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Modelling of Information Spreading in the Population of Taxpayers: Evolutionary Approach*

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Abstract Information technologies such as social networks and Internet allow to spread ideas, rumors, advertisements and information effectively and widely. Here we use this fact to describe two different approach of evaluation the impact of information on the members of tax system. We consider an impact of spreading information about future audits of the tax authorities in a population of taxpayers. It is assumed that all agents pay taxes, if they know that the probability of a tax audit is high. However some agents can hide their true income and then such behavior provokes the tax audit. Each agent adopts her behavior to the received information of future audits, which depends on the behavior of other agents.
Firstly, we model a process of propagation information as an epidemic process and combine it with game between tax authority and taxpayers. Secondly, we consider evolutionary game on network which define structured population of taxpayers and evaluate the impact of the spreading of information on the changes of population states over the time.
We formulate mathematical models, analyze the behavior of agents and corroborate all results with numerical simulations.

Keywords: tax audit, tax evasion, total tax revenue, information spreading, evolutionary game on networks.

1. Introduction

The system of tax control as a key element of fiscal system provides several tools to improve the collection of taxes. However if the budget is restricted then the tax authority should explore new approaches to stimulate the tax collection. One of the most effective method of struggle against the concealment of taxes is total tax audit, but this procedure is very expensive and can not be applied to whole population. Another way to enhance the collection of taxes is selective tax audit. It allows to check special subgroups of taxpayers from the population taking into account additional information about their incomes and propensity to risk (Gubar et al, 2015).

Previous research (Boure and Kumacheva, 2010; Chander and Wilde, 1998; Vasin and Morozov, 2005) have shown that often goals of taxpayers and tax authority are opposite. This fact forms a conflict situation which can be formulated

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as a game-theoretical problem. Thus in many studies authors combine statistical and game-theoretical approach together to describe the behavior of all members of the fiscal system (Chander and Wilde, 1998; Reinganum and Wilde, 1985; Sanchez and Sobel, 1993). Such complex scheme allows us to estimate behavior of taxpayers with assumption about their rationality and including our beliefs about behavior of tax authority.

Here we suppose that different shares of taxpayers demonstrate own propensity of risk. It is supposed that each taxpayer in the population demonstrates different risk propensity: to be risk-averse, to be risk-neutral or to be risk-loving. Risk-averse taxpayers tend to avoid risk and, hence, they pay taxes. Risk-loving taxpayers do not pay taxes even if a high risk (high probability) of a tax audit exists. The risk-neutral taxpayers are rational. It means that they evaluate the risk of tax auditing and if it is high enough, for example, this probability is bigger or equal to the threshold value, then risk-neutral taxpayers prefer to pay taxes. Since we have noticed that taxpayers and tax authority prefer to optimize own cost functions hence we can say that normally, the society should find a compromise between the opposite goals of population as single whole and separate individuals. It means that the goal of society is to increase the collected taxes to use them in social needs but at the same time individuals try to minimize payment to fiscal system. To analyze this social conflict we use known indexes such as Price of Anarchy and Price of Stability as well as we introduce new index of Social Welfare (Monderer and Shapley, 1996).

However, as we noticed above, total tax control is very expensive procedure, hence in recent studies several new methods have been developed (Antocia et al, 2014; Bloomquist, 2006; Chander and Wilde, 1998). For example, one approach offers to spread information about future audits over the population. This scheme can be considered as a useful tool to stimulate collection of task and decrease costs of tax control.

In current work we consider two different model which include the impact of information to the taxpayers decisions. Firstly, we assume that the process of propagation information resembles the process of spreading virus in epidemic process (Goffman and Newill, 1964). In this case we use classical Susceptible-Infected-Susceptible (SIS) model (Kandhway and Kuri, 2014; Kolesin et al, 2014) to describe the transition of information between Informed and Uninformed taxpayers. Then we combine the model of information spreading with game-theoretical model, which describes behavior of tax authority and taxpayers and reaction of agents on received information (Goffman and Newill, 1964; Gubar and Kumacheva and Zhitkova, 2015; Altman et al, 2014). Secondly, we model the same idea as an evolutionary game on structured population (Riehl and Cao, 2015; Altman et al, 2010). As we mentioned early nowadays modern information technologies can be successfully used to spread various types of information (Nekovee et al, 2007). So we suppose that the population of taxpayers can be described by the network where nodes are individual taxpayers and links define connections between them. We suppose that at the initial time moment tax authority throws an information about future tax audit into a small part of population of taxpayers. Economic agents communicate over the time period and information will spread. Received information can force an agent to change her strategy. This technics allows to improve the process of collection taxes with less costs. Propagation of information also initiates migration of economic agents between two subgroups: those who pay taxes and those who evade payments.

We consider a population of n homogeneous taxpayers, where every taxpayer can evade and declare his income less than its true level. In turn the tax authority can audit every taxpayer from the population. If the evasion is revealed, the taxpayer must pay his tax arrears and penalty. Tax authority is able to spread information about future audit. In this case can be informed or uninformed regardless of the risk propensity. We assume that each agent selects the best behavior that depends on the preferences of other participants and incorporate their beliefs or received information about probability of the future tax audit. It means that if taxpayer believes that a large part of the population pays taxes and this fact reduces the probability of tax audit, then a part of the taxpayers can deviate from the payment of taxes. Otherwise, if the most part of taxpayers attempt to evade the taxation then the optimal behavior is to pay taxes. Thereby we can say that the choice of each agent affects the state of population.

In current paper we estimate the reaction of taxpayers on information received from tax authority and circulated in population. We study several approaches and propose numerical experiments to support the theoretical results.

The paper is organized as follows. Section 2. presents the mathematical model of tax audit in classical formulation. Section 3. shows the dynamic model of tax control, which includes the knowledge about additional information. Its extension, the evolutionary model with network structure, is considered in Section 4. Numerical examples are presented in Section 5.

2. Game Theoretical Model of Interaction Between Tax Authority and Taxpayers with Information Dissemination

In current section we study the model of tax control based on the problem presented in (Boure and Kumacheva, 2010). There is a homogeneous set of n taxpayers, each of them has an income I_i, where $i = \overline{1,n}$. In the end of tax period a taxpayer i declares his income as D_i, $D_i \leq I_i$, $i = \overline{1,n}$. Let denote as ξ the tax rate, as π the penalty rate, which are measured in shares of amount of money. If the evasion is revealed as the result of the tax audit, then the tax evader should pay unpaid tax and the penalty, which depends on the evasion level $(\xi + \pi)(I_i - D_i)$, $i = \overline{1,n}$. The tax authority makes auditing with probability p_i and worth c_i.

2.1. Strategies of Players

Taxpayers have two strategies – to pay a tax in accordance to the true level of income (to declare $D_i = I_i$) or not to pay (to declare $D_i < I_i$). As it was previously obtained in the model (Boure and Kumacheva, 2010), the construction of further arguments depends on the ratio of the parameters ξ, π c_i, $i = \overline{1,n}$. The tax auditing of i-th taxpayer can be profitable or not for tax authority depending on whether the condition

$$(\xi + \pi)I_i \geq c_i \tag{1}$$

satisfied or not.

For the model described above the following results were obtained in (Boure and Kumacheva, 2010).

Proposition 1. *If for the taxpayer i ($i = \overline{1,n}$) the inequality (1) is fulfilled, the optimal strategy of tax authority (due to maximize its income) is the probability of tax audit*

$$p^* = \frac{\xi}{\xi + \pi} \tag{2}$$

for every $i = \overline{1,n}$. The optimal strategy of the taxpayer i is

$$D_i^*(p_i) = \begin{cases} 0, & \text{if } p_i < p^*, \\ I_i, & \text{if } p_i \geq p^*. \end{cases} \tag{3}$$

If for the i-th taxpayer the inequality (1) is not fulfilled, the optimal tax authority's strategy is $p_i = 0$ for every i. In this case the i-th taxpayer's optimal strategy is $D_i^*(I_i) = 0$.

Formally, behavior of taxpayers can be defined as strategies:

- y^1 – is to pay a tax in accordance to the true income level;
- y^2 – is not to pay (to pay $D_i = 0$ due to the Proposition 1).

The behavior of each taxpayer depends on many factors, for example one of them is the risk propensity of the taxpayer. All considered taxpayers possess one of the three statuses: risk-averse, risk-neutral and risk-loving. Risk-averse taxpayers prefer to pay taxes in accordance with their true level of income. They always choose the strategy y^1. Risk-loving are malicious evaders, despite of all external circumstances such as an audit, promised or conducted in reality. Their choice is the strategy y^2. Risk-neutral agents decide to pay or to evade (corresponding to the strategies y^1 or y^2) depending on two factors. One of them is the preferences of other participants of the system. Every taxpayer compares her own strategy with the strategy of others in order to estimate wether it is profitable to change strategy or not. The received information about probability of possible tax audits also impacts on the agent's behavior: as far as mentioned probability is high so the risk of penalty is high.

In the real life there are some practical difficulties in the application of the model. The first is that the tax authority often has no information about the relation of parameters in (1) for every i, $i = \overline{1,n}$. Thus, the value of individual probability of audit for each taxplayer is unknown. Therefore in the current tax period it is assumed that the tax authority intend to audit taxpayers with average value of probability p and average cost c.

Another difficulty is that the tax authority has a strongly limited budget. It is supposed not to be enough for the optimal auditing with probability p^* (from the equation (1)). Due to the mentioned fact the tax authority needs to find additional ways to stimulate taxpayer fees. One of these ways is the injection of information about future auditing (which possibly can be false) into the population of taxpayers. In our study we assume that "$p \geq p^*$". Cost of the information spreading we denote as $\tilde{c}$.

Thus, the tax authority has the following methods of influence on a population of taxpayers (strategies):

- x^1 – is to audit taxpayer, declared $D_i = 0$, with probability p;
- x^2 – is to spread information about future audits with probability through initial share of taxpayers $\nu_{inf}^0 = \nu_{inf}(t_0)$.

2.2. Dynamic Model of Spreading Information

As we introduce in section 1. we study two different approaches of influence of information to the model of tax control. In this section we consider a non-cooperative game $G = \langle N, X_i, U_i \rangle$, where N is the set of players, X_i is the strategy set of the player $i \in N$, U_i is the payoff function of the player $i \in N$.

We consider the population of the taxpayers as a community network (complete graph with the set of vertices N, dimension n). Suppose that at the initial moment of time $t = 0$ the tax authority can inject the information about future auditing. We suppose that the cost of spreading information for tax authority significantly differs from the audit cost. Hence at the initial moment this information divides the total population of taxpayers into two groups – Informed and Uninformed:

$$n = n_{inf}(t_0) + n_{noinf}(t_0).$$

The share of informed we define as follows:

$$\nu^0_{inf} = \nu_{inf}(t_0) = \frac{n_{inf}(t_0)}{n}.$$

Let the further transmission of information occurs according to the following scheme. Uninformed agents ($Noinf$) meet Informed agents (Inf) and receive the information about possible audit (infected by this information) and switch to the state of Informed. As soon as the information becomes irrelevant then Informed agent returns to the state Uninformed again (Altman et al, 2014). The process of transferring information resembles the scheme of transition viruses in epidemic model (Nekovee et al, 2007; Altman et al, 2014), so we can use the classical Susceptible-Infected-Susceptible model in our case. Fig.1 demonstrates the transitions between subgroups of population.

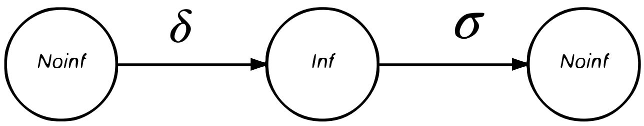

Fig. 1. Scheme of transmission of information form Uninformed to Informed.

Such process can be formalized by the next system of differential equations (Altman et al, 2014):

$$\frac{d\nu_k(n,t)}{dt} = -\sigma\nu_k(n,t) + \delta(1 - \nu_k(n,t)) \sum_{i=1}^{n} a_{k\,i}\, \nu_i(n,t), \tag{4}$$

where δ is a probability that Uninformed received information, (named information spreading rate), σ is a probability that information becomes obsolescent, (named the rate of forgetting information), $\nu_k(n,t)$ is the probability that the agent k received the information at the moment t of n participants of the population (it is also can be interpreted as the share of informed agents in the population). The value of the coefficient $a_{k\,i} = 1$, if there is a connection between taxpayers, and $a_{k\,i} = 0$, if there is no connection between taxpayers.

Following the method proposed in the (Altman et al, 2014), we proceed to consider stationary state of the system in which the probability $\nu_{k,\infty}(n)$ of the fact that the k-th agent is informed in the stationary state, does not depend on time:

$$\nu_{k,\infty}(n) = \lim_{t\to\infty} \nu_k(n,\, t).$$

In this case the stationary state of the system (4), obtained by the same scheme as in (Altman et al, 2014), becomes

$$\nu_{k,\infty}(n) = 1 - \frac{1}{1 + \tau \sum\limits_{i=1}^{n} a_{k\,i}\nu_{i,\,\infty}(n)}\ , \tag{5}$$

where $\tau = \dfrac{\delta}{\sigma}$ is the coefficient of information dissemination efficiency. This equation has two solutions:

- trivial, which is $\nu_{k,\,\infty}(n) = 0$;
- non-trivial, which corresponds to SIS-process.

If we exclude existing links from the population of n taxpayers ($a_{k\,i} = 1$), we obtain new population consists of $n - n_{inf}$ links. For each agent k in this population the next solution holds (in accordance to metastable state of the system):

$$\nu_{k,\infty}(n_{inf}) = \begin{cases} 1 - \dfrac{1}{\tau(n_{inf}-1)}, & \text{if } \tau \geq \dfrac{1}{n_{inf}-1} \\ 0, & \text{in opposite case.} \end{cases}$$

2.3. The game ¡¡Tax authority — one taxpayer¿¿

In this section we formulate a game between tax authority and one taxpayer. We assume that information has been propagated over the population and during the time population reaches its steady state. It means that we have a stable distribution of Informed and Uninformed taxpayers. Also we set an assumption that if any taxpayer receives information about future tax audit then she pays. We denote the share of those agents as n_{inf}. Here we consider a situation where taxpayer makes her decision in response to received information and incorporates the beliefs about the behavior of tax authority. Then we have a set of conflict situations in population between tax authority and individual agent k, which can be formulated as a bimatrix game. We suppose that the taxpayer from the population chooses the strategy y^1 and pays with probability ω, which depends on the share of informed agents, i. e. $\omega = \omega(\nu_{k,\infty}(n_{inf}))$. As it was discussed before, the population of taxpayers is heterogeneous in its relation to risk. The variable ω reflects this heterogeneity, that is ω characterizes how different part of the population will react on the received information. It's obvious that, due to the probability properties,

$$n_{inf} = n \cdot \omega.$$

The payoff matrix of presented game is:

	y^1	y^2
x^1	$(-pc + \xi I_i\omega; -\xi I_i\omega)$	$(-pc + p\,(\xi+\pi)I_i(1-\omega); -p\,(\xi+\pi)I_i(1-\omega))$
x^2	$(-\nu^0_{inf}\tilde{c} + \xi I_i\omega; -\xi I_i\omega)$	$(-\nu^0_{inf}\tilde{c}; 0)$

Analysis of the payoff matrix. In the case when $pc > \nu^0_{inf}\tilde{c}$ (the cost of auditing of the share of the population is greater than the cost of information injection ν^0_{inf} at the initial time moment), the strategy x^1 of the tax authority is dominated by the strategy x^2. If the opposite case is possible ($pc \le \nu^0_{inf}\tilde{c}$) and, besides, the probability of audit is $p \ge p^*$, the strategies profile (x^1, y^1) is the Nash Equilibrium. This equilibrium corresponds to a high probability of auditing and high cost of information dissemination. If the latter is higher than the cost of auditing we obtain that there is no need for consideration of the spreading of information because it is unprofitable. Therefore this model is similar to the previous one which was studied in (Boure and Kumacheva, 2010) and is not of interest for a detailed study here.

Further let's assume that the audit cost and information cost are related as follows:

$$\nu^0_{inf}\tilde{c} < p\,c. \tag{6}$$

Due to inequation (6) the strategy profile situation (x^1, y^1) is not an equilibrium, and thus (x^2, y^2) is the unique situation which can be equilibrium.

The taxpayer's strategy y^2 dominates the strategy y^1, if the auditing probability is

$$p \le \frac{\omega}{1-\omega}p^*. \tag{7}$$

The higher the probability of the strategy ¡¡to pay¿¿, which depends on the share of the informed, the more natural and obvious is the implementation of the inequality (7). If the inequation

$$p\,(c - (1-\omega)(\xi + \pi)I_i) > \nu^0_{inf}\tilde{c}, \tag{8}$$

is fullfilled, then (x^2, y^2) is Nash equilibrium. For the further analysis of the strategies the next cases of relation between parameters of the system are considered.

Case 1. Let the condition (1) is satisfied, then $c - (\xi + \pi)I_i \le 0$. If probability ω is small (ω is positive, close to zero, $\omega \to 0+$) then the expression $c - (1-\omega)(\xi+\pi)I_i$ is less or close to zero. Hence, we have

$$c - (1-\omega)(\xi + \pi)I_i < \frac{\nu^0_{inf}}{p}\tilde{c}.$$

Then the condition (8) is not fulfilled, and there is no equilibrium in the pure strategies in order to (x^2, y^2) is not an equilibrium.

Case 2. If the condition (1) is fulfilled, but the probability ω is high and tends to one from the left ($\omega \to 1-$), then the value of the expression $c - (1-\omega)(\xi + \pi)I_i$ is close to c, and the fulfillment of (8) is guaranteed by condition (6). In this case (x^2, y^2), indeed, is a Nash equilibrium.

Case 3. Now let (1) is not fulfilled, then the cost of auditing is high as well as $c-(\xi+\pi)I_i > 0$ and the expression $c-(1-\omega)(\xi+\pi)I_i$, is a strongly, nonnegative. But it is impossible to say whether (8) is satisfied or not. In this case, due to proposition 1, the auditing is not profitable when the costs of it is large, in other words if $p = 0$.

Thus, boundary cases are analyzed. It is obvious that (7), (8), and therefore the existing of equilibrium in pure strategies, essentially depends on the values of probability ω, defined by the dissemination of information in the population of agents.

The price of stability. In the studied game the best Nash equilibrium is the strategy profile (x^2, y^2). The value of the game is

$$V(x^2, y^2) = -\nu^0_{inf}\tilde{c} + 0 = -\nu^0_{inf}\tilde{c}.$$

Let's use the term of the Price of Stability given in (Nisan et al, 2007).

Definition 1. The Price of Stability (PoS) of a game is ratio between the value of the best Nash equilibrium to the value of the optimal solution:

$$PoS = \frac{\max\limits_{s \in S} SW(s)}{\max\limits_{s \in E} SW(s)}, \tag{9}$$

where SW is the Social Welfare function, S is the set of payoff, $E \subseteq S$ is the subset of the set of equilibrium strategies.

If parameters satisfy (6), then the strategy profile, which gives the highest value of the system's revenue, is Nash equilibrium (x^2, y^2). In this case the Price of Stability is

$$PoS = \frac{V(x^2, y^2)}{V(x^2, y^2)} = 1. \tag{10}$$

If the information spreading is not profitable and the inequation (6) is broken, the highes return of the system is reached if the tax authority chooses the strategy x^1. Then the Price of Stability is

$$PoS = \frac{V(x^2, y^2)}{-pc} = \frac{\nu^0_{inf}\tilde{c}}{pc}. \tag{11}$$

2.4. The game ¡¡Tax Authority — n Taxpayers¿¿

Now let's consider a game where the tax authority interacts to n taxpayers by the same scheme as we presented in subsection 2.3. As in the previous game the information about future tax audits has been propagated over the population of taxpayers and the system has reached its steady state. The information has been injected by the tax authority at the initial time moment. Analogously to model from (Altman et al, 2014) we define the payments of i-th taxpayer as

$$C_i = \xi I_i \omega(\nu_{j,\infty}(n)), \tag{12}$$

Payments of an audited taxpayer are:

$$H_i = p(\xi + \pi) I_i (1 - \omega(\nu_{j,\infty}(n))). \tag{13}$$

We denote as U_i the payments of player i and $U_{i\sigma_i}(n_{inf})$ the payoff of i-th taxpayer if the rate of obsolescence of information is σ_i. As in previous section we define the share of those who paid as n_{inf}. Then we construct the Social Welfare function for this system as:

$$SW(\nu) = \sum_{i=1}^{n} U_{i\sigma_i}(n_{inf}) = \sum_{i=1}^{n_{inf}} \xi I_i \omega(\nu_{j,\infty}(n)) + \sum_{k=1}^{n-n_{inf}} p(\xi + \pi) I_i (1 - \omega(\nu_{j,\infty}(n))). \tag{14}$$

We also assume that the income is uniformly distributed over the population of taxpayers. To simplify the model we use the average income $\overline{I}$ of the population (as it was introduced in (Gubar et al, 2015)) instead the exact value of income of the taxpayer i.

Then the Social Welfare function (14) is

$$SW(n_{inf}) = n_{inf}\overline{C} + (n - n_{inf})\overline{H}, \tag{15}$$

where $\overline{C}$ and $\overline{H}$ are the average values of (12) and (13) correspondingly.

Here we remind the definition of Price of Anarchy as in (Nisan et al, 2007).

Definition 2. Price of Anarchy (PoA) is a ratio between the value function of a Nash equilibrium and the optimal objective value function

$$PoA = \frac{\max\limits_{s \in S} SW(s)}{\min\limits_{s \in E} SW(s)}, \tag{16}$$

where SW is the Social Welfare function, S is the set of payoffs, $E \subseteq S$ is the subset of the equilibrium strategy set.

Based on this definition we obtain that the Price of Anarchy for the system with information spreading (4) – (14):

$$1 \leq PoA \leq \frac{1}{1 - (1 + \frac{1}{\tau})\frac{1}{n}}. \tag{17}$$

However, analyzing the coefficient of Price of Anarchy for considered game we faced with the problem how to estimate the relation between the price of individual and the total welfare. The results of the study we present in the next subsection.

Coefficient of Inter-Social Welfare. In this subsection we consider the modification of the index of Price of Anarchy and introduce a new coefficient which takes into account the difference between the individual and the total welfare.

Definition 3. Define as the Coefficient of Inter-Social Welfare(CoISW) a ratio between the summary of minimum payment of all taxpayers and maximum value of Social Welfare:

$$CoISW = \frac{\sum\limits_{i=1}^{n} \min U_k}{\max SW}. \tag{18}$$

To estimate this index, we follow the same scheme of the analysis as in the previous subsections.

Case 1. Let the condition (6) be justified. Due to the economic reason the tax authority chooses strategy x^2 – to inform taxpayers at the initial time moment $t = 0$. In this case the total income of tax authority R is defined as:

$$R = \left(\xi\overline{I}\omega(\nu_{j,\infty}(n)) - \nu^0_{inf}\tilde{c}\right) n. \tag{19}$$

If we consider equation (19) without the second term, which corresponds to budgetary expenses, then this is the Social Welfare function. This function reaches its maximum with the probability of tax payments in the case of the strategy ¡¡to pay¿¿:

$$\omega(\nu_{i,\infty}(n)) = 1.$$

Then, in this case, $n_{inf} = n$, and maximum value of SW is

$$\max_{n_{inf}} SW(n_{inf}) = SW(n) = \xi \overline{I} n. \tag{20}$$

Taxpayers prefer to minimize their tax payments and chose y^2. Then

$$\min U_i(x^2, \cdot) = U_i(x^2, y^2) = 0,$$

In the case described above Coefficient of Inter-Social Welfare is $CoISW = 0$.

Case 2. If the inequation (6) is not fulfilled, then it is more preferable to audit taxpayers than to spread information over the population. Thus the tax authority choose the strategy x^1 and audit taxpayers with probability p.

In this case the aggregated tax income is

$$R = \big(\xi \overline{I} \omega(\nu_{i,\infty}(n_{inf})) + p(\xi + \pi)\overline{I}(1 - \omega(\nu_{i,\infty}(n_{inf}))) - pc\big)\, n. \tag{21}$$

By analogy to the previous case, the last term in (21) is the budgetary expenses. Excluding this term from (21) we have the Social Welfare function:

$$SW(n_{inf}) = (pn(\xi + \pi) + n_{inf}(\xi - p(\xi + \pi)))\, \overline{I}. \tag{22}$$

The function (22) reaches its maximum if every audited taxpayer is evader ($n_{inf} = 0$) or if $\omega(\nu_{i,\infty}) = 0$. This fact is absolutely logical, if it is assumed that the probability to choose the strategy ¡¡to pay¿¿ tends to zero in the absence of information.

If the last assumption is incorrect and $\omega(\nu_{i,\infty}(n_{inf})) > 0$, then we should analyze the summands of (22) separately. It is obvious that the second term, which depends on n_{inf}, decreases if $p \in [0, p^*)$ and increases if $p > p^*$. However extremely small values of p essentially minimize the first term of relation (22) (collected taxes and fees). We obtain that SW function reaches its maximum when the second term is equal to zero that is when $p = p^*$. In both of two considered situations the equation (20) satisfied.

If the tax authority audits taxpayers with the probability $p \geq p^*$, then taxpayers minimize their payments if they honestly pay taxes. That is, $\min U_i = \xi I_i$ for each $i = \overline{1, n}$. In this case the Coefficient of Inter-Social Welfare is

$$CoISW = \frac{\xi \overline{I} n}{\xi \overline{I} n} = 1.$$

If the probability of audit is small($p < p^*$), then taxpayers try to minimize their payments and evade, thus $\min U_i = \xi I_i$ for each $i = \overline{1, n}$. Then the Coefficient of Inter-Social Welfare is

$$CoISW = \frac{p(\xi + \pi)\overline{i}N}{\xi \overline{i} N}.$$

After simplifying the model and taking into account that $p^* = \dfrac{\xi}{\xi + \pi}$, the previous equation becomes

$$CoISW = \frac{p}{p^*}.$$

It has an obvious interpretation. In this case the Coefficient of Inter-Social Welfare differs from its optimal value (which is equal to one) as many times as the actual probability of audit is less than its optimal value (p^*). When the value of auditing probability is $p = p^*$ the trade-off between personal and public welfare is reached.

In the section 2. we represent the game-theoretical model which describes interaction between tax authority and taxpayers taking into account information about future tax audit which was injected by the tax authority at the initial time moment and then circulates in population. We combine SIS model to describe the process of propagation information over the population of taxpayers with the analysis of different scenarios which realize into the system ¡¡tax authority-taxpayers¿¿.

Then in the next part of our paper we formulate an evolutionary model on the network which also includes agents' beliefs about possible tax audit. We will study a complicated model which merges a process of information spreading with evolutionary process in structured population of taxpayers in response to received information. We formulate the evolutionary game on the network and evaluate the value of initial information invasion. We also corroborate our theoretical results by numerical simulations.

3. The evolutionary model of Information Dissemination

Suppose that at the initial moment of time tax authority injects information about the audit. We also suppose that taxpayers can transfer this information to their neighbors and friends according to the network which define the structure of population. Hence the total population of the taxpayers is divided into subpopulations: Informed (n_{inf}) and Uninormed (n_{noinf}), also we can say that agents can have a propensity to perceive or not the information:

$$n = n_{inf}(t_0) + n_{noinf}(t_0).$$

In other words taxpayers can be inclined to receive information or not. Then each taxpayer can choose strategy to pay or not to pay taxes due to her true income level based on received information.

If one taxpayer from the subpopulation of those who inclined to perceive the information meets another one from this population, they will get the payoffs (U_{inf}, U_{inf}). In this case both of them know the same information and pay, hence their payoff is defined from the equation

$$U_{inf} = (1 - \xi)\overline{I}. \tag{23}$$

Similarly, if the taxpayer who does not perceive the information (and therefore wants to evade) meets the same taxpayer, they will get the payoffs (U_{ev}, U_{ev}), which are defined from the equation

$$U_{ev} = (1 - p)\overline{I} + p\,\overline{I}(1 - (\xi + \pi)),$$

or, the equivalent equation,

$$U_{ev} = (1 - p\,(\xi + \pi))\overline{I}. \tag{24}$$

We denote the taxpayer's propensity to perceive the information as α and consider the case when the uninformed taxpayer meets the informed taxpayer. As a result of such meeting, uninformed taxpayer obtains the information and should pay the payoff (3) with probability α if she believes in this information, or the payoff (4) if she does not believe.

As in classical evolutionary game the instant communications between taxpayers defines by two-players bimatrix game. For the cases, when taxpayers of different types meet each other, the matrix of payoffs can be written in the form:

	Inf	$Noinf$
Inf	$(U_{inf},\, U_{inf})$	$(U_{inf},\, \alpha U_{inf} + (1-\alpha)U_{ev})$
$Noinf$	$(\alpha U_{inf} + (1-\alpha)U_{ev},\, U_{inf})$	$(\alpha U_{inf} + (1-\alpha)U_{ev},\, \alpha U_{inf} + (1-\alpha)U_{ev})$

where Inf is the strategy of taxpayer if she is informed (she perceives the information) and $Noinf$ is the strategy not to be informed.

In this section, we consider a comparison of two modifications of the model of tax control based on the propensity to risk which demonstrate different taxpayers. The first case of the model does not include the process of information dissemination and in the absence of information risk-loving taxpayers do not pay. Risk-neutral taxpayers suppose that the probability of auditing is rather small ($p < p^*$), therefore they also evade. We define as risk-averse agents the agents who pay taxes. Let's denote by ν_p, ν_n, ν_a the shares of these subgroups ($\nu_p + \nu_n + \nu_a = 1$). Then the total tax revenue is

$$R_1 = N\left(\nu_a \xi \overline{I} + p\,(1-\nu_a)(\xi + \pi)\overline{I} - p\,c\right). \tag{25}$$

The second case takes into account the dissemination of information in the population of taxpayers. At the initial moment of time there is an information injection which is a share of informed taxpayers $\nu^0_{inf} = \nu_{inf}(t_0)$. The cost of unit of information is still $\widetilde{c}$. At the moment when the system reached its steady state ν_{inf} is the share of those who perceived information and paid taxes, ν_{ev} is the share of those who still evades. In this case the total tax revenue is

$$R_2 = N\left(\nu_{inf} \xi \overline{I} + p\,\nu_{ev}(\xi + \pi)\overline{I} - p\,c - \nu^0_{inf}\widetilde{c}\right). \tag{26}$$

4. The evolutionary model with network structure

In this section we suppose that agents transfer information not to a random opponent but then communicate with their neighbors and friends. In this case we can describe the possible links between agents using network.

Let $G = (N, L)$ denote an indirect network, where $N = \{1, \ldots, n\}$ is a set of economic agent and $L \subset N \times N$ is an edge set. Each edge in L represents two-player symmetric game between connected taxpayers. The taxpayers choose strategies from a binary set $X = \{A, B\}$ and receive payoffs according to the matrix of payoffs in section 3. Each instant time moment agents use a single strategy against all opponents and thus the games occurs simultaneously. We denote the strategy state by $x(T) = (x_1(t), \dots, x_n(t))^T, \; x_i(t) \in X$. Here $x_i(t) \in X$ is a strategy of taxpayer i, $i = \overline{1,n}$, at time moment t. Aggregated payoff of agent i will be defined as in (Riehl and Cao, 2015):

$$u_i = \omega_i \sum_{j \in M_i} a_{x_i(t), x_i(t)}, \tag{27}$$

where $a_{x_i(t), x_i(t)}$ is a component of payoff matrix, $M_i := \{j \in L : \{i,j\} \in L\}$ is a set of neighbors for taxpayer i, weighted coefficient $\phi_i = 1$ for cumulative payoffs and $\phi_i = \frac{1}{|M_i|}$ for averaged payoffs. Vector of payoffs of the total population is $u(t) = (u_1(t), \dots, u_n(t))^T$.

The state of population will be changed according to the rule, which is a function of the strategies and payoffs of neighboring agents:

$$x_i(t+i) = f(\{x_j(t), u_j(t) : j \in N_i \cup \{i\}\}). \tag{28}$$

Here we suppose that taxpayer changes her behavior if at least one neighbor has better payoff. As the example of such dynamics we can use the proportional imitation rule (Sandholm, 2010; Weibull, 1995), in which each agent chooses a neighbor randomly and if this neighbor received a higher payoff by using a different strategy, then the agent will switch with a probability proportional to the payoff difference. The proportional imitation rule can be presented as:

$$p\left(x_i(t+1) = x_j(t)\right) := \left[\frac{\lambda}{|M_i|}(u_j(t) - u_i(t))\right]_0^1 \tag{29}$$

for each agent $i \in L$ where $j \in M_i$ is a uniformly randomly chosen neighbor, $\lambda > 0$ is an arbitrary rate constant, and the notation $[z]_0^1$ indicates $\max(0, \min(1, z))$.

Now let's consider two cases of the rule described above.

- **Rule. Case 1.** *Initial distribution of agents is nonuniform.* When agent i receives an opportunity to revise her strategy then she considers her neighbors as one homogeneous player with aggregated payoff function. This payoff function is equal to mean value of payoffs of players included in homogeneous player. It is assumed that the agents meets with any neighbor with uniform probability, then mixed strategy of such homogeneous player is a vector of distribution of pure strategies of included players. If payoff function of homogeneous player is better then player i changes her strategy to the strategy of her more popular neighbor.
- **Rule. Case 2.** *Initial distribution of agents is uniform.* In this case agent i keeps her own strategy.

5. Numerical simulations

In this section we present numerical examples to support the approaches described in precious sections and demonstrate the influence of the structure of the network to the population of taxpayers in those series of experiments we will use the following structures of graphs: grid and random.

Firstly, we demonstrate the combined model with contains spreading of information based on SIS model and the game between taxpayers and tax authority. Secondly, we show simulations reffered to spreading information as an evolutionary game on the graph.

5.1. Information spreading in structured population

In this paragraph we present numerical simulation in population of taxpayers based on the model from section 2.2. Here we suppose that at the initial time moment $t = 0$ tax authority starts to spread information about possible tax audit over the population of taxpayers. To simplify calculations we define function $y(t)$

$$y(t) = \frac{\sum\limits_{i=1}^{N} v_i(t)}{N} \tag{30}$$

which is an average probability of transfer of information between taxpayers in population at time moment t.

As we presented above we formulate the process of spreading information as an epidemic process on the network. Let G_N be an undirected graph with $N = 30$ nodes, A is a connectivity symmetric matrix with binary coefficients $\{a_{ij}\}$:

$$a_{ij} = \begin{cases} 1, & \text{if node } i \text{ has a link with } j \\ 0, & otherwise. \end{cases}$$

Remark. If $i = j$ then $a_{ij} = 0$. Here, value y_0 is probability of receiving information at time moment $t = 0$.

In Figs. 2 — 7 we present the series of experiments varying coefficient σ with step 0.1 and fixed $\delta = 0.1$. In this case information spreads over the population of taxpayers, but value of function $y(t)$ decreases because of growth of parameter σ.

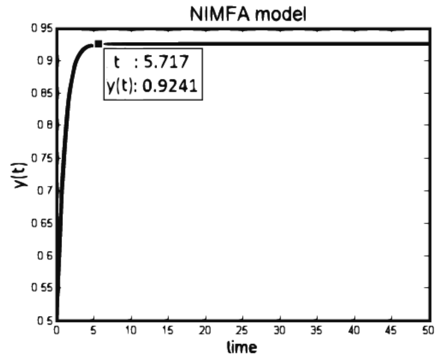

Fig. 2. Experiment 1. $\delta = 0.1$, $\sigma = 0.1$. Stationary state: $t = 5.717$, $y(t) = 092$.

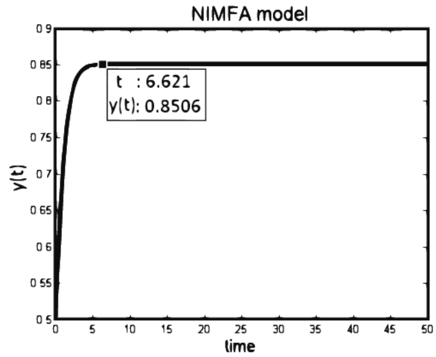

Fig. 3. Experiment 1. $\delta = 0.1$, $\sigma = 0.2$. Stationary state $t = 6.621$, $y(t) = 0.8506$.

From experiments we can see that system (4) reaches its stationary states inside the time interval $t \in [5.7,\ 8.4]$ but value of function $y(t)$ monotonically decreases.

If parameter σ is fixed but value of δ is decreased then we have that information spreads and behavior of function $y(t)$ changes, Figs.8 — 9 demonstrate this process.

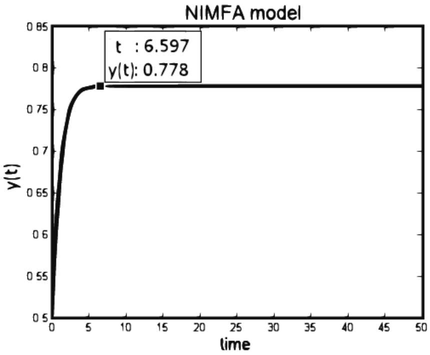

Fig. 4. Experiment 1. $\delta = 0.1$, $\sigma = 0.3$. Stationary state: $t = 6.537$, $y(t) = 0.778$.

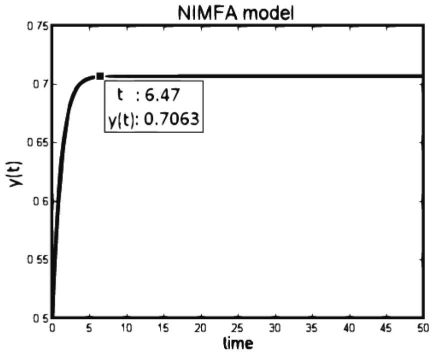

Fig. 5. Experiment 1. $\delta = 0.1$, $\sigma = 0.4$. Stationary state: $t = 6.47$, $y(t) = 0.7063$.

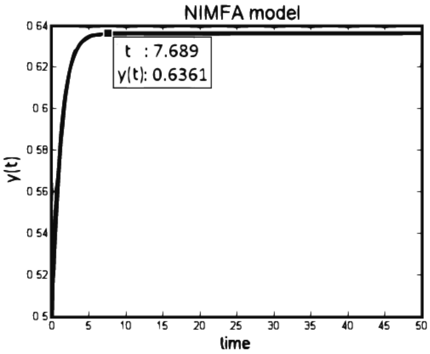

Fig. 6. Experiment 1. $\delta = 0.1$, $\sigma = 0.5$. Stationary state: $t = 7.689$, $y(t) = 0.6361$.

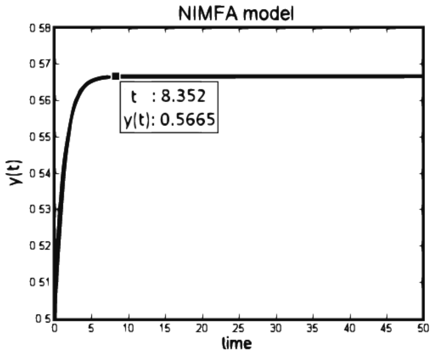

Fig. 7. Experiment 1. $\delta = 0.1$, $\sigma = 0.6$. Stationary state: $t = 8.352$, $y(t) = 0.5665$.

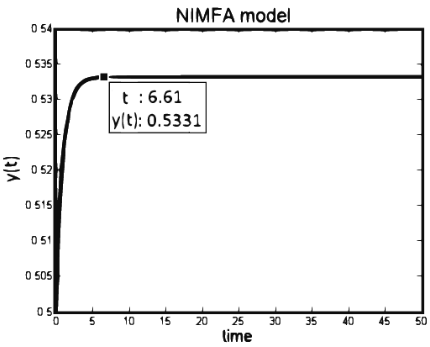

Fig. 8. Experiment 2. $\delta = 0.08$, $\sigma = 0.9$. Stationary state: $t = 6.61$, $y(t) = 0.5331$.

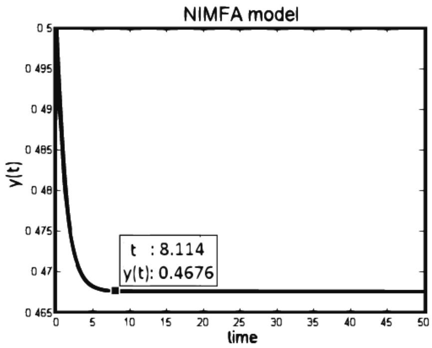

Fig. 9. Experiment 2. $\delta = 0.07$, $\sigma = 0.9$. Stationary state: $t = 8.114$, $y(t) = 0.4676$.

In the next series of experiments we fix the value of coefficients as $\delta = 0.1$, $\sigma = 0.8$ and estimate the effect of different initial number of informed taxpayers in population. In table 1 we collect information about number of links for each of taxpayer.

Table 1: The Matrix of links between agents

Node i	Number of links	Node i	Number of links	Node i	Number of links
1	16	11	8	21	15
2	15	12	19	22	11
3	15	13	16	23	12
4	17	14	16	24	15
5	22	15	13	25	10
6	14	16	11	26	7
7	14	17	15	27	13
8	15	18	15	28	10
9	15	19	14	29	15
10	13	20	10	30	17

In Figs. 10 — 13 we demonstrate behavior of the system which depends on different initial states. In this case we receive that stationary state of the system is reached if $y(t) = 0.4978$. We can also notice that a number of informed taxpayers at the initial time moment comes to steady state faster.

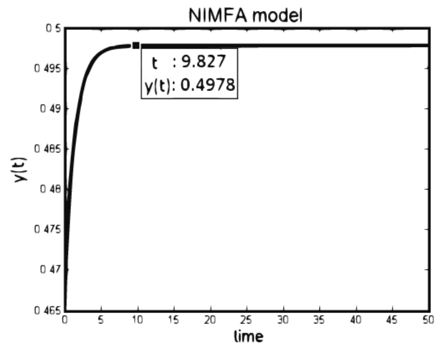

Fig. 10. Experiment 3. Number of informed nodes is 14 at $t = 0$.

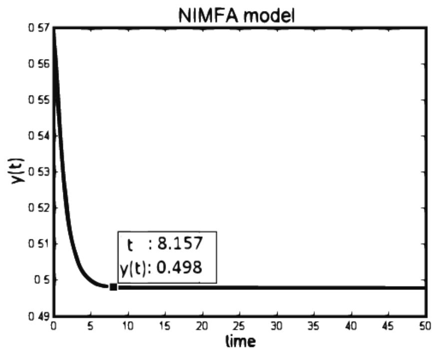

Fig. 11. Experiment 3. Number of informed nodes is 17 at $t = 0$.

Now, we can compute the value of Social Welfare Function SW, base on the known average probability of perceiving information about future tax audit

$$SW(y_f) = y_f \xi \overline{I} \tag{31}$$

where y_f is the value of the function $y(t)$ when the system reaches its steady state.

In Experiment 5 we change the initial number of informed taxpayers. In the next series of experiment we compare the behavior of function $y(t)$ which depends on different value of σ, δ and calculate SW function.

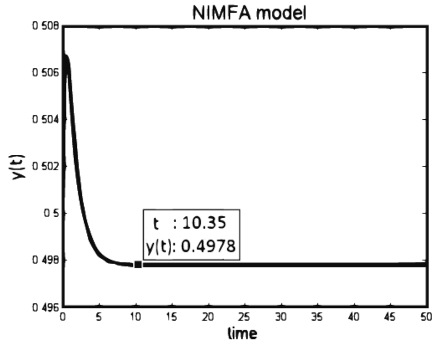

Fig. 12. Experiment 4. Number of informed nodes is 15 at $t = 0$.

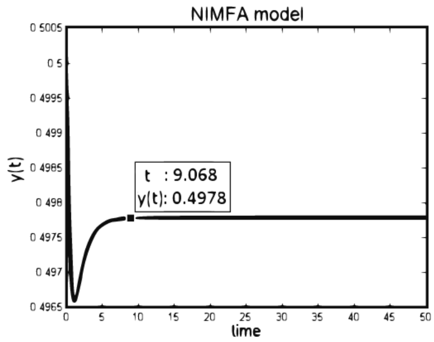

Fig. 13. Experiment 4. Number of informed nodes is 15 at $t = 0$.

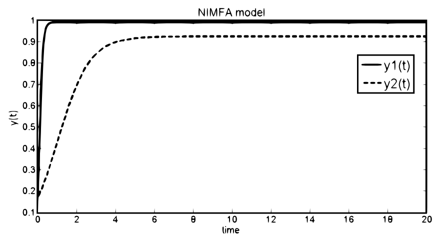

Fig. 14. Experiment 5. Number of informed nodes is 5 at $t = 0$. $y1(t) : \delta = 0.9$, $\sigma = 0.1$, stationary state $t = 1.499$, $y(t) = 0.9916$, $SW(y_f) = 5372.885$; $y2(t) : \delta = 0.1$, $\sigma = 0.1$, stationary state $t = 10.09$, $y(t) = 0.9247$, $SW(y_f) = 5010.395$.

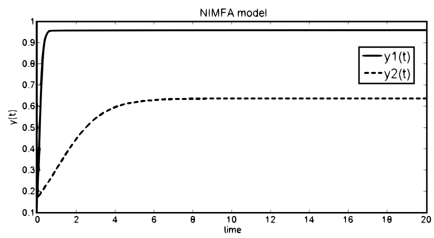

Fig. 15. Experiment 5. Number of informed nodes is 5 at $t = 0$. $y1(t) : \delta = 0.9$, $\sigma = 0.5$, stationary state $t = 1.28$, $y(t) = 0.958$, $SW(y_f) = 5190.827$; $y2(t) : \delta = 0.1$, $\sigma = 0.5$, stationary state $t = 11.95$, $y(t) = 0.6362$, $SW(y_f) = 3447.186$.

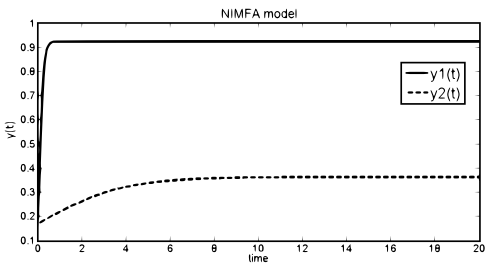

Fig. 16. Experiment 5. Number of informed nodes is 5 at $t = 0$. $y1(t) : \delta = 0.9$, $\sigma = 0.9$, stationary state $t = 1.121$, $y(t) = 0.9247$, $SW(y_f) = 5010.395$; $y2(t) : \delta = 0.1$, $\sigma = 0.9$, stationary state $t = 15.59$, $y(t) = 0.3625$, $SW(y_f) = 51964.17$.

From Figs. 14-15 we can notice that if value of δ growths then value of social welfare function SW also increases. However if at the same time value of σ is large

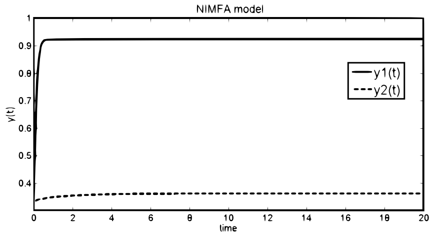

Fig. 17. Experiment 5. Number of informed nodes is 10 at $t = 0$. $y1(t) : \delta = 0.9$, $\sigma = 0.9$, stationary state $t = 1.286$, $y(t) = 0.9247$, $SW(y_f) = 5010.395$; $y2(t) : \delta = 0.1$, $\sigma = 0.9$, stationary state $t = 10.23$, $y(t) = 0.3625$, $SW(y_f) = 1964.17$.

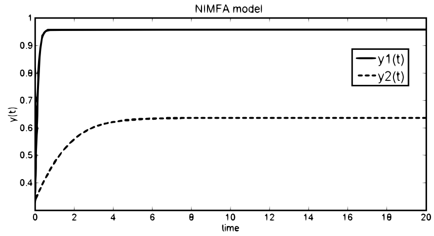

Fig. 18. Experiment 5. Number of informed nodes is 10 at $t = 0$. $y1(t) : \delta = 0.9$, $\sigma = 0.5$, stationary state $t = 1.548$, $y(t) = 0.958$, $SW(y_f) = 5190.827$; $y2(t) : \delta = 0.1$, $\sigma = 0.5$, stationary state $t = 10.54$, $y(t) = 0.6362$, $SW(y_f) = 3447.186$.

($\sigma = 0.9$) then value of SW function is less. So it means that taxpayers also take into anount importance of information.

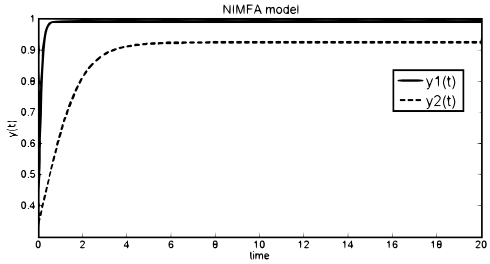

Fig. 19. Experiment 5.Number of informed nodes is 10 at $t = 0$. $y1(t) : \delta = 0.9$, $\sigma = 0.1$, stationary state $t = 1.662$, $y(t) = 0.9916$, $SW(y_f) = 5372.885$; $y2(t) : \delta = 0.1$, $\sigma = 0.1$, stationary state $t = 10.23$, $y(t) = 0.9247$, $SW(y_f) = 5010.395$.

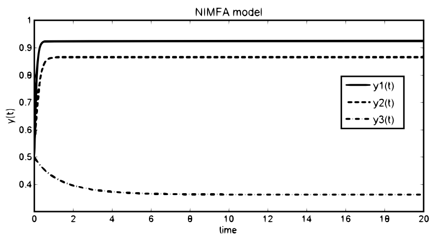

Fig. 20. Experiment 5. Number of informed nodes is 15 at $t = 0$. $y1(t) : \delta = 0.9$, $\sigma = 0.9$, stationary state $t = 0.8408$, $y(t) = 0.9247$, $SW(y_f) = 5010.395$; $y2(t) : \delta = 0.5$, $\sigma = 0.9$, stationary state $t = 1.795$, $y(t) = 0.8656$, $SW(y_f) = 4690.167$; $y3(t) : \delta = 0.1$, $\sigma = 0.9$, stationary state $t = 12.87$, $y(t) = 0.3626$, $SW(y_f) = 1964.17$.

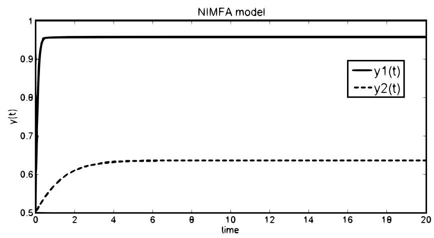

Fig. 21. Experiment 5. Number of informed nodes is 15 at $t = 0$. $y1(t) : \delta = 0.9$, $\sigma = 0.5$, stationary state $t = 0.8463$, $y(t) = 0.958$, $SW(y_f) = 5190.827$; $y2(t) : \delta = 0.1$, $\sigma = 0.5$, stationary state $t = 8.863$, $y(t) = 0.6362$, $SW(y_f) = 3447.186$.

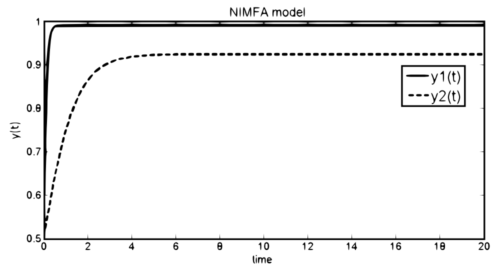

Fig. 22. Experiment 5. Number of informed nodes is 15 at $t = 0$. $y1(t) : \delta = 0.9$, $\sigma = 0.1$, stationary state $t = 1.009$, $y(t) = 0.9916$, $SW(y_f) = 5372.885$; $y2(t) : \delta = 0.1$, $\sigma = 0.1$, stationary state $t = 7.906$, $y(t) = 0.9247$, $SW(y_f) = 5010.395$.

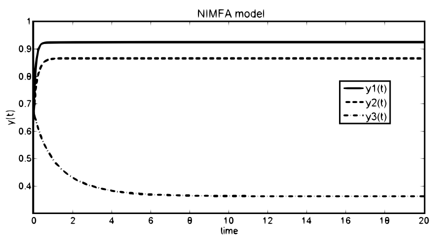

Fig. 23. Experiment 5. Number of informed nodes is 20 at $t = 0$. $y1(t) : \delta = 0.9$, $\sigma = 0.9$, stationary state $t = 0.9392$, $y(t) = 0.9247$, $SW(y_f) = 5010.395$; $y2(t) : \delta = 0.5$, $\sigma = 0.9$, stationary state $t = 2.238$, $y(t) = 0.8656$, $SW(y_f) = 4690.167$; $y3(t) : \delta = 0.1$, $\sigma = 0.9$, stationary state $t = 14.03$, $y(t) = 0.3626$, $SW(y_f) = 1964.17$.

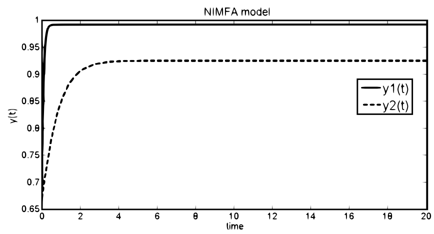

Fig. 24. Experiment 5. Number of informed nodes is 20 at $t = 0$. $y1(t) : \delta = 0.9$, $\sigma = 0.1$, stationary state $t = 0.8297$, $y(t) = 0.9916$, $SW(y_f) = 5372.885$;; $y2(t) : \delta = 0.1$, $\sigma = 0.1$, stationary state $t = 6.033$, $y(t) = 0.9247$, $SW(y_f) = 5010.395$.

Nevertheless from Figs. 14–25 we have observed that a number of initially informed taxpayers do not influence significantly the value of SW function. We can interpret that fact as the importance of information prevails over the initial number of informed agents in population. Perhaps, those experiments demonstrate one of the possible scenarios of minimization of coast of audit.

5.2. Evolutionary game on the graph

In this section we present series of experiments based on the evolutionary model of section 3.

We use the network G to define the structure of population and set following data for the network, process dynamics and matrix coefficients: size of population is $n = 30$, share of risk-averse taxpayers in population is $\nu_a = 17\%$ due to the psychological research (Niazashvili, 2007), $\frac{\lambda}{|M_i|} = 1$, tax and penalty rates are

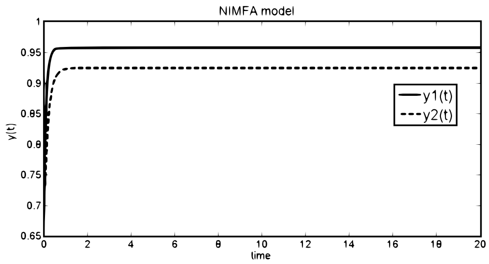

Fig. 25. Experiment 5. Number of informed nodes is 20 at $t = 0$. $y1(t)$: $\delta = 0.9$, $\sigma = 0.5$, stationary state $t = 1.13$, $y(t) = 0.958$, $SW(y_f) = 5190.827$; $y2(t)$: $\delta = 0.5$, $\sigma = 0.5$, stationary state $t = 1.691$, $y(t) = 0.9247$, $SW(y_f) = 5010.395$.

$\xi = 0.13$ due to the income tax rate in Russia (RF Tax Service, 2017), $\pi = 0.065$ (for bigger values of π, we obtain even bigger values of optimal audit probability p^*), the average taxpayers' income is $\overline{I} = 47908$ due to the value of average income (The web-site of the Russian Federation State Statistics Service, 2017), optimal value of the probability of audit is $p^* = 0.167$, unit cost of auditing is $c = 7455$ (minimum wage in St. Petersburg (The web-site of the Russian Federation State Statistics Service, 2017)), unit cost of information injection is $\widetilde{c} = 10\% = 745.5$, actual value of the probability of audit is $p = 0.1$.

Let the number of nodes in the population be $n = 30$. Then for the first model (which does not include the process of information dissemination) the value of total tax revenue (25) is $R_1 = 815617.78$. For the second model (which takes into account the dissemination of information) we compute the payoff functions of taxpayers: $U_{inf} = 41679.96$, $U_{ev} = 46973.79$.

Experiment 5. As a network G we use the random graph to define the structure of population and assume that probability of perception of information is $\alpha = 0.9$. The initial distribution (ν_{inf}, ν_{ev}) is $(19, 11)$ respectively.

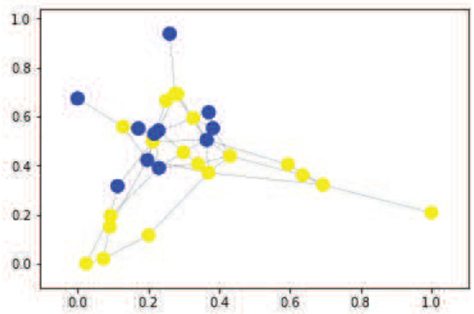

Fig. 26. Experiment 5: $\alpha = 0.9$. Initial state is $(\nu_{inf}, \nu_{ev}) = (19, 11)$. Strategy A corresponds to yellow dots on the graph and strategy B corresponds to blue dots.

From Fig.26 we obtain that agents who use strategy A (to perceive information) switch on strategy B (not to perceive information). This process starts if strategy B gives better payoff against strategy A, this fact occurs only if the most neighbors in i-th agent's environment are non-informed (see **Rule. Case 1**). In this case the stationary state of population is $(11, 19)$. The slight difference between initial and stationary state is caused the fact that if in initial state agents with strategy A have many connections with B-strategy agents, then they replace their strategy else they keep own behavior.

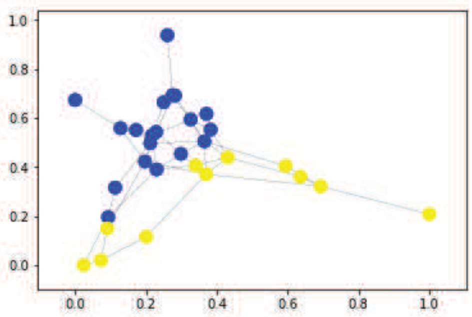

Fig. 27. Experiment 5: $\alpha = 0.9$. Final state is $(\nu_{inf}, \nu_{ev}) = (11, 19)$.

By using the initial distribution of Informed and Uniformed taxplayers (ν_{inf}, ν_{ev}) we compute the revenue of tax authority (26) for the second model: $R_2 = 2140450.62$. In this experiment we can see if the information which was injected at the initial time moment, significantly increases R, despite the share of informed taxpayers at the final moment has decreased (from 19 to 11).

In this case the second model is more effective, that is the dissemination of information is profitable.

Experiment 6. In the experiment 6 at random graph we use the next initial proportion of agents $(18, 12)$ and keep the same probability of perception of information $\alpha = 0.9$.

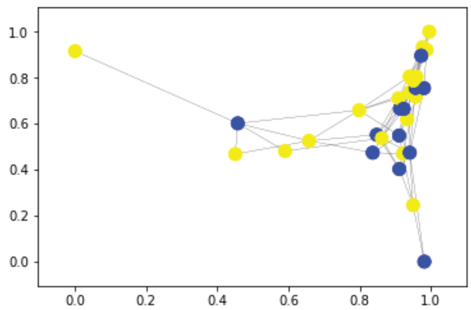

Fig. 28. Experiment 6: $\alpha = 0.9$. Initial state is $(\nu_{inf}, \nu_{ev}) = (18, 12)$.

In **Experiment 6** according to **Rule. Case 2** we show that total population aspires to stationary state $(0, 30)$ from the initial state $(18, 12)$, where the first and the second numbers correspond to agents with strategy A and B respectively.

This replacement occurs as well as at each stage of evolutionary process agents with strategy A have many connections with B-strategy agents who receive better payoff.

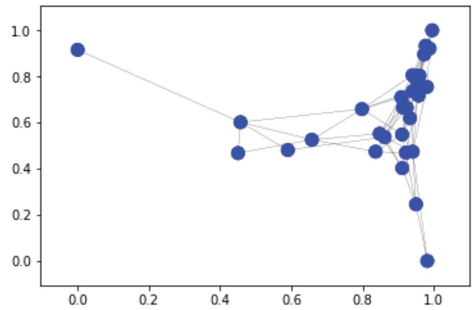

Fig. 29. Experiment 6: $\alpha = 0.9$. Final state is $(\nu_{inf}, \nu_{ev}) = (0, 30)$. Total tax revenue $R_2 = 415850.40$.

For this experiment the dissemination of information is unprofitable: R_2 decreases insufficiently because there is not enough agents who received information at the initial time moment, hence at the final moment there is no people in the system who will use the information and pay. Thus the aggregated income of the system is $R_2 = 415850.40$ which is two times less than R_1.

Experiment 7. This example demonstrates behavior of population taxpayers structured by random graph with low probability of perception information $\alpha = 0.1$.

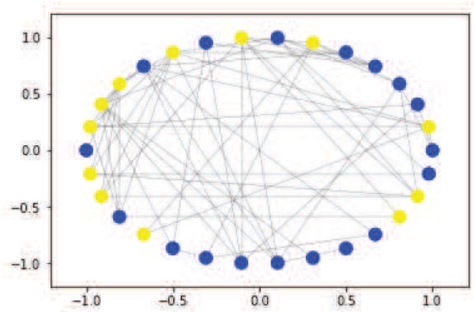

Fig. 30. Experiment 7: $\alpha = 0.1$. Initial state is $(\nu_{inf}, \nu_{ev}) = (12, 18)$.

In **Experiment 7** we receive that our population contains a mixture of A- and B-strategies agents, here stationary state is $(7, 23)$. Agents use strategy A who change it to B if they surround by B-strategy neighbors and else their hold own strategy if they have many connections with A-strategy taxpayers. Total tax revenue is $R_2 = 1661745.54$.

This experiment shows that the dissemination of information helps to increase the revenue of the tax authority, despite the fact that the share of informed agents has declined in comparison to the initial value. In this example we have significant

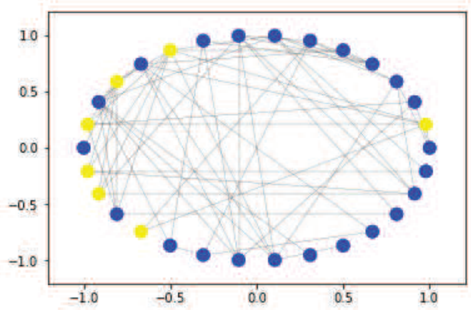

Fig. 31. Experiment 7: $\alpha = 0.1$. Final state is $(\nu_{inf}, \nu_{ev}) = (7, 23)$. Total tax revenue $R_2 = 1661745.54$.

number of agents, who received information at the initial time moment, therefore we obtain that the second model is more attractive, that is $R_2 > R_1$.

Now let's consider the series of experiments in which the number of nodes in the network (the size of population) is $n = 25$. Then for the first model the value of total tax revenue (25) is $R_1 = 679681.49$.

Experiment 8. – **Experiment 9.** show that if the probability of perception information is $\alpha = 0.1$ and the structure of graph is grid then for both variants of initial distribution of Uninformed and Informed taxpayer systems aspires to ¡¡pure¿¿ stationary state $(\nu_{inf}, \nu_{ev}) = (0, 25)$.

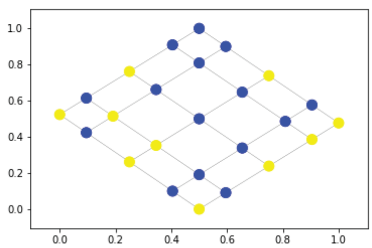

Fig. 32. Experiment 8. Grid. Probability of perception information $\alpha = 0.1$. $n = 25$. Initial state is $(\nu_{inf}, \nu_{ev}) = (10, 15)$.

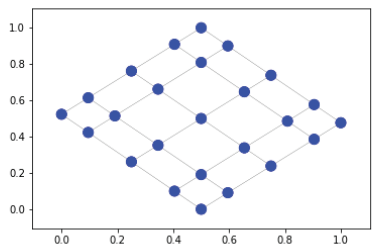

Fig. 33. Experiment 8. Grid. Probability of perception information $\alpha = 0.1$. $n = 25$. Stationary state is $(\nu_{inf}, \nu_{ev}) = (0, 25)$. Total tax revenue $R_2 = 454639.50$.

In both experiments we obtain that spreading of information is unprofitable due to the structure of the network.

Experiment 10. shows that for value of probability $\alpha = 0.1$ and the grid structure of graph during the time system aspires to ¡¡mixed¿¿ stationary state. Total tax revenue is $R_2 = 2930844.84$.

In **Experiment 11.** the value of perception information is $\alpha = 0.9$ and the structure of graph is grid, we have that for initial distribution of Uninformed and Informed taxpayers system aspires to ¡¡mixed¿¿ stationary state and location of informed taxpayers at initial time moment have a strong influence on the final state. In this case the aggregated revenue of the system is $R_2 = 978074.58$. If

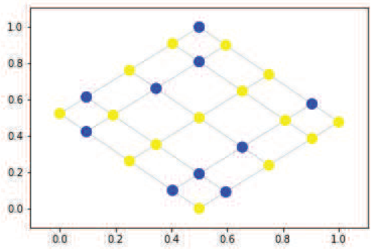

Fig. 34. Experiment 9. Grid.Probability of perception information $\alpha = 0.1$. $n = 25$. Initial state $(\nu_{inf}, \nu_{ev}) = (15, 10)$.

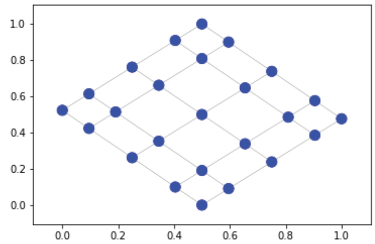

Fig. 35. Experiment 9. Grid. Probability of perception information $\alpha = 0.1$. $n = 25$. Stationary state is $(\nu_{inf}, \nu_{ev}) = (0, 25)$. Total tax revenue $R_2 = 342814.50$.

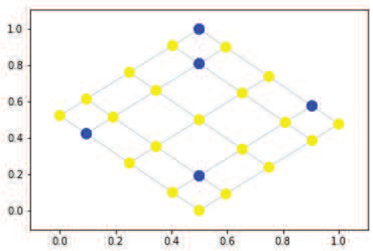

Fig. 36. Experiment 10. Grid. Probability of perception information $\alpha = 0.1$. $n = 25$. Initial state is $(\nu_{inf}, \nu_{ev}) = (20, 5)$.

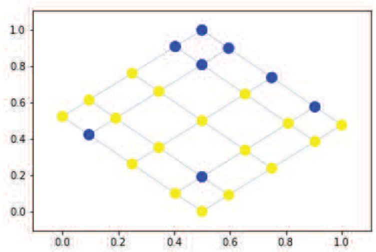

Fig. 37. Experiment 10. Grid. Probability of perception information $\alpha = 0.1$. $n = 25$. Stationary state is $(\nu_{inf}, \nu_{ev}) = (17, 8)$. Total tax revenue $R_2 = 2930844.84$.

the probability of perceiving information is high the number of agents who don't perceive the information is larger then in the previous case. But for the network with this structure R_2 is still bigger then R_1.

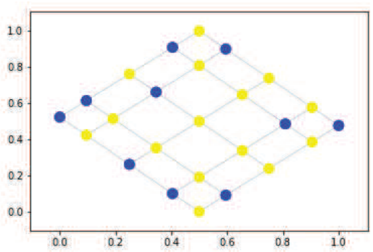

Fig. 38. Experiment 11. Grid. Probability of perception information $\alpha = 0.9$. $n = 25$. Initial state is $(\nu_{inf}, \nu_{ev}) = (15, 10)$.

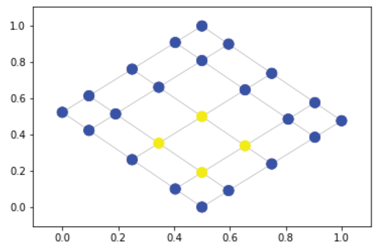

Fig. 39. Experiment 11. Grid. Probability of perception information $\alpha = 0.9$. $n = 25$. Stationary state is $(\nu_{inf}, \nu_{ev}) = (4, 21)$. Total tax revenue $R_2 = 978074.58$.

In **Experiment 12.** we also obtain that stationary state is ¡¡mixed¿¿ and very close to the initial distribution of Informed and Uninformed taxpayers.

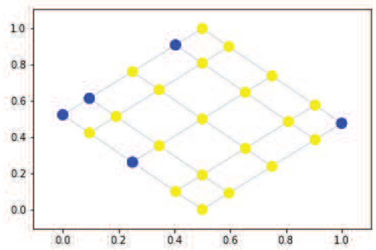

Fig. 40. Experiment 12. Grid. Probability of perception information $\alpha = 0.9$. $n = 25$. Initial state is $(\nu_{inf}, \nu_{ev}) = (20, 5)$.

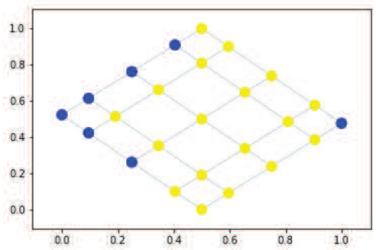

Fig. 41. Experiment 12. Grid. Probability of perception information $\alpha = 0.9$. $n = 25$. Stationary state is $(\nu_{inf}, \nu_{ev}) = (7, 18)$.

This example demonstrates that if the number of the agents who perceived information is large in a steady state, then the revenue of the system will increase significantly. We have the distribution $(20, 5)$ at the initial time moment and $(18, 7)$ in the steady state. That is if at the initial time moment the number of informed is equal or bigger then 15 we can see weak decrease of number of susceptible to information, but R_2 is almost 5 times bigger then R_1.

In **Experiment 13.** and **Experiment 14.** for random graph and probability $\alpha = 0.1$ as in previous cases we obtain that stationary state depends on the initial distribution of taxpayers and only if number of initially informed taxpayers prevails in population we have a ¡¡mixed¿¿ stationary state.

From experiments we obtain that of the number of susceptible to information taxplayers is zero then the total tax revenue of the system does not increase.

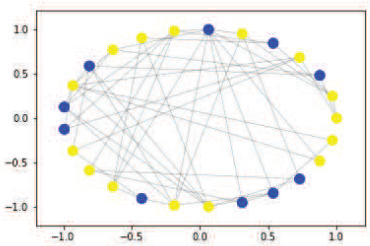

Fig. 42. Experiment 13. Random graph. Probability of perception information $\alpha = 0.1$. $n = 25$. Initial state is $(\nu_{inf}, \nu_{ev}) = (15, 10)$.

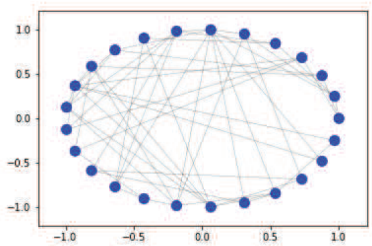

Fig. 43. Experiment 13. Random graph. Probability of perception information is $\alpha = 0.1$. $n = 25$. Stationary state $(\nu_{inf}, \nu_{ev}) = (0, 25)$. Total tax revenue $R_2 = 342814.5$.

In **Experiment 15.** and **Experiment 16.** for random graph and probability $\alpha = 0.9$ we have that stationary state is ¡¡mixed¿¿ for different initial distribution of Informed and Uninformed taxpayers. For the **Experiment 15.** $R_2 = 931084.56$, for the **Experiment 16.** $R_2 = 2566224.78$.

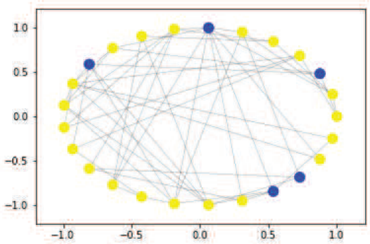

Fig. 44. Experiment 14. Random graph. Probability of perception information is $\alpha = 0.1$. $n = 25$. Initial state $(\nu_{inf}, \nu_{ev}) = (20, 5)$.

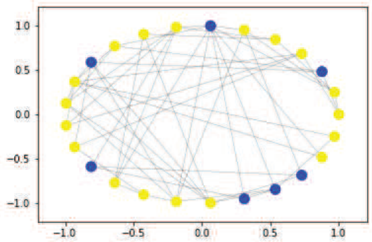

Fig. 45. Experiment 14. Random graph. Probability of perception information is $\alpha = 0.1$. $n = 25$. Stationary state $(\nu_{inf}, \nu_{ev}) = (18, 7)$. Total tax revenue $R_2 = 3089659.86$.

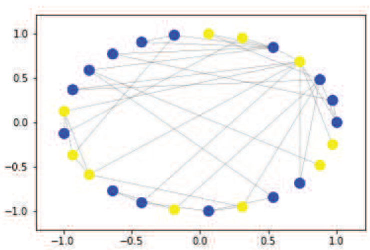

Fig. 46. Experiment 15. Random graph. Probability of perception information is $\alpha = 0.9$. $n = 25$. Initial state $(\nu_{inf}, \nu_{ev}) = (10, 15)$.

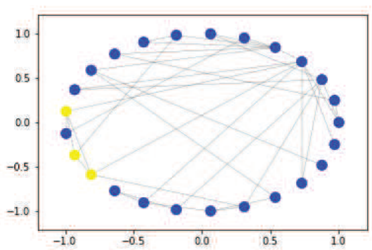

Fig. 47. Experiment 15. Random graph. Probability of perception information is $\alpha = 0.9$. $n = 25$. Stationary state $(\nu_{inf}, \nu_{ev}) = (3, 22)$. Total tax revenue R_2= 931084.56.

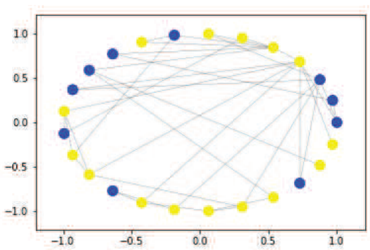

Fig. 48. Experiment 16. Random graph. Probability of perception information is $\alpha = 0.9$. $n = 25$. Initial state $(\nu_{inf}, \nu_{ev}) = (15, 10)$.

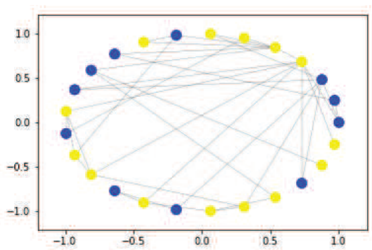

Fig. 49. Experiment 16. Random graph. Probability of perception information is $\alpha = 0.9$. $n = 25$. Stationary state $(\nu_{inf}, \nu_{ev}) = (14, 11)$. Total tax revenue R_2= 2566224.78.

Experiment 17. shows the case where final stationary state coincides with the initial distribution of taxpayers. In this case $R_2 = 3407289.90$.

Also from the series of experiments 6-17 we have that the expert influence upon the system demonstrate the next parameters: the probability of perception information and the initial distribution of Informed and Uninformed taxpayers. Those values increase total tax revenue significantly. As well as the structure of initial

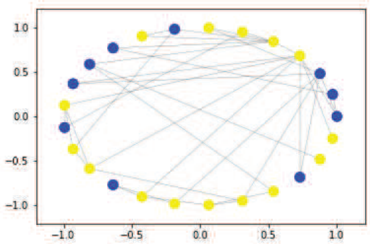

Fig. 50. Experiment 17.Random graph. Probability of perception information is $\alpha = 0.9$. $n = 25$. Initial state $(\nu_{inf}, \nu_{ev}) = (20, 5)$.

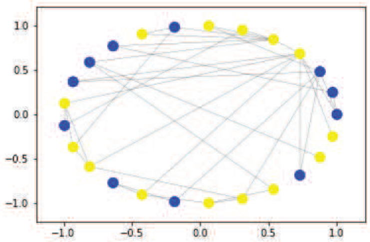

Fig. 51. Experiment 17. Random graph. Probability of perception information is $\alpha = 0.9$. $n = 25$. Stationary state $(\nu_{inf}, \nu_{ev}) = (20, 5)$. Total tax revenue R_2= 3407289.90.

distribution and initial location of Informed taxpayers impact on revenue of the system.

Thus from the simulations presented above it is obvious that the total revenue of the system R_2 depends on the structure of the network. In both cases if the probability α is high ($\alpha = 0.9$) or low ($\alpha = 0, 1$), we obtain that if there is enough number of the susceptible to the information in the network when the system comes to its steady state and the total tax revenue is larger then R_1 which computed for the model, which does not include the process of information spreading.

6. Conclusion

In this paper we have presented two approaches which combine inter-agent communications and the process of propagation of information. First, we have formulated the game-theoretical model of interaction between taxpayers and tax-authority which include the information spreading based on structured SIS model. The second approach uses evolutionary game on the network to illustrate the idea of using information in fiscal system In both cases we present mathematical formulations, analysis of the behavior of the system and present series of experiments.

We investigate the impact of information received from the tax authority on the decisions of taxpayers. We obtained that the final distribution of taxpayers who pay taxes depends on the network structure, risk-status and received information. We can see that propagation information about possible tax audit gives a positive effect for the total revenue of fiscal system and increases total amount of taxpayers who prefer to pay taxes honestly.

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