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Games with Incomplete Information on the Both Sides and with Public Signal on the State of the Game*

Misha Gavrilovich and Victoria Kreps

National Research University Higher School of Economics,St.Petersburg
St.Petersburg Institute for Economics and Mathematics, RAS

Abstract Supposing that Player 1's computational power is higher than that of Player 2, we give three examples of different kinds of public signal about the state of a two-person zero-sum game with symmetric incomplete information on both sides (both players do not know the state of the game) where Player 1 due to his computational power learns the state of the game meanwhile it is impossible for Player 2. That is, the game with incomplete information on both sides becomes a game with incomplete information on the side of Player 2. Thus we demonstrate that information about the state of a game may appear not only due to a private signal but as a result of a public signal and asymmetric computational resources of players.

Keywords: zero-sum game; incomplete information; asymmetry; finite automata.

1. Introduction

The literature on repeated games with incomplete information usually assumes that players have unlimited computational capacity. Since in practice this assumption does not hold, it is important to study whether and how its absence affects the predictions of the theory.

We consider zero-sum games of players with limited computational capacity, and discuss how these limitations may affect the information structure of the game. We show how difference in computation resources may give rise to informational asymmetry in an otherwise symmetric game.

Our model of limited computation resources is similar to the model of Abraham Neyman (Neyman, 1997; Neyman, 1998). The strategies available to players are limited to finite automata of different sizes.

Starting with the seminal papers by Rubinstein (Rubinstein, 1986) and by Abreu, Rubinstein (Abreu and Rubinstein, 1988) there appeared a number of papers on repeated games where strategies of players are implemented by finite automata. These papers investigate properties of the set of equilibrium payoffs under this assumption. For an abundant bibliography on the subject see Hernández, Solan (Hernández and Solan, 2016).

We are interested in another aspect. Supposing that Player 1's computational power is higher than that of Player 2, we give three examples of different kinds of public signal about the state of a two-person zero-sum game with symmetric incomplete information on both sides (both players do not know the state of game but know its probability) where Player 1 due to his computational power learns the state of the game meanwhile it would be impossible for Player 2.

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In our examples each player chooses a finite automaton. The both chosen automata are given a signal depending on the state of the game. Intuitively it is clear that higher computational power of Player 1 may let him "know" the state of the game meanwhile it would be impossible for Player 2. That is, if Player 1 "computes" the state of the game with help of his computational resources and Player 2 does not, the game with incomplete information on both sides becomes a game with incomplete information on the side of Player 2.

Thus we demonstrate that knowledge of the state of a game may arise not only due to a private information (a private signal) but as a result of a public signal and computational resources of players.

In the first two examples both chosen automata are given a signal consisting of a string of 1's whose length depends on the state of the game. In the first example a signal is deterministic. In the second example a signal is random.

In the third example both chosen automata are given a random signal consisting of a string of 0's and 1's. The state of the game is determined by the value of the bit (0 or 1) of a certain fixed distance from the end of the string.

In the first example where a signal is deterministic Player 2 gets no new information on the state of the game. In the second and third examples of random signals Player 2 reestimates the probability of the state. Hence players are faced with a game with incomplete information on the side of Player 2 where the posterior probability of the state known to both players is more accurate than the prior one.

2. Games under consideration

As our approach only deals with revealing the information about the state of the game before the game starts, the number of stages is irrelevant. So we are not concerned with repetition of a game and do not go beyond analysis of games which are played once.

We base our consideration on the classical setting of matrix games with incomplete information on one side and with incomplete information on both sides (see (Harsanyi, 1967-68; Aumann and Maschler, 1995)).

I. The case of symmetric incomplete information on both sides.

Let $\mathcal{A}(p)$ denote the matrix game with incomplete information on both sides given by two square payoff matrices A_1 and A_2. Before the game starts a chance move determines the "state of nature" $k \in K = \{1, 2\}$ and therefore the payoff matrix A_k: with probability p the matrix A_1 is played and with probability $1-p$ the matrix A_2 is played. Both players know the probability p and do not know the result of the chance move.

As a matter of fact in such a game with incomplete information on both sides players are faced with the matrix game given by payoff matrix $A(p) = pA_1 + (1-p)A_2$. We will denote the matrix game given by payoff matrix B by the same symbol B. The value $Val\mathcal{A}(p)$ is a continuous function on p over the interval $[0, 1]$, where $Val\mathcal{A}(0) = ValA_2$ and $Val\mathcal{A}(1) = ValA_1$ as equity of probability p to 0 or 1 means that players know what game is played: if $p = 0$ then it is A_2 and if $p = 1$ then it is A_1.

Note that the absence of information on a state of the game on the both sides may be profitable for one player and not profitable for another one. Consider a simple example: 2×2-matrices with $ValA_1 = ValA_2 = 0$

$$A_1 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \quad \text{and so} \quad A(p) = \begin{bmatrix} 0 & p \\ 1-p & 0 \end{bmatrix}.$$

It is easy to calculate that $Val\mathcal{A}(p) = p(1-p) > 0$. Hence in the game $Val\mathcal{A}(p)$ Player 1 can guarantee the positive payoff while his guaranteed payoff is only zero if both players know what game is played. If in matrices A_1 and A_2 the elements equal to 1 are replaced by elements equal to -1, then $Val\mathcal{A}(p) = -p(1-p)$ is negative. In this case it is Player 2 who gets a profit from the absence of information on the both sides.

II. The case of incomplete information on the side of Player 2.

Now consider the same game as in case I but Player 1 is informed on the result of the chance move but Player 2 is not. So Player 1 knows exactly what game is played. Player 2 has no such information. Player 2 knows that Player 1 is the informed player.

Let $\mathcal{A}^{asy}(p)$ denote this game with incomplete information on the side of Player 2. Naturally in this game Player 1 could guarantee himself not less than in the game $\mathcal{A}(p)$. In any case he may play as though he "forgives" the obtained information. But usually it is profitable for him to use his knowledge of the state of the game. Demonstrate it for the example of matrices A_1 and A_2 given in the case I. As Player 1 knows exactly what game is played he chooses the first row if it is A_1 and the second row if it is A_2. The best reply of Player 2 who has no information on the state is to choose the first column with probability p and the second column with probability $1-p$. Thus Player 1' guaranteed payoff ($1-p$ if A_1 is played or p if A_2 is played) is greater than his guaranteed payoff $p(1-p)$ in the game $\mathcal{A}(p)$ with lack of information on the both sides.

It is known (Aumann and Maschler, 1995) that the value $Val\mathcal{A}^{asy}(p)$ is a continuous piecewise linear concave function over $[0,1]$ and as in the previous case $Val\mathcal{A}^{asy}(0) = ValA_2$ and $Val\mathcal{A}^{asy}(1) = ValA_1$.

In this paper we consider a case in certain sense intermediate between I and II: both players do not know the state of the game but there is some additional information on this state besides its probability p.

For considerably complicated cases Gensbittel (for infinite action spaces)(Gensbittel, 2016) and Gensbittel, Oliu-Barton, Venel (for an evolution of states) (Gensbittel et al., 2014) deal with another intermediate informational structure: the informed player does not observe the state variable directly but receives a stochastic signal whose distribution depends on the state variable. The authors generalize several classical asymptotic results concerning zero-sum repeated games with incomplete information on one side.

III. The case of symmetric information, public signal and automata. Let $\mathcal{A}_f^{m,n}(p)$ denote a modification of game $\mathcal{A}(p)$ with incomplete information on both sides (both players do not know the matrix chosen). Here f is an injective function of the state of the game, $f(k)$, $k = 1, 2$. The codomain of the function f is the set of binary strings (i.e. strings consisting of symbols 0's and 1's) of arbitrary length. Function f is known to both players.

Each player chooses a finite automaton. Player 1 chooses an arbitrary automaton (Automaton 1) of size at most m, and Player 2 chooses an arbitrary automaton (Automaton 2) of size bounded by n where $m > n$. Note that the number of automata of size at most m is exponential in m, of order $m^{2m}2^{m+2m} = 2^{2m \log m + 3m}$. This rough estimate is obtained as follows. There are exactly two edges leaving each of m vertices; there are m possibilities for an edge leaving a given vertex. This gives m^{2m} possibilities for the choice of the edges of the automaton. There are m vertices and $2m$ edges, each labelled by either 0 or 1. This gives 2^{m+2m} possibilities to choose the labels of the edges.

Thus the number of possibilities for Player 1 to choose the automaton is of order $2^{2m\, log\, m+3m}$ while for Player 2 this number is $2^{2n\, log\, n+3n}$. This implies that Player 1 has exponentially more options than Player 2 if $m >> n$. Both players know the size of automaton of the opponent.

A meaningful strategy of choosing an automaton is as follows. A player runs each automaton of appropriate size on input $f(k)$, $k = 1, 2$ and chooses the one whose output is k if such exists. In our examples such an automaton exists for Player 1 but not for Player 2.

After the players have chosen their automata, the game sends the public signal $f(k)$, $k = 1, 2$. This signal is received by the chosen automata which compute their responses.

The output of Automaton i is interpreted by Player i as an indication towards the state of the game $\mathcal{A}(p)$. Payoffs of the players are determined accordingly.

In sections 4-5 we give examples of functions f and numbers m, n depending on f such that the size m of Player 1's automaton allows him to "compute" the state of the game but the size n of Player 2's automaton is not sufficient for this purpose. Hence Player 1 learns the state of the game and the game $\mathcal{A}_f^{m,n}(p)$ is turning to the game with incomplete information on the side of Player 2.

3. Automaton

For theory of finite automata see textbooks (Sakarovitch, 2009; Kobrinskii and Trakhtenbrot, 1965); here we give a quick overview and introduce the notation we use.

An automaton is represented by a connected labelled directed graph with a finite set of vertices

- One vertex is distinguished as a *initial* vertex v_0.
- Each edge of the graph is labelled by either 0 or by 1.
- Each vertex is labelled by either 0 or by 1.
- There are exactly two edges leaving each vertex,
  one labelled 0 and one labelled 1.
- There is no restriction how many edges enter a vertex.

In our context a label on a vertex represents the output of the automaton which is interpreted as a state of the game. As input the automaton receives a signal which is a binary string; labels on the edges correspond to the symbols of the binary string. Next we explain how an automaton computes.

The computation of the automata proceeds as follows: the automaton receives a string $s_1 \dots s_l$ of 0's and 1's from the game.

Intuitively, we think that the automaton reads symbols one by one, starts at the initial vertex v_0 and upon reading the symbol s_1, moves to the vertex v_1 by the unique edge labelled s_1 coming out of v_0. Then upon reading the symbol s_2, moves to the vertex v_2 by the unique edge labelled s_2 coming out of v_1 and so on... Thus there is a unique path of edges starting from the initial vertex v_0

$$v_0 \xrightarrow{s_1} v_1 \xrightarrow{s_2} v_2 \xrightarrow{s_3} \dots \xrightarrow{s_{l-1}} v_{l-1} \xrightarrow{s_l} v_l$$

such that the path from the vertex v_{i-1} to the vertex v_i is labelled by s_i, $1 \le i \le l$.

The output of the automaton is the label of the end vertex v_l of this path.

4. Results. Degenerate cases

In the first two examples the automaton is degenerate because the signal consists only of 1's and thus only edges labelled 1 matter. In the first case the signal is deterministic, in the second case it is random.

4.1. Example 1: degenerate deterministic case

Consider function $f(k) = 1^{kn!}$, $k = 1, 2$; here $1^{kn!}$ denotes the string of $1 \dots 1$ consisting of 1 repeated $kn!$ times.

Theorem 1. *For $m > n$ such that m does not divide $n!$, Player 1 deciphers the signal while Player 2 does not.*

Proof. The proof is based on the following observation. Consider computation of an automaton G with n vertices on a string of 1's, i.e., equivalently, the path $e_1 \dots e_l$ of edges in G labelled by 1. If $l > n$, then the path necessarily has a cycle, i.e. for some n' it holds $e_i = e_{i+n'}$ for all $i > n$.

Then the output of the automaton G with n vertices is the same for two strings 1^i and $1^{i+rn'}$, where r is an integer positive number. Thus, the output is the same for any two strings whose lengths are more than n and have the same reminders modulo n'.

As n' divides $n!$ for each $1 \le n' \le n$, we get that for any automaton of size at most n, the output is the same for the strings $f(1) = 1^{n!}$ and $f(2) = 1^{2n!}$.

This proves that the automaton of Player 2 can not distinguish the two strings $f(1) = 1^{n!}$ and $f(2) = 1^{2n!}$.

On the other hand, as m does not divide $n!$ by the hypothesis of the theorem, it is easy to construct an automaton of size m which distinguishes these two strings, as follows.

Namely, consider the automaton such that its edges labelled by 1 form a single cycle of size m. As m does not divide $n!$, the reminders modulo m of the lengths of $f(1)$ and $f(2)$ are different, the end-vertices of the paths corresponding to the two signals are different.

Now label them with different appropriate actions. Hence, this automaton correctly distinguishes the states of the game and therefore it is optimal for Player 1 to choose this automaton to be able to use the information about the state of the game.

Thus Player 1 knows the state of the game while Player 2 does not. $\square$

Corollary . Under hypothesis of Theorem 1 in the game $\mathcal{A}_f^{m,n}(p)$ the players are faced with the game $\mathcal{A}^{asy}(p)$ with incomplete information on the side of Player 2. Thus it may be said that the game $\mathcal{A}_f^{m,n}(p)$ is equivalent to the game $\mathcal{A}^{asy}(p)$.

4.2. Example 2: degenerate random case

Definition. We say that the game $\mathcal{A}_f^{m,n}(p)$ with incomplete information on both sides is ε-*equivalent* to the game $\mathcal{A}^{asy}(p)$ with incomplete information on the side of Player 2 if in the game $\mathcal{A}_f^{m,n}(p)$ the players are faced with the game $\mathcal{A}^{asy}(p')$ where $|p - p'| < \varepsilon$.

Now we consider the game $\mathcal{A}_f^{m,n}(p)$ where the function $f(k)$, $k \in K$ is random. For simplicity assume that m is prime.

The signal $f(1)$ is a string of 1's of a random length l, where l takes value uniformly among $m, 2m, \ldots, (m-1)m, m^2$.

The signal $f(2)$ is a string of 1's of a random length l, where l takes value uniformly among $m+1, 2m+1, \ldots, (m-1)m+1, m^2+1$.

Theorem 2. *For $m > 100n$ the game $\mathcal{A}_f^{m,n}(p)$ is ε-equivalent to the game $\mathcal{A}^{asy}(p)$ with $\varepsilon = 0.05$.*

Remark. If we replace the condition $m > 100n$ by $m > Cn$ for an integer positive constant C, then $\varepsilon = 5/C$.

Proof. By the definition of the signal f we get that the reminders modulo m of the lengths of $f(1)$ and $f(2)$ are different. As in the previous proof, Player 1 may pick an automaton which runs through a cycle of length exactly m and hence distinguishes the $f(1)$ and $f(2)$. Hence Player 1 learns what game is played.

We will show that, regardless of prior probability p, Player 2 is able to correctly guess the state of the game with probability at most $1/2 + 0.01$. Indeed, as before, any automaton that Player 2 is allowed to choose has the property that there is $n' < n$ such that its output depends only on the length of the input signal modulo n'. Look at the two sequences $m, \ldots, m^2$ and $m+1, \ldots m^2+1$ modulo n'. By assumption m is prime, hence n' and m are coprime. Hence both sequences consist of a cycle of reminders n' repeated several times, one last cycle may be not complete. Therefore each reminder modulo n' is repeated either $[m/n']$ times or $[m/n'] + 1$ times.

Now assume that $p \geq 1/2$. The case $p \leq \frac{1}{2}$ is analogous.

Let us see whether Player 2 can reestimate the prior probability p based on his knowledge of the reminder modulo n' of the length of the input signal. If this reminder appears the same number of times in both sequences, i.e. the probability that if occurs is the same for both $f(1)$ and $f(2)$, then Player 2 can not reestimate the prior probability.

If the reminder occurs $[m/n'] + 1$ times in one sequence and $[m/n']$ times in the other one, then the probabilities that it occurs for $f(1)$, respectively $f(2)$, differ at most by factor $([m/n'] + 1)/[m/n'] < 1.01$.

Hence, the probabilities α_1, respectively α_2, that the automaton of Player 2 outputted 1 for $f(1)$, respectively 2 for $f(2)$, differ at most by factor $([m/n'] +$

$1)/[m/n'] < 1.01$, as probability α_1, respectively α_2, is the sum of the probabilities of the signals that make the automaton outputs 1, respectively 2.

Thus

$$\alpha_1 \leq 1.01\alpha_2 \quad \text{and} \quad \alpha_2 \leq 1.01\alpha_1.$$

Now use Bayes formula to reestimate the prior probability p of state 1

$$p' = \frac{\alpha_1 p}{\alpha_1 p + \alpha_2(1-p)} = p\frac{\alpha_1}{\alpha_1 + (\alpha_2 - \alpha_1)(1-p)} \leq$$

$$\leq p\frac{\alpha_1}{\alpha_1 - 0.01\alpha_1(1-p)} < 1.05p.$$

So it holds $p' < 1.05p$.

Hence in the game $\mathcal{A}_f^{m,n}(p)$ players are faced with the game $\mathcal{A}^{asy}(p')$ where $|p-p'| < \epsilon$. Thus the game $\mathcal{A}_f^{m,n}(p)$ is 0.05-equivalent to the game $\mathcal{A}^{asy}(p)$. $\square$

5. Result. Non-degenerate random case

In the games above we considered signals consisting of only one symbol repeated many times. Much shorter signals suffice for the same effect (namely, that Player 1 can differentiate between the states but Player 2 can not) if one considers signals using at least two different symbols.

Here we consider the game $\mathcal{A}_f^{m,n}(p)$ where a random signal $f(k), k \in K$ consists of a binary string of both 0's and 1's.

For simplicity assume that $m = 2^L$ for some integer L.

To define $f(1)$, consider the probability distribution over the set of binary strings of length $L \leq l < 2L$ such that the probability of a string $s_1 \dots s_l$ is 0 if $s_{l-L} = 1$, and is $2^{-l}/L$ if $s_{l-L} = 0$. For this distribution the probability of a string having size l is $1/L$. The signal $f(1)$ takes value according to this distribution.

Similarly, to define $f(2)$, consider the probability distribution over the set of binary strings of length $L \leq l < 2L$ such that the probability of a string $s_1 \dots s_l$ is 0 if $s_{l-L} = 0$, and is $2^{-l}/L$ if $s_{l-L} = 1$. As before, for this distribution the probability of a string having size l is $1/L$. The signal $f(2)$ takes value according to this distribution.

Note that for the uniform distribution on binary strings of length l such that $L \leq l < 2L$ the probability that a string has length l is equal to 2^{L-l}.

Observe that the signal described above is significantly shorter than in Theorem 2, namely signals of length $l < 2L = 2\log_2 m$ are shorter than signals of length m.

Theorem 3. *Fix an $0 < \varepsilon \leq 0.1$ and an integer L. Let $m > 2^{2\varepsilon L}L$ and $n < exp(2\varepsilon^2 L)$. The game $\mathcal{A}_f^{m,n}(p)$, is ε-equivalent to the game $\mathcal{A}^{asy}(p)$.*

Remark. The hypothesis of Theorem 3 assumes that m is substantially larger that n. For example, one may take $L = 1000$, $\varepsilon = 0.1$ and $n = e^{19}$. Then the theorem requirements $m = 2^{1000} > 2^{200}1000$, $n = e^{19} < e^{20} = exp(2\varepsilon^2 L)$ are fulfilled and $m = 2^{1000} \approx 10^{300}$, $n = e^{19} \approx 10^8$.

To prove the theorem we need the following lemmas.

Lemma 1 *Let S be a set of strings of length L. Pick randomly and uniformly both an integer number $k < L$ and a string $s_1 \dots s_L$ from S. Let p_S be the probability that $s_k = 1$. Let $\varepsilon > 0$. If $p_S \geq 1/2 + \varepsilon$ then the size of S is bounded above by*

$$2^L \cdot 10exp(-2\varepsilon^2 L).$$

Remark. One way to construct such a set where $p_S = 1/2 + \varepsilon$ is to take the set of all strings of length L starting with 1...1 repeated $[2\varepsilon L]$ times where $[2\varepsilon L]$ denotes the least integer not less than $2\varepsilon L$. Note that

$$|S| = 2^{(-2\varepsilon+1)L} < 2^L \cdot 10exp(-2\varepsilon^2 L)$$

as $\varepsilon < 1$.

Proof. Let S be a set such that $p_S \geq 1/2 + \varepsilon$. The inequality $2^{(-2\varepsilon+1)L} < 2^L \cdot 10exp(-2\varepsilon^2 L)$ implies we may assume that $|S| > 2^{(-2\varepsilon+1)L}$. Now split S into three disjoint sets:

$$S = S_1 \cup S_2 \cup S_3$$

where S_1 is the subset of strings containing more that $(1/2 + 2\varepsilon)L$ occurrences of 1's; S_2 is the subset of strings containing not more than $(1/2 + 2\varepsilon)L$ but more than $(1/2 + 1/2\varepsilon)L$ occurrences of 1's. Finally, S_3 is the subset of strings containing not more than $(1/2 + 1/2\varepsilon)L$ occurrences of 1's.

We have the following equality:

$$|S| = |S_1| + |S_2| + |S_3|.$$

Estimating the number of k's such that $s_k = 1$ among strings $s_1 \dots s_L \in S$, we also get

$$\varepsilon|S| \leq |S_1| + 2\varepsilon|S_2| + \varepsilon/2|S_3|.$$

Then

$$\varepsilon(|S_2| + |S_3|) \leq 3\varepsilon|S_2| + \varepsilon/2|S_1|$$

and thus

$$\varepsilon/2|S_3| \leq 3\varepsilon|S_2| \quad \text{and} \quad |S_3| \leq 6|S_2|.$$

Finally, by the Chernoff bound (see for example (Hagerup, 1990))

$$|S_2| < 2^L \cdot exp(-(2\varepsilon)^2 L/2) = 2^L \cdot exp(-2\varepsilon^2 L),$$

$$|S_1| < 2^L \cdot exp(-(8\varepsilon)^2 L/2) = 2^L \cdot exp(-4\varepsilon^2 L).$$

Hence

$$|S| \leq 2^L[exp(-4\varepsilon^2 L) + exp(-2\varepsilon^2 L) + 6exp(-2\varepsilon^2 L)] \leq 2^L \cdot 10exp(-2\varepsilon^2 L),$$

thereby proving the lemma. □

Remark. Note that there is another proof of the lemma using entropy bounds (see for example (Borda, 2011)).

Now let us estimate how often an automaton of size n may correctly guess the state of the game, i.e. what is the probability that it outputs 1 when receiving signal $f(1)$.

Lemma 2 *An automaton of size $n < 2^L exp(-2\varepsilon^2 L)$ guesses correctly with probability at most $\varepsilon + 1/2$.*

Proof. The automaton has at most n states (vertices) $v_1, ..., v_n$. Let V_i be the set of strings of length exactly L such that the automaton is in state v_i after reading the string. Let $p_i = 1/2 + \varepsilon_i$ be the probability the automaton guesses correctly after reading an input string whose first L bits is in V_i. The automaton output depends on v_i and the last $l - L$ bits of the input string. Note that these last bits are irrelevant for the correctness of the output.

At least for some choice of the rest of the string the conditional probability of success is at least p_i. Without loss of generality let this string be 1...1. Then we see by Lemma 1 that the size of $|V_i| \leq exp((1 - 2\varepsilon_i^2)L)$.

The overall probability of success is at most

$$1/2 + \varepsilon \leq \sum_{1 \leq i \leq n} p_i |V_i| \leq \sum_{1 \leq i \leq n} p_i \cdot exp(-2\varepsilon_i^2 L).$$

We have to estimate the number of summands. First notice that we may disregard summands where $\varepsilon_i > 2\varepsilon$ as we need at least 2^ε of them to get $\varepsilon/2$. By a calculation similar to the calculation above, the proportion of summands where $\varepsilon_i < \varepsilon/2$ cannot be more than 5/6. This implies that we need a number of summands of order $exp(-2\varepsilon^2 L)$. This completes the proof of the lemma. $\square$

Lemma 3 *There is an automaton of size of order $L2^{2\varepsilon L}$ which guesses correctly with probability at least $\varepsilon + 1/2$.*

Proof. The automaton is constructed as follows. Let $l = [2\varepsilon L]$ be the least integer not less than $2[\varepsilon L]$. It has states $v_{s_1 \ldots s_i}$ where $s_1 \ldots s_i$, $1 \leq i \leq l$ runs through strings of 0 and 1's of length at most l.

Vertices $v_{s_1 \ldots s_i}$ and $v_{s_1 \ldots s_i s_{i+1}}$ are connected by an edge labelled s_{i+1}. Further there are states $v^k_{s_1 \ldots s_l}$, $l \leq k \leq L + l$;

for $l \leq k < L + l$ both edges leaving $v^k_{s_1 \ldots s_l}$ go to $v^{k+1}_{s_1 \ldots s_l}$; both edges leaving $v^{L+l}_{s_1 \ldots s_l}$ go to the state itself. For $L < k \leq L + l$, the state $v^k_{s_1 \ldots s_l}$ is labelled by s_{k-L}; for $1 \leq k \leq L$ the state $v^k_{s_1 \ldots s_l}$ is labelled by 1. By construction this automaton guesses correctly if the input string has length at most $L + l$. If the string is longer it always outputs 1, which is correct with probability 1/2. The probability of an input string being of length at most $L + l$ is l/L. Hence, the total probability of a correct guess is then at least $2l/L + (1 - 2l/L) \leq 2\varepsilon + (1 - 2\varepsilon)/2 = 1/2 + \varepsilon$. $\square$

Proof of the Theorem 3. The structure of the proof is similar to that of Theorem 2. We use similar estimates on the posterior probabilities.

By Lemma 3 Player 1 can choose an automaton which always decodes the signal correctly and thus Player 1 knows the state of the game.

By Lemma 2 the automaton chosen by Player 2 guesses correctly with probability at most $1/2 + \varepsilon$. Now let us calculate the posterior probabilities of state 1 of the game.

Assume Player 2's automaton pointed out to state 1 whose probability is p. Using Bayes formula the posterior probability p' of state 1 can be calculated as follows:

$$p' = \frac{p(\frac{1}{2} + \varepsilon)}{p(\frac{1}{2} + \varepsilon) + (1 - p)(\frac{1}{2} - \varepsilon)} = p \frac{\frac{1}{2} + \varepsilon}{\frac{1}{2} - \varepsilon + 2\varepsilon p}.$$

It is easy to verify that $p \leq p' \leq (1 + \varepsilon)p$.

Assume Player 2's automaton pointed out to state 2 whose probability is $1-p$. Similarly we get $1-p \leq 1-p' \leq (1+\varepsilon)(1-p)$. Hence $(1-p') \leq (1+\varepsilon)(1-p)$ and thus $p' \geq (1+\varepsilon)p - \varepsilon \geq (1-\varepsilon)p$ for $p \geq 1/2$. Hence the game is ε-equivalent to game $\mathcal{A}^{asy}(p')$. $\square$

Conclusions

We consider zero-sum games with incomplete information on both sides with a public signal about the state of the game. Supposing that Player 1's computational power is higher than that of Player 2, we give three examples of different kinds of public signal where Player 1 learns the state of the game meanwhile itis impossible for Player 2. Thus we show that a player may receive informationabout the state of a game due to a public signal and his computational resource.

Note that boundedness of players's computational resources is equivalent (in a certain sense) to considering effectively computable strategies only. Hence we demonstrate that such a restriction may change the information structure of the game.

We hope to use this effect to shed some light on the open problem of existence of the value of stochastic games formulated in (Mertens, 1986) (see also (Mertens et al., 2015)). Introduced by Shapley(Shapley, 1953) stochastic games model dynamic interactions in which the current state of the game depends on the behavior of the players. These games are games with complete information — players know the current state of the game. We plan to construct an example of a stochastic game for which the solution does not exist in the class of effectively computable strategies.

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