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Existence of Stable Coalition Structures in Four-person Games*

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Abstract Cooperative games with coalition structures are considered and the principle of coalition structure stability with respect to some cooperative solution concepts is determined. This principle is close to the concept of Nash equilibrium. The existence of a stable coalition structure with respect to the Shapley value and the equal surplus division value for the cases of two- and three-person games is proved. In this paper, the problem of existence of a stable coalition structure with respect to the Shapley value and the equal surplus division value for the case of four-person games with special characteristic function is examined.

Keywords: coalition structure, stability, the Shapley value, the ES-value, four-player cooperative games.

1. Introduction

When the nature of the game allows to form not only grand coalition but also smaller coalitions, we may consider the games with coalition structures and find cooperative solutions according to the structure. In the paper we consider single-valued solution concepts such as the Shapley value (Shapley, 1953) and the ES-value (Driessen and Funaki, 1991). The Shapley value modified for the games with coalition structure is introduced in (Aumann and Dreze, 1974). If players may form different coalitions, hence different coalition structures, an essential question arises: which coalition structure is stable in some sense? By stability one may assume different properties of a coalition structure. By stable coalition structure we mean a partition of players which is stable against individual deviations of any player. Deviating from a coalition structure a player has several options: (i) to become a singleton, (ii) to join another coalition from the coalition structure. If these deviations are not profitable for any player, we call the coalition structure stable. We also assume that in all coalitions belonging to the structure, the players' payoffs are defined according to the same cooperative solution. Therefore, we determine stable coalition structure with respect to a cooperative solution like the Shapley value and the ES-value.

Some ideas of stability concepts of coalition structures are introduced in the papers (Ha er in ger, 2001, Hart and Kurz, 1983). Following (Carraro, 1997) the stable coalition structure should satisfy three basic properties: (i) internal stability,

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i. e. each player looses if she leaves her coalition becoming a singleton, (ii) external stability, i. e. each player-singleton looses if she joins any coalition or another singleton, and, finally, (iii) intracoalitional stability, i. e. each player from a coalition looses if she leaves her coalition and joins another one. Here we may find some similarities with the Nash equilibrium concept.

In (Sedakov et al., 2013) it is proved that in three-player games stable coalition structure always exists with respect to the Shapley value and the ES-value. A special game of cost reduction in which characteristic function has a specific form is considered in (Pari li na and Se da kov, 2014a). For the game the stable coalition structures are found. Communication between players may be also restricted by a graph structure. Therefore, the definition of a stable coalition structure should take this into account. The problem of stability of coalition structures in the games with a given communication structure is examined in (Pari li na and Se da kov, 2014b).

In the present paper we consider several special cases of characteristic function in four-player games and find the conditions for which there exists a stable coalition structure with respect to the Shapley value and the ES-value. There are some cases when there are no stable coalition structures which proves that in general case in four-player games the stable coalition structure does not always exist.

The paper has the following structure. In Section 2 the model of the game with coalition structure is given. The definition of stable coalition structure with respect to a solution concept is introduced. In Sections 3–6 we introduce some special cases of characteristic functions of four-player cooperative games and find the conditions for which at least one stable coalition structure exists or there are no stable coalition structures with respect to the Shapley value and the ES-value. We conclude in Section 7.

2. Game with coalition structure

2.1. Basic definitions

Cooperative game is a tuple (N, v) where N is a set of players and $v : 2^N \to R$ is a characteristic function defined for every nonempty set $S \subseteq N$ called coalition. We suggest that grand coalition N should be formed and then players from N allocate the total payoff $v(N)$ according to some solution concept. We also suppose that the characteristic function might not be supperadditive, i. e. there exist at least two disjoint coalitions $S, T \subset N$ such as $v(S \cup T) < v(S) + v(T)$. Therefore, in general not only the grand coalition but smaller ones can be formed. It can take place when some players get larger payoff if they form a smaller coalition. Therefore, we allow formation of not only grand coalition, and consider games with coalition structure.

Definition 1. Coalition structure π is a partition $\{B_1, \ldots, B_m\}$ of set N, i. e. $B_1 \cup \ldots \cup B_m = N$, and $B_i \cap B_j = \emptyset$ for all $i, j = 1, \ldots, m, i \neq j$.

Denote a game with player set N, characteristic function v and coalition structure π by (N, v, π).

Definition 2. A profile $x^\pi = (x_1^\pi, \ldots, x_n^\pi) \in R^n$ is a payoff distribution in the game (N, v, π) with coalition structure π if the efficiency condition, i. e. $\sum_{i \in B_j} x_i^\pi = v(B_j)$ holds for all coalitions $B_j \in \pi$, $j = 1, \ldots, m$.

Definition 3. A payoff distribution x^π is an allocation in the game (N, v, π) with coalition structure π if the individual rationality condition, i. e. $x_i^\pi \geq v(\{i\})$ holds for any player $i \in N$.

Denote the coalition partition $\pi_{-B_i} = \pi \setminus B_i \subset \pi$ by π_{-B_i}, and the coalition which contains player $i \in N$ by $B(i) \in \pi$.

2.2. Stable coalition structures

Defining a stable coalition structure we use an approach which takes into account the player's payoff as a member of her coalition. The player compares his payoff according to the current coalition structure with the payoffs that she can obtain if he deviates from his coalition and other players do not deviate. She can change coalition structure becoming a singleton or joining another coalition from the current coalition structure. And if any player cannot increase her payoff by the way describing above, the coalition structure is stable. More formally we use the following definition of stability.

Definition 4. (Sedakov et al., 2013) Coalition structure $\pi = \{B_1, \ldots, B_m\}$ is said to be stable with respect to a single-valued cooperative solution concept if for any player $i \in N$ the inequality

$$x_i^{\pi} \geq x_i^{\pi'} \text{ holds for all } B_j \in \pi \cup \emptyset,\ B_j \neq B(i).$$

Here x^{π} and $x^{\pi'}$ are two payoff distributions calculated according to the chosen cooperative solution concept for games (N, v, π) and (N, v, π') with coalition structures π, π' respectively, where $\pi' = \{B(i) \setminus \{i\}, B_j \cup \{i\}, \pi_{-B(i) \cup B_j}\}$.

The stability concept from Definition 4 is similar to the Nash equilibrium concept. Consider stable coalition structure π and calculate player i's payoff according to the some cooperative solution concept like the Shapley value. Now imagine that player i has the following set of strategies: to stay in a current coalition, to become a singleton or to join any other existing coalition in the coalition structure. If each player compares his payoff x_i^{π}, $i \in N$ with all the possible payoffs that he can obtain using one of the above mentioned strategies (when all other players do not deviate) and finds out that he cannot get larger payoff, then the current players' strategies form the Nash equilibrium. In other words, the current coalition structure is stable with respect to the chosen cooperative solution concept.

As single-valued cooperative solution concepts we can consider concepts as the Shapley value (Shapley, 1953), the ES-value.

3. The case of symmetric two- and three-player coalitions

3.1. The ES-value as a cooperative solution

In the game (N, v, π) with coalition structure $\pi = \{B_1, \ldots, B_m\}$, we can choose the ES-value $\psi^{\pi} = (\psi_1^{\pi}, \ldots, \psi_n^{\pi})$ as the cooperative solution. And its components are calculated as follows:

$$\psi_i^{\pi} = v(\{i\}) + \frac{v(B(i)) - \sum\limits_{j \in B(i)} v(\{j\})}{|B(i)|} \tag{1}$$

for any $i \in N$.

Consider four-person cooperative games with characteristic function $v(\cdot)$ determined by the following way:

$$v(\{1,2,3,4\}) = c, v(\{1\}) = v(\{2\}) = v(\{3\}) = v(\{4\}) = 0,$$

$$v(\{1,2\}) = v(\{1,3\}) = v(\{1,4\}) = v(\{2,3\}) = v(\{2,4\}) = v(\{3,4\}) = c_1, \quad (2)$$
$$v(\{1,2,3\}) = v(\{1,2,4\}) = v(\{1,3,4\}) = v(\{2,3,4\}) = c_2.$$

The ES-values calculated for all possible coalition structures and "Stable if" conditions are represented in Table 1.

Table 1. The ES-value for a four-player game determined by (2) and "Stable if" conditions.

π	ψ_1^π	ψ_2^π	ψ_3^π	ψ_4^π	"Stable if" condition
$\pi_1 = \{\{1,2,3,4\}\}$	$c/4$	$c/4$	$c/4$	$c/4$	$c \geq 0$
$\pi_2 = \{\{1\},\{2,3,4\}\}$	0	$c_2/3$	$c_2/3$	$c_2/3$	$\begin{cases} c_2 \geq \max\{0, 3c_1/2\} \\ c \leq 0 \end{cases}$
$\pi_3 = \{\{2\},\{1,3,4\}\}$	$c_2/3$	0	$c_2/3$	$c_2/3$	$\begin{cases} c_2 \geq \max\{0, 3c_1/2\} \\ c \leq 0 \end{cases}$
$\pi_4 = \{\{3\},\{1,2,4\}\}$	$c_2/3$	$c_2/3$	0	$c_2/3$	$\begin{cases} c_2 \geq \max\{0, 3c_1/2\} \\ c \leq 0 \end{cases}$
$\pi_5 = \{\{4\},\{1,2,3\}\}$	$c_2/3$	$c_2/3$	$c_2/3$	0	$\begin{cases} c_2 \geq \max\{0, 3c_1/2\} \\ c \leq 0 \end{cases}$
$\pi_6 = \{\{1,2\},\{3,4\}\}$	$c_1/2$	$c_1/2$	$c_1/2$	$c_1/2$	$c_1 \geq \max\{0, 2c_2/3\}$
$\pi_7 = \{\{1,3\},\{2,4\}\}$	$c_1/2$	$c_1/2$	$c_1/2$	$c_1/2$	$c_1 \geq \max\{0, 2c_2/3\}$
$\pi_8 = \{\{1,4\},\{2,3\}\}$	$c_1/2$	$c_1/2$	$c_1/2$	$c_1/2$	$c_1 \geq \max\{0, 2c_2/3\}$
$\pi_9 = \{\{1\},\{2\},\{3,4\}\}$	0	0	$c_1/2$	$c_1/2$	$\begin{cases} c_1 \geq 0 \text{ and } c_1 \leq 0 \\ c_2 \leq 0 \end{cases}$
$\pi_{10} = \{\{1\},\{3\},\{2,4\}\}$	0	$c_1/2$	0	$c_1/2$	$\begin{cases} c_1 \geq 0 \text{ and } c_1 \leq 0 \\ c_2 \leq 0 \end{cases}$
$\pi_{11} = \{\{1\},\{4\},\{2,3\}\}$	0	$c_1/2$	$c_1/2$	0	$\begin{cases} c_1 \geq 0 \text{ and } c_1 \leq 0 \\ c_2 \leq 0 \end{cases}$
$\pi_{12} = \{\{1,2\},\{3\},\{4\}\}$	$c_1/2$	$c_1/2$	0	0	$\begin{cases} c_1 \geq 0 \text{ and } c_1 \leq 0 \\ c_2 \leq 0 \end{cases}$
$\pi_{13} = \{\{1,3\},\{2\},\{4\}\}$	$c_1/2$	0	$c_1/2$	0	$\begin{cases} c_1 \geq 0 \text{ and } c_1 \leq 0 \\ c_2 \leq 0 \end{cases}$
$\pi_{14} = \{\{1,4\},\{2\},\{3\}\}$	$c_1/2$	0	0	$c_1/2$	$\begin{cases} c_1 \geq 0 \text{ and } c_1 \leq 0 \\ c_2 \leq 0 \end{cases}$
$\pi_{15} = \{\{1\},\{2\},\{3\},\{4\}\}$	0	0	0	0	$c_1 \leq 0$

There are two obvious results:

1. From Table 1 and calculations, we can find that the coalition structures $\pi_9 = \{\{1\},\{2\},\{3,4\}\}$, $\pi_{10} = \{\{1\},\{3\},\{2,4\}\}$, $\pi_{11} = \{\{1\},\{2,3\},\{4\}\}$, $\pi_{12} = \{\{1,2\},$ $\{3\},\{4\}\}$, $\pi_{13} = \{\{1,3\},\{2\},\{4\}\}$, $\pi_{14} = \{\{1,4\},\{2\},\{3\}\}$ are always unstable with any c or $c_i, i = 1,2$. Because for stability we get $c_1 \geq 0$ and $c_1 \leq 0$ at the same time, which is possible if $c_1 = 0$.
2. Notice that if $c \geq 0$, then coalition structure $\pi_1 = \{\{1,2,3,4\}\}$ is always stable with respect to the ES-value for any $c_i, i = 1,2$. And we can also notice that if $c_1 \leq 0$, then coalition structure $\pi_{15} = \{\{1\},\{2\},\{3\},\{4\}\}$ is always stable with respect to the ES-value.

There is an analysis of another cases:

1. Let $c_1, c_2 \geq 0$ and $c \geq 0$, then coalition structures π_1, π_6, π_7, π_8 may be stable with respect to the ES-value. And for all c, $c_i \geq 0$, $i = 1, 2$, coalition structures π_1 is always stable. But coalition structure π_6, π_7, π_8 are stable when $c_1 \geq \frac{2}{3}c_2$.
2. Let $c_1 \geq 0$, $c_2 \leq 0$ and $c \geq 0$, then coalition structures π_1, π_6, π_7, π_8 are always stable with respect to the ES-value.
3. Let $c_1 \leq 0$, $c_2 \geq 0$ and $c \geq 0$, then coalition structures π_1, π_{15} are always stable with respect to the ES-value.
4. Let $c_1 \leq 0$, $c_2 \leq 0$ and $c \geq 0$, then coalition structures π_1, π_{15} are always stable with respect to the ES-value.
5. Let $c_1, c_2 \geq 0$ and $c \leq 0$, then coalition structures π_2, π_3, π_4, π_5, π_6, π_7, π_8 may be stable with respect to the ES-value. But coalition structures π_2, π_3, π_4, π_5 are stable when $c_2 \geq \frac{3}{2}c_1$ and coalition structures π_6, π_7, π_8 are stable when $c_1 \geq \frac{2}{3}c_2$. Therefore, the solutions of these seven systems of inequalities cover the whole first quadrant constructed by c_1, c_2 when $c_1, c_2 \geq 0, c \leq 0$.
6. Let $c_1 \geq 0$, $c_2 \leq 0$ and $c \leq 0$, then coalition structures π_6, π_7, π_8 are always stable with respect to the ES-value.
7. Let $c_1 \leq 0$, $c_2 \geq 0$ and $c \leq 0$, then coalition structures π_2, π_3, π_4, π_5, π_{15} are always stable with respect to the ES-value.
8. Let $c_1 \leq 0$, $c_2 \leq 0$ and $c \leq 0$, then coalition structures π_{15} is stable with respect to the ES-value.

Therefore, we prove the following proposition.

Proposition 1. *Let characteristic function be given by* (2). *In this case, there always exists at least one stable coalition structure with respect to the ES-value.*

3.2. The Shapley value as a cooperative solution

In the game (N, υ, π) with coalition structure $\pi = \{B_1, \ldots, B_m\}$, we can choose the Shapley value $\phi^\pi = (\phi^\pi_1, \ldots, \phi^\pi_n)$ as the cooperative solution. And its components are calculated as follows:

$$\phi^\pi_i = \sum_{\substack{i \in S \\ S \subseteq B(i)}} \frac{(|B(i)| - |S|)!(|S| - 1)!}{|B(i)|!} [v(S) - v(S \setminus \{i\})] \tag{3}$$

for any $i \in N$.

Consider four-person cooperative games with characteristic function $\upsilon(\cdot)$ determined by (2). The Shapley value calculated for all possible coalition structures and “Stable if" conditions are represented in Table 2.

There are two obvious results:

1. From Table 2 and calculations, we can find that the coalition structures $\pi_9 = \{\{1\}, \{2\}, \{3, 4\}\}$, $\pi_{10} = \{\{1\}, \{3\}, \{2, 4\}\}$, $\pi_{11} = \{\{1\}, \{2, 3\}, \{4\}\}$, $\pi_{12} = \{\{1, 2\},$
$\{3\}, \{4\}\}$, $\pi_{13} = \{\{1, 3\}, \{2\}, \{4\}\}$, $\pi_{14} = \{\{1, 4\}, \{2\}, \{3\}\}$ are always unstable with any c or $c_i, i = 1, 2$. Because for stability we get $c_1 \geq 0$ and $c_1 \leq 0$ at the same time, which is possible if $c_1 = 0$.

Table 2. The Shapley value for a four-player game determined by (2) and "Stable if" conditions.

π	ψ_1^π	ψ_2^π	ψ_3^π	ψ_4^π	"Stable if" condition
$\pi_1 = \{\{1,2,3,4\}\}$	$c/4$	$c/4$	$c/4$	$c/4$	$c \geq 0$
$\pi_2 = \{\{1\},\{2,3,4\}\}$	0	$c_2/3$	$c_2/3$	$c_2/3$	$\begin{cases} c_2 \geq \max\{0, 3c_1/2\} \\ c \leq 0 \end{cases}$
$\pi_3 = \{\{2\},\{1,3,4\}\}$	$c_2/3$	0	$c_2/3$	$c_2/3$	$\begin{cases} c_2 \geq \max\{0, 3c_1/2\} \\ c \leq 0 \end{cases}$
$\pi_4 = \{\{3\},\{1,2,4\}\}$	$c_2/3$	$c_2/3$	0	$c_2/3$	$\begin{cases} c_2 \geq \max\{0, 3c_1/2\} \\ c \leq 0 \end{cases}$
$\pi_5 = \{\{4\},\{1,2,3\}\}$	$c_2/3$	$c_2/3$	$c_2/3$	0	$\begin{cases} c_2 \geq \max\{0, 3c_1/2\} \\ c \leq 0 \end{cases}$
$\pi_6 = \{\{1,2\},\{3,4\}\}$	$c_1/2$	$c_1/2$	$c_1/2$	$c_1/2$	$c_1 \geq \max\{0, 2c_2/3\}$
$\pi_7 = \{\{1,3\},\{2,4\}\}$	$c_1/2$	$c_1/2$	$c_1/2$	$c_1/2$	$c_1 \geq \max\{0, 2c_2/3\}$
$\pi_8 = \{\{1,4\},\{2,3\}\}$	$c_1/2$	$c_1/2$	$c_1/2$	$c_1/2$	$c_1 \geq \max\{0, 2c_2/3\}$
$\pi_9 = \{\{1\},\{2\},\{3,4\}\}$	0	0	$c_1/2$	$c_1/2$	$\begin{cases} c_1 \geq 0 \text{ and } c_1 \leq 0 \\ c_2 \leq 0 \end{cases}$
$\pi_{10} = \{\{1\},\{3\},\{2,4\}\}$	0	$c_1/2$	0	$c_1/2$	$\begin{cases} c_1 \geq 0 \text{ and } c_1 \leq 0 \\ c_2 \leq 0 \end{cases}$
$\pi_{11} = \{\{1\},\{4\},\{2,3\}\}$	0	$c_1/2$	$c_1/2$	0	$\begin{cases} c_1 \geq 0 \text{ and } c_1 \leq 0 \\ c_2 \leq 0 \end{cases}$
$\pi_{12} = \{\{1,2\},\{3\},\{4\}\}$	$c_1/2$	$c_1/2$	0	0	$\begin{cases} c_1 \geq 0 \text{ and } c_1 \leq 0 \\ c_2 \leq 0 \end{cases}$
$\pi_{13} = \{\{1,3\},\{2\},\{4\}\}$	$c_1/2$	0	$c_1/2$	0	$\begin{cases} c_1 \geq 0 \text{ and } c_1 \leq 0 \\ c_2 \leq 0 \end{cases}$
$\pi_{14} = \{\{1,4\},\{2\},\{3\}\}$	$c_1/2$	0	0	$c_1/2$	$\begin{cases} c_1 \geq 0 \text{ and } c_1 \leq 0 \\ c_2 \leq 0 \end{cases}$
$\pi_{15} = \{\{1\},\{2\},\{3\},\{4\}\}$	0	0	0	0	$c_1 \leq 0$

2. Notice that if $c \geq 0$, then coalition structure $\pi_1 = \{\{1,2,3,4\}\}$ is always stable with respect to the Shapley value for any $c_i, i = 1,2$. And we can also notice that if $c_1 \leq 0$, then coalition structure $\pi_{15} = \{\{1\},\{2\},\{3\},\{4\}\}$ is always stable with respect to the Shapley value.

We analyse all possible cases:

1. Let $c_1, c_2 \geq 0$ and $c \geq 0$, then coalition structures π_1, π_6, π_7, π_8 may be stable with respect to the Shapley value. And for all c, $c_i \geq 0, i = 1,2$, coalition structures π_1 is always stable. But coalition structures π_6, π_7, π_8 are stable when $c_1 \geq \frac{2}{3}c_2$.
2. Let $c_1 \geq 0$, $c_2 \leq 0$ and $c \geq 0$, then coalition structures π_1, π_6, π_7, π_8 are always stable with respect to the Shapley value.
3. Let $c_1 \leq 0$, $c_2 \geq 0$ and $c \geq 0$, then coalition structures π_1, π_{15} are always stable with respect to the Shapley value.
4. Let $c_1 \leq 0$, $c_2 \leq 0$ and $c \geq 0$, then coalition structures π_1, π_{15} are always stable with respect to the Shapley value.

5. Let $c_1, c_2 \geq 0$ and $c \leq 0$, then coalition structures π_2, π_3, π_4, π_5, π_6, π_7, π_8 may be stable with respect to the Shapley value. But coalition structures π_2, π_3, π_4, π_5 are stable when $c_2 \geq \frac{3}{2}c_1$ and coalition structures π_6, π_7, π_8 are stable when $c_1 \geq \frac{2}{3}c_2$. Therefore, the solutions of these seven systems of inequalities cover the whole first quadrant constructed by c_1, c_2 when $c_1, c_2 \geq 0, c \leq 0$.
6. Let $c_1 \geq 0$, $c_2 \leq 0$ and $c \leq 0$, then coalition structures π_6, π_7, π_8 are always stable with respect to the Shapley value.
7. Let $c_1 \leq 0$, $c_2 \geq 0$ and $c \leq 0$, then coalition structures π_2, π_3, π_4, π_5, π_{15} are always stable with respect to the Shapley value.
8. Let $c_1 \leq 0$, $c_2 \leq 0$ and $c \leq 0$, then coalition structures π_{15} is stable with respect to the Shapley value.

Proposition 2. *Let characteristic function be given by (2). In this case, there always exists at least one stable coalition structure with respect to the Shapley value.*

4. The case of symmetric two-player coalitions and non-symmetric three player coalitions

4.1. The ES-value as a cooperative solution

In the game (N, v, π) with coalition structure $\pi = \{B_1, \dots, B_m\}$, the components of the ES-value are defined as (1). Consider the cooperative games with characteristic function $v(\cdot)$ determined by the following way:

$$v(\{1,2,3,4\}) = c, v(\{1\}) = v(\{2\}) = v(\{3\}) = v(\{4\}) = 0,$$

$$v(\{1,2\}) = v(\{1,3\}) = v(\{1,4\}) = v(\{2,3\}) = v(\{2,4\}) = v(\{3,4\}) = c_1, \qquad (4)$$

$$v(\{1,2,3\}) = v(\{1,2,4\}) = c_2, v(\{1,3,4\}) = v(\{2,3,4\}) = c_3.$$

The ES-values calculated for all possible coalition structures and "Stable if" condition are represented in Table 3.

There are two obvious results:

1. From Table 3 and calculations, we can find that the coalition structure $\pi_9 = \{\{1\},\{2\},\{3,4\}\}$, $\pi_{10} = \{\{1\},\{3\},\{2,4\}\}$, $\pi_{11} = \{\{1\},\{2,3\},\{4\}\}$, $\pi_{12} = \{\{1,2\},$
$\{3\},\{4\}\}$, $\pi_{13} = \{\{1,3\},\{2\},\{4\}\}$, $\pi_{14} = \{\{1,4\},\{2\},\{3\}\}$ are always unstable with any c or $c_i, i = 1,2,3$, because the existence requires $c_1 \geq 0$ and $c_1 \leq 0$ at the same time, which is possible only if $c_1 = 0$.
2. Notice that if $c \geq 0$, then coalition structure $\pi_1 = \{\{1,2,3,4\}\}$ is always stable with respect to the ES value for any $c_i, i = 1,2,3$. And we can also notice that if $c_1 \leq 0$, then coalition structure $\pi_{15} = \{\{1\},\{2\},\{3\},\{4\}\}$ is always stable with respect to the ES value.

Analysis of the results for these cases:

1. Consider the case when $c \geq 0$. We can find that coalition structure π_1 is always stable for any $c_i, i = 1,2,3$. When $c_1 \geq 0$, $c_2 \geq 0$, $c_3 \geq 0$, from Fig. 3, we can conclude that the coalition structures π_6, π_7, π_8 are stable when solutions of these three systems of inequalities cover the first octant in region *II*. When $c_1 \geq 0$, $c_2 \geq 0$, $c_3 \leq 0$, from Fig. 3, we can conclude that coalition structures π_6, π_7, π_8 are stable when solutions of these three systems of inequalities cover

Table 3. The ES-value for a four-player game determined by (4) and "Stable if" conditions.

π	ψ_1^π	ψ_2^π	ψ_3^π	ψ_4^π	"Stable if" condition
$\pi_1 = \{\{1,2,3,4\}\}$	$c/4$	$c/4$	$c/4$	$c/4$	$c \geq 0$
$\pi_2 = \{\{1\},\{2,3,4\}\}$	0	$c_3/3$	$c_3/3$	$c_3/3$	$\begin{cases} c_3 \geq \max\{0, 3c_1/2\} \\ c \leq 0 \end{cases}$
$\pi_3 = \{\{2\},\{1,3,4\}\}$	$c_3/3$	0	$c_3/3$	$c_3/3$	$\begin{cases} c_3 \geq \max\{0, 3c_1/2\} \\ c \leq 0 \end{cases}$
$\pi_4 = \{\{3\},\{1,2,4\}\}$	$c_2/3$	$c_2/3$	0	$c_2/3$	$\begin{cases} c_2 \geq \max\{0, 3c_1/2\} \\ c \leq 0 \end{cases}$
$\pi_5 = \{\{4\},\{1,2,3\}\}$	$c_2/3$	$c_2/3$	$c_2/3$	0	$\begin{cases} c_2 \geq \max\{0, 3c_1/2\} \\ c \leq 0 \end{cases}$
$\pi_6 = \{\{1,2\},\{3,4\}\}$	$c_1/2$	$c_1/2$	$c_1/2$	$c_1/2$	$c_1 \geq \max\{0, 2c_2/3, 2c_3/3\}$
$\pi_7 = \{\{1,3\},\{2,4\}\}$	$c_1/2$	$c_1/2$	$c_1/2$	$c_1/2$	$c_1 \geq \max\{0, 2c_2/3, 2c_3/3\}$
$\pi_8 = \{\{1,4\},\{2,3\}\}$	$c_1/2$	$c_1/2$	$c_1/2$	$c_1/2$	$c_1 \geq \max\{0, 2c_2/3, 2c_3/3\}$
$\pi_9 = \{\{1\},\{2\},\{3,4\}\}$	0	0	$c_1/2$	$c_1/2$	$\begin{cases} c_1 \geq 0 \text{ and } c_1 \leq 0 \\ c_3 \leq 0 \end{cases}$
$\pi_{10} = \{\{1\},\{3\},\{2,4\}\}$	0	$c_1/2$	0	$c_1/2$	$\begin{cases} c_1 \geq 0 \text{ and } c_1 \leq 0 \\ c_2 \leq 0 \\ c_3 \leq 0 \end{cases}$
$\pi_{11} = \{\{1\},\{4\},\{2,3\}\}$	0	$c_1/2$	$c_1/2$	0	$\begin{cases} c_1 \geq 0 \text{ and } c_1 \leq 0 \\ c_2 \leq 0 \\ c_3 \leq 0 \end{cases}$
$\pi_{12} = \{\{1,2\},\{3\},\{4\}\}$	$c_1/2$	$c_1/2$	0	0	$\begin{cases} c_1 \geq 0 \text{ and } c_1 \leq 0 \\ c_2 \leq 0 \end{cases}$
$\pi_{13} = \{\{1,3\},\{2\},\{4\}\}$	$c_1/2$	0	$c_1/2$	0	$\begin{cases} c_1 \geq 0 \text{ and } c_1 \leq 0 \\ c_2 \leq 0 \\ c_3 \leq 0 \end{cases}$
$\pi_{14} = \{\{1,4\},\{2\},\{3\}\}$	$c_1/2$	0	0	$c_1/2$	$\begin{cases} c_1 \geq 0 \text{ and } c_1 \leq 0 \\ c_2 \leq 0 \\ c_3 \leq 0 \end{cases}$
$\pi_{15} = \{\{1\},\{2\},\{3\},\{4\}\}$	0	0	0	0	$c_1 \leq 0$

the fifth octant in region *II*. When $c_1 \geq 0$, $c_2 \leq 0$, $c_3 \geq 0$, from Fig. 3, we can conclude that coalition structures π_6, π_7, π_8 are stable when solutions of these three systems of inequalities cover the fourth octant in region *II*. When $c_1 \geq 0$, $c_2 \leq 0$, $c_3 \leq 0$, coalition structures π_6, π_7, π_8 are always stable, because from Fig. 3, we can find that the solutions of these three systems of inequalities cover the whole eighth octant in region *II*.

2. When $c \geq 0$ and $c_1 \leq 0$, we can conclude that coalition structures π_1 and π_{15} are always stable for any $c_i, i = 2, 3$.
3. When $c \leq 0$. When $c_1 \geq 0$, $c_2 \geq 0$, $c_3 \geq 0$, the coalition structures π_2, π_3 are stable when solutions of these two systems of inequalities cover the first octant in region *I* from Fig. 1, coalition structures π_4, π_5 are stable when solutions cover the first octant in region *I* from Fig. 2 and coalition structures π_6, π_7, π_8

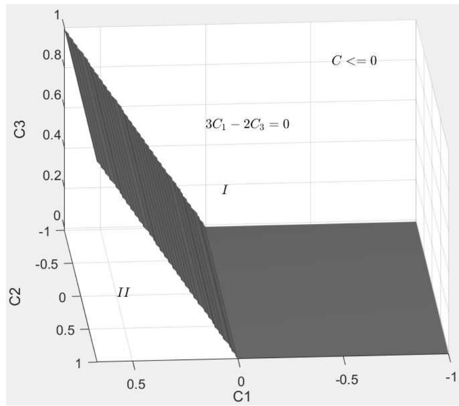

Fig. 1. Coalition structures π_2,π_3 are stable in region I with respect to the ES-value.

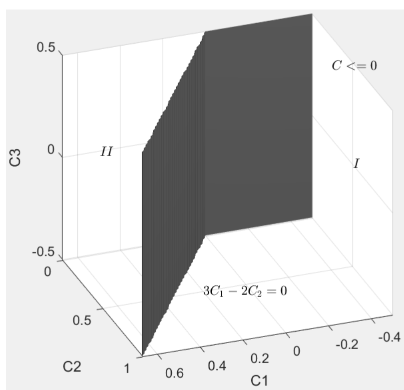

Fig. 2. Coalition structures π_4, π_5 are stable in region I with respect to ES-value.

are stable when solutions cover the first octant in region II from Fig. 3. And we can find that the solutions of these seven systems of inequalities cover the whole first octant constructed by c_1, c_2, c_3. Therefore, there always exists stable coalition structure when $c \leq 0$ and $c_1 \geq 0$, $c_2 \geq 0$, $c_3 \geq 0$.

4. When $c \leq 0$ and $c_1 \geq 0$, $c_2 \geq 0$, $c_3 \leq 0$, the coalition structures π_4, π_5 are stable when solutions of these two systems of inequalities cover the fifth octant in region I from Fig. 2 and coalition structures π_6, π_7, π_8 are stable when solutions cover the fifth octant in region II from Fig. 3. And we can find that the solutions of these five systems of inequalities cover the whole fifth octant constructed by c_1, c_2, c_3. Therefore, there always exists stable coalition structure when $c \leq 0$ and $c_1 \geq 0$, $c_2 \geq 0$, $c_3 \leq 0$.

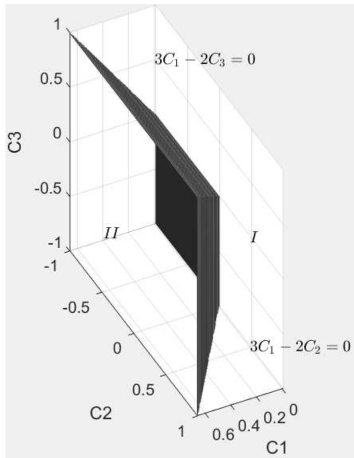

Fig. 3. Coalition structures π_6, π_7, π_8 are stable in region *II* with respect to ES-value.

5. When $c \leq 0$ and $c_1 \geq 0$, $c_2 \leq 0$, $c_3 \geq 0$, we can conclude that coalition structures π_2, π_3 are stable when solutions of these two systems of inequalities cover the fourth octant in region *I* from Fig. 1 and coalition structures π_6, π_7, π_8 are stable when solutions cover the fourth octant in region *II* from Fig. 3. And we can find that the solutions of these five systems of inequalities cover the whole fourth octant constructed by c_1, c_2, c_3. Therefore, there always exists stable coalition structure when $c \leq 0$ and $c_1 \geq 0$, $c_2 \leq 0$, $c_3 \geq 0$.
6. When $c \leq 0$ and $c_1 \geq 0$, $c_2 \leq 0$, $c_3 \leq 0$. From Fig. 3, we can conclude that coalition structures π_6, π_7, π_8 are always stable because the solutions of these three systems of inequalities cover the whole eighth octant in region *II*.
7. Consider the case when $c \leq 0$. And we can find that when $c_1 \leq 0$, coalition structure π_{15} is always stable for any $c_i, i = 2, 3$. When $c_1 \leq 0$, $c_2 \geq 0$, $c_3 \geq 0$, we can conclude that coalition structures π_2, π_3 are stable when solutions of these two systems of inequalities cover the second octant in region *I* from Fig. 1 and coalition structures π_4, π_5 are stable when solutions cover the second octant in region *I* from Fig. 2. When $c_1 \leq 0$, $c_2 \geq 0$, $c_3 \leq 0$, from Fig. 2, we obtain that coalition structures π_4, π_5 are stable when solutions of two systems of inequalities cover the sixth octant in region *I*. When $c_1 \leq 0$, $c_2 \leq 0$, $c_3 \geq 0$, from Fig. 1, the coalition structures π_2, π_3 are stable when solutions of two systems of inequalities cover the third octant in region *I*. And when $c_1 \leq 0$, $c_2 \leq 0$, $c_3 \leq 0$, the only coalition structure π_{15} is always stable.

Proposition 3. *Let characteristic function be given by* (4). *In this case, there always exists at least one stable coalition structure with respect to the ES-value.*

4.2. The Shapley value as a cooperative solution

In the game (N, v, π) with coalition structure $\pi = \{B_1, \ldots, B_m\}$, the components of the Shapley value are defined as (3). Consider the cooperative games with characteristic function $v(\cdot)$ determined by (4). The Shapley value calculated for all possible coalition structures is represented Table 6 (see Appendix) and “Stable if" conditions are represented in Table 4.

Table 4. The “Stable if" conditions respected to Shapley value for a four-player game determined by (4).

π	“Stable if" condition
$\pi_1 = \{\{1,2,3,4\}\}$	$\begin{cases} 3c + 2c_2 - 2c_3 \geq 0 \\ 3c - 2c_2 + 2c_3 \geq 0 \end{cases}$
$\pi_2 = \{\{1\},\{2,3,4\}\}$	$\begin{cases} 3c + 2c_2 - 2c_3 \leq 0 \\ 3c_1 \leq 2c_3 \\ c_3 \geq 0 \end{cases}$
$\pi_3 = \{\{2\},\{1,3,4\}\}$	$\begin{cases} 3c + 2c_2 - 2c_3 \leq 0 \\ 3c_1 \leq 2c_3 \\ c_3 \geq 0 \end{cases}$
$\pi_4 = \{\{3\},\{1,2,4\}\}$	$\begin{cases} 3c - 2c_2 + 2c_3 \leq 0 \\ 3c_1 \leq 2c_2 \\ c_2 \geq 0 \end{cases}$
$\pi_5 = \{\{4\},\{1,2,3\}\}$	$\begin{cases} 3c - 2c_2 + 2c_3 \leq 0 \\ 3c_1 \leq 2c_2 \\ c_2 \geq 0 \end{cases}$
$\pi_6 = \{\{1,2\},\{3,4\}\}$	$c_1 \geq \max\{0, 2c_2/3, 2c_3/3\}$
$\pi_7 = \{\{1,3\},\{2,4\}\}$	$c_1 \geq \max\{0, 2c_2/3, 2c_3/3\}$
$\pi_8 = \{\{1,4\},\{2,3\}\}$	$c_1 \geq \max\{0, 2c_2/3, 2c_3/3\}$
$\pi_9 = \{\{1\},\{2\},\{3,4\}\}$	$\begin{cases} c_1 \geq 0 \text{ and } c_1 \leq 0 \\ c_3 \leq 0 \end{cases}$
$\pi_{10} = \{\{1\},\{3\},\{2,4\}\}$	$\begin{cases} c_1 \geq 0 \text{ and } c_1 \leq 0 \\ c_2 \leq 0 \\ c_3 \leq 0 \end{cases}$
$\pi_{11} = \{\{1\},\{4\},\{2,3\}\}$	$\begin{cases} c_1 \geq 0 \text{ and } c_1 \leq 0 \\ c_2 \leq 0 \\ c_3 \leq 0 \end{cases}$
$\pi_{12} = \{\{1,2\},\{3\},\{4\}\}$	$\begin{cases} c_1 \geq 0 \text{ and } c_1 \leq 0 \\ c_2 \leq 0 \end{cases}$
$\pi_{13} = \{\{1,3\},\{2\},\{4\}\}$	$\begin{cases} c_1 \geq 0 \text{ and } c_1 \leq 0 \\ c_3 \leq 0 \end{cases}$
$\pi_{14} = \{\{1,4\},\{2\},\{3\}\}$	$\begin{cases} c_1 \geq 0 \text{ and } c_1 \leq 0 \\ c_2 \leq 0 \\ c_3 \leq 0 \end{cases}$
$\pi_{15} = \{\{1\},\{2\},\{3\},\{4\}\}$	$c_1 \leq 0$

There are two obvious results:

1. From Table 4 and calculations, we can find that the coalition structures $\pi_9 = \{\{1\},\{2\},\{3,4\}\}$, $\pi_{10} = \{\{1\},\{3\},\{2,4\}\}$, $\pi_{11} = \{\{1\},\{2,3\},\{4\}\}$, $\pi_{12} = \{\{1,2\},$

$\{3\}, \{4\}\}$, $\pi_{13} = \{\{1,3\}, \{2\}, \{4\}\}$, $\pi_{14} = \{\{1,4\}, \{2\}, \{3\}\}$ are always unstable with any c or $c_i, i = 1,2,3$, because the stability requires $c_1 \geq 0$ and $c_1 \leq 0$ at the same time, which is possible only if $c_1 = 0$.

2. Notice that if $c_1 \leq 0$, then coalition structure $\pi_{15} = \{\{1\}, \{2\}, \{3\}, \{4\}\}$ is always stable with respect to the Shapley value.

We analyse these cases:

1. Consider the case when $c \geq 0$. When $c_1 \geq 0$, $c_2 \geq 0$, $c_3 \geq 0$, we can observe that solutions of the eight systems of inequalities cover the whole first octant constructed by c_1, c_2, c_3. From Fig. 4, we can conclude that the coalition structure π_1 is stable in region *I*; coalition structures π_2, π_3 are stable in region *II*; coalition structures π_4, π_5 are stable in region *III* and coalition structures π_6, π_7, π_8 are stable in region *IV*.

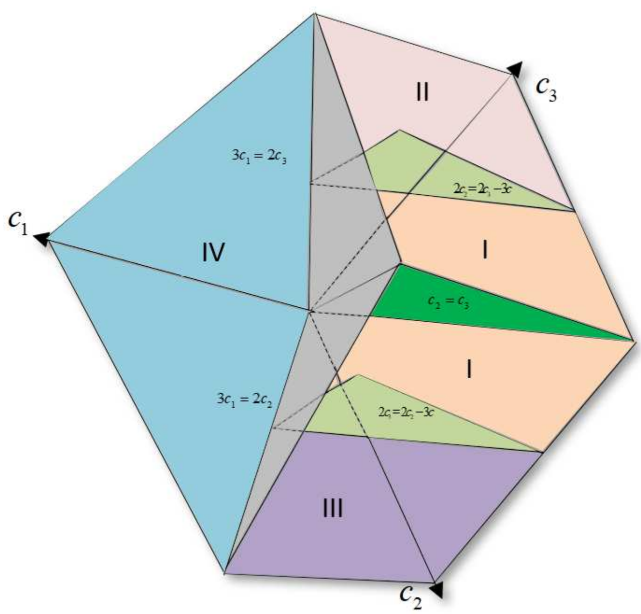

Fig. 4. Case $c \geq 0$ and $c_1, c_2, c_3 \geq 0$.

2. When $c \geq 0$ and $c_1 \geq 0$, $c_2 \geq 0$, $c_3 \leq 0$, we can observe that solutions of the six systems of inequalities cover the whole fifth octant constructed by c_1, c_2, c_3. From Fig. 5, we can conclude that coalition structure π_1 is stable in region *III*; coalition structures π_4, π_5 are stable in region *I* and coalition structures π_6, π_7, π_8 are stable in region *II*.
3. When $c \geq 0$ and $c_1 \geq 0$, $c_2 \leq 0$, $c_3 \geq 0$, we can observe that solutions of the six systems of inequalities cover the whole fourth octant constructed by c_1, c_2, c_3. From Fig. 6, we can conclude that coalition structure π_1 is stable in region *III*;

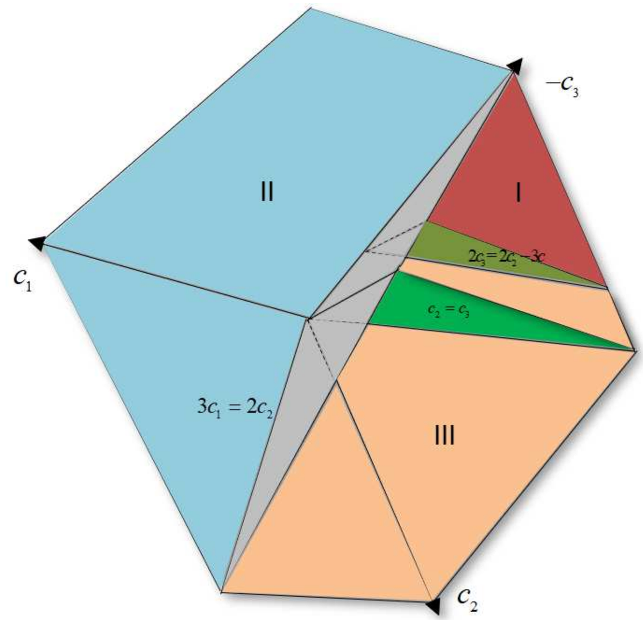

Fig. 5. Case $c \geq 0$ and $c_1, c_2 \geq 0, c_3 \leq 0$.

coalition structures π_2, π_3 are stable in region I and coalition structures π_6, π_7, π_8 are stable in region II.

4. When $c \geq 0$ and $c_1 \geq 0$, $c_2 \leq 0$, $c_3 \leq 0$, from the "Stable if" condition, we can observe that coalition structures π_6, π_7, π_8 are always stable, because the solutions of these three systems of inequalities cover the whole eighth octant constructed by c_1, c_2, c_3.
5. Consider the case when $c \leq 0$, the analysis is very similar to the the cases when $c \geq 0$ described above. When $c_1 \geq 0$, $c_2 \geq 0$, $c_3 \geq 0$, we can conclude that coalition structures π_2, π_3, π_4, π_5, π_6, π_7, π_8 are stable. And the solutions of these seven systems of inequalities cover the whole first octant constructed by c_1, c_2, c_3. When $c_1 \geq 0$, $c_2 \geq 0$, $c_3 \leq 0$, we can conclude that coalition structures π_4, π_5, π_6, π_7, π_8 are stable. And the solutions of these five systems of inequalities cover the whole eighth octant constructed by c_1, c_2, c_3. When $c_1 \geq 0$, $c_2 \leq 0$, $c_3 \geq 0$, we can conclude that coalition structures π_2, π_3, π_6, π_7, π_8 are stable when solutions of these five systems of inequalities cover the whole fourth octant constructed by c_1, c_2, c_3. When $c_1 \geq 0$, $c_2 \leq 0$, $c_3 \leq 0$, we can conclude that coalition structures π_6, π_7, π_8 are always stable because the solutions of these three systems of inequalities cover the whole eighth octant.
6. When $c_1 \leq 0$, the coalition structure π_{15} is always stable for any c and $c_i, i = 2, 3$.

By the above calculation we prove the following proposition.

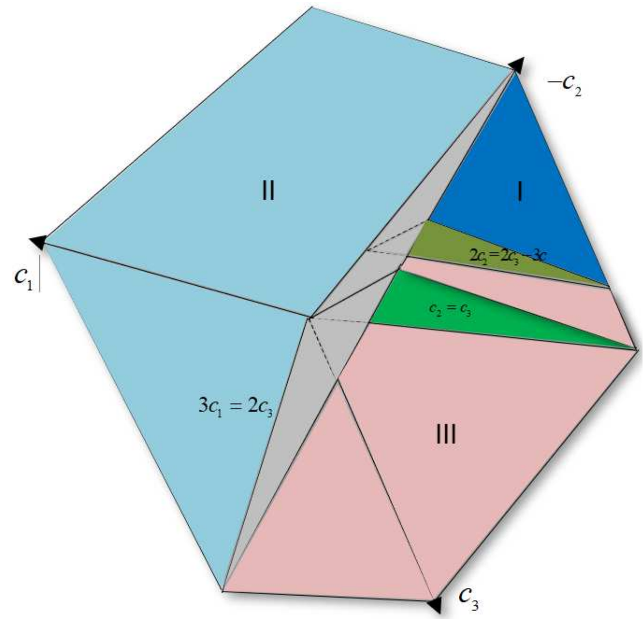

Fig. 6. Case $c \geq 0$ and $c_1, c_3 \geq 0, c_2 \leq 0$.

Proposition 4. *Let characteristic function be given by* (4)*. In this case, there always exists at least one stable coalition structure with respect to the Shapley value.*

5. The case of symmetric three-player coalitions and non-symmetric two-player coalitions

5.1. The ES-value as a cooperative solution

In the game (N, v, π) with coalition structure $\pi = \{B_1, \ldots, B_m\}$, the components of the ES-value are defined as (1). Consider the cooperative games with characteristic function $v(\cdot)$ determined by the following way:

$$v(\{1,2,3,4\}) = c, v(\{1\}) = v(\{2\}) = v(\{3\}) = v(\{4\}) = 0,$$

$$v(\{1,2\}) = v(\{1,3\}) = v(\{1,4\}) = c_1, v(\{2,3\}) = v(\{2,4\}) = v(\{3,4\}) = c_2, \quad (5)$$

$$v(\{1,2,3\}) = v(\{1,2,4\}) = v(\{1,3,4\}) = v(\{2,3,4\}) = c_3.$$

The ES values calculated for all possible coalition structures and "Stable if" condition are represented in Table 5.

There is an obvious result. From Table 5, we notice that if $c \geq 0$, then coalition structure $\pi_1 = \{\{1,2,3,4\}\}$ is always stable with respect to the ES-value for any $c_i, i = 1,2,3$.

There are another cases:

Table 5. The ES-value for a four-player game determined by (5) and "Stable if" condition.

π	ψ_1^π	ψ_2^π	ψ_3^π	ψ_4^π	"Stable if" condition
$\pi_1 = \{\{1,2,3,4\}\}$	$c/4$	$c/4$	$c/4$	$c/4$	$c \geq 0$
$\pi_2 = \{\{1\},\{2,3,4\}\}$	0	$c_3/3$	$c_3/3$	$c_3/3$	$\begin{cases} c_3 \geq \max\{0, 3c_1/2\} \\ c \leq 0 \end{cases}$
$\pi_3 = \{\{2\},\{1,3,4\}\}$	$c_3/3$	0	$c_3/3$	$c_3/3$	$\begin{cases} c_3 \geq \max\{0, 3c_1/2, 3c_2/2\} \\ c \leq 0 \end{cases}$
$\pi_4 = \{\{3\},\{1,2,4\}\}$	$c_3/3$	$c_3/3$	0	$c_3/3$	$\begin{cases} c_3 \geq \max\{0, 3c_1/2, 3c_2/2\} \\ c \leq 0 \end{cases}$
$\pi_5 = \{\{4\},\{1,2,3\}\}$	$c_3/3$	$c_3/3$	$c_3/3$	0	$\begin{cases} c_3 \geq \max\{0, 3c_1/2, 3c_2/2\} \\ c \leq 0 \end{cases}$
$\pi_6 = \{\{1,2\},\{3,4\}\}$	$c_1/2$	$c_1/2$	$c_2/2$	$c_2/2$	$\begin{cases} c_1 \geq \max\{0, 2c_3/3\} \\ c_2 \geq \max\{0, 2c_3/3\} \end{cases}$
$\pi_7 = \{\{1,3\},\{2,4\}\}$	$c_1/2$	$c_2/2$	$c_1/2$	$c_2/2$	$\begin{cases} c_1 \geq \max\{0, 2c_3/3\} \\ c_2 \geq \max\{0, 2c_3/3\} \end{cases}$
$\pi_8 = \{\{1,4\},\{2,3\}\}$	$c_1/2$	$c_2/2$	$c_2/2$	$c_1/2$	$\begin{cases} c_1 \geq \max\{0, 2c_3/3\} \\ c_2 \geq \max\{0, 2c_3/3\} \end{cases}$
$\pi_9 = \{\{1\},\{2\},\{3,4\}\}$	0	0	$c_2/2$	$c_2/2$	$\begin{cases} c_1 \leq 0 \\ c_2 \geq 0 \\ c_3 \leq 0 \end{cases}$
$\pi_{10} = \{\{1\},\{3\},\{2,4\}\}$	0	$c_2/2$	0	$c_2/2$	$\begin{cases} c_1 \leq 0 \\ c_2 \geq 0 \\ c_3 \leq 0 \end{cases}$
$\pi_{11} = \{\{1\},\{4\},\{2,3\}\}$	0	$c_2/2$	$c_2/2$	0	$\begin{cases} c_1 \leq 0 \\ c_2 \geq 0 \\ c_3 \leq 0 \end{cases}$
$\pi_{12} = \{\{1,2\},\{3\},\{4\}\}$	$c_1/2$	$c_1/2$	0	0	$\begin{cases} c_1 \geq 0 \\ c_2 \leq 0 \\ c_3 \leq 0 \end{cases}$
$\pi_{13} = \{\{1,3\},\{2\},\{4\}\}$	$c_1/2$	0	$c_1/2$	0	$\begin{cases} c_1 \geq 0 \\ c_2 \leq 0 \\ c_3 \leq 0 \end{cases}$
$\pi_{14} = \{\{1,4\},\{2\},\{3\}\}$	$c_1/2$	0	0	$c_1/2$	$\begin{cases} c_1 \geq 0 \\ c_2 \leq 0 \\ c_3 \leq 0 \end{cases}$
$\pi_{15} = \{\{1\},\{2\},\{3\},\{4\}\}$	0	0	0	0	$\begin{cases} c_1 \leq 0 \\ c_2 \leq 0 \end{cases}$

1. Let $c \geq 0$. When $c_1 \geq 0$, $c_2 \geq 0$, $c_3 \geq 0$, from Fig. 9, we can conclude that the coalition structures π_6, π_7, π_8 are stable when solutions of these three systems of inequalities cover the first octant in region II. When $c_1 \geq 0$, $c_2 \geq 0$, $c_3 \leq 0$, from Table 5, we can conclude that coalition structures π_6, π_7, π_8 are always stable. When $c_1 \geq 0$, $c_2 \leq 0$, $c_3 \geq 0$, only coalition structure π_1 is always stable.

When $c_1 \geq 0$, $c_2 \leq 0$, $c_3 \leq 0$, from Table 5, we can observe that coalition structures π_{12}, π_{13}, π_{14} are always stable.

2. Continue to consider the case when $c \geq 0$. When $c_1 \leq 0$, $c_2 \geq 0$, $c_3 \geq 0$, from Table 5, we know that only coalition structures π_1 is stable. When $c_1 \leq 0$, $c_2 \geq 0$, $c_3 \leq 0$, from Table 5, we know that coalition structures π_9, π_{10}, π_{11} are stable. When $c_1 \leq 0$, $c_2 \leq 0$, $c_3 \geq 0$ and $c_1 \leq 0$, $c_2 \leq 0$, $c_3 \leq 0$, from Table 5, we know that coalition structure π_{15} is stable.

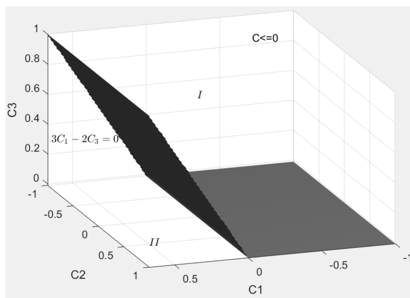

Fig. 7. Coalition structure π_2 is stable in region I with respect to the ES-value.

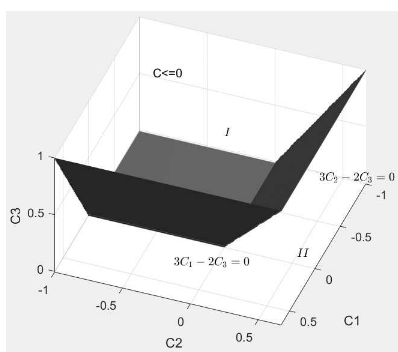

Fig. 8. Coalition structures π_3,π_4,π_5 are stable II with respect to ES-value.

3. Consider the case when $c \leq 0$. When $c_1 \geq 0$, $c_2 \geq 0$, $c_3 \geq 0$, from Fig. 10, we can find that coalition structures π_2 is stable when solutions of these two systems of inequalities cover the region *III, IV*; coalition structures π_3, π_4, π_5 are stable when solutions cover region IV and coalition structures π_6, π_7, π_8 are stable when solutions cover the region *II.* However, we can find that the solutions of these seven systems of inequalities cannot cover the whole first octant. Because

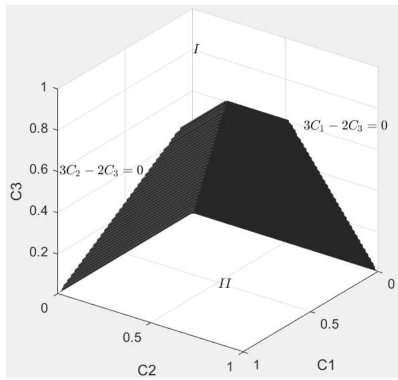

Fig. 9. Coalition structures π_6,π_7,π_8 are stable II with respect to ES-value.

there doesn't exist any stable coalition structure in region I in Fig. 10 which is the set $\{c \leq 0, 3c_1 \geq 2c_3, 3c_2 \leq 2c_3, c_i \geq 0, i = 1, 2, 3\}$.

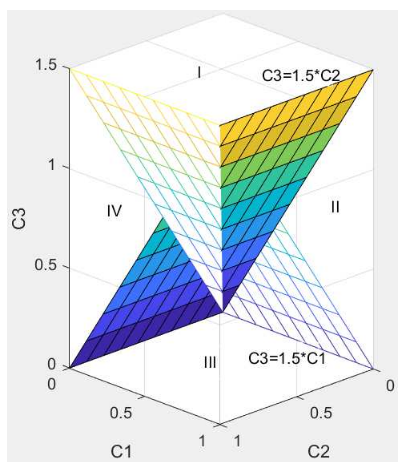

Fig. 10. Case $c \leq 0$ and $c_1, c_2, c_3 \geq 0$.

4. When $c \leq 0$ and $c_1 \geq 0$, $c_2 \geq 0$, $c_3 \leq 0$, from Table 5, we can conclude that coalition structures π_6, π_7, π_8 are always stable because solutions of these three systems of inequalities cover the whole fifth octant constructed by c_1, c_2, c_3. Therefore, there always exists stable coalition structure when $c \leq 0$ and $c_1 \geq 0$, $c_2 \geq 0$, $c_3 \leq 0$.
5. When $c \leq 0$ and $c_1 \geq 0$, $c_2 \leq 0$, $c_3 \geq 0$, from Table 5 and Fig. 11, we can conclude that coalition structures π_2, π_3, π_4, π_5 are stable when solutions of

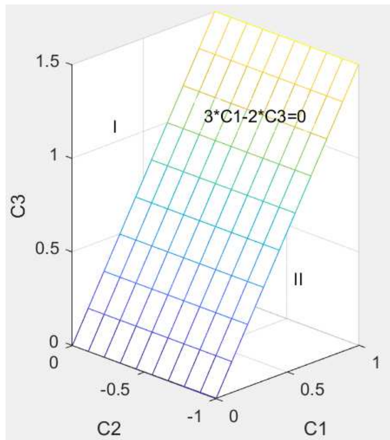

Fig. 11. Case when $c \leq 0$ and $c_1, c_3 \geq 0, c_2 \leq 0$.

these four systems of inequalities cover the fourth octant in region *I*. However, we can find that the solutions of these four systems of inequalities cannot cover the whole fourth octant. Because there doesn't exists any stable coalition structure in region *II* in Fig. 11 which is the set $\{c \leq 0, 3c_1 \geq 2c_3, c_1 \geq 0, c_2 \leq 0, c_3 \geq 0\}$.

6. Continue to consider the case $c \leq 0$ and $c_1 \geq 0$, $c_2 \leq 0$, $c_3 \leq 0$. From Table 5, we can conclude that coalition structures π_{12}, π_{13}, π_{14} are always stable because the solutions of these three systems of inequalities cover the whole eighth octant constructed by c_1, c_2, c_3.
7. Let $c \leq 0$. When $c_1 \leq 0, c_2 \geq 0, c_3 \geq 0$, from Fig. 7, we can observe that coalition structures π_2 is always stable when solutions of this system of inequalities cover the whole second octant in region I. When $c_1 \leq 0$, $c_2 \geq 0$, $c_3 \leq 0$, from Table 5, we can find that coalition structures π_9, π_{10}, π_{11} are always stable where solutions of these three systems of inequalities cover the whole sixth octant. When $c_1 \leq 0$, $c_2 \leq 0$, $c_3 \geq 0$, we can conclude that coalition structure π_2 is stable from Fig. 7, coalition structures π_3, π_4, π_5 are stable from Fig. 8 and we can find that the solutions of these four systems of inequalities cover the whole third octant. When $c_1 \leq 0$, $c_2 \leq 0$, $c_3 \leq 0$, only coalition structure π_{15} is always stable.

The following proposition is proved.

Proposition 5. *Let characteristic function be given by* (5). *In this case, at least one stable coalition structure always exists excluding the cases: (i)* $\{c \leq 0, 3c_1 \geq 2 \times c_3, 3c_2 \leq 2c_3, c_i \geq 0, i = 1, 2, 3\}$ *and (ii)* $\{c \leq 0, 3c_1 \geq 2c_3, c_1 \geq 0, c_2 \leq 0, c_3 \geq 0\}$.

5.2. The Shapley value as a cooperative solution

In the game (N, v, π) with coalition structure $\pi = \{B_1, \ldots, B_m\}$, the components of the Shapley value are defined as (3). Consider the cooperative game with characteristic function $v(\cdot)$ determined by (5). The Shapley value calculated for all possible coalition structures are represented in Table 7 (see Appendix) and "Stable if" conditions are represented in Table 8 (see Appendix).

In this case, there are many possible sets of combinations of parameters for which there are no stable coalition structures and all of the results are obtained by calculations. Here, we find the stable coalition structures which can satisfy the conditions of c, c_1, c_2, c_3. Maybe there also exists another coalition structure which is stable in the same range but we focus on existence result.

First, we describe the sets which always have stable coalition structures:

1. When $c_1 \geq 0$, $c_2 \geq 0$, $c_3 \leq 0$, coalition structures π_6, π_7, π_8 are always stable for any c. When $c_1 \geq 0$, $c_2 \leq 0$, $c_3 \leq 0$, coalition structures π_{12}, π_{13} are always stable for any c. When $c_1 \leq 0$, $c_2 \geq 0$, $c_3 \leq 0$, coalition structures π_9, π_{10}, π_{11} are always stable for any c. When $c_1 \leq 0$, $c_2 \leq 0$, $c_3 \geq 0$ and $c_1 \leq 0$, $c_2 \leq 0$, $c_3 \leq 0$, coalition structure π_{15} is always stable for any c.
2. Consider the case $c_1 \leq 0$, $c_2 \geq 0$, $c_3 \geq 0$. When $c \geq 0$, coalition structures π_1, π_2 are stable and the solutions of these two systems of inequalities cover the whole second octant constructed by c_1, c_2, c_3. When $c \leq 0$, coalition structure π_2 is always stable.

Second, we consider the regions where there are no stable coalition structures:

1. Consider the case when $c \geq 0$ and $c_1 \geq 0$, $c_2 \geq 0$, $c_3 \geq 0$, from Table 8 and calculations, we can conclude that coalition structures π_1, π_2, π_3, π_4, π_5, π_6, π_7, π_8 can be stable in the first octant constructed by c_1, c_2, c_3. But the solutions of these eight systems of inequalities cannot cover the whole first octant. Therefore, there doesn't always exist stable coalition structure when $c \geq 0$, $c_1 \geq 0$, $c_2 \geq 0$, $c_3 \geq 0$.
2. Continue to consider the case $c \geq 0$. When $c_1 \geq 0$, $c_2 \leq 0$, $c_3 \geq 0$, from Table 8 and calculations, we can conclude that coalition structures π_1, π_3, π_4, π_5, π_{12}, π_{13} can be stable in the fourth octant constructed by c_1, c_2, c_3. But the solutions of these six systems of inequalities cannot cover the whole fourth octant. Therefore, there doesn't always exist stable coalition structure when $c \geq 0$, $c_1 \geq 0$, $c_2 \leq 0$, $c_3 \geq 0$.
3. Similarly, consider the case when $c \leq 0$ and $c_1 \geq 0$, $c_2 \geq 0$, $c_3 \geq 0$, from Table 8 and calculations, we can conclude that coalition structures π_2, π_3, π_4, π_5, π_6, π_7, π_8 can be stable in the first octant constructed by c_1, c_2, c_3. But the solutions of these seven systems of inequalities cannot cover the whole first octant. Therefore, there doesn't always exist stable coalition structure when $c \leq 0$, $c_1 \geq 0$, $c_2 \geq 0$, $c_3 \geq 0$.
4. Consider the case when $c \leq 0$ and $c_1 \geq 0$, $c_2 \leq 0$, $c_3 \geq 0$, from Table 8 and calculations, we can conclude that coalition structures π_2, π_3, π_4, π_5, π_{12}, π_{13} can be stable in the fourth octant constructed by c_1, c_2, c_3. But the solutions of these six systems of inequalities cannot cover the whole fourth octant. Therefore, there doesn't always exist stable coalition structure when $c \leq 0$ and $c_1 \geq 0$, $c_2 \leq 0$, $c_3 \geq 0$.

Thus we prove Proposition 6.

Proposition 6. *Let characteristic function be given by* (5). *In this case, the stable coalition structure always exists with respect to the Shapley value excluding the cases: (i)* $\{c_1 \geq 0, c_2 \geq 0, c_3 \geq 0,$ *any* $c\}$, *and (ii)* $\{c_1 \geq 0, c_2 \leq 0, c_3 \geq 0,$ *any* $c\}$.

6. The case of non-symmetric two-player and three-player coalitions

For this case, we only consider the ES-value as a cooperative solution and find the stable coalition structures with respect to the ES-value which can cover the whole region. Maybe other coalition structure can be stable in the same region but we focus on existence result. And we also find the region for which there doesn't exist any stable coalition structure.

In the game (N, v, π) with coalition structure $\pi = \{B_1, \dots, B_m\}$, the components of the ES-value are defined as (1). Consider the cooperative games with characteristic function $v(\cdot)$ determined by the following way:

$$v(\{1,2,3,4\}) = c, v(\{1\}) = v(\{2\}) = v(\{3\}) = v(\{4\}) = 0,$$

$$v(\{1,2\}) = v(\{1,3\}) = v(\{1,4\}) = c_1, v(\{2,3\}) = v(\{2,4\}) = v(\{3,4\}) = c_2, \quad (6)$$

$$v(\{1,2,3\}) = v(\{1,2,4\}) = c_3, v(\{1,3,4\}) = v(\{2,3,4\}) = c_4.$$

The ES-values calculated for all possible coalition structures and "Stable if" condition are represented in Table 9 (see Appendix).

In this case, there exist many sets of parameters for which there doesn't exist stable coalition structure and all of the results are obtained by calculations. Here, we introduce the stable coalition structures. Maybe there also exists another coalition structure which is stable in the same range but we focus on the existence result.

From Table 9, we notice that there is an obvious result, which is the following. If $c \geq 0$, coalition structure $\pi_1 = \{\{1,2,3,4\}\}$ is always stable with respect to the ES-value for any $c_i, i = 1,2,3,4$. Therefore, we should consider the case when $c \leq 0$.

First, we find the sets for which there always exist stable coalition structures:

1. Consider the case when $c \leq 0$. When $c_1 \geq 0$, $c_2 \geq 0$, $c_3 \leq 0$, $c_4 \geq 0$, from calculations and Table 9, we can conclude that the coalition structures π_2 and π_6 are stable where the solutions of these two systems of inequalities cover the whole fifth octant constructed by c_1, c_2, c_3. When $c_1 \geq 0$, $c_2 \geq 0$, $c_3 \leq 0$, $c_4 \leq 0$, the coalition structures π_7, π_8 are always stable. When $c_1 \geq 0$, $c_2 \leq 0$, $c_3 \leq 0$, $c_4 \geq 0$, the coalition structure π_{12} is always stable. When $c_1 \geq 0$, $c_2 \leq 0$, $c_3 \leq 0$, $c_4 \leq 0$, the coalition structures π_{12}, π_{13}, π_{14} are always stable.
2. Continue to consider the case when $c \leq 0$. When $c_1 \leq 0$, $c_4 \geq 0$, we can conclude that the coalition structures π_2 is always stable for any $c_i, i = 2,3$. When $c_1 \leq 0$, $c_2 \geq 0$, $c_4 \leq 0$, we obtain that the coalition structures π_9 is always stable for any c_3. When $c_1 \leq 0$, $c_2 \leq 0$, $c_4 \leq 0$, the coalition structures π_{15} is always stable for any c_3.

Second, we introduce the regions for which there are no stable coalition structures:

1. Let $c \leq 0$ and $c_1 \geq 0$, $c_2 \geq 0$, $c_3 \geq 0$, $c_4 \leq 0$. By calculations and Table 9, we conclude that coalition structures π_4, π_5, π_6, π_7, π_8 can be stable. However, the solutions of these five systems of inequalities cannot cover the whole region. That is, there doesn't exist any stable coalition structure when the region is constructed by the set $\{c \leq 0, c_1 \geq 0, c_2 \geq 0, c_3 \geq 0, c_4 \leq 0, 2c_3 \leq 3c_1, 2c_3 \geq 3 \times c_2\}$.

2. When $c \leq 0$ and $c_1 \geq 0$, $c_2 \leq 0$, $c_3 \geq 0$, $c_4 \geq 0$, by calculation and Table 9, we easily prove that coalition structures π_2, π_3, π_4, π_5 can be stable. However, the solutions of these four systems of inequalities do not cover the whole region. That is, there doesn't exist any stable coalition structure when the region is $L_1 \setminus L_2$, where $L_1 = \{c \leq 0, c_1 \geq 0, c_2 \leq 0, c_3 \geq 0, c_4 \geq 0\}$ and $L_2 = \{c \leq 0, c_1 \geq 0, c_2 \leq 0, c_3 \geq 0, c_4 \geq 0, 2c_4 \geq 3c_1, 2c_3 \geq 3c_1\}$.

3. When $c \leq 0$ and $c_1 \geq 0$, $c_2 \leq 0$, $c_3 \geq 0$, $c_4 \leq 0$, by calculations and Table 9, we obtain that coalition structures π_4, π_5 can be stable. However, the solutions of these two systems of inequalities do nor cover the whole region. That is, there doesn't exist any stable coalition structure when the region is $L_3 \setminus L_4$, where $L_3 = \{c \leq 0, c_1 \geq 0, c_2 \leq 0, c_3 \geq 0, c_4 \leq 0\}$ and $L_4 = \{c \leq 0, c_1 \geq 0, c_2 \leq 0, c_3 \geq 0, c_4 \leq 0, 2c_3 \geq 3c_1, 2c_3 \geq 3c_2\}$.

4. Specially, when $c \leq 0$ and $c_1 \geq 0$, $c_2 \geq 0$, $c_3 \geq 0$, $c_4 \geq 0$, by calculations and Table 9, we obtained that the stable coalition structures cannot cover the whole region. But it is more difficult to describe the exact region. Therefore, we denote L_5 is the region which doesn't exist stable coalition structure when $c \leq 0$ and $c_1 \geq 0$, $c_2 \geq 0$, $c_3 \geq 0$, $c_4 \geq 0$.

The following result is proved.

Proposition 7. *Let characteristic function be given by* (6)*. In this case, the coalition structure always exists excluding the cases: (i)* $\{c \leq 0, c_1 \geq 0, c_2 \geq 0, c_3 \geq 0, c_4 \leq 0, 2c_3 \leq 3c_1, 2c_3 \geq 3c_2\}$*, (ii)* $L_1 \setminus L_2$*, (iii)* $L_3 \setminus L_4$*, and (iv)* L_5*.*

7. Conclusion

In the paper we consider the class of four-player games with coalition structures and find the conditions of existence of stable coalition structures with respect to the ES-value and the Shapley value for special characteristic functions. The definition of stable coalition structure is similar to the Nash equilibrium meaning that there is no player who may benefit from deviation from the coalition which is a part of stable coalition structure.

The existence of stable coalition structure with respect to the ES-value and the Shapley value in three-player games has been proved in (Sedakov et al., 2013). In case of four-player games the problem is more difficult. Therefore we consider some special cases of characteristic function and find the conditions for which at least one stable coalition structure exists. We also prove that for some conditions there are no stable coalition structures with respect to the ES-value and the Shapley value. This result proves that in general case in four-player games there does not always exist stable coalition structure with respect to the ES-value and the Shapley value.

Appendix

Table 6. The Shapley value for a four-player game determined by (4).

π	ψ_1^π	ψ_2^π	ψ_3^π	ψ_4^π
$\pi_1 = \{\{1,2,3,4\}\}$	$\frac{3c+2c_2-2c_3}{12}$	$\frac{3c+2c_2-2c_3}{12}$	$\frac{3c-2c_2+2c_3}{12}$	$\frac{3c-2c_2+2c_3}{12}$
$\pi_2 = \{\{1\},\{2,3,4\}\}$	0	$c_3/3$	$c_3/3$	$c_3/3$
$\pi_3 = \{\{2\},\{1,3,4\}\}$	$c_3/3$	0	$c_3/3$	$c_3/3$
$\pi_4 = \{\{3\},\{1,2,4\}\}$	$c_2/3$	$c_2/3$	0	$c_2/3$
$\pi_5 = \{\{4\},\{1,2,3\}\}$	$c_2/3$	$c_2/3$	$c_2/3$	0
$\pi_6 = \{\{1,2\},\{3,4\}\}$	$c_1/2$	$c_1/2$	$c_1/2$	$c_1/2$
$\pi_7 = \{\{1,3\},\{2,4\}\}$	$c_1/2$	$c_1/2$	$c_1/2$	$c_1/2$
$\pi_8 = \{\{1,4\},\{2,3\}\}$	$c_1/2$	$c_1/2$	$c_1/2$	$c_1/2$
$\pi_9 = \{\{1\},\{2\},\{3,4\}\}$	0	0	$c_1/2$	$c_1/2$
$\pi_{10} = \{\{1\},\{3\},\{2,4\}\}$	0	$c_1/2$	0	$c_1/2$
$\pi_{11} = \{\{1\},\{4\},\{2,3\}\}$	0	$c_1/2$	$c_1/2$	0
$\pi_{12} = \{\{1,2\},\{3\},\{4\}\}$	$c_1/2$	$c_1/2$	0	0
$\pi_{13} = \{\{1,3\},\{2\},\{4\}\}$	$c_1/2$	0	$c_1/2$	0
$\pi_{14} = \{\{1,4\},\{2\},\{3\}\}$	$c_1/2$	0	0	$c_1/2$
$\pi_{15} = \{\{1\},\{2\},\{3\},\{4\}\}$	0	0	0	0

Table 7. The Shapley value for a four-player coalition game determined by (5).

π	ψ_1^π	ψ_2^π	ψ_3^π	ψ_4^π
$\pi_1 = \{\{1,2,3,4\}\}$	$\frac{c+c_1-c_2}{4}$	$\frac{3c-c_1+c_2}{12}$	$\frac{3c-c_1+c_2}{12}$	$\frac{3c-c_1+c_2}{12}$
$\pi_2 = \{\{1\},\{2,3,4\}\}$	0	$c_3/3$	$c_3/3$	$c_3/3$
$\pi_3 = \{\{2\},\{1,3,4\}\}$	$\frac{c_1-c_2+c_3}{3}$	0	$\frac{2c_3-c_1+c_2}{6}$	$\frac{2c_3-c_1+c_2}{6}$
$\pi_4 = \{\{3\},\{1,2,4\}\}$	$\frac{c_1-c_2+c_3}{3}$	$\frac{2c_3-c_1+c_2}{6}$	0	$\frac{2c_3-c_1+c_2}{6}$
$\pi_5 = \{\{4\},\{1,2,3\}\}$	$\frac{c_1-c_2+c_3}{3}$	$\frac{2c_3-c_1+c_2}{6}$	$\frac{2c_3-c_1+c_2}{6}$	0
$\pi_6 = \{\{1,2\},\{3,4\}\}$	$c_1/2$	$c_1/2$	$c_1/2$	$c_1/2$
$\pi_7 = \{\{1,3\},\{2,4\}\}$	$c_1/2$	$c_1/2$	$c_1/2$	$c_1/2$
$\pi_8 = \{\{1,4\},\{2,3\}\}$	$c_1/2$	$c_1/2$	$c_1/2$	$c_1/2$
$\pi_9 = \{\{1\},\{2\},\{3,4\}\}$	0	0	$c_2/2$	$c_2/2$
$\pi_{10} = \{\{1\},\{3\},\{2,4\}\}$	0	$c_2/2$	0	$c_2/2$
$\pi_{11} = \{\{1\},\{4\},\{2,3\}\}$	0	$c_2/2$	$c_2/2$	0
$\pi_{12} = \{\{1,2\},\{3\},\{4\}\}$	$c_1/2$	$c_1/2$	0	0
$\pi_{13} = \{\{1,3\},\{2\},\{4\}\}$	$c_1/2$	0	$c_1/2$	0
$\pi_{14} = \{\{1,4\},\{2\},\{3\}\}$	$c_1/2$	0	0	$c_1/2$
$\pi_{15} = \{\{1\},\{2\},\{3\},\{4\}\}$	0	0	0	0

Table 8. The “Stable if" conditions for the Shapley value in four-player game determined by (5).

π	“Stable if" condition
$\pi_1 = \{\{1,2,3,4\}\}$	$\begin{cases} c + c_1 - c_2 \geq 0 \\ 3c - c_1 + c_2 \geq 0 \end{cases}$
$\pi_2 = \{\{1\},\{2,3,4\}\}$	$\begin{cases} c + c_1 - c_2 \leq 0 \\ c_3 \geq \max\{0, 3c_1/2\} \end{cases}$
$\pi_3 = \{\{2\},\{1,3,4\}\}$	$\begin{cases} 3c - c_1 + c_2 \leq 0 \\ c_1 - c_2 + c_3 \geq 0 \\ 2c_3 - c_1 - 2c_2 \geq 0 \\ 2c_3 - c_1 + c_2 \geq 0 \end{cases}$
$\pi_4 = \{\{3\},\{1,2,4\}\}$	$\begin{cases} 3c - c_1 + c_2 \leq 0 \\ c_1 - c_2 + c_3 \geq 0 \\ 2c_3 - c_1 - 2c_2 \geq 0 \\ 2c_3 - c_1 + c_2 \geq 0 \end{cases}$
$\pi_5 = \{\{4\},\{1,2,3\}\}$	$\begin{cases} 3c - c_1 + c_2 \leq 0 \\ c_1 - c_2 + c_3 \geq 0 \\ 2c_3 - c_1 - 2c_2 \geq 0 \\ 2c_3 - c_1 + c_2 \geq 0 \end{cases}$
$\pi_6 = \{\{1,2\},\{3,4\}\}$	$\begin{cases} c_1 \geq \max\{0, 2c_3/3\} \\ c_1 + 2c_2 - 2c_3 \geq 0 \\ c_2 \geq 0 \end{cases}$
$\pi_7 = \{\{1,3\},\{2,4\}\}$	$\begin{cases} c_1 \geq \max\{0, 2c_3/3\} \\ c_1 + 2c_2 - 2c_3 \geq 0 \\ c_2 \geq 0 \end{cases}$
$\pi_8 = \{\{1,4\},\{2,3\}\}$	$\begin{cases} c_1 \geq \max\{0, 2c_3/3\} \\ c_1 + 2c_2 - 2c_3 \geq 0 \\ c_2 \geq 0 \end{cases}$
$\pi_9 = \{\{1\},\{2\},\{3,4\}\}$	$\begin{cases} c_1 \leq 0 \\ c_2 \geq 0 \\ c_3 \leq 0 \end{cases}$
$\pi_{10} = \{\{1\},\{3\},\{2,4\}\}$	$\begin{cases} c_1 \leq 0 \\ c_2 \geq 0 \\ c_3 \leq 0 \end{cases}$
$\pi_{11} = \{\{1\},\{4\},\{2,3\}\}$	$\begin{cases} c_1 \leq 0 \\ c_2 \geq 0 \\ c_3 \leq 0 \end{cases}$

$\pi_{12} = \{\{1,2\},\{3\},\{4\}\}$	$\begin{cases} c_1 \geq 0 \\ c_2 \leq 0 \\ 2c_3 - c_1 + c_2 \leq 0 \end{cases}$
$\pi_{13} = \{\{1,3\},\{2\},\{4\}\}$	$\begin{cases} c_1 \geq 0 \\ c_2 \leq 0 \\ 2c_3 - c_1 + c_2 \leq 0 \end{cases}$
$\pi_{14} = \{\{1,4\},\{2\},\{3\}\}$	$\begin{cases} c_1 \geq 0 \\ c_2 \leq 0 \\ 2c_3 - c_1 + c_2 \leq 0 \\ c_1 - c_2 + c_3 \leq 0 \end{cases}$
$\pi_{15} = \{\{1\},\{2\},\{3\},\{4\}\}$	$\begin{cases} c_1 \leq 0 \\ c_2 \leq 0 \end{cases}$

Table 9. The ES-value for a four-player game determined by (6) and the "Stable if" conditions.

π	ψ_1^π	ψ_2^π	ψ_3^π	ψ_4^π	"Stable if" condition
$\pi_1 = \{\{1,2,3,4\}\}$	$c/4$	$c/4$	$c/4$	$c/4$	$c \geq 0$
$\pi_2 = \{\{1\},\{2,3,4\}\}$	0	$c_4/3$	$c_4/3$	$c_4/3$	$\begin{cases} c_4 \geq \max\{0, 3c_1/2\} \\ c \leq 0 \end{cases}$
$\pi_3 = \{\{2\},\{1,3,4\}\}$	$c_4/3$	0	$c_4/3$	$c_4/3$	$\begin{cases} c_4 \geq \max\{0, 3c_1/2, 3c_2/2\} \\ c \leq 0 \end{cases}$
$\pi_4 = \{\{3\},\{1,2,4\}\}$	$c_3/3$	$c_3/3$	0	$c_3/3$	$\begin{cases} c_3 \geq \max\{0, 3c_1/2, 3c_2/2\} \\ c \leq 0 \end{cases}$
$\pi_5 = \{\{4\},\{1,2,3\}\}$	$c_3/3$	$c_3/3$	$c_3/3$	0	$\begin{cases} c_3 \geq \max\{0, 3c_1/2, 3c_2/2\} \\ c \leq 0 \end{cases}$
$\pi_6 = \{\{1,2\},\{3,4\}\}$	$c_1/2$	$c_1/2$	$c_2/2$	$c_2/2$	$\begin{cases} c_1 \geq \max\{0, 2c_4/3\} \\ c_2 \geq \max\{0, 2c_3/3\} \\ c \geq 0 \end{cases}$
$\pi_7 = \{\{1,3\},\{2,4\}\}$	$c_1/2$	$c_2/2$	$c_1/2$	$c_2/2$	$\begin{cases} c_1 \geq \max\{0, 2c_3/3, 2c_4/3\} \\ c_2 \geq \max\{0, 2c_3/3, 2c_4/3\} \end{cases}$
$\pi_8 = \{\{1,4\},\{2,3\}\}$	$c_1/2$	$c_2/2$	$c_2/2$	$c_1/2$	$\begin{cases} c_1 \geq \max\{0, 2c_3/3, 2c_4/3\} \\ c_2 \geq \max\{0, 2c_3/3, 2c_4/3\} \end{cases}$
$\pi_9 = \{\{1\},\{2\},\{3,4\}\}$	0	0	$c_2/2$	$c_2/2$	$\begin{cases} c_1 \leq 0 \\ c_2 \geq 0 \\ c_4 \leq 0 \end{cases}$

$\pi_{10} = \{\{1\},\{3\},\{2,4\}\}$	0	$c_2/2$	0	$c_2/2$	$\begin{cases} c_1 \leq 0 \\ c_2 \geq 0 \\ c_3 \leq 0 \\ c_4 \leq 0 \end{cases}$
$\pi_{11} = \{\{1\},\{4\},\{2,3\}\}$	0	$c_2/2$	$c_2/2$	0	$\begin{cases} c_1 \leq 0 \\ c_2 \geq 0 \\ c_3 \leq 0 \\ c_4 \leq 0 \end{cases}$
$\pi_{12} = \{\{1,2\},\{3\},\{4\}\}$	$c_1/2$	$c_1/2$	0	0	$\begin{cases} c_1 \geq 0 \\ c_2 \leq 0 \\ c_3 \leq 0 \end{cases}$
$\pi_{13} = \{\{1,3\},\{2\},\{4\}\}$	$c_1/2$	0	$c_1/2$	0	$\begin{cases} c_1 \geq 0 \\ c_2 \leq 0 \\ c_3 \leq 0 \\ c_4 \leq 0 \end{cases}$
$\pi_{14} = \{\{1,4\},\{2\},\{3\}\}$	$c_1/2$	0	0	$c_1/2$	$\begin{cases} c_1 \geq 0 \\ c_2 \leq 0 \\ c_3 \leq 0 \\ c_4 \leq 0 \end{cases}$
$\pi_{15} = \{\{1\},\{2\},\{3\},\{4\}\}$	0	0	0	0	$\begin{cases} c_1 \leq 0 \\ c_2 \leq 0 \end{cases}$

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