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On Competition in the Telecommunications Market*

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Abstract The paper investigates the process of competition in the market of telecommunication services between three firms: the leader, the challenger and the follower. In this work we construct a model of competition between three players in the form of a multistage non-zero sum game. As a solution of the game we find a subgame perfect equilibrium. We illustrate the results with an example for three companies working on the Saint-Petersburg telecommunications market.

Keywords: telecommunications market, non-zero sum game, multistage game, subgame perfect equilibrium

1. Introduction

In the paper competition between three companies in the telecommunications market is investigated. All firms have different types: the leader, the challenger and the follower. The leader is a company that prevails in the market and acts in three main directions:

- expansion of the market by attracting new customers and finding new areas;
- increasing its market share in the current telecommunication market;
- protecting its business from attacks by using defensive strategies.

The challenger firm is a company that does not lag far behind the leader of the market and tries to become the leader by using attacking strategies. This firm uses strategies aimed to expand its market share, but those that do not cause active opposition to competitors.

In Stackelberg's monograph (H. Von Stackelberg, 1952) the competition on the market is presented by a multistage decision-making model. At the first stage, the decision is made by the leader, and at the next stage, taking into account the decision of the leader, the decision is made by the company-follower. At the same time, when making decisions, each of the firms pursues its own goal. In paper (Beresnev and Suslov, 2010) authors propose an algorithm for constructing an approximate solution of such a problem.

In this paper, we consider a more complicated problem. At the first stage, the leader and the challenger make decisions about which telecommunication services and at what prices to offer subscribers. At this stage, some customers make a choice in favor of the first or second company. At the next stage, the follower, taking into account the choice of competitors, decides on what services it would be better to offer to potential customers. At the same time, the follower tries to keep its customers

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and, if possible, to attract a part of the competitors customers. At this stage, the remaining customers make their own choices.

We assume that each customer must choose one of the telecommunication services (tariff). If a customer decides to stay at his tariff with his company, we believe that he chooses the appropriate service from the relevant company. The leader and the challenger aim to maximize their profits by attracting some of the competitor's customers. The purpose of the follower is to maximize its profit and save the customers.

We formalize this problem of competition in the telecommunications market between the leader and the challenger as a nonzero-sum game in normal form. Then we consider a multistage game where this nonzero-sum game is realized as the first stage. The second stage is made by the follower. As a solution of this game we consider a subgame perfect equilibrium (SPE) (Hellwig and Leininger, 1987, D.W.K. Yeung, L.A. Petrosyan and N.A. Zenkevich, 2009, Nessah and Tian, 2014). Some examples are given and discussed in the paper.

The obtained solution allows each company to develop a long-term strategy to maximize its summing payoff. In the future, it would be interesting to study a strongly time consistency (Petrosyan, 1993) of the solution.

2. The game

To formalize the problem of competition in the telecommunication market we introduce the following assumptions:

- firms are informed on subscriber preferences which are created taking into account service prices;
- under the profit we will understand the difference between the price of the service and the unit costs for it; the profit can only be positive;
- the income from the sale of a certain service is determined by the number of subscribers who have decided to use this service, and its price;
- for the price of the service we accept the total cost of services, which the subscriber should pay per month;
- Under a telecommunication service we understand a certain tariff consisting of a package of services, for example, a tariff consisting of v minutes for all outgoing calls, b gigabytes of Internet and z outgoing SMS messages. Further, the number of outgoing SMS messages is omitted from consideration, since to date SMS messages have been replaced by so-called messengers;
- as practice shows, in the realities of the modern world, the unit costs for tariffs, in which the main emphasis is on the volume of Internet traffic in comparison with tariffs, in which the main emphasis is on the number of minutes for outgoing calls, is much less. Since quite often subscribers use the Internet to make calls, and the demand for such tariffs is higher, telecommunication operators set the price for Internet tariffs higher;
- we assume that the unit costs for the same services type for different tariffs within the same operator are equal;
- let the fixed costs within the same operator are equal for all of the offered tariffs;

- fixed costs for larger companies are higher than their competitors, and unit costs are lower;
- we also make the assumption that subscribers are informed about what services the players can offer.

We denote F_1 by the leader, F_2 is the challenger and F_3 is the follower. Let $N = \{F_1, F_2, F_3\}$ be the set of players – telecommunication companies, which provide services on the market.

Let $I = \{1, \ldots, m\}$ be the set of services (tariffs) that are offered on the telecommunications market. Each element of $i_r \in I$ is some specific type of service. This service will be called the service type i_r, offered by some firm.

Denote by I_1, I_2 and I_3 subsets of I, which contain the offered services, respectively, by the leading firm, the challenger firm and the follower firm. Assume that $I_1 \cup I_2 \cup I_3 = I$ and $I_1 \cap I_2 \cap I_3 = \emptyset$.

Let the following quantities are known:

- c_i^k is the price of service i for the player F_k, where $i \in I_k$ and $k \in \{1, 2, 3\}$;
- a_i is the unit costs of service i;
- f_k is the fixed costs (i.e. costs that are not depending on the volume of services) for the player's service F_k, $k \in \{1, 2, 3\}$. At the same time, the fixed costs are constant.

We denote by $J = \{1, \ldots, n\}$ the set of subscribers using the services offered on the market. Each element $j \in J$ is a certain subscriber (customer). We assume, that the subscriber chooses an offered by one of the firms service based on his internal preferences, which are specified by splitting the set J into two subsets J_T and J_P. J_T includes subscribers, who mainly think about low price, when choosing an operator. In turn, J_T is divided into J_{T_1}, which consists of subscribers, for whom the most important thing along with the price is the number of minutes for outgoing calls within the tariff, and J_{T_2}, consisting of those who, along with the price pays attention to the volume of Internet traffic provided within the tariff. The subset J_P contains "conservative" subscribers. They are subscribers which can not change an operator, because it is a problem for various reasons, for example, corporate users.

We suppose that $J = J_T \cup J_P$, $J_T \cap J_P = \emptyset$. We have

$$J^0 = J = J_1^0 \cup J_2^0 \cap J_3^0. \tag{1}$$

Expression (1) describes the distribution of the set of subscribers between the players at the initial stage of the game. Let the following relations hold:

$$|J_1^0 \cap J_T| \geq |J_2^0 \cap J_T| > |J_3^0 \cap J_T|,$$
$$|J_1^0 \cap J_P| > |J_2^0 \cap J_P| > |J_3^0 \cap J_P|.$$

We assume that subscribers from the set $J_P \cap J_k^0$ always choose player k, where $k \in \{1, 2, 3\}$, and the service that the operator offers at the moment, regardless of the offered tariff.

By the strategy of player F_k, where $k \in \{1, 2, 3\}$, we define the pair $s_k^{i_r} = (c_{i_r}^k, i_r)$, $i_r \in I_k$. We denote the set of strategies of F_k by $S_k = \{s_k^{i_r} : \ i_r \in I_k\}$. We assume that the firm's strategies are developed, based on the results of SWOT analysis, and are aimed at minimizing the risks associated with the identified weaknesses of each firms, and aimed to use the identified opportunities.

We introduce the preference relationships for the subscriber $j \in J$ of services offered by firms F_1 and F_2. Obviously, the player's strategy is characterized by the following indicators: the price c_i^k, the number of minutes v_i^k, the number of gigabytes of mobile Internet b_i^k.

For subscriber $j \in J \in J_{T_2}$ we say that the pair $(c^1_{i_l}, i_l)$ is preferable to the pair $(c^2_{i_p}, i_p)$, i.e. $(c^1_{i_l}, i_l) \succ (c^2_{i_p}, i_p)$, if at least one of the following conditions holds:

1. $c^1_{i_l} < c^2_{i_p}$ and $b^1_{i_l} > b^2_{i_p}$;
2. $c^1_{i_l} = c^2_{i_p}$ and $b^1_{i_l} > b^2_{i_p}$;
3. $c^1_{i_l} < c^2_{i_p}$ and $b^1_{i_l} = b^2_{i_p}$.

For other cases:

1. if $c^1_{i_l} < c^2_{i_p}$ and $b^1_{i_l} < b^2_{i_p}$, then $(c^1_{i_l}, i_l) \succ (c^2_{i_p}, i_p)$, if $j \in J_2^0 \cap J_{T_2}$ and $(c^1_{i_l}, i_l) \prec (c^2_{i_p}, i_p)$, if $j \in J_1^0 \cap J_{T_2}$.
2. if $c^1_{i_l} > c^2_{i_p}$ and $b^1_{i_l} > b^2_{i_p}$, then $(c^1_{i_l}, i_l) \succ (c^2_{i_p}, i_p)$, if $j \in J_2^0 \cap J_{T_2}$ and $(c^1_{i_l}, i_l) \prec (c^2_{i_p}, i_p)$, if $j \in J_1^0 \cap J_{T_2}$.
3. if $c^1_{i_l} = c^2_{i_p}$ and $b^1_{i_l} = b^2_{i_p}$, then $(c^1_{i_l}, i_l) \succ (c^2_{i_p}, i_p)$, if $j \in J_2^0 \cap J_{T_2}$ and $(c^1_{i_l}, i_l) \prec (c^2_{i_p}, i_p)$, if $j \in J_1^0 \cap J_{T_2}$.
4. if one of the previous three conditions is fulfilled, but $j \notin J_1^0 \cap J_{T_2}$ and $j \notin J_2^0 \cap J_{T_2}$, then $j \in J_{nd}$, where J_{nd} is a set of subscribers who can equally choose the services of both compared firms.

For subscriber $j \in J \cap J_{T_1}$ we say that the pair $(c^1_{i_l}, i_l)$ is preferable to the pair $(c^2_{i_l}, i_p)$, i.e. $(c^1_{i_l}, i_l) \succ (c^2_{i_p}, i_p)$, if at least one of the following conditions holds:

1. $c^1_{i_l} < c^2_{i_p}$ and $v^1_{i_l} > v^2_{i_p}$;
2. $c^1_{i_l} = c^2_{i_p}$ and $v^1_{i_l} > v^2_{i_p}$;
3. $c^1_{i_l} < c^2_{i_p}$ and $v^1_{i_l} = v^2_{i_p}$.

For other cases:

1. if $c^1_{i_l} < c^2_{i_p}$ and $v^1_{i_l} < v^2_{i_p}$, then $(c^1_{i_l}, i_l) \succ (c^2_{i_p}, i_p)$, if $j \in J_2^0 \cap J_{T_1}$ and $(c^1_{i_l}, i_l) \prec (c^2_{i_p}, i_p)$, if $j \in J_1^0 \cap J_{T_1}$.
2. if $c^1_{i_l} > c^2_{i_p}$ and $v^1_{i_l} > v^2_{i_p}$, then $(c^1_{i_l}, i_l) \succ (c^2_{i_p}, i_p)$, if $j \in J_2^0 \cap J_{T_1}$ and $(c^1_{i_l}, i_l) \prec (c^2_{i_p}, i_p)$, if $j \in J_1^0 \cap J_{T_1}$.
3. if $c^1_{i_l} = c^2_{i_p}$ and $v^1_{i_l} = v^2_{i_p}$, then $(c^1_{i_l}, i_l) \succ (c^2_{i_p}, i_p)$, if $j \in J_2^0 \cap J_{T_1}$ and $(c^1_{i_l}, i_l) \prec (c^2_{i_p}, i_p)$, if $j \in J_1^0 \cap J_{T_1}$.
4. if one of the previous three conditions is fulfilled, but $j \notin J_1^0 \cap J_{T_2}$ and $j \notin J_2^0 \cap J_{T_1}$, then $j \in J_{nd}$, where J_{nd} is a set of subscribers who can equally choose the services of both compared firms.

Next, we look, if $|J_{nd}| = 2k$, where $k \in Z$, then a half of the subscribers from J_{nd} choose the firm F_1, and the other half choose the firm F_2. If $|J_{nd}| = 2k + 1$, where $k \in Z$, then the firm F_1 (firm with higher market position) chooses one subscriber $j \in J_{nd}$ more.

To determine the preference of services offered by firms F_2 and F_3, we proceed similarly. To determine the preference of services offered by the firms F_1 and F_3, we proceed similarly.

First, the subscriber chooses two of the three firms to compare their services for preference based on the value $\frac{c_i^k}{v_i^k}$ for subscribers $j \in J \cap J_{T_1}$ and the value $\frac{c_i^k}{b_i^k}$ for subscribers $j \in J \cap J_{T_2}$. He chooses those two firms for which this value is less. It should be noted that when the subscriber compares the leader company and the challenger company, he compares the corresponding values for one strategy chosen by each player, since at the first step of the game, a bi-matrix game is played between the leader firm and the challenger firm. For the third company, the subscriber compares all possible strategies, since the follower firm can choose any of them, and if for all of the theoretically possible strategies it turns out that the value $\frac{c_i^k}{v_i^k}$ for subscribers $j \in J \cap J_{T_1}$ and the value $\frac{c_i^k}{b_i^k}$ for subscribers $j \in J \cap J_{T_2}$, respectively, is not the greatest one, then the follower firm becomes one of the two firms, which services are compared to the preference by the subscriber j.

We introduce the switching function $V_j(s_k^{i_r})$.

$$V_j(s_k^{i_r}) = \begin{cases} 1, \text{ if } \ i_r \text{ is the preferred service for subscriber } j; \\ 0, \text{ otherwise,} \end{cases}$$

i.e., the function characterises the preference for the subscriber $j \in J$ of the service $i_r \in I_k$ offered by the player F_k, compared to all other types of services that are offered on the market. For regular subscribers, i.e. for $j \in J \cap J_P$:

$$V_j(s_k^i) = 1 \text{ for all } i \in I_k, \ k \in \{1, 2, 3\}.$$

We assume that the services are selected by the subscriber for a month in advance. Let introduce the value $g_j(s_k^{i_r}) = (c_{i_r}^k - a_{i_r})$, which characterises the profit of a company F_k from the subscriber j when the firm uses strategy $s_k^{i_r}$. The i_r service, which offers a larger amount of Internet traffic is designated for i_r^{gb}, and the service, which offers a greater number of minutes for outgoing calls for i_r^{mnt}. Taking account of assumption 6 6, we obtain the following inequality

$$g_j(s_k^{i_r^{gb}}) \geq g_j(s_k^{i_r^{mnt}}) > 0,$$

for $k \in \{1, 2, 3\}$, $i_r \in I_k$, $j \in J$.

Denote by $G_k(s_k^{i_r})$ the total profit of firm k from customers $j \in J_P \cap J_k^0$, which choose the service i_r, i.e.

$$G_k(s_k^{i_r}) = \sum_{j \in J_P \cap J_k^0} g_j(s_k^{i_r}),$$

where $k \in \{1, 2, 3\}$ and $i_r \in I_k$. Then the payoff function of the leader firm we can define by the following way:

$$H_1(s_1^{i_l}, s_2^{i_p}, J^0) = -f_1 + \sum_{j \in J_T \cap J^0} g_j(s_1^{i_l}) \times V_j(s_1^{i_l}) \times (1 - V_j(s_2^{i_p})) + G_1(s_1^{i_l}),$$

where $i_l \in I_1$, $i_p \in I_2$.

The payoff function of the leader firm expresses the amount of profit of this player taking into account changes in income due to the loss and acquisition of subscribers. The payoff function for the challenger firm is the following

$$H_2(s_1^{i_l}, s_2^{i_p}, J^0) = -f_2 + \sum_{j \in J_T \cap J^0} g_j(s_2^{i_p}) \times V_j(s_2^{i_p}) \times (1 - V_j(s_1^{i_l})) + G_2(s_2^{i_p}),$$

where $i_l \in I_1$, $i_p \in I_2$.

The payoff function of the challenger firm expresses the amount of profit of this player taking into account changes in income due to the loss and acquisition of subscribers. The payoff function for the follower firm is

$$H_3(s_1^{i_l}, s_2^{i_p}, s_3^{i_s}, J^0) = -f_3 +$$
$$+ \sum_{j \in J_T \cap J^0} g_j(s_3^{i_s}) \times V_j(s_3^{i_s}) \times (1 - V_j(s_1^{i_l})) \times (1 - V_j(s_2^{i_p})) + G_3(s_3^{i_s}),$$

where $i_l \in I_1$, $i_p \in I_2$, $i_s \in I_3$.

The payoff function of the follower firm expresses the total profit of this player taking into account changes in income due to the loss and acquisition of subscribers.

$$V_j(s_1^{i_l}) + V_j(s_2^{i_p}) + V_j(s_3^{i_s}) \leq 1, \ i_l \in I_1, \ i_p \in I_2, \ i_s \in I_3, \ j \notin J_{nd}. \qquad (2)$$

Inequality (2) shows that it might not be that for the subscriber $j \notin J_{nd}$ two services are the most preferable at the same time compared to each other. For subscribers $j \in J_{nd}$, in general, this situation can not be either, since, in fact, according to condition 4 from the block "other cases" when determining the preference of services, the subscriber $j \in J_{nd}$ chooses only one company.

The leader in the game is determined by the number of subscribers available to the company at the beginning of the game. At the end of the game, in the case of equality of the subscribers number in several companies, the leader is determined by the amount of total profit. Since, ceteris paribus, follower firm can both play along with the leader firm and play along with the challenger firm, for definiteness, we assume that player F_3 plays along to player F_1.

Thus, the competition on the telecommunications market can be formalised in the form of non-zero sum game Γ:

$$\Gamma = < N, S_1, S_2, S_3, H_1, H_2, H_3 > .$$

How is the game going?

1. First, the leader firm F_1 and the challenger firm F_2 put forward their tariffs (services) in the first step of the game, i.e. they choose the strategies. Thus, previously two players play a non-antagonistic bi-matrix game.
2. In the next step, taking into account the choice of players F_1 and F_2, follower firm F_3 makes a decision.
3. Each player seeks to maximise their win function.

Schematically, the game in expanded form for a one-stage case, when the set of strategies of each player contains two strategies, is shown in figure 1.

Let us explain the game shown in figure 1. In position 1 the leader company F_1 and the challenger firm F_2 choose one of two strategies, operating simultaneously and independently of each other, playing a bi-matrix game. In positions 2, 3, 4, 5, depending on the choice of players F_1 and F_2, the follower firm F_3 makes its choice

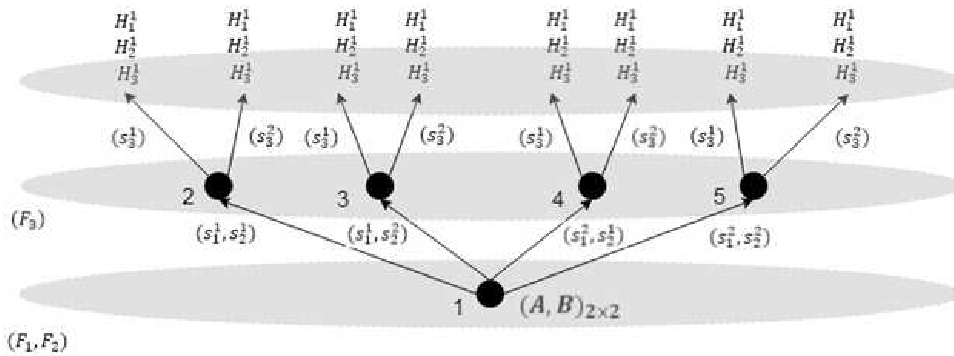

Fig. 1.

choosing between s_3^1 or s_3^2 strategies. At the same time, if both of these strategies give him the same gain, when choosing his strategy, this player plays along with the firm-leader.

3. Nash equilibrium

We assume that for the same strategy for the same player the value $g_j(s_k^{i_r})$ will be greater for the strategy $s_k^{i_r}$, if it offers a greater volume of the services, that is, if i_1 and i_2 are "Internet" tariffs, than the value of $g_j(s_k^{i_1})$ will be greater if the service i_1 offers a much larger package of Internet traffic. This is due to the fact that when the volume of the service increases, the price for it increases, while the unit costs according to the assumptions for services of the same type are the same.

Let $S_1 = \{s_1^1, \dots, s_1^m\}$, $S_2 = \{s_2^1, \dots, s_2^n\}$, $S_3 = \{s_3^1, \dots, s_3^n\}$ be the sets of strategies of F_1, F_2, F_3, correspondingly .

Suppose there is a payoff matrix $(A, B)_{m\times n}$ for firms F_1 and F_2.

We search for a subgame perfect equilibrium from the end of the game. Suppose that players F_1 and F_2 have chosen their strategies (s_1^{*l}, s_2^{*p}) and have already played the first stage of the game. Let us compare two arbitrary strategies $s_3^{i_1}$ and $s_3^{i_2}$ of the company F_3 and the payoff functions for these strategies:

$$H_3(s_1^{*l}, s_2^{*p}, s_3^{i_1}, J_0) = -f_3 + \sum_{j\in J_T\cap J^0} g_j(s_3^{i_1}) \times V_j(s_3^{i_1}) \times (1 - V_j(s_1^{*l})) \times (1 - V_j(s_2^{*p})) + G_3(s_3^{i_1}), \tag{3}$$

$$H_3(s_1^{*l}, s_2^{*p}, s_3^{i_2}, J_0) = -f_3 + \sum_{j\in J_T\cap J^0} g_j(s_3^{i_2}) \times V_j(s_3^{i_2}) \times (1 - V_j(s_1^{*l})) \times (1 - V_j(s_2^{*p})) + G_3(s_3^{i_2}), \tag{4}$$

where:

$$G_3(s_3^{i_1}) = \sum_{j\in J_P\cap J_3^0} g_j(s_3^{i_1}),$$

$$G_3(s_3^{i_2}) = \sum_{j\in J_P\cap J_3^0} g_j(s_3^{i_2}).$$

Suppose that the set $J_P \cap J_3^0$ contains w_3 subscribers, then we can rewrite the previous conditions in the form of:

$$G_3(s_3^{i_1}) = w_3 \times g_{j\in J_P\cap J_3^0}(s_3^{i_1}),$$

$$G_3(s_3^{i_2}) = w_3 \times g_{j\in J_P\cap J_3^0}(s_3^{i_2}),$$

where $w_3 > 0$.

Then, we get that in order for the strategy $s_3^{i_1}$ to be no worse for the player F_3 than the strategy $s_3^{i_2}$, it is required that the following inequality be executed:

$$\begin{aligned}
&\sum_{j\in J_T\cap J^0} g_j(s_3^{i_1}) \times V_j(s_3^{i_1}) \times (1 - V_j(s_1^{*l})) \times (1 - V_j(s_2^{*p}))+\\
&w_3 \times g_{j\in J_P\cap J_3^0}(s_3^{i_1})-\\
&\sum_{j\in J_T\cap J^0} g_j(s_3^{i_2}) \times V_j(s_3^{i_2}) \times (1 - V_j(s_1^{*l})) \times (1 - V_j(s_2^{*p}))-\\
&w_3 \times g_{j\in J_P\cap J_3^0}(s_3^{i_2}) \geq 0.
\end{aligned} \tag{5}$$

Converting expression (5), we have

$$\begin{aligned}
&g_{j\in J_T\cap J^0}(s_3^{i_1}) \left(\sum_{j\in J_T\cap J^0} V_j(s_3^{i_1}) \times (1 - V_j(s_1^{*l})) \times (1 - V_j(s_2^{*p})) \right)\\
&+w_3 \times g_{j\in J_P\cap J_3^0}(s_3^{i_1})-\\
&-g_{j\in J_T\cap J^0}(s_3^{i_2}) \times \left(\sum_{j\in J_T\cap J^0} V_j(s_3^{i_2}) \times (1 - V_j(s_1^{*l})) \times (1 - V_j(s_2^{*p})) \right) -\\
&-w_3 \times g_{j\in J_P\cap J_3^0}(s_3^{i_2}) \geq 0.
\end{aligned} \tag{6}$$

Note that $g_{j\in J_T\cap J^0}(s_3^{i_1}) = g_{j\in J_P\cap J_3^0}(s_3^{i_1})$ and $g_{j\in J_T\cap J^0}(s_3^{i_2}) = g_{j\in J_P\cap J_3^0}(s_3^{i_2})$, since the value of g_j does not depend on the set of the subscriber, and depends only on the player's strategy (choosing service). We introduce the following notation:

$$g_{j\in J_T\cap J^0}(s_3^{i_1}) = g(s_3^{i_1}),$$
$$g_{j\in J_T\cap J^0}(s_3^{i_2}) = g(s_3^{i_2}).$$

Then, expression (6) can be written as:

$$g(s_3^{i_1}) \times \left(\sum_{j\in J_T\cap J^0} V_j(s_3^{i_1}) \times (1 - V_j(s_1^{*l})) \times (1 - V_j(s_2^{*p})) + w_3 \right) \geq$$

$$g(s_3^{i_2}) \times \left(\sum_{j\in J_T\cap J^0} V_j(s_3^{i_2}) \times (1 - V_j(s_1^{*l})) \times (1 - V_j(s_2^{*p})) + w_3 \right).$$

Given that $g(s_3^{i_2}) > 0$, $w_3 > 0$ and $\sum\limits_{j \in J_T \cap J^0} V_j(s_3^{i_1}) \times (1 - V_j(s_1^{*l}) \times (1 - V_j(s_2^{*p})) \geq 0$, we obtain that the strategy $s_3^{i_1}$ is more profitable for the player F_3 when the following condition is fulfilled:

$$\frac{g(s_3^{i_1})}{g(s_3^{i_2})} \geq \frac{\sum\limits_{j \in J_T \cap J^0} V_j(s_3^{i_2}) \times (1 - V_j(s_1^{*l})) \times (1 - V_j(s_2^{*p})) + w_3}{\sum\limits_{j \in J_T \cap J^0} V_j(s_3^{i_1}) \times (1 - V_j(s_1^{*l})) \times (1 - V_j(s_2^{*p})) + w_3}. \quad (7)$$

Next, consider two arbitrary strategies $s_1^{i_1}$ and $s_1^{i_2}$ of player F_1 and the payoff function for them. Let the player F_2 use a fixed strategy s_2^{*p}. Write the payoff function of the first player for strategies $s_1^{i_1}$ and $s_1^{i_2}$.

$$H_1(s_1^{i_1}, s_2^{*p}, J^0) = -f_1 + \sum_{j \in J_T \cap J^0} g_j(s_1^{i_1}) \times V_j(s_1^{i_1}) \times (1 - V_j(s_2^{*p})) + G_1(s_1^{i_1}), \quad (8)$$

$$H_1(s_1^{i_2}, s_2^{*p}, J^0) = -f_1 + \sum_{j \in J_T \cap J^0} g_j(s_1^{i_2}) \times V_j(s_1^{i_2}) \times (1 - V_j(s_2^{*p})) + G_1(s_1^{i_2}), \quad (9)$$

where

$$G_1(s_1^{i_1}) = \sum_{j \in J_P \cap J_1^0} g_j(s_1^{i_1})$$

$$G_1(s_1^{i_2}) = \sum_{j \in J_P \cap J_1^0} g_j(s_1^{i_2}).$$

Let set $J_P \cap J_1^0$ contains w_1 subscribers, then the previous two expressions can be written as:

$$G_1(s_1^{i_1}) = w_1 \times g_{j \in J_P \cap J_1^0}(s_1^{i_1}),$$

$$G_1(s_1^{i_2}) = w_1 \times g_{j \in J_P \cap J_1^0}(s_1^{i_2}),$$

where $w_1 > 0$.

Thus, in order for the strategy $s_1^{i_1}$ to be no worse for player F_1 in comparison with the strategy $s_1^{i_2}$, the following condition must be fulfilled:

$$\begin{aligned}&\sum_{j \in J_T \cap J^0} g_j(s_1^{i_1}) \times V_j(s_1^{i_1}) \times (1 - V_j(s_2^{p*})) + w_1 \times g_{j \in J_P \cap J_1^0}(s_1^{i_1}) - \\ &\sum_{j \in J_T \cap J^0} g_j(s_1^{i_2}) \times V_j(s_1^{i_2}) \times (1 - V_j(s_2^{p*})) \\ &-w_1 \times g_{j \in J_P \cap J_1^0}(s_1^{i_2}) \geq 0.\end{aligned} \quad (10)$$

Transforming inequality (10), we get

$$g_{j\in J_T\cap J^0}(s_1^{i_1})\times\left(\sum_{j\in J_T\cap J^0}V_j(s_1^{i_1})\times(1-V_j(s_2^{p*}))\right)+w_1\times g_{j\in J_P\cap J_1^0}(s_1^{i_1})-$$
$$g_{j\in J_T\cap J^0}(s_1^{i_2})\times\left(\sum_{j\in J_T\cap J^0}V_j(s_1^{i_2})\times(1-V_j(s_2^{p*}))\right)- \tag{11}$$
$$w_1\times g_{j\in J_P\cap J_1^0}(s_1^{i_2})>0,$$

where

$$g_{j\in J_T\cap J^0}(s_1^{i_1})=g_{j\in J_P\cap J_1^0}(s_1^{i_1}),$$

$$g_{j\in J_T\cap J^0}(s_1^{i_2})=g_{j\in J_P\cap J_1^0}(s_1^{i_2}),$$

since g_j does not depend on the set to which the subscribers j belongs, but depends only on the strategy. We introduce the following notation:

$$g_{j\in J_T\cap J^0}(s_1^{i_1})=g(s_1^{i_1}),$$
$$g_{j\in J_T\cap J^0}(s_1^{i_2})=g(s_1^{i_2}),$$

then, expression (17) can be written as:

$$g(s_1^{i_1})\times\left(\sum_{j\in J_T\cap J^0}V_j(s_1^{i_1})\times(1-V_j(s_2^{*p}))+w_1\right)\geq$$
$$\geq g(s_1^{i_2})\times\left(\sum_{j\in J_T\cap J^0}V_j(s_1^{i_2})\times(1-V_j(s_2^{*p}))+w_1\right).$$

Given that $g(s_1^{i_2})>0$, $w_1>0$ and $\sum\limits_{j\in J_T\cap J^0}V_j(s_2^{i_1})\times(1-v_j(s_1^{*l}))\geq 0$, we obtain that the strategy $s_1^{i_1}$ is more preferable for the player F_1 than the strategy $s_1^{i_2}$ when the following condition is fulfilled:

$$\frac{g(s_1^{i_1})}{g(s_1^{i_2})}\geq\frac{\sum\limits_{j\in J_T\cap J^0}V_j(s_1^{i_2})\times(1-V_j(s_2^{*p}))+w_1}{\sum\limits_{j\in J_T\cap J^0}V_j(s_1^{i_1})\times(1-V_j(s_2^{*p}))+w_1} \tag{12}$$

Next, consider two arbitrary strategies $s_2^{i_1}$ and $s_2^{i_2}$ of player F_2 and the payoff function for these strategies. Let the player F_1 use his fixed strategy s_1^{*l}. Carrying out reflections analogous to the case for firm F_1, we obtain a condition, when the strategy $s_2^{i_1}$ is more profitable for the player F_2 than the strategy $s_2^{i_2}$, i.e.

$$\frac{g(s_2^{i_1})}{g(s_2^{i_2})}\geq\frac{\sum\limits_{j\in J_T\cap J^0}V_j(s_2^{i_2})\times(1-V_j(s_1^{*l}))+w_2}{\sum\limits_{j\in J_T\cap J^0}V_j(s_2^{i_1})\times(1-V_j(s_1^{*l}))+w_2}, \tag{13}$$

where $g(s_2^{i_2}) > 0$, $w_2 > 0$ and $\sum_{j \in J_T \cap J^0} V_j(s_2^{i_1}) \times (1 - V_j(s_1^{*l})) \geq 0$.

Thus, in a two-stage non-zero sum game $\Gamma =< N, S_1, S_2, S_3, H_1, H_2, H_3 >$, the strategies of the players s_1^*, s_2^*, s_3^* lead to a subgame perfect equilibrium if the following conditions are satisfied:

$$\frac{g(s_1^*)}{g(s_1^{i_2})} \geq \frac{\sum\limits_{j \in J_T \cap J^0} V_j(s_1^{i_2}) \times (1 - V_j(s_2^*)) + w_1}{\sum\limits_{j \in J_T \cap J^0} V_j(s_1^*) \times (1 - V_j(s_2^*)) + w_1},$$

$$\frac{g(s_2^*)}{g(s_2^{i_2})} \geq \frac{\sum\limits_{j \in J_T \cap J^0} V_j(s_2^{i_2}) \times (1 - V_j(s_1^*)) + w_2}{\sum\limits_{j \in J_T \cap J^0} V_j(s_2^*) \times (1 - V_j(s_1^*)) + w_2},$$

$$\frac{g(s_3^*)}{g(s_3^{i_2})} \geq \frac{\sum\limits_{j \in J_T \cap J^0} V_j(s_3^{i_2}) \times (1 - V_j(s_1^*)) \times (1 - V_j(s_2^*)) + w_3}{\sum\limits_{j \in J_T \cap J^0} V_j(s_3^*) \times (1 - V_j(s_1^*)) \times (1 - V_j(s_2^*)) + w_3},$$

for $\forall s_1^{i_2} \in \{S_1\}$, $\forall s_2^{i_2} \in \{S_2\}$, $\forall s_3^{i_2} \in \{S_3\}$, where $w_k = |J_P \cap J_k^0|$, $k \in \{1, 2, 3\}$. These inequalities can be rewritten as:

$$g(s_1^*) \geq \frac{\sum\limits_{j \in J_T \cap J^0} V_j(s_1^{i_2}) \times (1 - V_j(s_2^*)) + w_1}{\sum\limits_{j \in J_T \cap J^0} V_j(s_1^*) \times (1 - V_j(s_2^*)) + w_1} \times g(s_1^{i_2}),$$

$$g(s_2^*) \geq \frac{\sum\limits_{j \in J_T \cap J^0} V_j(s_2^{i_2}) \times (1 - V_j(s_1^*)) + w_2}{\sum\limits_{j \in J_T \cap J^0} V_j(s_2^*) \times (1 - V_j(s_1^*)) + w_2} \times g(s_2^{i_2}), \tag{14}$$

$$g(s_3^*) \geq \frac{\sum\limits_{j \in J_T \cap J^0} V_j(s_3^{i_2}) \times (1 - V_j(s_1^*)) \times (1 - V_j(s_2^*)) + w_3}{\sum\limits_{j \in J_T \cap J^0} V_j(s_3^*) \times (1 - V_j(s_1^*)) \times (1 - V_j(s_2^*)) + w_3} \times g(s_3^{i_2}),$$

for $\forall s_1^{i_2} \in \{S_1\}$, $\forall s_2^{i_2} \in \{S_2\}$, $\forall s_3^{i_2} \in \{S_3\}$, where $w_k = |J_P \cap J_k^0|$, $k \in \{1, 2, 3\}$.

So, we have proved the following theorem.

Theorem 1. *In a non-zero sum two-stage game $\Gamma =< N, S_1, S_2, S_3, H_1, H_2, H_3 >$ the strategies s_1^*, s_2^*, s_3^* lead to a subgame perfect equilibrium if inequalities (15) are fulfilled.*

Proof. The proof follows from the construction.

It can be noted that the ratio of payoffs from the use of the strategy (service), leading to the subgame perfect equilibrium in the game for player F_k, to the amount of a payoff from the use of any other strategy should be not less than the ratio of the number of all subscribers who have chosen the services of player F_k when using any other strategy, to the number of all subscribers who have chosen the service leading to the subgame perfect equilibrium.

4. Example

We determine the strategies of the players taking into account the results of the SWOT-analysis (Bogomolova, 2004), that has been conducted with using a real data set for three companies working on the Saint-Petersburg telecommunications market.

First, we assume that $I_1 = \{1, 2\}$, $I_2 = \{3, 4\}$, $I_3 = \{5, 6\}$.

- Tariff 1 contains 200 minutes of outgoing calls, 2 Gigabytes of Internet traffic. Fixed costs f_1^1 are equal to 70, the unit cost a_1 is equal to 60.
- Tariff 2 contains 100 minutes of outgoing calls, 6 Gigabytes of Internet traffic. Fixed costs f_2^1 are 70, the unit cost a_2 is 50.
- Tariff 3 contains 200 minutes of outgoing calls, 3 Gigabytes of Internet traffic. Fixed costs f_3^2 are equal to 60, the unit cost a_3 is equal to 70.
- Tariff 4 contains 150 minutes of outgoing calls, 5 Gigabytes of Internet traffic. Fixed costs f_4^2 equal to 60, the unit cost a_4 equals to 60.
- Tariff 5 contains 150 minutes of outgoing calls, 4 Gigabytes of Internet traffic. Fixed costs f_5^3 equal to 50, the unit cost a_5 is 70.
- Tariff 6 contains 100 minutes of outgoing calls, 7 Gigabytes of Internet traffic. Fixed costs f_6^3 equal to 50, the unit cost a_6 equals to 60.

That is, the size of the fixed costs takes into account that for leading companies the fixed costs are relatively less per. They depend on a number of staff.

Let $J = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17\}$. Divide J_T into two sets J_{T_1} and J_{T_2}. J_{T_1} includes the customers for whom the determining factor when choosing a service is the number of minutes per month for outgoing phone calls. J_{T_2} includes the customers, for whom the volume of monthly Internet traffic is important along with the price. Thus, taking into account tariffs 1-6, we have $J_{T_1} = \{1, 2, 3, 4, 5\}$, $J_{T_2} = \{6, 7, 8, 9\}$.

The set J_P includes customers $10, 11, 12, 1314, 15, 16, 17$.

Let $J_1^0 \cap J_P = \{10, 11, 12, 13\}$, $J_2^0 \cap J_P = \{14, 15, 16\}$, $J_3^0 \cap J_P = \{17\}$.

Assume that

$$J_1^0 \cap J_T = \{1, 4, 6, 9\},\ J_2^0 \cap J_T = \{2, 5, 7\},\ J_3^0 \cap J_T = \{3, 8\}.$$

Let us move on to the strategy sets: $S_1 = \{s_1^1, s_1^2\}$, $S_2 = \{s_2^1, s_2^2\}$, $S_3 = \{s_3^1, s_3^2\}$,

$$s_1^1 = (300, 1),\ s_1^2 = (330, 2),$$

$$s_2^1 = (310, 3),\ s_2^2 = (320, 4),$$

$$s_3^1 = (320, 5),\ s_3^2 = (340, 6).$$

Firm	Strategy	Tariff	Fixed costs	Unit costs	Minutes	Gigabytes	Price
Leader	S_1^1	1	70	60	200	2	300
	S_1^2	2	70	50	100	6	330
Challenger	S_2^1	3	60	70	200	3	310
	S_2^2	4	60	60	150	5	320
Follower	S_3^1	5	50	70	150	4	320
	S_3^2	6	50	60	200	7	340

We calculate the players payoffs for all situations in this game. Let us make clear how the players payoffs in an arbitrary situation are determined. Consider the situation in which player F_1 uses strategy s_1^1, player F_2 uses s_2^1, player F_3 uses s_3^1, so we have situation (s_1^1, s_2^1, s_3^1).

Determine what services the two firms customerss $j \in J \cap J_{T_1}$ and $j \in J \cap J_{T_2}$ will be compared to the preference. To do this, we calculate the values that characterize the ratio of the cost of services to their volume. For subscribers $j \in J \cap J_{T_1}$:

- player F_1 has $\frac{300}{200}$;
- player F_2 has $\frac{310}{200}$;
- player F_3 has $\frac{320}{150}$ or $\frac{340}{100}$.

Thus, subscribers of $j \in J \cap J_{T_1}$ will compare the service preference of the first and second players. Similarly, for subscribers $j \ inJ \cap J_{T_2}$:

- player F_1 has $\frac{300}{2}$;
- player F_2 $\frac{310}{3}$;
- player F_3 has $\frac{320}{4}$ or $\frac{340}{7}$.

Thus, subscribers of $j \in J \cap J_{T_2}$ will compare the service preference of the second and third players.

Then $H_1(s_1^1, s_2^1) = 2090$, $H_2(s_1^1, s_2^1) = 1140$, $H_3(s_1^1, s_2^1, s_3^1) = 700$, as the first player will be chosen by customers $1, 2, 3, 4, 5, 10, 11, 12, 13$; second player will be chosen by customers $6, 8, 14, 15, 16$; the third player will be chosen by customers $7, 9, 17$. So, we can write down in the table results of distribution of subscribers depending on strategies:

Strategy profile	Customers of firm F_1	Customers of firm F_2	Customers of firm F_3	Payoffs (H_1, H_2, H_3)
s_1^1, s_2^1, s_3^1	$\{1, 2, 3, 4, 5, 10, 11, 12, 13\}$	$\{6, 8, 14, 15, 16\}$	$\{7, 9, 17\}$	$(2090, 1140, 700)$
s_1^2, s_2^1, s_3^1	$\{7, 8, 10, 11, 12, 13\}$	$\{1, 2, 3, 4, 5, 14, 15, 16\}$	$\{6, 9, 17\}$	$(1610, 1860, 700)$
s_1^1, s_2^2, s_3^1	$\{1, 2, 3, 4, 5, 10, 11, 12, 13\}$	$\{6, 7, 8, 9, 14, 15, 16\}$	$\{17\}$	$(2090, 1760, 200)$
s_1^2, s_2^2, s_3^1	$\{7, 8, 10, 11, 12, 13\}$	$\{1, 2, 3, 4, 5, 14, 15, 16\}$	$\{17\}$	$(1610, 2540, 200)$
s_1^1, s_2^1, s_3^2	$\{1, 2, 3, 4, 5, 10, 11, 12, 13\}$	$\{6, 8, 14, 15, 16\}$	$\{7, 9, 17\}$	$(2090, 1140, 790)$
s_1^2, s_2^1, s_3^2	$\{7, 8, 10, 11, 12, 13\}$	$\{1, 2, 3, 4, 5, 14, 15, 16\}$	$\{6, 9, 17\}$	$(1610, 1860, 790)$
s_1^1, s_2^2, s_3^2	$\{1, 2, 3, 4, 5, 10, 11, 12, 13\}$	$\{6, 8, 14, 15, 16\}$	$\{7, 9, 17\}$	$(2090, 1240, 790)$
s_1^2, s_2^2, s_3^2	$\{7, 8, 10, 11, 12, 13\}$	$\{1, 2, 3, 4, 5, 6, 9, 14, 15, 16\}$	$\{17\}$	$(1610, 2540, 230)$

The game with the players payoffs is presented in figure 2.

Now let us find a subgame perfect equilibrium in this two-stage game. To do this, we consider the game from the end. In nodes $2, 3, 4,\ \ 5$, player F_3 makes his decision. Accordingly, in all of these nodes, he chooseC< the strategy s_3^2, because it gives to him the highest payoff. Depending on the strategy selected by the third players, in node 1 a bimatrix game is played between the first and the second players. The corresponding payoff matrix is presented in table:

	s_2^1	s_2^2
s_1^1	$(2090, 1140)$	$(2090, 1240)$
s_1^2	$(1610, 1860)$	$(1610, 2540)$

In that game firms F_1 and F_2 choose their strategies simultaneously. The solution of this bimatrix game is the situation (s_1^1, s_2^2) with payoffs $(2090, 1240)$. Note that in nodes $2, 3, 4, 5$ player F_3 chooses the strategy s_3^2. All this forms a subgame perfect equilibrium which can be written as $[s_1^1; s_2^2; s_3^2, s_3^2, s_3^2, s_3^2]$. Figure 2 shows the constructed subgame perfect equilibrium.

5. Conclusion

In conclusion, we would like to note that using various intellectual response techniques for the process of competition is a large and interesting range of tasks, the

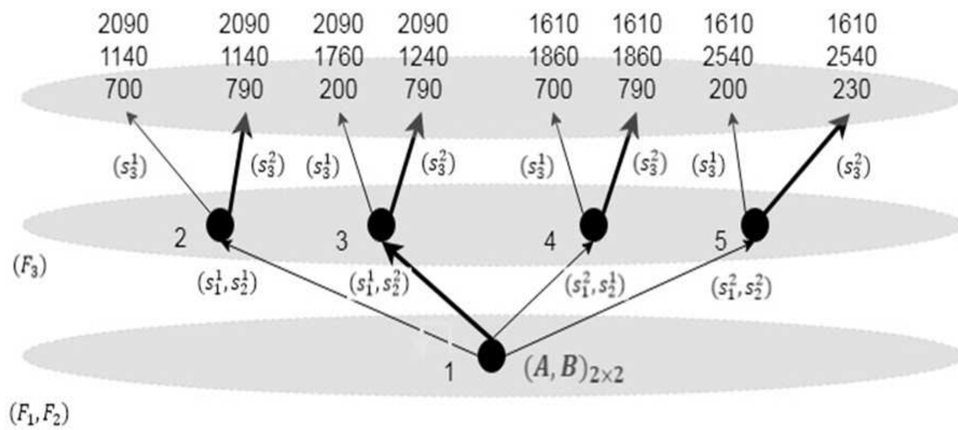

Fig. 2.

relevance of which does not decrease over time, but on the contrary, in the conditions of increasing globalisation, opening of new markets, growth of production capacities and production volumes is increasing, thereby increasing the scientific and practical value of the works devoted to this subject.

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