-4)

One of the most doubtful issues of the Base-3 models is assumption that determines the number of players. Certainly, specialists in game theory will understand to this assumption is likely. However, it met serious objections from the experts in the field of political science and international economy.

The models Base-3 is based on a compromise between properties of the researched object and the opportunity to apply the mathematical tools. The increase in the number of players makes the model more appropriate. The model is complicated at the same time. There are arisen difficulties in the construction of the characteristic function in addition. With a large number of players their possible coalitions in fact never occur, and therefore their potential utility can be evaluated only in a hypothetical manner.

As a consequence, the results obtained from the analysis of models with a large number of players are equally ambiguous and debatable, as is the case $n = 3$. Also need to add that in the transition to cooperative games with number of players more than 4 we lose the possibility of constructing geometrical interpretations (for games possible three-dimensional illustration, however, in terms of practical application, they are rather decorative character).

The increase in the number of players does not lead to an increase in the quality of the results. Moreover in the case of large n we get the muddy mathematical construction. However, the transition from $n = 3$ to $n = 4$ can be carried out relatively easy and clear.

Consider the model, conventionally referred to as Base-4. In this model the player 3 is splitting into players 3 and 4 (West-I, West-II). As the player West-I we will continue to treat the United States and its allies. West-II could be treated as country's traditionally assigned to the Western world, which in the long term can be more clear form their own system of goals and interests that are not directly correlated neither with the interests of the United States, Russia or China.

The authors did not insist on the inevitability of the transformation of the system players. We are quite assume other scenarios of the appearance of the fourth player. However, they have obvious advantages with regard to the our scenario.

In addition, it is reasonable and realistic looks assumption 1 and 2 players in these models will need to be treated not as Russia and China, as well as Russia and its allies and China, as well as the country focused on its support.

The characteristic feature of the game Base-4 is submitted in Table 6.

Table 6. The characteristic feature of the game Base-4

i	$v(i)$	S	$v(S)$	S	$v(S)$	S	$v(S)$
1	0	{1,2}	0.8	{1,2,3}	1	{1,2,3,4}	1
2	0.2	{1,3}	0.9	{1,2 4}	1		
3	0.4	{1,4}	0.3	{1,3,4}	1		
4	0	{2,3}	0.3	{2,3,4}	1		
		{2,4}	0.5				
		{3,4}	0.7				

Source: conditional data

Shapley value for games Base-4 takes the form

$$Sh^{(f)} = (\frac{29}{120}, \frac{29}{120}, \frac{43}{120}, \frac{19}{120}). \tag{13}$$

Its distinguishing feature is the "equalization of influence" of the players 1 and 2, in spite of the initial differences in their ability defined intrinsic function. See Table. 3. It can be seen as one of the effects caused by the appearance of "fourth power" - player West-II.

Nucleolus for this game takes the from

$$N^{(f)} = (0.308, 0.197, 0.308, 0.187). \tag{14}$$

Note, the nucleolus is imputation, which reached the lexicographical minimum maximum exesses (coalition $S \neq \emptyset, I$). When $N^{(f)}$

$$\max_{S\neq\emptyset,I}\{\min_x\{e(S,x)\}\} = 0.3077.$$

This excess is reached for a coalition formed by the players 1, 2, 4. In other words, that this coalition is the most dissatisfied with the fact that it offers imputation $N(f)$. At the same time, we can express some doubts as to the adequacy of this decision for the game nucleolus Base-4. First of all, it concerns the ratio of shares Player 1 and Player 2. It is understandable if guided solely by the criterion of minimizing the dissatisfaction most offended coalition. However, it is extremely difficult to accept, if we take into account the actual practice of international relations. Thus, with regard to the concept of the game nucleolus Base-4 we can be regarded as a theoretical guide.

5. Development of models of interaction centers of political influence

Returning to the issues related to the problems of construction of characteristic functions in cooperative games, describing the relationship between the centers of power and political influence in the modern world. As already mentioned, a methodology based on the principle of influence as an opportunity to lock can be seen (with all the reservations) as an acceptable only in the initial stages of the study focused on obtaining generalizing, qualitative conclusions.

In this connection, it is natural looks the question: how are alternative approaches? From our point as one of the possible alternatives is an approach, involving an assessment of the impact of the forces and their potential coalitions on the basis of the portfolio, composed of the currencies of these countries.

The first task, the solution of which depends the success or lack of success of this approach is to determine the principles and rules on which data should be formed of the currency portfolio.

To date, there are a number of serious studies that yielded important and interesting results with respect to the laws of dynamics of multi-currency portfolios and, in particular, with respect to the so-called currency with minimal volatility. See in particular (Hovanov et al., 2004; Hovanov, 2005; Hovanov, 2005; Bubenko and Hovanov, 2012).

One of the problems that we face when used as a base for the construction of the characteristic function of the minimum income in the currency volatility, built on the basis of a portfolio of exchange members of the coalition is the stochastic nature of the data. A constructive solution to this problem is associated with the transition from deterministic to stochastic cooperative games.

In the modern theory of cooperative games it has developed several approaches to the definition of stochastic cooperative game. One of the first studies in this direction was the work (Charnes, 1977; Charnes, 1973). Also worth mentioning is the series of works on the subject (Yeung and Petrosyan, 2006; Yeung and Petosyan, 2004; Suijs and al., 1999).

In this article a stochastic cooperative game (SCG) we mean a pair $\Gamma = (I, \tilde{v})$ where

- $I = \{1..n\}$ - the set of participants;
- $\tilde{v}(\{S\})$- random variables with known distribution density $p_{\tilde{v}(\{S\})}(x)$ which interpreted as revenues (utility payments), the corresponding coalitions $S \subset I$.

This approach to the task of stochastic cooperative games were previously presented in the paper Konyukhovskiy (2012).

Imputation stochastic cooperative game will be called the vector $x(\alpha) \in R^n$ satisfying for conditions

(a)

$$(\forall i \in I) P\{x_i(\alpha) \geq \tilde{v}(\{i\})\} \geq \alpha \tag{15}$$

stochastic analog of individual rationality;

(b)

$$P\{\sum_{i=1..n} x_i(\alpha) \leq \tilde{v}(\{I\})\} \geq \alpha \tag{16}$$

stochastic analog of the group rationality.

Note that the condition (15) sets that the share prescribed by delay $x(\alpha)$ for i^{th} player has to be greater or equal than the random value of his personal gain with a probability of not less than α . In accordance with (15) i^{th} component of the vector division $x(\alpha)$ compared with the α-quantile $F_{\tilde{v}(\{i\})}(x)$ (distribution function of the random variable). For compactness subsequent expressions we introduce the notation

$$v_\alpha(i) = F^{(-1)}_{\tilde{v}(\{i\})}(\alpha) \tag{17}$$

for some players and i,

$$v_\alpha(S) = F^{(-1)}_{\tilde{v}(\{S\})}(\alpha) \tag{18}$$

for some coalition $S \subset I$. Then (15) can be written as

$$(\forall i \in I) x_i(\alpha) \geq v_\alpha(\{i\}) \tag{19}$$

The possibility of transformation from condition (15) to (19) follows from the properties of non-decreasing distribution functions. Indeed, the condition $x_i(\alpha) \geq \tilde{v}(\{i\})$ is satisfied for a certain level of probability α , will be carried out for all $\alpha' < \alpha$.

In the classical cooperative games under group rationality means the need for full distribution utility large (complete) coalition within the division. In a modification of the stochastic game (16) means that the big (full) coalition is able to win with a probability of not less than α , to ensure the realization of the imputation $x(\alpha)$. Note that the condition (16) is equivalent to

$$P\{\sum_{i=1..n} x_i(\alpha) \geq \tilde{v}(\{I\})\} \leq 1 - \alpha \tag{20}$$

From (20), denoting $v_\alpha(\{I\}) = F^{(-1)}_{\tilde{v}(\{I\})}(\alpha)$ the αquantile of the distribution function $F^{(-1)}_{\tilde{v}(\{I\})}(\alpha)$, we obtain $\sum_{i=1..n} x_i(\alpha) \leq v_{1-\alpha}(\{I\})$.

We emphasize quite a significant difference. If the normal condition of cooperative games group rationality is defined as the strict equality and thus defines a hyperplane in m-dimensional space, the approach proposed here it is in the form of inequality and defines loosely in half-dimensional space. Thus, the nature of vectors x that satisfy the definition (15) - (16) differs from the nature of imputations in their classical interpretation. Sometimes for naming such objects use the term distributions (allocations).

As a result, the system conditions, which determines imputation stochastic game, takes the form

(a)

$$(\forall i \in I) x_i(\alpha) \geq v_\alpha(\{i\}) \tag{21}$$

(b)

$$\sum_{i=1..m} x_i(\alpha) \leq v_{1-\alpha}(\{I\}) \tag{22}$$

For values $v_\alpha(\{i\})$ in modern risk management usually apply the term value at risk (VaR). In this connection it may be noted advantages of the approach (21) - (22). Namely, it is logically consistent with the concept of VaR. This potentially opens up opportunities for a meaningful interpretation of the results of subsequent studies of the properties of this class of games and concepts of finding solutions.

When modeling the interaction of centers of influence in the modern world as a base for building the characteristic functions we can use the basket of the currencies of the countries forming the corresponding coalition. The potential income provided by the currency basket is a random value.

Due to the fact that the actual implementation of the yield occurs under conditions of uncertainty in the simulation it is expedient to use random values. Accordingly, for the simulation of these processes should be used models based on stochastic games. For example, the income received under the basket for a coalition of players S, we can assume a random variable $\tilde{v}(\{S\})$ with the density $p_{\tilde{v}(\{s\})}(x)$. Of course, this applies to both the coalitions formed by individual players, and to complete the coalition.

In practice, modeling income currency basket can be used normally distributed random variables or random variables with asymmetric triangular distribution.

An important specific feature of stochastic cooperative games is a substantial modification in them the concept superadditivity. For ordinary (non-stochastic) cooperative games

$$v(\{S \cup T\}) = v(\{S\}) + v(\{T\})$$

(equality between the sum of utilities coalitions S and T and utility of united coalition $S \cup T$) means meaninglessness of association. At the same time in the stochastic game similar amount

$$\tilde{v}^+(\{S \cup T\}) = \tilde{v}(\{S\}) + \tilde{v}(\{T\})$$

is also a random variable, for which

$$v_\alpha^+(\{S \cup T\}) \neq v_\alpha(\{S\}) + v_\alpha(\{T\})$$

in general.

The problems of relationships between quantile sums and the sum of the quantile most concern to the field of probability theory than the theory of games. See, for example, (Watson and Gordon, 1986; Liu and David, 1989).

In terms of assessing the impact of cross-national coalition stated above property means that when used as a tool for modeling stochastic cooperative games, even a simple inter-coalition agreements summation income (utility) when combined currency baskets may bring additional effect. Accordingly, there is an opportunity for an adequate modelling of the effects of the emergence of cross-country coalition. In particular, can be distinguished the following types of coalitions.

- Basket of currencies united coalition $S \cup T$ is characterized by simple summation of revenues $\tilde{v}^+(\{S \cup T\}) = \tilde{v}(\{S\}) + \tilde{v}(\{T\})$ it new qualities are determined by differences between VaR of united coalition and sum of VAR's of participants.
- When merging coalitions S and T their union in a coalition $S \cup T$ occurs basket of currencies, the income of which is described by some new random variable $\tilde{v}(\{S \cup T\})$ with the density $p_{\tilde{v}(\{S \cup T\})}(x)$ is not directly related to the amount of $\tilde{v}(\{S\}) + \tilde{v}(\{T\})$

The second option allows the association to reflect the substantial effects of combining for specific coalitions.

In such models there is another area of analysis of rationality imputations. Indeed, in the case of meaningful association (the second type of coalition), the share of utility $x(\{Q\})$ which imputation x prescripts for coalition Q can be compared not only $v_\alpha(\{Q\}) = F^{-1}_{\tilde{v}(\{Q\})}(\alpha)$ (VaR of utility Q), but with the sum of VAR's of player included in the coalition Q

$$\tilde{v}^+(\{S\}) = \sum_{i \in S} \tilde{v}(\{i\})) \tag{23}$$

Moreover, there are possible comparisons $v_\alpha(\{Q\})$ with $v_\alpha(\{S\}) + v_\alpha(\{T\})$ where S, T - subsets Q $(Q = S \cap T, S \cup T = \emptyset)$. Generally, such comparisons can be conducted over all possible partitions of each set $S \subset I$ into subsets.

As a result, we can get a essential conclusions about preferred forms of cross-national associations, and that the impact on which they can objectively claim.

6. Closing

The methods of the modern theory of cooperative games can act as effective tools for modeling and analysis of processes of redistribution of political and economic influence among the world's centers of power.

The given examples in this paper largely suggest that the cooperative game models provide some internally consistent logic to explain the trend, according to which the relationship has evolved, and there was a redistribution of influence between the world centers of power in recent decades.

The condition for the successful development of co-operative models of interaction between power centers, the improvement of methods of construction of characteristic functions in the direction of improving the adequacy of accounting objective interests (utilities) countries and coalitions.

It is very important to forth researches aimed at improving of mathematical tools used in the models. This applies particularly to development potential concepts of the solutions for stochastic cooperative games.

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