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Competitive and Cooperative Behavior in Distribution Networks*

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Abstract This paper considers the problem of cooperation in supply networks. The model is based on distribution network, which includes several manufactures, single distributor and multi retailers, operated and competed in consumer markets that are functioning according to the Cournot model with the linear demand. All participants in a chain are trying to maximize their profit. A multi-stage hierarchic game was carried out. At the first step, we construct the competitive solution for such supply network as the perfect Nash equilibrium in the multi-step hierarchical game in the closed form. At the second step, we construct the cooperative solution for the network, where winnings of all participants in the found perfect Nash equilibrium are considered as the status quo point. Cooperative decision we calculate in the form of the weighted Nash bargaining solution, which comes down to the solution of a separable nonlinear programming problem. Numerical example for the network shows that cooperative decision is more profitable than competitive decision for all participants.

Keywords: distribution network, competitive and cooperative decisions, multi-stage hierarchical game, perfect Nash equilibrium, weighted Nash bargaining solution

1. Introduction

In the last several years, the evolution of supply chain management recognized that a business process consists of several decentralized firms and operational decisions of these different entities influence each other's profit, and thus the profit of the whole supply chain. With this understanding came a great deal of interest in modeling and understanding the impact of strategic operational decisions of the various players in supply chains. To effectively model and analyze decision making in such multi-person situation where the outcome depends on the choice made by every party, game theory is a natural choice. Researchers in supply chain management now use tools from game theory and economics to understand, predict, and help managers to make strategic operational decisions in complex multiagent supply chain systems (Nagarajan and Sosic, 2006).

This paper considers the problem of cooperation in distribution network, which includes several manufactures, single distributor and multi retailers, operated and competed in consumer markets that are functioning according to the Cournot model with the linear demand. Two types of behavior were considered in the research. As the competitive behavior in this paper we recognize the perfect Nash equilibrium

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solution, and as the cooperative behavior – the weighted Nash bargaining solution. At the first step, we construct the competitive solution for such supply network as the perfect Nash equilibrium in the multi-step hierarchical game in closed form. At the second step, we construct the cooperative solution for the network, where winnings of all participants in the found perfect Nash equilibrium are considered as a point of the status quo. As cooperative decision we calculate the weighted Nash bargaining solution, which comes down to the solution of a separable nonlinear programming problem with concave payoff function, which has a unique solution.

This study can be divided into two main logical parts. The first one provides theoretical results; the second one presents the computational results of the case of Russian distribution network. The research organized as follows: theoretical background in cooperative games in supply chain management. The next two parts are devoted to description and formulation of the problem and finding equilibrium and cooperative solutions. Further, the case of Russian distribution network is considered. The next part provides comparative analysis of the results. Conclusion and limitations of the study and directions for future research are presented in the final part.

2. Theoretical background

Supply chains (SC) have been characterized as organizational networks that are linked through upstream and downstream processes and activities that produce value in the form of products and services delivered to the hands of the ultimate customer (Christopher, 1998). Chopra & Meindl defined the term of supply chain management as follow: supply chain management involves the management of flows between and among stages in a supply chain to maximize total profitability (Chopra and Meindl, 2001). Handfield and Nichols (1999) defined supply chain management as the integration of activities through improved supply chain relationships, to achieve a sustainable competitive advantage. As can be seen from the above definitions, all of them have more or less in common that supply chains are based on cooperation in order to generate a benefit. Some authors claim that, in the future, competition will take place between supply chains rather than between individual companies.

Supply chains have nowadays more and more complex structures, and may involve partners from different domains, size, countries, therefore of different cultures. In that context, the performance of the partnership can be assessed through technical criteria (Ounnar et al., 2007), but is also concerned with behavioral issues (Möllering, 2003). According to Sepulveda Rojas and Frein (2008), cooperation is the following level of the relationship: companies are more tightly tied together, sharing more information than they would even in an extended armlength relationship. In case of cooperation, there are fewer suppliers and longer-term supplier–customer relationships. Cooperation is therefore an upper level of relationship, determined by the degree of information sharing.

Many researchers have taken multiple perspectives and have developed many theories to understand the activities involved in inter-organizational cooperation. Since the emergence of international cooperation and the development of vertical disintegration, managers have paid more attention to inter-firm spanning activities than to the optimization of interior processes (Buhman et al., 2005; Chen and Paulraj, 2004). The common objective of academics and practitioners is to determine how a firm can achieve a sustainable competitive advantage. As a concept for coordinating information and material between companies, supply chain management has a significant potential in creating competitive advantage for the companies involved. The great potential of supply chain management for competitiveness has often been mentioned in the literature (Chopra & Meindl, 2001). The main advantages that can be derived from choosing the right supply chain are an improvement in efficiency, e.g. due to high turns of inventory, or an increase in market responsiveness, e.g. by shorter lead time (Fisher, 1997). Another important benefit is to fight cooperatively against a phenomenon commonly referred to as the "bullwhip" effect which was first observed by logistic executives at P&G concerning disposable diapers (Lee et al., 1997; Forrester, 1958). By cooperation across the participants of supply chain, the bullwhip effect can be mitigated. In that sense, supply chain management is currently a major issue within the academic discussion.

In order to generate advantages, contracts for vertical cooperation are established within supply chains. Cooperative interactions in a supply chain have been comprehensively researched in the past. Cachon and Larivier (2005) investigated several types of supply chain contracts to promote cooperation between a manufacturer and a retailer. Li et al. (2000), Huang and Li (2001), and Zhang et al. (2012) discussed cooperative advertising models in a manufacturer-retailer supply chain and investigated the effect of cooperation on investment effort levels. Leng and Parlar (2009) analyzed how the cooperative effect would influence cost savings from a supply chain with a manufacturer, a distributor and a retailer. The above studies aim at the issues of cooperation in forward supply chains. Even though it has been widely discussed in the academic literature, there is still a lack of applied rational methodologies analyzing supply chain management.

There is a shortage of research in cooperative models in supply chains. Operational research models are mathematical instruments to solve decision problems. Most of them deal with one decision maker situations. However, in real world, it is very common that the result of decisions depends also on other decision makers' choices, i.e. in the real world many decision situations are interactive. Operations management focused on single-firm analysis in the past. Its goal was to provide managers with suitable tools to improve the performance of their firms. Nowadays, business decisions are dominated by the globalization of markets and should consider the increasing competition among firms. Further, more and more products reach the customer through supply chains that are composed of independent firms. Following these trends, research in supply chain has shifted its focus from single-firm analysis to multi-firm analysis, in particular to improving the efficiency and performance of supply chains under decentralized control. The main characteristics of such chains are that the firms in the chain are independent actors who try to optimize their individual objectives, and that the decisions taken by a firm do also affect the performance of the other parties in the supply chain. These interactions among firms' decisions ask for alignment and coordination of actions and, therefore, game theory is very well suited to deal with these interactions.

There is an increasing number of documents that apply tools, techniques, and models from game theory to supply chain problems. The authors discuss both non-cooperative and cooperative game theory in static and dynamic settings. Additionally, Cachon (1998) reviewed competitive supply chain inventory management, and Cachon (2003) reviewed and extends the supply chain literature on the management of incentive conflicts with contracts. Papers using cooperative game theory to study supply chain management are scarce, but the use of cooperative games in this context is becoming more popular. Nagarajan and Sosic (2008) reviewed and extended the problem of bargaining and negotiations in supply chain relationships. A very recent survey on applications of cooperative game theory to supply chain management, the so-called supply chain collaboration, is Meca and Timmer (2008). Thus, one challenging field within operations research is that of game theoretical models in operations research.

Game theoretic models of supply chains can be classified into non-cooperative (Cachon and Netessine, 2004) or cooperative (Slikker and Nouweland, 2001; Nagarajan and Sosic, 2008). The cooperative game studies intrachain relationships, which have three issues: what coalitions will form; how the outcome be divided; and whether the outcomes are stable and robust (Nagarajan and Sosic, 2008). Cooperative games may further be classified into coalitional form, alliance and negotiation game theoretic models. The coalitional form game assumes that there is a defined set of players, a combination of which form a coalition. The members in the coalition collectively generate a value that is independent from non-members or other coalitions. The feasible outcomes represent the total set of all possible outcomes that players may realize. Players may select their respective coalition from the set of feasible outcomes such that each player's respective payoff is maximized (Xue, 1998). However, there is a dynamic process of coalition formation. Once a player joins a coalition, he may join or form an alternative coalition with a higher payoff. This deviation process continues until a stable equilibrium is reached (Konishi and Ray, 2003). Alternatively, members may initially decide on and successfully form a coalition and subsequently negotiate allocation rules with chain members. These are alliance models. In one-echelon horizontal alliances, Gerchak and Gupta (1991) studied the cost allocation for centralized inventory between horizontal retailers, while Hartman and Dror (1996) studied centralized inventory between stores. Meca et al. (2004) studied a single inventory model with n retailers to develop a proportional rule to allocate joint-ordering costs. Plambeck and Taylor (2004) studied a two-echelon supply chain where a manufacturer negotiates and efficiently allocates its capacity among n buyers. Leng and Parlar (2009) analyzed the allocation of cost savings from sharing demand information in a three-echelon supply chain that includes a supplier, manufacture and retailer. Besides negotiating allocation rules, players in a successful coalition may negotiate the terms of trade, called negotiation models.

In this paper, we specifically investigate the problem of cooperation in distribution network. We constructed the perfect Nash equilibrium solution as a competitive behavior. Using the obtained solution as a point of the status quo, we constructed the weighted Nash bargaining solution as a cooperative solution in distribution network.

3. Description and formulation of the problem

Let us look at the tree-like graph $G_1 = (X_1, F_1)$ where X_1 is a set of nodes and F_1 is a function of alternatives (Petrosyan et al., 2014). The root node of this tree can be named as x^*. Also let us look at the graph $G_2 = (X_2, F_2)$ such that:

1. There is unique node $x^* \in X_2$ such that $F_2(x^*) = \emptyset$;

2. For all $x \in X_2 \backslash x^* : |F_2(x)| = 1$, where $|F_2(x)|$ means a cardinality of the set $F_2(x)$.

The example of such a graph with the root node $x^* = x_{11}$ is depicted on the Fig.1.

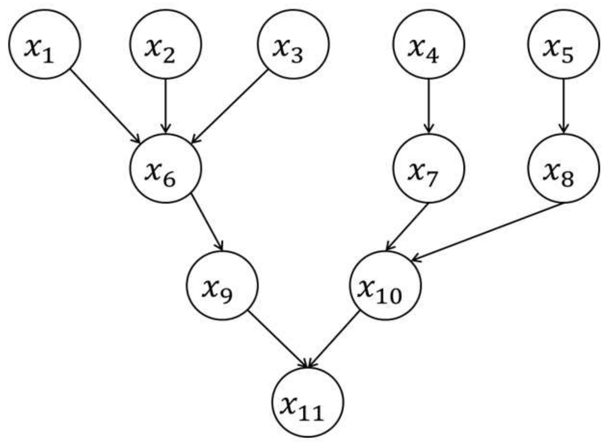

Fig. 1. The graph with the root node $x^* = x_{11}$.
Source: Authors' own

Consider the graph $G = (X, F)$, where

$$X = X_1 \cup X_2;$$

$$F = \begin{cases} F_1(x), & x \in X_1 \backslash x^*; \\ F_2(x), & elsewhere. \end{cases}$$

We will say that the graph G has an hourglasses structure. The example of such a graph is depicted below (Fig. 2).

In the set of nodes X let us define the set $\overline{X}$ of final nodes: $\overline{X} = \{x \in X \mid F(x) = \emptyset\}$. Then in the set of nodes $X \backslash \overline{X}$ we define the sets of $X_1, \ldots, X_l$ in the following way:

$$X_1 = \{x \mid \nexists y \in X : F(y) = x\}; \tag{1}$$

$$X_{k+1} = \bigcup_{x \in X_k} (F(x) \backslash \overline{X}), \text{ if } \bigcup_{x \in X_k} (F(x) \backslash \overline{X}) \neq \emptyset, \quad k = 1, 2, \ldots, l-1;$$

$$X_l = \overline{X}.$$

Definition 1. Subset of nodes $X_i \subset X$, $i = 1, \ldots, l$ will be named as the set of nodes of the level i.

We will denote the nodes x from the set X as x_j^i, where the upper index is equal to the number of the level X_i this node is situated and the lower index to the order number of this node in the set X_i. Also by m_i we will understand the number of the nodes of the level i, i.e. $m_i = |X_i|$.

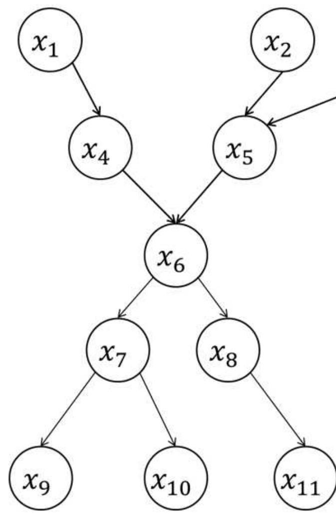

Fig. 2. The example of an hourglasses supply chain.
Source: Authors' own

Definition 2. We will say that the decomposition $X_1, \ldots, X_l$ of the set X, which was defined under the rule (1), is defining the supply chain with the hourglasses structure.

Definition 3. The sector of the node $x_j^i \in X \backslash X_l$ is the set of nodes $F(x_j^i)$.

The set S_j^i is the set of paired indexes of these nodes that are in the sector of the node x_j^i: $S_j^i = \{(k,h) \mid x_h^k \in F(x_j^i)\}$.

Assume that every node x_j^i, $i = 1, \ldots, l$, $j = 1, \ldots, m_i$ of a supply chain consists of a finite set of elements $\{x_{jk}^i\}_{k=1}^{n_{ij}}$ for which the set of numbers is defined $\{v_{ijk}\}_{k=1}^{n_{ij}}$, $v_{ijk} \geq 0$, where n_{ij} is a number of elements. This set of elements is a group of competitive firms that are producing and consuming the homogeneous product as well as having the different production costs (the production power is meant to be unrestricted). For each firm x_{jk}^i let us define the variable $q_{ijk} \geq 0$ that is characterizing the production quantity of this firm as well as the integrated quantity of the homogeneous product that was produced by all firms $\{x_{jk}^i\}_{k=1}^{n_{ij}}$ in the node x_j^i let us call $Q_{ij} = \sum_{k=1}^{n_{ij}} q_{ijk}$.

Then for the sector of each node in a supply chain, the following condition is considered to be fulfilled:

$$Q_{ij} = \sum_{k=1}^{n_{ij}} q_{ijk} = \sum_{(r,h) \in S_j^i} Q_{rh} = \sum_{(r,h) \in S_j^i} \sum_{t=1}^{n_{rh}} q_{rht}. \quad (2)$$

That means that there is no deficit or surplus of production in the supply chain. For each node $x_j^i \in X$ let us work in the variable p_{ij} that is equivalent the prices according to that firm are selling the unit of the produced good. It is considered that for the every of the final nodes $x_j^l \in X_l$ there is the following linear demand function prescribed

$$p_{lj} = a_{lj} - b_{lj} Q_{lj}, \tag{3}$$

where $a_{lj} > 0,\ b_{lj} > 0$ are known parameters.

Definition 4. The set $(\{q_{ijk}\}_{i,j,k}, \{p_{ij}\}_{i,j})$ is defining the commodity flow d in the supply chain.

Definition 5. A flow d will be named feasible if the conditions (2) are satisfied and $p_{lj} > 0,\ Q_{lj} > 0,\ \ j = 1, \ldots, m_l$.

Let the set D is the set of all feasible flows in a supply chain. For each firm let us define the profit function as the following:

$$\pi_{ijk} = \begin{cases} q_{1jk}\left(p_{1j} - v_{1jk}\right), & if\ i = 1; \\ q_{ljk}\left(a_{lj} - b_{lj}Q_{lj} - p_{rh} - v_{ljk}\right), & if\ i = l; \\ q_{ijk}\left(p_{ij} - p_{rh} - v_{ijk}\right), & elsewhere; \end{cases}$$

where $p_{rh} : x_j^i \in S_h^r$.

We arrange the set of nodes X of a supply chain: in the first places is a root nodes, then the nodes of the second level, then the third level, fourth level and up to the final inclusively, i.e. we will receive the arranged system $\{x_1^1,\ x_2^1,\ \ldots,\ x_1^2,\ x_2^2,\ \ldots,\ x_{m_l}^l\}$. This arranged set of all nodes (let us denote it with N) of supply chain we will consider as the set of players. The set $U_{ij} = \{u_{ij}\}$ the strategy of the player x_j^i will be considered as the set of all the possible vectors $u_{ij} \in D$, where:

$$u_{ij} = \left[\begin{array}{c} \left(q_{ij1}, \ldots, q_{ijn_{ij}}, p_{ij}\right) \in D,\ x_j^i \in N,\ i = 1, \ldots,\ l-1,\ \ j = 1, \ldots,\ m_{ij} \\ \left(q_{lj1}, \ldots, q_{ljn_{lj}}\right) \in D,\ \ x_j^l \in N,\ \ \ \ j = 1, \ldots,\ m_{ij}. \end{array} \right. \tag{4}$$

We assume that each of the supply chains participants is acting independently from each other and exclusively in favor of his own interests. Such model and corresponding solution will be named decentralized.

Definition 6. The feasible flow d^* will be called optimal if it is fulfilled:

$$\pi_{ijk}\left(d^*\right) \geq \pi_{ijk}\left(d^{ij}\right) \text{ for all } i, j, k,$$

where d^{ij} is the flow that was created by the deviation of the strategy u_{ij} of the player x_j^i.

In the terms of game theory, the optimal solution is equal to Nash equilibrium in a multi-step hierarchical game with the complete information $\Gamma = \left\langle N, \{U_{ij}\}_{i,j},\ \{\pi_{ijk}\}_{i,j,k} \right\rangle$ on the graph G.

4. Behavior models in distribution network

In this section, theoretical statements of problems for competitive and cooperative solutions are formulated. As a model of competitive behavior we consider the perfect Nash equilibrium solution, as a model of cooperative behavior we consider the weighted Nash bargaining solution.

4.1. Nash equilibrium in a multilevel decentralized model

The search for an optimal solution will be carried out with consideration of the final nodes. Let us analyze the revenue function of the firm k from the node x_j^l :

$$\pi_{ljk} = q_{ljk}\left(p_{lj} - p_{it} - v_{ljk}\right), \quad p_{it} : (l,j) \in S_t^i. \tag{5}$$

Let us substitute in the revenue formula (5) the formula for the variable p_{ij}, using the equation (3):

$$\pi_{ljk} = q_{ljk}\left(a_{lj} - b_{lj}Q_{lj} - p_{it} - v_{ljk}\right). \tag{6}$$

Having done (5)–(6) for all $k = 1, \ldots, n_{ij}$ and having applied the necessary maximum condition:

$$\frac{\partial \pi_{ljk}}{\partial q_{ljk}} = 0, \quad k = \overline{1, n_{lj}}, \tag{7}$$

we will come to the following system:

$$\begin{pmatrix} 2 & 1 & 1 & \cdots & 1 \\ 1 & 2 & 1 & & 1 \\ \vdots & & & \ddots & \vdots \\ 1 & 1 & 1 & \cdots & 2 \end{pmatrix} * \begin{pmatrix} q_{lj1} \\ q_{lj2} \\ \vdots \\ q_{ljn_{lj}} \end{pmatrix} = \begin{pmatrix} \frac{1}{b_{lj}}\left(a_{lj} - p_{it} - v_{lj1}\right) \\ \frac{1}{b_{lj}}\left(a_{lj} - p_{it} - v_{lj2}\right) \\ \vdots \\ \frac{1}{b_{lj}}\left(a_{lj} - p_{it} - v_{ljn_{lj}}\right) \end{pmatrix}. \tag{8}$$

The system (8) is solvable due to it has the non-singular matrix

$$\begin{pmatrix} 2 & 1 & 1 & \cdots & 1 \\ 1 & 2 & 1 & & 1 \\ \vdots & & & \ddots & \vdots \\ 1 & 1 & 1 & \cdots & 2 \end{pmatrix}_{[n_{lj} \times n_{lj}]}.$$

The unique solution is:

$$\begin{pmatrix} q_{lj1} \\ q_{lj2} \\ \vdots \\ q_{ljn_{lj}} \end{pmatrix} = \begin{pmatrix} \frac{n_{lj}}{n_{lj}+1} & \frac{-1}{n_{lj}+1} & \cdots & \frac{-1}{n_{lj}+1} \\ \vdots & & \ddots & \vdots \\ \frac{-1}{n_{lj}+1} & \frac{-1}{n_{lj}+1} & \cdots & \frac{n_{lj}}{n_{lj}+1} \end{pmatrix} * \begin{pmatrix} \frac{1}{b_{lj}}\left(a_{lj} - p_{it} - v_{lj1}\right) \\ \frac{1}{b_{lj}}\left(a_{lj} - p_{it} - v_{lj2}\right) \\ \vdots \\ \frac{1}{b_{lj}}\left(a_{lj} - p_{it} - v_{ljn_{lj}}\right) \end{pmatrix},$$

after the multiplication:

$$\begin{pmatrix} q_{lj1} \\ q_{lj2} \\ \vdots \\ q_{ljn_{lj}} \end{pmatrix} = \begin{pmatrix} \frac{1}{b_{lj}(n_{lj}+1)}\left(a_{lj} - \left(p_{it} + n_{lj}v_{lj1} - \sum\limits_{h=2}^{n_{lj}} v_{ljh}\right)\right) \\ \frac{1}{b_{lj}(n_{lj}+1)}\left(a_{lj} - \left(p_{it} + n_{lj}v_{lj2} - \sum\limits_{\substack{h=1 \\ h \neq 2}}^{n_{lj}} v_{ljh}\right)\right) \\ \vdots \\ \frac{1}{b_{lj}(n_{lj}+1)}\left(a_{lj} - \left(p_{it} + n_{lj}v_{ljn_{lj}} - \sum\limits_{h=1}^{n_{lj}-1} v_{ljh}\right)\right) \end{pmatrix}. \tag{9}$$

For the node x_j^l the following equation holds true as well:

$$Q_{lj} = \sum_{k=1}^{n_{lj}} q_{ljk} = \frac{n_{lj}(a_{lj} - p_{it}) - \sum_{k=1}^{n_{lj}} v_{ljk}}{b_{lj}\left(n_{lj}+1\right)}. \tag{10}$$

Let us fulfill the same analogical operations (5)–(10) for all the final nodes $x_j^l \in X_l$.

Now let us analyze the firm k from x_j^{l-1}. Its revenue function has the following form:

$$\pi_{(l-1)jk} = q_{(l-1)jk}\left(p_{(l-1)j} - p_{it} - v_{(l-1)jk}\right), \quad k = \overline{1, n_{(l-1)j}}, \tag{11}$$

where $p_{it} : (l-1, j) \in S_t^i$.
Taking into account that the node x_j^{l-1} composes a sector, then from the condition (2) let us have the formula:

$$\sum_{k=1}^{n_{(l-1)j}} q_{(l-1)jk} = Q_{(l-1)j} = \sum_{h:(l,h)\in S_j^{l-1}} Q_{lh} =$$

$$= \sum_{h:(l,h)\in S_j^{l-1}} \frac{n_{lh}\left(a_{lh} - p_{(l-1)j}\right) - \sum\limits_{r=1}^{n_{lh}} v_{lhr}}{b_{lh}\left(n_{lh}+1\right)},$$

from that it is possible to express the variable $p_{(l-1)j}$ in explicit form:

$$p_{(l-1)j} = f_{(l-1)j}\left(q_{(l-1)j1}, \ldots, q_{(l-1)jn_{(l-1)j}}\right) =$$

$$= \frac{-\sum\limits_{k=1}^{n_{(l-1)j}} q_{(l-1)jk} + \sum\limits_{h:(l,h)\in S_j^{l-1}} \frac{n_{lh}a_{lh} - \sum\limits_{r=1}^{n_{lh}} v_{lhr}}{b_{lh}(n_{lh}+1)}}{\sum\limits_{h:(l,h)\in S_j^{l-1}} \frac{n_{lh}}{b_{lh}(n_{lh}+1)}}. \tag{12}$$

Let us substitute (12) in the revenue formulas (11)

$$\pi_{(l-1)jk} = q_{(l-1)jk}\left(f_{(l-1)j} - p_{it} - v_{(l-1)jk}\right), \quad k = \overline{1, n_{(l-1)j}}, \tag{13}$$

and let us apply the maximum condition of necessity to the formulas (13):

$$\frac{\partial \pi_{(l-1)jk}}{\partial q_{(l-1)jk}} = \left(f_{(l-1)j} - p_{it} - v_{(l-1)jk}\right) +$$

$$+ q_{(l-1)jk} \frac{-1}{\sum\limits_{h:(l,h)\in S_j^{l-1}} \frac{n_{lh}}{b_{lh}(n_{lh}+1)}} = 0, \quad k = \overline{1, n_{(l-1)j}},$$

or in the matrix form:

$$\begin{pmatrix} 2\,1\;1 & \cdots & 1 \\ 1\,2\;1 & & 1 \\ \vdots & \ddots & \vdots \\ 1\,1\;1 & \cdots & 2 \end{pmatrix} * \begin{pmatrix} q_{(l-1)j1} \\ q_{(l-1)j2} \\ \vdots \\ q_{(l-1)jn_{(l-1)j}} \end{pmatrix} =$$

$$= \begin{pmatrix} \sum\limits_{h:(l,h)\in S_j^{l-1}} \frac{1}{b_{lh}(n_{lh}+1)} \left(n_{lh}a_{lh} - n_{lh}p_{it} - n_{lh}v_{(l-1)j1} - \sum\limits_{r=1}^{n_{lh}} v_{lhr} \right) \\ \sum\limits_{h:(l,h)\in S_j^{l-1}} \frac{1}{b_{lh}(n_{lh}+1)} \left(n_{lh}a_{lh} - n_{lh}p_{it} - n_{lh}v_{(l-1)j2} - \sum\limits_{r=1}^{n_{lh}} v_{lhr} \right) \\ \vdots \\ \sum\limits_{h:(l,h)\in S_j^{l-1}} \frac{1}{b_{lh}(n_{lh}+1)} \left(n_{lh}a_{lh} - n_{lh}p_{it} - n_{lh}v_{(l-1)jn_{(l-1)j}} - \sum\limits_{r=1}^{n_{lh}} v_{lhr} \right) \end{pmatrix}. \quad (14)$$

This system is solvable as its matrix is non-singular. As a result, (14) could be solved in a one-valued way in relation to the variables $q_{(l-1)jk}$, $k = \overline{1, n_{(l-1)j}}$:

$$q_{(l-1)jk} = \frac{1}{n_{(l-1)j}+1} \left[\sum_{h:(l.h)\in S_j^{l-1}} \frac{1}{b_{lh}(n_{lh}+1)} \left(n_{lh}a_{lh} - n_{lh}p_{it} - \sum_{r=1}^{n_{lh}} v_{lhr} - \right.\right.$$

$$\left.\left. - n_{(l-1)j} n_{lh} v_{(l-1)jk} + n_{lh} \sum_{\substack{e=1 \\ e\neq k}}^{n_{(l-1)j}} v_{(l-1)je} \right) \right], \; k = \overline{1, n_{(l-1)j}}.$$

There are could be further calculated the value of $Q_{(l-1)j}$:

$$Q_{(l-1)j} = \sum_{k=1}^{n_{(l-1)j}} q_{(l-1)jk} = \frac{1}{n_{(l-1)j}+1} \left[\sum_{h:(l.h)\in S_j^{l-1}} \frac{1}{b_{lh}\,(n_{lh}+1)} * \right.$$

$$\left. * \left(n_{(l-1)j} \left(n_{lh}a_{lh} - n_{lh}p_{it} - \sum_{r=1}^{n_{lh}} v_{lhr} \right) - n_{lh} \sum_{k=1}^{n_{(l-1)j}} v_{(l-1)jk} \right) \right]. \quad (15)$$

Let us repeat the process (11)–(15) for all the remained nodes x_i^{l-1} from the level $l-1$.

Then in the similar way we will analyze the nodes x_t^i from sets X_i, $i = (l-2)$, $(l-3)$, …, $t-1$, where t is a number of level the node x^* is situated.

Let us proceed to the analysis of the node x^*, which after denotation is equal to x_1^t. The revenue function of a firm t from this node as follows:

$$\pi_{t1k} = q_{t1k} \left(p_{t1} - \sum_{(t,1)\ni S_h^i} p_{ih} - v_{t1k} \right), \quad k = 1, \ldots, n_{t1}\ .$$

From the previous step we have explicit form for the variable p_{t1} (denote it as $f_{t1}(q_{t11},\ldots,\ q_{t1n_{t1}},\ldots)$, it is easy to see that f_{t1} — is a linear function of its variables) and we can substitute it to the formula above:

$$\pi_{t1k} = q_{t1k}\left(f_{t1}\left(q_{t11},\ldots,\ q_{t1n_{t1}},\ldots\right) - \sum_{(t,1)\in S_h^i} p_{ih} - v_{t1k}\right), \quad k = 1,\ \ldots,\ n_{t1}.$$

As before we apply the necessary maximum condition and solve the obtained system in relation to variables q_{t1k}, $\quad k = \overline{1, n_{t1}}$. Then we also calculate the integrated quantity Q_{t1}. Using the condition (2) we obtain the following system of equations:

$$Q_{t1} = Q_{ih}, \quad i, h : (t,1) \in S_h^i,$$

from which we can express variables p_{ih}, but in this case they depend both from quantity and from each other.

Next, we proceed to the analysis of the nodes from X_{t-1}. Let us consider the following systems:

$$\begin{cases} \pi_{(t-1)jk} = q_{(t-1)jk}\left(p_{(t-1)j} - \sum\limits_{(t-1,j)\in S_h^i} p_{ih} - v_{(t-1)jk}\right), \\ k = \overline{1, n_{(t-1)j}} \end{cases} \quad j = \overline{1, m_{(t-1)}}.$$

Substituting the corresponding variable $p_{(t-1)j}$ by its explicit form, we apply the necessary maximum condition for each system and solve the obtained systems in relation to variables $q_{(t-1)jk}$, $\quad k = \overline{1, n_{(t-1)j}}$, $j = \overline{1, m_j}$. After that we calculate $Q_{(t-1)j}$, substitute these expressions to the expressions for $p_{(t-1)j}$ and solve the system, getting $p_{(t-1)j}$ depending only on prices of suppliers' nodes.

Acting the same way, we move up level by level towards to the root nodes. For these nodes the revenue functions are:

$$\begin{cases} \pi_{1jk} = q_{1jk}\left(p_{1j} - v_{1jk}\right), \quad k = 1,\ \ldots,\ n_{1j}\ ; \\ j = \overline{1, m_1}. \end{cases}$$

As before we substitute p_{1j} by its explicit form, obtained earlier, apply the necessary maximum condition and solve the system, getting solutions, which depend only on known parameters of the chain. Moving back from the root nodes to the final ones we will found values for all variables of quantity and prices.

4.2. The weighted Nash bargaining solution

Suppose we have a multi-level distribution supply chain $G = (X, F)$ with a centralized model of behavior of participants, i.e. all participants in this chain join the coalition and act centrally to achieve a common goal. We will consider the weighted Nash function as the objective function.

Let there is a game in the normal form, i.e. a set $\Gamma = \langle N, \{U_l\}_{l\in N}, \{H_l\}_{l\in N}\rangle$, where $N = \{1,\ 2,\ \ldots,\ n\}$ – a non-empty set of players, U_l – the set of strategies of the player l, H_l – payoff function of player l, defined on the Cartesian product of sets $\{U_l\}_{l\in N}$ players' strategies $Y = \prod_{l\in N} Y_l$, $H_l : Y \to R$ (Grossman, Hart, 1983).

Definition 7. A weighted Nash bargaining solution for the game with weights $\alpha_1,\ \alpha_2,\ \ldots,\ \alpha_n:\ \alpha_i > 0 \quad \forall i = \overline{1,n},\ \ \sum_{i=1}^{n} \alpha_i = 1$, we will call a vector such $y' = (y'_1,\ y'_2, \ldots,\ y'_n) \in Y$, which maximize function:

$$\arg\max_{y_1,\ y_2,\ldots,\ y_n} \prod_{i=1}^{n} \left(H_i\left(y_1,\ y_2,\ \ldots,\ y_n\right) - \theta_i\right)^{\alpha_i} = y'. \tag{16}$$

The point $\theta = (\theta_1, \ldots,\ \theta_n)$, where $\theta_i,\ i = \overline{1,n}$ are known parameters, is also called "status quo" point for the problem (16).

As a set of players, we take an ordered set of graph nodes, as a set of strategies – a set U_{ij}, and as the functions of winning – profit function. The status quo point will be the value of the profit function on the decentralized solution of the same supply chain (denote it by π^*). Then the weighted Nash bargaining solution of this cooperative game will be the solution of the following optimization problem:

$$\max_{q_{ijh},p_{ij}} \left(\prod_{i=1}^{l} \prod_{j=1}^{m_i} \prod_{k=1}^{n_{ij}} \left(\pi_{ijk}\left(q_{ij1}, \ldots, q_{ijn_{ij}}, v_{ij1}, \ldots, v_{ijn_{ij}}, p_{ij}, p_{th}\right) - \pi^*_{ijk}\right)^{\alpha_{ijk}} \right) \tag{17}$$

$$p_{th} : (i,j) \in S_h^t;$$
$$\pi_{ijk} \geq \pi^*_{ijk}, \quad i = \overline{1,l}, \quad j = \overline{1,m_i}, k = \overline{1,n_{ij}};$$

$$p_{lj} = a_{lj} - b_{lj} \sum_{k=1}^{n_{lj}} q_{ljk}, \quad j = \overline{1,m_l}; \tag{18}$$

$$\sum_{r=1}^{n_{th}} q_{thr} = \sum_{i,j:(i,j)\in S_h^t} \sum_{k=1}^{n_{ij}} q_{ijk}, \quad t,h:\ x_h^t \notin X_l; \tag{19}$$

$$q_{ijk} \geq 0, \quad i = \overline{1,l}, \quad j = \overline{1,m_i}, k = \overline{1,n_{ij}};$$
$$p_{lj} \geq 0, \quad j = \overline{1,m_i}.$$

where α_{ijk} – given weights, such that:

$$\alpha_{ijk} > 0, \quad i = \overline{1,l}, \quad j = \overline{1,m_i}, \quad k = \overline{1,n_{ij}},$$
$$\sum_{i=1}^{l} \sum_{j=1}^{m_i} \sum_{k=1}^{n_{ij}} \alpha_{ijk} = 1.$$

The existence and uniqueness of the solution are proved by the fact that the Nash multiplication is a continuous convex function, and the constraints set a compact, hence, by the Weierstrass theorem, the maximum of the function exists and is unique.

5. Competitive and cooperation behavior in GTM distribution network

This section considers the case of the Russian distribution network. Solutions for competitive and cooperative behavior models are explored.

5.1. GTM network description

We will use "GTM" as a name of distributor and distribution network. The data for the research was provided by the GTM distribution company operating in Russia. The company is presented in more than 150 cities in different regions of Russia from the North-West to the Far East. As a major player in the market, the company has its own intra-organizational supply chain network including 8 distribution centers. There are more than 60 sales departments with full category B warehouses. The number of employees is nearly six thousand. The number of suppliers having a valid contract is more than 600 by the end of 2016. Among suppliers, there are more than 400 manufacturers. The main suppliers of the company are manufacturers representing electrical industry divided into six parts, namely: Cable production; Industrial electrical equipment; Lighting products; Installation electrical equipment; Safety systems and Fasteners and Plumbing.

For the research one district of distribution company was selected. We limited the network to four suppliers, which are manufactures of industrial electrical equipment, distributor's center and retailers, operated in Central Region of Russia. Each manufacture (supplier) supply only one product which is used to form a portfolio. Costs of each manufacture and the share of its product in portfolio are presented in the table 1.

Table 1. Input data for computational results. Suppliers.

Suppliers	Supplier's costs (cost price), Rub.	Share of the product in portfolio
Supplier 1.1	5 742	0.222
Supplier 1.2	2 441	0.148
Supplier 1.3	11 399	0.444
Supplier 1.4	14 010	0.185

The table 2 shows the costs that the distributor incurs for the purchase of products from suppliers and the costs that the distributor incurs for the organization of logistics.

Table 2. Input data for computational results. Distributor.

Suppliers	Distributor's costs (cost price), Rub.	Distributor's logistic costs per unit of portfolio
Distributor 2.1	48 778	1 340.411

In each region, there are a certain number of retailers. Logistics costs per unit of production vary for each retailer. For ease of computation, we have accepted that demand is a linear function. Demand function was constructed for each region based on data from previous periods. The retailers compete according to the Cournot model. The data presents in the table below.

Based on the data presented in the tables, we construct the structure of the network. The model includes sixteen nodes: four manufactures, one distribution center and eleven regions with compete retailers. The structure of GTM's supply network is presented on the Fig. 3.

Table 3. Input data for computational results. Retailers.

Region	Retailers	Logistic costs for 1 portfolio, Rub.	Demand function
3.1_Belgorod region	BelRet_.3.1.1	923	P = 59 104-0.17Q
	BelRet_3.1.2	818	
	BelRet_3.1.3	263	
3.2_Vladimir region	VladRet_3.2.1	947	$P = 60399 - 0.02Q$
	VladRet_3.2.2	554	
	VladRet_3.2.3	1 060	
	VladRet_3.2.4	1 046	
	VladRet_3.2.5	439	
	VladRet_3.2.6	583	
	VladRet_3.2.7	647	
	VladRet_3.2.8	735	
3.3_Voronezh region	VorRet_3.3.1	513	$P = 59866 - 0.08Q$
	VorRet_3.3.2	820	
	VorRet_3.3.3	1 125	
	VorRet_3.3.4	671	
	VorRet_3.3.5	800	
3.4_Kaluga region	KalRet_3.4.1	847	$P = 60488 - 2.06Q$
	KalRet_3.4.2	794	
3.5_Kursk region	KurRet_3.5.1	463	$P = 64469 - 0.72Q$
	KurRet_3.5.2	278	
	KurRet_3.5.3	253	
3.6_Lipetsk region	LipRet_3.6.1	1 265	$P = 61802 - 0.38Q$
	LipRet_3.6.2	805	
	LipRet_3.6.3	739	
	LipRet_3.6.4	1 237	
	LipRet_3.6.5	521	
	LipRet_3.6.6	919	
3.7_Orel region	OreRet_3.7.1	543	$P = 61120 - 1.44Q$
	OreRet_3.7.2	934	
3.8_Ryazan region	RyazRet_3.8.1	338	$P = 61364 - 0.23Q$
	RyazRet_3.8.2	229	
	RyazRet_3.8.3	470	
	RyazRet_3.8.4	183	
	RyazRet_3.8.5	620	
3.9_Tambov region	TamRet_3.9.1	201	$P = 62133 - 1.90Q$
	TamRet_3.9.2	515	
3.10_Tula region	TulRet_3.10.1	609	$P = 58236 - 0.07Q$
	TulRet_3.10.2	652	
	TulRet_3.10.3	736	
3.11_Yaroslavl region	YarRet_3.11.1	341	$P = 61773 - 1.66Q$
	YarRet_3.11.2	607	
	YarRet_3.11.3	608	

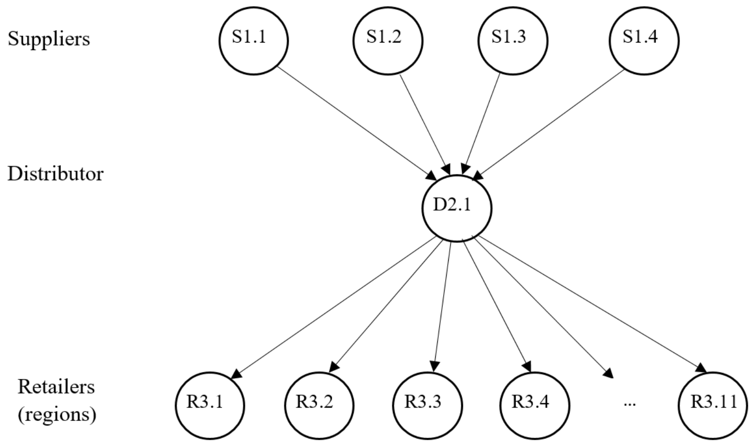

Fig. 3. The GTM network structure
Source: Authors' own

Distributor takes the goods from the supplier's warehouse, so on the supplier-distributor branch logistics costs are incurred by distributor. Retailers incur the logistics costs on distributor – retailer branches. The distributor determines the volume of product order from the manufacturer. The distributor forms a portfolio of suppliers ' products and further supplies retailers only in the amount of at least one portfolio. Partial delivery of the portfolio is not allowed. This scheme of supply network is designed in order to provide a presence of products of all four suppliers in the markets.

5.2. Perfect Nash equilibrium

We present the results of competitive solution (the perfect Nash equilibrium) only for one node of retailers of GTM network. The whole computations are presented in the Appendix. The first region was selected for presenting results. We construct profit functions for end vertices, i.e. for retailers.

$$\pi_{311} = q_{311}\left(p_{31} - p_{21} - 923\right);$$

$$\pi_{312} = q_{312}\left(p_{31} - p_{21} - 818\right);$$

$$\pi_{313} = q_{313}\left(p_{31} - p_{21} - 263\right);$$

In these functions, we substitute expressions for market prices, using the demand functions, and apply the necessary maximum condition:

$$\begin{cases} 58\ 181 - 0,17\ \left(q_{311} + q_{312} + q_{313}\right) - p_{21} - 0,17\ q_{311} = 0; \\ 58\ 286 - 0,17\ \left(q_{311} + q_{312} + q_{313}\right) - p_{21} - 0,17\ q_{313} = 0; \\ 58\ 841 - 0,17\ \left(q_{311} + q_{312} + q_{313}\right) - p_{21} - 0,17\ q_{313} = 0. \end{cases}$$

We solve systems with respect to quantity variables:

$$q_{311} = 84\ 435,2941 - 1,4706\ p_{21};$$

$$q_{312} = 85\ 052,9412 - 1,4706\ p_{21};$$

$$q_{313} = 88\ 317,6471 - 1,4706\ p_{21}.$$

Thereby, the quantity for distributor is:

$$q_{211} = Q_{21} = 4\ 660\ 598,4499 - 78,4969\ p_{21}.$$

So, we get that:

$$p_{21} = 59\ 373,0052 - 0,0127\ q_{211}. \quad (20)$$

Distributor's profit function:

$$\pi_{211} = q_{211}\left(p_{21} - p_{11} - p_{12} - p_{13} - p_{14} - 1340\right).$$

Let's put an expression in it (20):

$$\pi_{211} \approx q_{211}\left(59\ 373 - 0.0127\ q_{211}\ - p_{11} - p_{12} - p_{13} - p_{14} - 1340\right)$$

and apply the necessary condition of the maximum:

$$58\ 033,0052 - p_{11} - p_{12} - p_{13} - p_{14} - 0,0255\ q_{211} = 0.$$

from which we get:

$$q_{211} = 2\ 277\ 706,3057 - 39,2485\ \left(p_{11} + p_{12} + p_{13} + p_{14}\right).$$

Using the condition of absence of shortage and surplus, we have a ratio:

$$q_{111} = 0,222 * q_{211} \approx 505\ 650,7999 - 8.7132\ \left(p_{11} + p_{12} + p_{13} + p_{14}\right);$$

$$q_{112} = 0,148 * q_{211} \approx 337\ 100,5333 - 5,8088\ \left(p_{11} + p_{12} + p_{13} + p_{14}\right);$$

$$q_{113} = 0,444 * q_{211} \approx 1\ 011\ 301,5997 - 17,4263\ \left(p_{11} + p_{12} + p_{13} + p_{14}\right);$$

$$q_{114} = 0,185 * q_{211} \approx 421\ 375,6666 - 7,261\ \left(p_{11} + p_{12} + p_{13} + p_{14}\right).$$

From each equality we express variables p_{11}, p_{12}, p_{13} and p_{14}:

$$p_{11} = 58033,0052 - p_{12} - p_{13} - p_{14} - 0,1148 q_{111}; \quad (21)$$

$$p_{12} = 58033,0052 - p_{11} - p_{13} - p_{14} - 0,1148 q_{121};$$

$$p_{13} = 58033,0052 - p_{11} - p_{12} - p_{14} - 0,1148 q_{131};$$

$$p_{14} = 58033,0052 - p_{11} - p_{12} - p_{13} - 0,1148 q_{141}.$$

Construct the profit functions for suppliers:

$$\pi_{111} = q_{111}\left(p_{11} - 5742\right);$$

$$\pi_{121} = q_{121}\left(p_{21} - 2\ 441\right);$$

$$\pi_{131} = q_{131}\left(p_{31} - 11\ 399\right);$$

$$\pi_{141} = q_{141}\left(p_{14} - 14\ 010\right)$$

and substitute them in (21):

$$\pi_{111} = q_{111}\,(58\ 033,0052 - p_{12} - p_{13} - p_{14} - 0,1148\ q_{111} - 5742)\,;$$

$$\pi_{121} = q_{121}\,(58\ 033,0052 - p_{11} - p_{13} - p_{14} - 0,1148\ q_{211} - 2441)\,;$$

$$\pi_{131} = q_{131}\,(58\ 033,0052 - p_{11} - p_{12} - p_{14} - 0,1148\ q_{211} - 11399)\,;$$

$$\pi_{141} = q_{141}\,(58\ 033,0052 - p_{11} - p_{12} - p_{13} - 0,1148\ q_{211} - 14010)\,.$$

Apply the necessary condition of the maximum:

$$52\ 291,0052 - p_{12} - p_{13} - p_{14} - 0,2295\ q_{111} = 0;$$

$$55\ 592,0052 - p_{11} - p_{13} - p_{14} - 0,3443\ q_{121} = 0;$$

$$46\ 634,0052 - p_{11} - p_{12} - p_{14} - 0,1148\ q_{131} = 0;$$

$$44\ 023,0052 - p_{11} - p_{12} - p_{13} - 0,2755\ q_{141} = 0.$$

Then, we solve equations with respect to quantity:

$$q_{111} = 277\ 809,9205 - 4,3566\ \ (p_{12} + p_{13} + p_{14})\,;$$

$$q_{121} = 161\ 460,6596 - 2,9044\ \ (p_{11} + p_{13} + p_{14})\,;$$

$$q_{131} = 406\ 329,5007 - 8,7132\ \ (p_{11} + p_{12} + p_{14})\,;$$

$$q_{141} = 159\ 824,7678 - 3,6305\ \ (p_{11} + p_{12} + p_{13})\,.$$

Substitute the values obtained in equality (21):

$$p_{11} = 31\ 887,5026 - 0,5\ \ (p_{12} + p_{13} + p_{14})\,;$$

$$p_{12} = 30\ 237,0026 - 0,5\ \ (p_{11} + p_{13} + p_{14})\,;$$

$$p_{13} = 34\ 716,0026 - 0,5\ \ (p_{11} + p_{12} + p_{14})\,;$$

$$p_{14} = 36\ 021,5026 - 0,5\ \ (p_{11} + p_{12} + p_{13})\,.$$

Solving the system, we get:

$$p_{11} \approx 10\ 630,2010;$$

$$p_{12} \approx 7\ 329;$$

$$p_{13} \approx 16\ 287;$$

$$p_{14} \approx 18\ 898.$$

Next, find the values of all other variables. The obtained values of all other variables are presented in the Table 4.

The results, presented in the table 4, show the solution of a non-cooperative game involving network members in which each member of the network is assumed to know the equilibrium strategies of the other members, and no member has anything to gain by changing only their own strategy. Obtained results reflect the performance of network participants in condition of competitive behavior. These results are used as a status quo point for cooperative game.

Table 4. The perfect Nash equilibrium solution

Node	Equilibrium solution		
	Quantity	Price	Profit
x_{11}	$q_{111} \approx 42\ 592$	$p_{11} \approx 10\ 630$	$\pi_{111} \approx 208\ 196\ 659$
x_{12}	$q_{112} \approx 28\ 394$	$p_{12} \approx 7\ 329$	$\pi_{121} \approx 138\ 797\ 773$
x_{13}	$q_{113} \approx 85\ 183$	$p_{13} \approx 16\ 287$	$\pi_{131} \approx 416\ 393\ 318$
x_{14}	$q_{114} \approx 35\ 493$	$p_{14} \approx 18\ 898$	$\pi_{141} \approx 173\ 497\ 216$
x_{21}	$q_{211} \approx 191\ 854$	$p_{21} \approx 56\ 929$	$\pi_{211} \approx 468\ 911\ 394$
x_{31}	$q_{311} \approx 716$ $q_{312} \approx 1\ 334$ $q_{313} \approx 4\ 599$	$p_{31} \approx 57\ 974$	$\pi_{311} \approx 87\ 229$ $\pi_{312} \approx 302\ 508$ $\pi_{313} \approx 3\ 595\ 120$
x_{32}	$q_{321} \approx 7\ 545$ $q_{322} \approx 7\ 195$ $q_{323} \approx 1\ 895$ $q_{324} \approx 2\ 595$ $q_{325} \approx 32\ 945$ $q_{326} \approx 25\ 745$ $q_{327} \approx 22\ 545$ $q_{328} \approx 18\ 145$	$p_{32} \approx 58\ 027$	$\pi_{321} \approx 1\ 138\ 533$ $\pi_{322} \approx 1\ 035\ 353$ $\pi_{323} \approx 71\ 819$ $\pi_{324} \approx 134\ 678$ $\pi_{325} \approx 21\ 707\ 426$ $\pi_{326} \approx 13\ 256\ 074$ $\pi_{327} \approx 10\ 165\ 517$ $\pi_{328} \approx 6\ 584\ 802$
x_{33}	$q_{331} \approx 7\ 892$ $q_{332} \approx 4\ 054$ $q_{333} \approx 242$ $q_{334} \approx 5\ 917$ $q_{335} \approx 4\ 304$	$p_{33} \approx 58\ 073$	$\pi_{331} \approx 4\ 982\ 523$ $\pi_{332} \approx 1\ 315\ 030$ $\pi_{333} \approx 4\ 680$ $\pi_{334} \approx 2\ 800\ 744$ $\pi_{335} \approx 1\ 482\ 205$
x_{34}	$q_{341} \approx 430$ $q_{342} \approx 456$	$p_{34} \approx 58\ 662$	$\pi_{341} \approx 381\ 380$ $\pi_{342} \approx 428\ 353$
x_{35}	$q_{351} \approx 2320$ $q_{352} \approx 2577$ $q_{353} \approx 2612$	$p_{35} \approx 59\ 062$	$\pi_{351} \approx 3\ 875\ 903$ $\pi_{352} \approx 4\ 781\ 901$ $\pi_{353} \approx 4\ 911\ 625$
x_{36}	$q_{361} \approx 565$ $q_{362} \approx 1\ 776$ $q_{363} \approx 1\ 950$ $q_{364} \approx 639$ $q_{365} \approx 2\ 523$ $q_{366} \approx 1\ 476$	$p_{36} \approx 58\ 408$	$\pi_{361} \approx 121\ 499$ $\pi_{362} \approx 1\ 198\ 554$ $\pi_{363} \approx 1\ 444\ 446$ $\pi_{364} \approx 155\ 227$ $\pi_{365} \approx 2\ 419\ 561$ $\pi_{366} \approx 827\ 832$
x_{37}	$q_{371} \approx 935$ $q_{372} \approx 663$	$p_{37} \approx 58\ 818$	$\pi_{371} \approx 1\ 258\ 819$ $\pi_{372} \approx 633\ 835$
x_{38}	$q_{381} \approx 3\ 078$ $q_{382} \approx 3\ 552$ $q_{383} \approx 2\ 504$ $q_{384} \approx 3\ 752$ $q_{385} \approx 1\ 852$	$p_{38} \approx 57\ 974$	$\pi_{381} \approx 2\ 178\ 481$ $\pi_{382} \approx 2\ 901\ 055$ $\pi_{383} \approx 1\ 441\ 749$ $\pi_{384} \approx 3\ 236\ 995$ $\pi_{385} \approx 788\ 468$
x_{39}	$q_{391} \approx 933$ $q_{392} \approx 768$	$p_{39} \approx 58\ 902$	$\pi_{391} \approx 1\ 653\ 304$ $\pi_{392} \approx 1\ 119\ 384$
x_{310}	$q_{3101} \approx 3\ 100$ $q_{3102} \approx 2\ 486$ $q_{3103} \approx 1\ 286$	$p_{310} \approx 57\ 754$	$\pi_{3101} \approx 672\ 848$ $\pi_{302} \approx 432\ 633$ $\pi_{3103} \approx 115\ 776$
x_{311}	$q_{3111} \approx 758$ $q_{3112} \approx 598$ $q_{3113} \approx 598$	$p_{311} \approx 58\ 528$	$\pi_{3111} \approx 954\ 904$ $\pi_{3112} \approx 594\ 034$ $\pi_{3113} \approx 592\ 838$

5.3. Weighted Nash bargaining solution

To find a cooperative solution, we used the MATLAB application package. We used the extremum search function of the constraint function based on the method of sequential quadratic programming, which is an iterative method for constrained nonlinear optimization. It is one of the most effective methods for nonlinearly constrained optimization problems. The method generates steps by solving quadratic subproblems; it can be used both in line search and trust-region frameworks. Sequential quadratic programming is appropriate for small and large problems and it is well suited to solving problems with significant nonlinearities.

To find a cooperative solution, weights were assigned to each member of the network. According to the total number of vertices equal to 16, the first node level (suppliers) was assigned the weight of 8/16, the second one (distributor) - 4/16, the third one (retailers) - 4/16. The weight of each supplier is the weight of the first level nodes (8/16) multiplied by the share of its product in the portfolio. The weight of each retailer is the weight of the nodes of the third level (4/16) divided by 8 (the number of nodes at the level) and the number of retailers at the node. Retailers have the least weight per retailer. This is because before retailers cooperate with a distributor, they need to come to an agreement within the region. Thus, the weight of each individual retailer in cooperation with the distributor is not as important as the weight value of the region.

Let's formulate and solve the following optimization problem:

$$
\begin{aligned}
\max_{q_{ijk},\quad p_{ij}} \big[& (q_{311}(p_{31}-p_{21}-923)-87\ 361)^{0.019} * \ (q_{312}(p_{31}-p_{21}-818)-302\ 576)^{0.019} * \\
& *(q_{313}(p_{31}-p_{21}-263)-3\ 594\ 527)^{0.019} * (q_{321}(p_{32}-p_{21}-947)-1\ 137\ 958)^{0.0007} * \\
& *(q_{322}(p_{32}-p_{21}-554)-3\ 915\ 571)^{0.0007} * (q_{323}(p_{32}-p_{21}-1\ 060)-72\ 430)^{0.0007} * \\
& *(q_{324}(p_{32}-p_{21}-1046)-134\ 873)^{0.0007} * (q_{325}(p_{32}-p_{21}-439)-21\ 711\ 969)^{0.0007} * \\
& *(q_{326}(p_{32}-p_{21}-583)-13\ 244\ 049)^{0.0007} * (q_{327}(p_{32}-p_{21}-647)-10\ 166\ 800)^{0.0007} * \\
& *(q_{328}(p_{32}-p_{21}-735)-6\ 592\ 280)^{0.0007} * (q_{331}(p_{33}-p_{21}-513)-4\ 985\ 510)^{0.0011} * \\
& *(q_{332}(p_{33}-p_{21}-820)-1\ 316\ 821)^{0.0011} * (q_{333}(p_{33}-p_{21}-1\ 125)-4\ 603)^{0.0011} * \\
& *(q_{334}(p_{33}-p_{21}-671)-2\ 801\ 497)^{0.0011} * (q_{335}(p_{33}-p_{21}-800)-1\ 480\ 311)^{0.0011} * \\
& *(q_{341}(p_{34}-p_{21}-847)-381\ 524)^{0.0028} * (q_{342}(p_{34}-p_{21}-794)-428\ 195)^{0.0028} * \\
& *(q_{351}(p_{35}-p_{21}-463)-3\ 875\ 142)^{0.0019} * (q_{35}(p_{35}-p_{21}-278)-4\ 780\ 888)^{0.0019} * \\
& *(q_{353}(p_{35}-p_{21}-253)-4\ 910\ 880)^{0.0019} * (q_{361}(p_{36}-p_{21}-1\ 265)-121\ 471)^{0.0009} * \\
& *(q_{362}(p_{36}-p_{21}-805)-1\ 198\ 779)^{0.0009} * (q_{363}(p_{36}-p_{21}-739)-1\ 444\ 701)^{0.0009} * \\
& *(q_{364}(p_{36}-p_{21}-1\ 237)-155\ 266)^{0.0009} * (q_{365}(p_{36}-p_{21}-521)-2\ 419\ 493)^{0.0009} * \\
& *(q_{366}(p_{36}-p_{21}-919)-828\ 314)^{0.0009} * (q_{371}(p_{37}-p_{21}-543)-1\ 259\ 126)^{0.0028} * \\
& *(q_{372}(p_{37}-p_{21}-934)-633\ 835)^{0.0028} * (q_{381}(p_{38}-p_{21}-338)-2\ 177\ 757)^{0.0011} * \\
& *(q_{382}(p_{38}-p_{21}-229)-2\ 899\ 771)^{0.0011} * (q_{383}(p_{38}-p_{21}-470)-1\ 442\ 984)^{0.0011} * \\
& *(q_{384}(p_{38}-p_{21}-183)-3\ 236\ 514)^{0.0011} * (q_{385}(p_{38}-p_{21}-620)-789\ 291)^{0.0011} * \\
& *(q_{391}(p_{39}-p_{21}-201)-1\ 653\ 011)^{0.0028} * (q_{392}(p_{39}-p_{21}-515)-1\ 119\ 008)^{0.0028} * \\
& *(q_{3101}(p_{310}-p_{21}-609)-673\ 995)^{0.0019} * (q_{3102}(p_{310}-p_{21}-652)-433\ 140)^{0.0019} * \\
& *(q_{3103}(p_{310}-p_{21}-736)-115\ 445)^{0.0019} * (q_{3111}(p_{311}-p_{21}-341)-954\ 613)^{0.0019} *
\end{aligned}
$$

$$*(q_{3112}\,(p_{311}-p_{21}-607)-594\;287)^{0.0019}*(q_{3113}\,(p_{311}-p_{21}-608)-592\;804)^{0.0019}*$$
$$*(q_{211}\,(p_{21}-p_{11}-p_{12}-p_{13}-p_{14}-1\;340)-4\;688\;322\;609)^{0.25}*$$
$$*(q_{111}\,(p_{11}-5\;742)-208\;187\;438)^{0.1526}*(q_{121}\,(p_{12}-2\;441)-138\;792\;459)^{0.1017}*$$
$$*(q_{131}\,(p_{13}-11\;399)-416\;350\;845)^{0.3053}*(q_{141}\,(p_{14}-14\;010)-173\;503\;931)^{0.1272}];$$

$p_{31}=59\;104-0,17Q_{31}$; $p_{35}=64\;469-0,72Q_{35}$; $p_{39}=62\;133-1,9Q_{39}$;

$p_{32}=60\;399-0,02Q_{32}$; $p_{36}=61\;802-0,38Q_{36}$; $p_{310}=58\;236-0,07Q_{310}$;

$p_{33}=59\;866-0,08Q_{33}$; $p_{37}=61\;120-1,44Q_{37}$; $p_{311}=61\;773-1,66Q_{311}$;

$p_{34}=60\;488-2,06Q_{34}$; $p_{38}=61\;364-0,23Q_{38}$;

$$Q_{31}+\dots+Q_{311}=Q_{21};$$
$$Q_{11}=0,222Q_{21};\quad Q_{13}=0,444Q_{21}$$
$$Q_{12}=0,148Q_{21};\quad Q_{14}=0,185Q_{21};$$
$$q_{ijk}\ge 0,\quad p_{ij}\ge 0.$$

Values of variables and profits of all firms are given in the Table 5.

Table 5. The weighted Nash bargaining solution

Node	Equilibrium solution		
	Quantity	Price	Profit
x_{11}	$q_{111}\approx 47\;794$	$p_{11}\approx 10\;158$	$\pi_{111}\approx 211\;069\;495$
x_{12}	$q_{112}\approx 31\;863$	$p_{12}\approx 6\;853$	$\pi_{121}\approx 140\;580\;514$
x_{13}	$q_{113}\approx 95\;588$	$p_{13}\approx 16\;268$	$\pi_{131}\approx 465\;388\;411$
x_{14}	$q_{114}\approx 39\;828$	$p_{14}\approx 18\;654$	$\pi_{141}\approx 184\;974\;523$
x_{21}	$q_{211}\approx 215\;288$	$p_{21}\approx 56\;487$	$\pi_{211}\approx 468\;322\;609$
x_{31}	$q_{311}\approx 672$ $q_{312}\approx 1\;412$ $q_{313}\approx 5\;369$	$p_{31}\approx 57\;837$	$\pi_{311}\approx 287\;401$ $\pi_{312}\approx 751\;483$ $\pi_{313}\approx 5\;836\;947$
x_{32}	$q_{321}\approx 9\;125$ $q_{322}\approx 8\;717$ $q_{323}\approx 2\;674$ $q_{324}\approx 3\;443$ $q_{325}\approx 38\;646$ $q_{326}\approx 30\;460$ $q_{327}\approx 26\;690$ $q_{328}\approx 21\;520$	$p_{32}\approx 57\;574$	$\pi_{321}\approx 1\;275\;419$ $\pi_{322}\approx 4\;647\;429$ $\pi_{323}\approx 72\;681$ $\pi_{324}\approx 140\;907$ $\pi_{325}\approx 25\;042\;313$ $\pi_{326}\approx 15\;333\;081$ $\pi_{327}\approx 11\;741\;319$ $\pi_{328}\approx 7\;580\;734$
x_{33}	$q_{331}\approx 9\;450$ $q_{332}\approx 5\;874$ $q_{333}\approx 390$ $q_{334}\approx 7\;089$ $q_{335}\approx 5\;170$	$p_{33}\approx 57\;708$	$\pi_{331}\approx 6\;699\;543$ $\pi_{332}\approx 1\;958\;995$ $\pi_{333}\approx 37\;520$ $\pi_{334}\approx 3\;903\;870$ $\pi_{335}\approx 2\;177\;095$

x_{34}	$q_{341} \approx 316$ $q_{342} \approx 358$	$p_{34} \approx 59\ 099$	$\pi_{341} \approx 558\ 523$ $\pi_{342} \approx 650\ 770$
x_{35}	$q_{351} \approx 1033$ $q_{352} \approx 1321$ $q_{353} \approx 1360$	$p_{35} \approx 61\ 795$	$\pi_{351} \approx 5\ 003\ 330$ $\pi_{352} \approx 6\ 645\ 457$ $\pi_{353} \approx 6\ 876\ 004$
x_{36}	$q_{361} \approx 108$ $q_{362} \approx 1\ 386$ $q_{363} \approx 1\ 581$ $q_{364} \approx 115$ $q_{365} \approx 2\ 228$ $q_{366} \approx 1\ 048$	$p_{36} \approx 59\ 345$	$\pi_{361} \approx 172\ 321$ $\pi_{362} \approx 2\ 845\ 741$ $\pi_{363} \approx 3\ 351\ 998$ $\pi_{364} \approx 185\ 716$ $\pi_{365} \approx 5\ 207\ 520$ $\pi_{366} \approx 2\ 032\ 443$
x_{37}	$q_{371} \approx 810$ $q_{372} \approx 588$	$p_{37} \approx 59\ 107$	$\pi_{371} \approx 1\ 681\ 812$ $\pi_{372} \approx 992\ 292$
x_{38}	$q_{381} \approx 3\ 153$ $q_{382} \approx 3\ 709$ $q_{383} \approx 2\ 482$ $q_{384} \approx 3\ 944$ $q_{385} \approx 1\ 723$	$p_{38} \approx 57\ 911$	$\pi_{381} \approx 3\ 425\ 495$ $\pi_{382} \approx 4\ 433\ 475$ $\pi_{383} \approx 2\ 370\ 941$ $\pi_{384} \approx 4\ 897\ 086$ $\pi_{385} \approx 1\ 387\ 695$
x_{39}	$q_{391} \approx 1\ 115$ $q_{392} \approx 949$	$p_{39} \approx 58\ 212$	$\pi_{391} \approx 1\ 698\ 889$ $\pi_{392} \approx 1\ 147\ 671$
x_{310}	$q_{3101} \approx 3\ 971$ $q_{3102} \approx 3\ 225$ $q_{3103} \approx 1\ 771$	$p_{310} \approx 57\ 608$	$\pi_{3101} \approx 2\ 037\ 296$ $\pi_{3102} \approx 1\ 515\ 255$ $\pi_{3103} \approx 682\ 368$
x_{311}	$q_{3111} \approx 480$ $q_{3112} \approx 407$ $q_{3113} \approx 407$	$p_{311} \approx 59\ 625$	$\pi_{3111} \approx 1\ 343\ 314$ $\pi_{3112} \approx 1\ 030\ 373$ $\pi_{3113} \approx 1\ 029\ 138$

The results of the weighted Nash bargaining solution presented in the table 5 reflect the possible performance of all participants in the cooperative behavior. For each of the network participants the value of profit in terms of cooperation is calculated. The price is set for the region in which the cooperation of retailers is carried out. For each of the retailers, the sales volume is calculated in terms of cooperation behavior maximizing their profits. To find a cooperative solution that will increase the profit of each participant relative to the equilibrium, the Nash equilibrium solution is taken as the status quo point. Thus, the network participants get profit better or at least not worse than in the equilibrium solution. This statement is well illustrated by the presented results. Table 5 shows that each of the network participants, including the manufacturer and distributor, and not just retailers, gained a profit value higher or at least not worse than in equilibrium.

6. Comparative analysis of competitive and cooperative behavior

In this study, we considered the perfect Nash equilibrium solution as competitive behavior and the weighted Nash bargaining solution as a cooperative solution. To find a cooperative solution, the Nash equilibrium solution was taken as the status quo point.

The Nash equilibrium solution reflects the results of the decision of a non-cooperative game involving in which each player is assumed to know the equilibrium strategies of the other players, and no player has anything to gain by changing only their own strategy. According to this the algorithm for foundation the optimal solution, which in terms of game theory is Nash equilibrium in a multistep hierarchical game with complete information on a graph G with an hourglasses structure was provided.

At the first step of the research, the perfect Nash equilibrium solution was found as a competitive behavior of participants within the network. In order to improve the results of the Nash equilibrium solution, i.e. to increase the profit of each participant in the chain, cooperation is necessary. In general, cooperation allows achieving better results from interaction, than in the case where companies operate independently.

To find a cooperative solution that will increase the profit of each participant relative to the equilibrium we included all participant in coalition and the Nash equilibrium solution was taken as the status quo point. Thus, the network participants get profit better or at least not worse than in the equilibrium solution.

Since retailers are working in the region markets, the solution was found for each region separately. The comparative analysis of retailers' profits for each region is presented in the table 6.

Table 6. The comparative analysis of retailers' profits for competitive and cooperative behavior

Regions (nodes)	The **profit** of retailers obtained in case of competitive behavior	The **profit** of retailers obtained in case of cooperative behavior	The deviation of cooperative solution from competitive solution
3.1_Belgorod region	3 984 464	6 875 831	72.6%
3.2_Vladimir region	56 975 930	65 833 884	15.5%
3.3_Voronezh region	10 589 442	14 777 023	39.5%
3.4_Kaluga region	809 719	1 209 308	49.3%
3.5_Kursk region	13 566 910	18 524 790	36.5%
3.6_Lipetsk region	6 168 024	13 795 648	123.7%
3.7_Orel region	1 892 961	2 674 104	41.3%
3.8_Ryazan region	10 546 316	16 514 692	56.6%
3.9_Tambov region	2 772 019	2 846 560	2.7%
3.10_Tula region	1 222 580	4 234 919	246.4%
3.11_Yaroslavl region	2 141 704	3 402 825	58.9%
Total	**110 670 069**	**150 689 584**	**36.2%**

The obtained results show that each of the network retailers gained a profit value higher or at least not worse than in equilibrium. At the same time, we see that the obtained results do not deviate from the equilibrium solution so much as to talk about their unattainability. For instance, the profit of all retailers of Belgorod region increased by 72.6%. For Ryazan region the retailers' profit in case of cooperative solution increased by 56.6%. At the same time the results of cooperative solution in Tambov region is higher than the results of competitive solution only by 2.7%. It is the worse result however it is still better than the results obtained in condition of competitive behavior. The only minus of such results is that the motivation for cooperation of Tambov retailers will be low for effective and stable relationships. The total profit of all retailers in the network is higher for cooperative solution by 36.2% that can be considered as a motivation factor for all retailers to cooperate and maintain stable relationships within the network.

The sale price of the portfolio in the market is also set for the region. As retailers compete according to Cournot model, the optimal quantity for each retailer in the region was found. The comparative analysis of obtained price and quantities for competitive and cooperative behaviors is presented in table 7 and table 8.

Table 7. The comparative analysis of retailers' volumes for competitive and cooperative behavior

Regions (nodes)	The **quantity** of retailers obtained in case of competitive behavior	The **quantity** of retailers obtained in case of cooperative behavior	The deviation of cooperative solution from competitive solution
3.1_Belgorod region	6 649	7 454	12.1%
3.2_Vladimir region	118 610	141 274	19.1%
3.3_Voronezh region	22 409	26 973	20.4%
3.4_Kaluga region	886	674	-23.9%
3.5_Kursk region	7 509	3 714	-50.5%
3.6_Lipetsk region	8 930	6 465	-27.6%
3.7_Orel region	1 598	1 398	-12.5%
3.8_Ryazan region	14 736	15 011	1.9%
3.9_Tambov region	1 700	2 064	21.4%
3.10_Tula region	6 872	8 967	30.5%
3.11_Yaroslavl region	1 954	1 294	-33.8%
Total	191 854	215 288	12.2%

Table 8. The comparative analysis of retailers' prices for competitive and cooperative behavior

Regions (nodes)	The **price** of retailers obtained in case of competitive behavior	The **price** of retailers obtained in case of cooperative behavior	The deviation of cooperative solution from competitive solution
3.1_Belgorod region	57 974	57 837	-0.2%
3.2_Vladimir region	58 027	57 574	-0.8%
3.3_Voronezh region	58 073	57 708	-0.6%
3.4_Kaluga region	58 662	59 099	0.7%
3.5_Kursk region	59 062	61 795	4.6%
3.6_Lipetsk region	58 409	59 345	1.6%
3.7_Orel region	58 818	59 107	0.5%
3.8_Ryazan region	57 975	57 911	-0.1%
3.9_Tambov region	58 902	58 212	-1.2%
3.10_Tula region	57 755	57 608	-0.3%
3.11_Yaroslavl region	58 529	59 625	1.9%

As can be seen from table 8, the price remained unchanged in all regions. Deviations for all regions are minor. The increase in efficiency was mainly due to the redistribution of volume between the network participants in such a way that the total costs were minimal and with a slight increase in volume (by 12%), the growth of total profit was significant.

The optimal price and quantity were found for distributor and then – for each supplier. Thus, we understand that the profit gained from the equilibrium solution is better result or at least not worse for each of the participants of the presented chain than in the conditions of competition.

The obtained results for suppliers and distributor are presented in the table 9 and table 10 respectively. The result reflects only the deviation in profits for competitive and cooperative solutions. This is due to the fact that demand is formed in the final nodes, i.e. retailers. The distributor forming a portfolio provides only distribution of volumes between retailers, without affecting the total demand.

Table 9. The comparative analysis of competitive and cooperative behavior of suppliers

Suppliers	The **supplier's profit** obtained in case of competitive behavior	The **supplier's profit** obtained in case of cooperative behavior	The deviation of cooperative solution from competitive solution
Supplier_1.1	208 187 437	211 069 495	1.4%
Supplier_1.2	138 792 459	140 580 514	1.3%
Supplier_1.3	416 350 844	465 388 410	11.8%
Supplier_1.4	173 503 931	184 974 523	6.6%

Table 10. The comparative analysis of competitive and cooperative behavior of distributor

Distributor	The **distributor's profit** obtained in case of competitive behavior	The **distributor's profit** obtained in case of cooperative behavior	The deviation of cooperative solution from competitive solution
Distributor_2.1	184 974 523	468 832 610	147.5%

The results presented in table 9 show that a cooperative solution is better than a competitive solution for only two suppliers. For the other two suppliers, the cooperative solution, we can say, is no worse than the Nash equilibrium. In general, all suppliers benefit from cooperative behavior, however, the motivation for cooperation of the latter two suppliers will be higher. This means that in the process of coalition formation, these suppliers are more likely to take a positive decision to join the coalition while the other two suppliers will take a neutral position. Table 10 clearly shows that the distributor benefits significantly from cooperation. Its profit in terms of cooperative behavior is growing by 47.5%. With a slight change in price, the total number of purchased and sold products in the cooperative solution increased by 12%, but the greatest effect was given by changes in the distribution of products between retailers.

7. Conclusion and limitations of the research

The research has two main results. First, we construct the perfect Nash equilibrium for such supply network as the competitive solution in the multi-step hierarchical game in closed form. Second, we construct a cooperative behavior for the network and found the unique weighted Nash bargaining solution with the perfect Nash equilibrium as a point of the status quo. The weighted Nash bargaining solution comes down to the solution of a separable nonlinear programming problem with concave payoff function.

The obtained results show that cooperative behavior is more profitable than competitive one for all participants. The supply chain profit in conditions of cooperative behavior is higher than the profit in conditions of competitive behavior by 31.57%. The distributor has the greatest increase in profit in terms of cooperative behavior. It means that distributor is motivated for organization of cooperation most. In the network considered in this study, the distributor has one of the most important roles. The distributor has relationships with suppliers, retailers, and acts as a focal company of the network. The total profit of all retailers in the network is higher for cooperative solution by 36.2%. It means that most of retailers are motivated to cooperate with distributor. At the same time, the total of suppliers is higher for cooperative behavior only by 6.96%. On the one hand, it can be considered that suppliers are not motivated enough for cooperation, but on the other hand if suppliers refuse to join a coalition, the distributor may revise the portfolio. In this case, suppliers who have not joined the coalition may not only make a profit worse than in case of equilibrium, but even lower.

Therefore, we considered the network in two behavior models: competitive behavior and cooperative behavior. The results show that all members of supply network are motivated for coalition formation. This result is important for several reasons. First, it confirms that cooperation in supply networks gets better results for each member of the network and for the whole network. Second, it shows quite clearly that in distribution networks the organization of cooperation in the network is the responsibility of the distributor as a focal company, because the distributor has a greater motivation and greater weight than, for instance, retailers with similar motivation. Moreover, the obtained results are essential for future research in supply networks and research in coalition formation problem.

This study has some limitations, which are imposed on the one hand by the research methods used, on the other hand – the study model. The first one is that the price of products within the region is the same for all retailers. This situation is often typical for chain stores of one company. However, in conditions of high competition, such situation can be observed. Another limitation of the study is the assumption that the demand function is linear. This assumption is made for ease of calculation and display of the results of the study. To calculate demand functions in each region were used the data about sales of previous periods. The next limitation of the study is that one product is selected. In our case, this product is the portfolio that is formed by distributor. This limitation is related to the chosen method of finding the solution. On the one hand, this limitation makes it possible to draw conclusions only within one product network, on the other hand, we understand that the distributor, having a significant weight in the network, can form a certain assortment matrix (portfolio) for interaction with retailers. In this case, consideration of multi-product networks becomes possible.

As a direction for future research, the coalition formation problem can be considered. To find the cooperative solution we included all participant of supply network in coalition. The results of a cooperative solution are better than the results of a decentralized solution, in that case, the question with which of the participants of the coalition will be the most profitable arises.

Appendix

The computations of the perfect Nash equilibrium for GTM network

We construct profit functions for end vertices, i.e. for retailers.

$$\pi_{311} = q_{311}\,(p_{31} - p_{21} - 923)\,;$$
$$\pi_{312} = q_{312}\,(p_{31} - p_{21} - 818)\,;$$
$$\pi_{313} = q_{313}\,(p_{31} - p_{21} - 263)\,;$$
$$\pi_{321} = q_{321}\,(p_{32} - p_{21} - 947)\,;$$
$$\pi_{322} = q_{322}\,(p_{32} - p_{21} - 554)\,;$$
$$\pi_{323} = q_{323}\,(p_{32} - p_{21} - 1\ 060)\,;$$
$$\pi_{324} = q_{324}\,(p_{32} - p_{21} - 1\ 046)\,;$$
$$\pi_{325} = q_{325}\,(p_{32} - p_{21} - 439)\,;$$
$$\pi_{326} = q_{326}\,(p_{32} - p_{21} - 583)\,;$$
$$\pi_{327} = q_{327}\,(p_{32} - p_{21} - 647)\,;$$
$$\pi_{328} = q_{328}\,(p_{32} - p_{21} - 735)\,;$$

$$\pi_{331} = q_{331}\,(p_{33} - p_{21} - 513)\,;$$
$$\pi_{332} = q_{332}\,(p_{33} - p_{21} - 820)\,;$$
$$\pi_{333} = q_{333}\,(p_{33} - p_{21} - 1\ 125)\,;$$
$$\pi_{334} = q_{334}\,(p_{33} - p_{21} - 671)\,;$$
$$\pi_{335} = q_{335}\,(p_{33} - p_{21} - 800)\,;$$

$$\pi_{341} = q_{341}\,(p_{34} - p_{21} - 847)\,;$$
$$\pi_{342} = q_{342}\,(p_{34} - p_{21} - 794)\,;$$

$$\pi_{351} = q_{351}\,(p_{35} - p_{21} - 463)\,;$$
$$\pi_{352} = q_{35}\,(p_{35} - p_{21} - 278)\,;$$
$$\pi_{353} = q_{353}\,(p_{35} - p_{21} - 253)\,;$$

$$\pi_{361} = q_{361}\,(p_{36} - p_{21} - 1\ 265)\,;$$
$$\pi_{362} = q_{362}\,(p_{36} - p_{21} - 805)\,;$$
$$\pi_{363} = q_{36}\,(p_{36} - p_{21} - 739)\,;$$
$$\pi_{364} = q_{364}\,(p_{36} - p_{21} - 1\ 237)\,;$$
$$\pi_{365} = q_{365}\,(p_{36} - p_{21} - 521)\,;$$
$$\pi_{366} = q_{366}\,(p_{36} - p_{21} - 919)\,;$$

$$\pi_{371} = q_{371}\,(p_{37} - p_{21} - 543)\,;$$
$$\pi_{372} = q_{372}\,(p_{37} - p_{21} - 934)\,;$$

$$\pi_{381} = q_{381}\,(p_{38} - p_{21} - 338)\,;$$
$$\pi_{382} = q_{382}\,(p_{38} - p_{21} - 229)\,;$$
$$\pi_{383} = q_{383}\,(p_{38} - p_{21} - 470)\,;$$
$$\pi_{384} = q_{384}\,(p_{38} - p_{21} - 183)\,;$$
$$\pi_{385} = q_{385}\,(p_{38} - p_{21} - 620)\,;$$

$$\pi_{391} = q_{391}\,(p_{39} - p_{21} - 201)\,;$$
$$\pi_{392} = q_{392}\,(p_{39} - p_{21} - 515)\,;$$

$$\pi_{3101} = q_{3101}\,(p_{310} - p_{21} - 609)\,;$$
$$\pi_{3102} = q_{3102}\,(p_{310} - p_{21} - 652)\,;$$
$$\pi_{3103} = q_{3103}\,(p_{310} - p_{21} - 736)\,;$$

$$\pi_{3111} = q_{3111}\,(p_{311} - p_{21} - 341)\,;$$
$$\pi_{3112} = q_{3112}\,(p_{311} - p_{21} - 607)\,;$$
$$\pi_{3113} = q_{3113}\,(p_{311} - p_{21} - 608)\,.$$

In these functions, we substitute expressions for market prices, using the demand functions, and apply the necessary maximum condition:

$$\begin{cases} 58\ 181 - 0,17\ \ (q_{311} + q_{312} + q_{313}) - p_{21} - 0,17q_{311} = 0; \\ 58\ 286 - 0,17\ \ (q_{311} + q_{312} + q_{313}) - p_{21} - 0,17\ q_{313} = 0; \\ 58\ 841 - 0,17\ \ (q_{311} + q_{312} + q_{313}) - p_{21} - 0,17\ q_{313} = 0; \end{cases}$$

$$\begin{cases} 59\ 452 - 0,02\ \ (q_{321} + \cdots + q_{328}) - p_{21} - 0,02\ q_{321} = 0; \\ 59\ 445 - 0,02\ \ (q_{321} + \cdots + q_{328}) - p_{21} - 0,02\ q_{322} = 0; \\ 59\ 339 - 0,02\ \ (q_{321} + \cdots + q_{328}) - p_{21} - 0,02\ q_{323} = 0; \\ 59\ 353 - 0,02\ \ (q_{321} + \cdots + q_{328}) - p_{21} - 0,02\ q_{324} = 0; \\ 59\ 960 - 0,02\ \ (q_{321} + \cdots + q_{328}) - p_{21} - 0,02\ q_{325} = 0; \\ 59\ 816 - 0,02\ \ (q_{321} + \cdots + q_{328}) - p_{21} - 0,02\ q_{326} = 0; \\ 59\ 752 - 0,02\ \ (q_{321} + \cdots + q_{328}) - p_{21} - 0,02\ q_{327} = 0; \\ 59\ 664 - 0,02\ \ (q_{321} + \cdots + q_{328}) - p_{21} - 0,02\ q_{328} = 0; \end{cases}$$

$$\begin{cases} 59\ 353 - 0,08\ \ (q_{331} + q_{332} + q_{333} + q_{334} + q_{335}) - p_{21} - 0,08\ q_{331} = 0; \\ 59\ 046 - 0,08\ \ (q_{331} + q_{332} + q_{333} + q_{334} + q_{335}) - p_{21} - 0,08\ q_{332} = 0; \\ 58\ 741 - 0,08\ \ (q_{331} + q_{332} + q_{333} + q_{334} + q_{335}) - p_{21} - 0,08\ q_{333} = 0; \\ 59\ 195 - 0,08\ \ (q_{331} + q_{332} + q_{333} + q_{334} + q_{335}) - p_{21} - 0,08\ q_{334} = 0; \\ 59\ 066 - 0,08\ \ (q_{331} + q_{332} + q_{333} + q_{334} + q_{335}) - p_{21} - 0,08\ q_{335} = 0; \end{cases}$$

$$\begin{cases} 59\ 641 - 2,06\ \ (q_{341} + q_{342}) - p_{21} - 2,06\ q_{341} = 0; \\ 59\ 694 - 2,06\ \ (q_{341} + q_{342}) - p_{21} - 2,06\ q_{342} = 0; \end{cases}$$

$$\begin{cases} 64\ 006 - 0,72\ \ (q_{351} + q_{352} + q_{353}) - p_{21} - 0,72\ q_{351} = 0; \\ 64\ 191 - 0,72\ \ (q_{351} + q_{352} + q_{353}) - p_{21} - 0,72\ q_{352} = 0; \\ 64\ 216 - 0,72\ \ (q_{351} + q_{352} + q_{353}) - p_{21} - 0,72\ q_{353} = 0; \end{cases}$$

$$\begin{cases} 60\ 537 - 0,38\ \ (q_{361} + q_{362} + q_{363} + q_{364} + q_{365} + q_{366}) - p_{21} - 0,38\ q_{361} = 0; \\ 60\ 997 - 0,38\ \ (q_{361} + q_{362} + q_{363} + q_{364} + q_{365} + q_{366}) - p_{21} - 0,38\ q_{362} = 0; \\ 61\ 063 - 0,38\ \ (q_{361} + q_{362} + q_{363} + q_{364} + q_{365} + q_{366}) - p_{21} - 0,38\ q_{363} = 0; \\ 60\ 565 - 0,38\ \ (q_{361} + q_{362} + q_{363} + q_{364} + q_{365} + q_{366}) - p_{21} - 0,38\ q_{364} = 0; \\ 61\ 281 - 0,38\ \ (q_{361} + q_{362} + q_{363} + q_{364} + q_{365} + q_{366}) - p_{21} - 0,38\ q_{365} = 0; \\ 60\ 883 - 0,38\ \ (q_{361} + q_{362} + q_{363} + q_{364} + q_{365} + q_{366}) - p_{21} - 0,38\ q_{366} = 0; \end{cases}$$

$$\begin{cases} 60\ 577 - 1,44\ \ (q_{371} + q_{372}) - p_{21} - 1,44\ q_{371} = 0; \\ 60\ 186 - 1,44\ \ (q_{371} + q_{372}) - p_{21} - 1,44\ q_{372} = 0; \end{cases}$$

$$\begin{cases} 61\ 026 - 0,23\ \ (q_{381} + q_{382} + q_{383} + q_{384} + q_{385}) - p_{21} - 0,23\ q_{381} = 0; \\ 61\ 135 - 0,23\ \ (q_{381} + q_{382} + q_{383} + q_{384} + q_{385}) - p_{21} - 0,23\ q_{382} = 0; \\ 60\ 894 - 0,23\ \ (q_{381} + q_{382} + q_{383} + q_{384} + q_{385}) - p_{21} - 0,23\ q_{383} = 0; \\ 61\ 181 - 0,23\ \ (q_{381} + q_{382} + q_{383} + q_{384} + q_{385}) - p_{21} - 0,23\ q_{384} = 0; \\ 60\ 744 - 0,23\ \ (q_{381} + q_{382} + q_{383} + q_{384} + q_{385}) - p_{21} - 0,23\ q_{385} = 0; \end{cases}$$

$$\begin{cases} 61\ 932 - 1,9\ \ (q_{391} + q_{392}) - p_{21} - 1,9\ q_{391} = 0; \\ 61\ 618 - 1,9\ \ (q_{391} + q_{392}) - p_{21} - 1,9\ q_{392} = 0; \end{cases}$$

$$\begin{cases} 57\ 627 - 0,07\ \ (q_{3101} + q_{3102} + q_{3103}) - p_{21} - 0,07\ q_{3101} = 0; \\ 57\ 584 - 0,07\ \ (q_{3101} + q_{3102} + q_{3103}) - p_{21} - 0,07\ q_{3102} = 0; \\ 57\ 500 - 0,07\ \ (q_{3101} + q_{3102} + q_{3103}) - p_{21} - 0,07\ q_{3103} = 0; \end{cases}$$

$$\begin{cases} 61\ 432 - 1,66\ \ (q_{3111} + q_{3112} + q_{3113}) - p_{21} - 1,66\ q_{3111} = 0; \\ 61\ 166 - 1,66\ \ (q_{3111} + q_{3112} + q_{3113}) - p_{21} - 1,66\ q_{3112} = 0; \\ 61\ 165 - 1,66\ \ (q_{3111} + q_{3112} + q_{3113}) - p_{21} - 1,66\ q_{3113} = 0. \end{cases}$$

We solve systems with respect to quantity variables:

$$q_{311} = 83\ 435,2941 - 1,4706\ p_{21};$$

$$q_{312} = 85\ 052,9412 - 1,4706\ p_{21};$$

$$q_{313} = 88\ 317,6471 - 1,4706\ p_{21};$$

$$q_{321} = 323\ 816,3334 - 5,5556\ p_{21};$$

$$q_{322} = 323\ 466,6667 - 5,5556\ p_{21};$$

$$q_{323} = 318\ 166,6667 - 5,5556\ p_{21};$$

$$q_{324} = 318\ 866,667 - 5,5556\ p_{21};$$

$$q_{325} = 349\ 216,6667 - 5,5556\ p_{21};$$

$$q_{326} = 342\ 016,6667 - 5,5556\ p_{21};$$

$$q_{327} = 338\ 816,6667 - 5,5556\ p_{21};$$

$$q_{328} = 334\ 416,6667 - 5,5556\ p_{21};$$

$$q_{331} = 126\ 493,75 - 2,0833\ p_{21};$$

$$q_{332} = 122\ 656,25 - 2,0833\ p_{21};$$

$$q_{333} = 118\ 843,75 - 2,0833\ p_{21};$$

$$q_{334} = 124\ 518,75 - 2,0833\ p_{21};$$

$$q_{335} = 122\ 906,25 - 2,0833\ p_{21};$$

$$q_{341} = 9\ 642,0712 - 0,1618\ p_{21};$$

$$q_{342} = 9\ 667,7994 - 0,1618\ p_{21};$$

$$q_{351} = 22\ 087,1528 - 0,3472\ p_{21};$$

$$q_{352} = 22\ 344,0972 - 0,3472\ p_{21};$$

$$q_{353} = 22\ 378,8194 - 0,3472\ p_{21};$$

$$q_{361} = 21\ 967,2932 - 0,3759\ p_{21};$$

$$q_{362} = 23\ 177,8196 - 0,3759\ p_{21};$$

$$q_{363} = 23\ 351,5038 - 0,3759\ p_{21};$$

$$q_{364} = 22\ 040,9774 - 0,3759\ p_{21};$$

$$q_{365} = 23\ 925,1880 - 0,3759\ p_{21};$$

$$q_{366} = 22\ 877,8196 - 0,3759\ p_{21};$$

$$q_{371} = 14\ 112,963 - 0,2315\ p_{21};$$

$$q_{372} = 13\ 841,4352 - 0,2315\ p_{21};$$

$$q_{381} = 44\ 330,4348 - 0,7246\ p_{21};$$

$$q_{382} = 44\ 804,3478 - 0,7246\ p_{21};$$

$$q_{383} = 43\ 756,5217 - 0,7246\ p_{21};$$

$$q_{384} = 45\ 004,3478 - 0,7246\ p_{21};$$

$$q_{385} = 43\ 104,3478 - 0,7246\ p_{21};$$

$$q_{391} = 10\ 920,3509 - 0,1754\ p_{21};$$

$$q_{392} = 10\ 755,0877 - 0,1754\ p_{21};$$

$$q_{3101} = 206\ 417,8571 - 3,5714\ p_{21};$$

$$q_{3102} = 205\ 803,5714 - 3,5714\ p_{21};$$

$$q_{3103} = 204\ 603,5714 - 3,5714\ p_{21};$$

$$q_{3111} = 9\ 332,0783 - 0,1506\ p_{21};$$

$$q_{3112} = 9\ 171,8374 - 0,1506\ p_{21};$$

$$q_{3111} = 9\ 171,2349 - 0,1506\ p_{21}.$$

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