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Key Player and Key Leader in a Distribution Network Game*

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Abstract Identification of positions which play a key role like the key player and key leader has attracted the interest of scholars in the organization studies, strategic management, and marketing literature. While most of the previous studies have paid attention to this issue in the social network analysis realm at the individual level, the organization level has remained underinvestigated especially when there is no room for linear mathematical analysis due to complexity of interactions among players.

Therefore, this research addresses this issue to identify the key player and key leader in a real distribution network (DN) game consist of 31 distribution centers (DCs) in Iran which can assist managers for making a decision about investments and DN design tasks.

For this purpose, after proposing a payoff function based on the contribution of players (DCs) in investment within the network to enhance their profit, we develop the formulation of the key leader and key player at organization level in the supply network context using Katz-Bonacich centrality and intercentrality, respectively.

We have used Nash equilibrium and Stackelberg solution models on the fundament of game theory to calculate two indexes concerning key leader and key player problems.

The results of this research can contribute to the literature to find leadership in the supply network level of analysis using game theory in complex situations.

Keywords: Distribution network game, key player, key leader, Katz-Bonacich centrality, intercentrality, Nash equilibrium, Stackelberg solution.

1. Introduction

In organization studies and strategic management, the investigation of positions and relationships in supply chains and networks analysis have attracted the interest of many scholars with respect to their influences on such issues as performance enhancement (Kotabe et al., 2002; Gulati, 1999; Jensen, 2003; Kim et al., 2011), innovation adoption (Feldman and Audretsch, 1998; Burt, 1980; Ibarra, 1993; Kim et al., 2011), and other benefits (see for example: Burt, 2001; Kim et al., 2011; Zaheer and Bell, 2005; Bellamy et al., 2014) that affect supply networks decision makers in supply chain management realm (Kemppainen and Vepsalainen, 2003).

Supply chain management (SCM) as a broad management philosophy is an evolution of logistics management which was concentrated on purchasing and transportation. It consists of facilities and distribution centers (DCs) that perform the

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functions of material purchasing, the transformation of these materials into semi-finished and finished products, and the distribution of them between players in the network and finally to the end consumers under a cooperative or non-cooperative relation (Chandra and Kumar, 2000; Donner et al., 2008; Kemppainen and Vepsalainen, 2003). Improvement of SCM is a critical task that shows the benefits of individual companies derives from the boost of performance and profitable growth of the supply chain as a whole through concentration on supply chain objectives, design, modeling, and implementation of best practices in supply chain (Kemppainen and Vepsalainen, 2003).

The relatively recent incorporation of the term "network" into supply chain management research as supply networks represents a pressing need to view supply chains as a network for all players to gain improved performance, operational efficiencies, and ultimately sustainable competitiveness (Kotabe et al., 2002). A supply network analysis approach allows us to better understand the operations of supply networks, both at the individual firm level and network level for the development of the supply network as a body. Therefore, for this purpose in this context, it becomes imperative to study and analyze the network structure to identify positions which play key roles (Wasserman and Faust, 1994; Kemppainen and Vepsalainen, 2003). This is a critical task for supply network decision makers to prevent incorrect decisions that may cause performance decline or even network failure and also can provide an applicable task in the marketing field in order to find best alternatives to investment (Herring, 1999).

Here three aspects of SCM including different activities, relations, and objectives will be explained in order to articulate the problem (Donner et al., 2008).

Firstly, SCM encompasses the management of different activities of members at different levels associated with moving goods between suppliers and the end users through the manufacturing, transportation, and end-of-life products' destination (Donner et al., 2008; Mahdiraji et al., 2014). These decisions are strategic (e.g., investments), tactical (e.g., the destination of products end-of-life) as well as operational (e.g., suppliers selection, etc.) which with synergy determine the network costs (Neto et al., 2008; Mahdiraji et al., 2014).

Second, many autonomous companies work in supply networks to reduce their individual costs and achieve their goals (Proff, 2006) under either cooperation or non-cooperation relationships. The aim of non-cooperation is to make independent and self-interested strategic decisions by individuals in the network to increase their payoff without considering the effect on others, while cooperation aims to improve participants' collective welfare as a payoff imputation. Thus, one must choose which theory to employ depending on one's interest and the relevant problem (Simchi-Levi et al., 2004).

Third, the objective of supply chain management is making decisions at a different level of the supply network especially strategic decisions about activities (e.g., collaborative investments (Donner et al., 2008)) to reach the goal of supply network (Mahdiraji et al., 2014). This goal which is mostly based on network performance in case of decreasing/ increasing overall costs/ profits must be coordinated for maximum effectiveness, efficiency, and productivity as the consequences of all activities in a dynamic environment. This may require modification of network members as some of the members in this regard do not deserve to continue networking. Hence, in this regard, we may face some applicable questions like which positions play key roles as key player and the key leader in the network, which players cannot be removed from the network, and which players are more beneficial to invest in. Then, effective decisions about the network activities (Mahdiraji et al., 2014; Neto et al., 2008; Wang and Liu, 2016) to find solutions for these questions are critical issues under investigation in this research.

For solving these issues we will consider a real problem concerning a large distribution network as part of a supply network belong to a popular online store consists of 31 DCs in Iran. The problem is to identify critical positions which play key roles in the network. For this purpose, the problem is the uncertainty about the flows, the diversity of products, the volume of orders, the complexity of interactions, the influence of one's decision on others' decisions, and generally speaking, all assumptions that take the problem out from a linear mathematical modeling framework and bring it into a dynamic and complex situation dependent on each player's decision in the network. Also, the decision of players affects the decision of others based on the externality effects and interactions between all players.

We consider this problem as a game in which players should decide about their level of investment in themselves to establish the logistics network in the supply network with high performance. We suppose some particular assumptions for all players to determine their contribution level in the given network structure (routes between DCs); the amount of investment in purchasing vehicles assigned to each route between DCs. This decision relies on each player's decision based on the complementary strategic situation which we will consider in the payoff function to incentivize all players to invest more.

More description of the method selection, context, and subject as leadership will be provided in the following sections. The remainder of this paper is organized as follow. In section 2, we review some relevant literature. Section 3 gives the methodology and proposes a model to illustrate the main ideas of this paper. Section 4 introduces the results of the calculations concerning our proposed model and case study. Finally, we draw some concluding remarks in section 5.

2. Distribution network game in the literature

2.1. Supply networks

Supply network is a multi-layered concept, as a network of suppliers, manufacturers, distribution centers, and retailers organized to produce and distribute the right products, to the right customer, in the right quantity, in the right condition, at the right place, at the right time, and at the right cost to minimize total costs (Coyle et al., 2016), which is not quite different from the notion of supply chain management (SCM) (Seyedhosseini et al., 2018).

The competition between firms from the one side in the same supply chain and on the other side between supply chains from the perspective of the network has been increasingly acknowledged by both academy and industry (Wu et al., 2018). This type of research is relatively rich in recent years, so the literature in this research stream is relatively abundant and mature, both in the theoretical system and model methods. For example, a review of competition situation and related practices was given in the research of Fahimi et al. (2017) and Wu et al. (2018).

In short, competition can provide advantages for the supply chains, and it is not always an unpleasant situation in the networks. From the form of network perspective, centralized supply chains cannot be considered ever as the best performance provider, and decentralized supply chains also can provide better performance under a specific situation such as uncertainty and non-cooperation. Therefore, competition plays a critical role in the determination of optimal structure and output. Finally, as a remedy in a fierce competition situation, sharing the profit through some contracts between players can facilitate the coordination in networks (Giannoccaro, 2018).

To remain competitive, companies must seek new solutions for important SCM issues such as production planning, route planning, load and capacity planning, and distribution network design (DND). DND is very important to deliver products as quick as possible and with the lowest price because faster product availability is key to sales and market share growth. In DND, supply network managers as main decision makers should know about the relations, supply network objectives, and the importance or roles of all players in the network. Because this decision is related to the performance, competitive advantage, and network survival (Wu et al., 2018; Kim et al., 2011; Bellamy et al., 2014).

In the supply network context, there are two critical positions which we should identify them as the key player and key leader to avoid undesirable destructive decisions about them. It is safe to say that no two supply networks are exactly alike, and a participant's role may vary in each network. According to literature (Zhou and Chen, 2015; Zhou and Chen, 2016; Ballester et al., 2006) key player and key leader can be defined based on primarily assumptions about the network structure, relations, the objectives such as the maximally connected to all other nodes, the impact on the network cohesion (the structural importance of members), the members' characteristics, the network centrality etc. (Borgatti and Everett, 2006; Friedrich et al., 2016; Kim et al., 2011).

There is a dearth of research and empirical studies, which have paid attention to the concept of leadership at an organizational level in the logistics network context. This is due to the complexity of both disciplines, which should be considered in a dynamic situation. Here, we will use centrality measurement on the fundament of both organizational theories especially the theory of power and power in social network analysis and its conceptualization in the supply network context. The term "power" is most popular as an interrelated feature with leadership to describe how an organization dominates over other players in the network. It can be defined differently based on the definition of performance and network objective in each problem (Daugherty, 2011; Jia et al., 2018; Gosling et al., 2016).

According to the mentioned main objective of each supply network, organizations should pay attention to enhance their profit as the main feature of performance. For this purpose, all players try to increase their profits in supply networks with investment in themselves to remain competitive and gain a valuable position in networks. Investments are critically important in modern logistics, where the adoption of quick-response demand systems and flexible manufacturing approaches has increased the need for coordination across different players in the supply chains. In general, cooperative investment on competitive investments under profit sharing as mentioned before as a remedy for fierce competition can be characterized as either cost reduction or as demand enhancement, but as shown in the literature (Belderbos et al., 2004; Wang and Liu, 2016; Ehie and Olibe, 2010; Gilbert and Cvsa, 2003), cost reduction or profit enhancement is of the main objective (Wang and Liu, 2016), and continuous investments in physical assets and innovation are necessary for competitive success (Bourreau et al., 2018) and increase performance (Gilbert and Cvsa, 2003; Werth and Boeert, 2013). However each player has an additional incentive to invest in order to improve its status quo, the benefit for a competitive and profit sharing investment is hybrid because it offers direct benefits as selfish motivation and indirect benefits as spillovers and externalities (Katz and Shapiro, 1985; Che and Hausch, 1999). While research on the effectiveness of cooperative investment is evident, little attention has been given to the distribution networks between all DCs in a competition relation. An investment in logistics infrastructure that benefits supply network and consumers who use it is essential, without which, a logistics infrastructure or a supply network is not sustainable (Kogan and Tapiero, 2009).

2.2. Centralized and decentralized supply networks

In order to understand the interaction between players in a network, it is indispensable to know about two main forms of supply network structures; centralized and decentralized supply networks.

Centralized supply network can contain a unique control center, which entirely controls and coordinates the whole set of players and echelons in the supply network through the managing all the information and decisions between different supply chains which work together (Chankong and Haimes, 2008).

A centralized decision-making system ignores the independence of its members. Thus, most of the supply chain systems are decentralized (Wang et al. 2004).

Decentralized supply network involves several self-interested players as decision-making units (DMU) in different echelons, each in different or same supply chains which works together. In other words, each player makes its own decision. In the decentralized architecture of supply chains, there is no DMU controlling all players. Thus, it cannot be guaranteed that the local decisions of players will converge to a global optimum solution of the supply chain. In order to solve the conflict problems, the players who establish a partnership, exchange information of transaction orders and feedback decisions to negotiate on their decisions. The partnership of players will break down when they cannot find a converged solution in their negotiations (Bernstein and Nagarajan, 2012).

2.3. Negotiation and interaction

In economics and political science, negotiation has a special place to be used in different ways. It can demonstrate an interaction process (communication) for resolving conflicts that may lead to competition (in which, players arrange their individual activities in a coherent manner) (Moulin and Chaib-Draa, 1996) or cooperation (in which, players work together to achieve a common objective) (Pynadath and Ttambe, 2002) in a wide variety of multi-player domains. These conflicts can be over a share of joint resources (commodities, services, time, money, etc.), tasks or document allocations, buyer-seller prices, etc. The main question is to know how rational autonomous players will choose their negotiation strategies (Costantini et al., 2013).

According to the literature there are three main negotiation methods including heuristic methods (Costantini et al., 2013), Game theory-based methods including cooperative and non-cooperative (competitive) game (Simchi-Levi et al., 2004), and argumentation-based methods (Sierra et al., 1998).

Here game theory can provide the best decision made by a given player in a supply chain. In other word, powerful strategies are offered by game theory with taking into account the possible decisions of others.

About the kind of interactions, in the real world application, the interaction of players leads to a wide variety of complexity dynamics. This complexity arises due to non-linear player interactions. The behavior of such non-linear systems can be chaotic and unpredictable. Complex adaptive systems (CAS) in the natural world and cyber-physical systems (CPS) in man-made systems are examples of such player interactions. However, a key issue in such models is to understand the dynamics of player interactions. But evolutionary game theory can solve this problem and can be utilized as an appropriate method for these problems (Nagarajan and Sosic, 2008).

Here also, game theory provides a formal analytical framework with a set of mathematical tools to study the complex horizontal or vertical interactions among rational firms (Nagarajan and Sosic, 2008). Therefore, in this research also game theory is an appropriate method for considering negotiation and interaction in a multi-player system.

2.4. Game theoretical modeling

As competition puts pressure on supply chains, it presents opportunities for new approaches such as the game theory approach for solving the transshipment problem (Donner et al., 2008).

The game theory is a powerful tool that effectively models and analyzes the strategic decision making between self-interested economic players, where a player makes a choice by taking into account the others' choices in a multi-player situation, and the outcome depends on the choice made by every player. Researchers in supply chain management now use tools from game theory and economics to understand, predict, and help managers to make strategic operational decisions in complex multi-player supply chain networks. Loosely speaking, game theory models situations where players make decisions to maximize their own utility while taking into account that other players are doing the same and that decisions made by players affect each other's utilities. In the game-theoretic analysis, researchers usually attempt to determine the optimal strategy by analyzing the interaction as a game between a set of players and seeking its equilibrium (Harsanyi, 1956; von Stengel, 2002). Assuming further that participants behave according to the assumptions of rational-choice theory, this approach can guide the design of the interaction mechanism itself, and thus force such players to behave in certain ways (Hu and Fukushima, 2015). There is a broad division of game theory into two approaches: the cooperative (Driessen, 1988) and the non-cooperative (Ritzberger, 2002) approaches. These approaches, though vary in their theoretical content and the methodology used in their analysis, are really just two different ways of looking at the same problem. Different basic concepts in game theory are available for example in the study by von Neumann and Morgenstern (1944) as summarized the basic concepts. It also has been widely developed since 1950 when John Nash introduced the well-known concept of Nash equilibrium in non-cooperative games (Nash, 1951; 1950; Cachon and Netessine, 2006; Nagarajan and Soŝić, 2008; Mahdiraji et al., 2014).

Considering the context of this research and interactions between players in the proposed case study, the field of supply chain management has seen, in recent years, a wide variety of research papers that employ different types of games to model interaction between players. For an excellent survey and state of art techniques, we refer you to Cachon and Netessine (2006).

In complicated problems with conflicting optimizations objective functions, game theory is a possible way to formulate the payoff function of one player considering the impact of other players by their decisions in a network to reach optimal decisions for all players not a single one. Here the knowledge structure of one player from other players is a complicated part of the model that can provide different types of games. From the one hand, the game can be categorized as incomplete information if players do not have well knowledge including strategies and payoff functions about each other (Petrosyan and Zenkevich, 2016).

On the other hand, the game can be categorized as a complete information game if all players have full knowledge of the set of players in the network (Kline, 2015). This category also can be defined in two ways about the knowledge structure. The game may be classified as perfect information game if a group of players decides about their actions with delay under the impact of having knowledge about the decisions of another group of players in every stage of the game. Otherwise, the game will be classified as an imperfect information game in the network (Petrosyan and Zenkevich, 2016).

In the game theory considering the interaction logic of players we will face with two most basic classification used in the game theory as cooperative (e.g., glove game and balanced game) (Branzei et al., 2008) and non-cooperative games (e.g., prisoner's dilemma and Cournot competition) (Lambertini, 1997).

On the one hand, when players make decisions autonomously the results of decisions can be formulated through non-cooperative game theory in which players are self-interested about their profits without any enforceable contracts outside of those modeled in the game. Therefore, this game is not defined based on lack of cooperation but is based on self-regulation or self-enforcement in any cooperation (Ritzberger, 2002)

On the other hand, when the purpose of the model is to find a set of payoffs for all players in an alliance to provide collective welfare through collective rationality, in which the players in a boundary of a binding contract think about the profit of the group as a whole (Branzei et al., 2008; von Neumann and O. Morgenstern, 1944; Nash, 1951; Suzumura, 1992).

Therefore, in the non-cooperative game we deal with individual self-interested players while in the cooperative game we should think about groups or coalitions and collective purpose. In both models, the aim is to reach a set of strategies for all players with which we can have optimal payoffs for all partners in the network. Here, there are two models that predict these strategies for players in a game including Nash equilibrium and Stackelberg solution (Petrosyan and Zenkevich, 2016).

2.5. Nash equilibrium and Stackelberg solution

Nash games (Nash, 1950; 1951) model competitive behavior among a set of autonomous players that all players are assumed to know the objective functions of other players and make decisions to choose their own strategies at the same time by taking into account the strategies of other players. A Nash equilibrium is a set of strategies in which each individual player has chosen an optimal strategy given the strategies chosen by the other players (Leyffer and Munson, 2010; Hu and Fukushima, 2015). So in Nash equilibrium all players are in a position of the same level, however in some real-world situation, when there is a single dominant firm, the market must be modeled as a Stackelberg (single-leader-follower) solution (von Stackelberg, 1952), in which the dominant firm, the leader, has the ability to decide the quantities or price to maximizes its profit subject to all other firms, the followers, who make their decisions after observing the decision of the leader in a competitive equilibrium (Leyffer and Munson, 2010; Hu and Fukushima, 2015).

According to the purpose of this research, we are looking for key roles in a set of interrelated players. In this regard, leader-follower games can provide sufficient ground for identifying leaders as critical positions in the networks.

In Stackelberg (single-leader–follower) game, while the leader anticipates the responses of the followers and commits to a strategy to optimize the upper-level problem, the remaining followers react to the selected strategy to optimize the lower-level problems jointly by competing among themselves. Here, the reaction of the followers is a Nash equilibrium parameterized by the decision variables for the leader. The leader chooses an optimal strategy knowing how the followers will react (Leyffer and Munson, 2010; Hu and Fukushima, 2015).

Between these two extremes is the multi-leader-follower game that has multiple dominant firms and a number of followers. Multi-leader-follower games can be further differentiated into those in which the follower responses are constrained to be identical for each leader and those in which the followers are allowed to respond differently to each leader (Leyffer and Munson, 2010).

Single-leader-follower games and multi-leader-follower games issue has been studied in depth in some duopoly (i.e., just one rival shows reactions to the newcomer) (Lederer, 1986; Goyal and Joshi, 2003; Wei and Hansen, 2007; Zhou and Chen, 2015; Zhou and Chen, 2016; Ballester et al., 2006) and oligopoly markets (Friedman, 1977; Hamilton and Slutsky, 1990; Nagurney, 2010; von Stengel and Zamir, 2010; Hu and Fukushima, 2015), respectively.

Identifying the key player in a network is one of the primary uses of supply network analysis (Wasserman and Faust, 1994). There are some metrics to show how the networks were organized such as network density, centralization, and complexity (Kim et al., 2011). The concept of centrality including degree centrality, closeness centrality, and betweenness centrality according to the literature is a fundamental criteria for measuring the importance of nodes in a network based on their impact on others and their power (Goyal, 2007; Freeman; 1979; Borgatti and Everett, 2006; Borgatti and Li, 2009; Zhou and Chen, 2015; Zhou and Chen, 2016; Ballester et al., 2006).

In recent studies related to key player and key leader in social networks the weighted Katz–Bonacich Centrality and intercentrality (Bonacich, 1987; Katz and Shapiro, 1985; Bonacich and Lloyd, 2001; Zhou and Chen, 2015; Zhou and Chen, 2016; Ballester et al., 2006) have been the main criteria, which also in this research will be considered for formulation the positions of key leader and key player, respectively.

We measure key leader on the fundament of Katz-Bonacich centrality and the key player on the fundament of intercentrality as an organization in the supply network context. Bonacich centrality fails to internalize all the network payoff externalities which players exert on each other, whereas the intercentrality measure internalizes them all and takes into account a player's own centrality and its contribution to the centrality of others (Abraham et al., 2010). Intuitively, need to capture not only a player's activity level (proportional to Katz-Bonacich centrality) but the player's contribution to others' centralities as well; for more study, different measures of network centralities have been introduced in Wasserman and Faust (1994).

The Katz-Bonacich centrality of a player counts the number of paths that stem from that player exponentially discounted based on the length of paths. The intercentrality counts the total number of such paths that hit the player; it is the sum of the player's Katz-Bonacich centrality and the player's contribution to every other players' Katz-Bonacich centrality (Ballester et al., 2006; Zhou and Chen, 2015; Zhou and Chen, 2016). But, we consider it through using a weighted graph as an increasing form with a length of the path to show the Butterfly effect "bullwhip effect" in the network (Osborn et al., 2002; Wycisk et al., 2008), which was not considered in the recent works. This effect describes how tiny initial shifts can result in chaotic and extreme events along the supply network due to dynamical processes. Illustrating strong interdependencies among the players in a supply network regarding each decision and action by an individual player that will affect the others. This is intuitively related to the equilibrium behavior because the paths capture all possible feedback (Osborn et al., 2002; Wycisk et al., 2008).

3. Model development

3.1. Model description

In supply networks, transportation networks and distribution centers (DC) can make a distribution network, which is intermediary that facilitates the physical flows of merchandises between sellers and buyers (Lai et al., 2002). Hence, the distribution network presented in this section is a considerable case study in the supply networks based on the road transportation mode. Indeed, investment in this network (the infrastructure, vehicles, and operations) by each DC is considered as a strategic decision to increase profits in a non-cooperative interaction which considered as a distribution network game. The importance of this decision to decrease transit time, inventory costs; security and availability rise, and environmental impact can be studied in the literature (e.g., Coyle et al., 2016). Therefore, total profit can be considered as measuring the performance of DCs.

The complexity of supply network structures with different relations; mutual interaction or undirected graphs, a hierarchical structure or directed graphs, and weighted graphs, are critical for application and adaptation of a suitable theory to understand the behaviors of such networks. Identification of positions which play a key role such as key players and key leaders in networks is critical task which is under investigation in this research considering a weighted graph form, the theory of power based on Katz–Bonacich Centrality and intercentrality, contribution of players on the fundament of their level of investments, and in supply network context altogether are the contribution of this research.

Online stores tend to face hypercompetitive business environments. Due to increasing competition, these companies are systematically pushed to search for growth opportunities in the market and to get to market before their competitors. Therefore, investments in the distribution networks are necessary in order to accelerate delivery of orders and to be an agile organization in the network. Investments in distribution infrastructures are important to maintain an agile organization in a supply network. Externally, supplier efficiency is extremely critical to supply network performance (Mahdiraji et al., 2014). Requirements for large investments in distribution infrastructure, which is reflected in the increased strategic importance

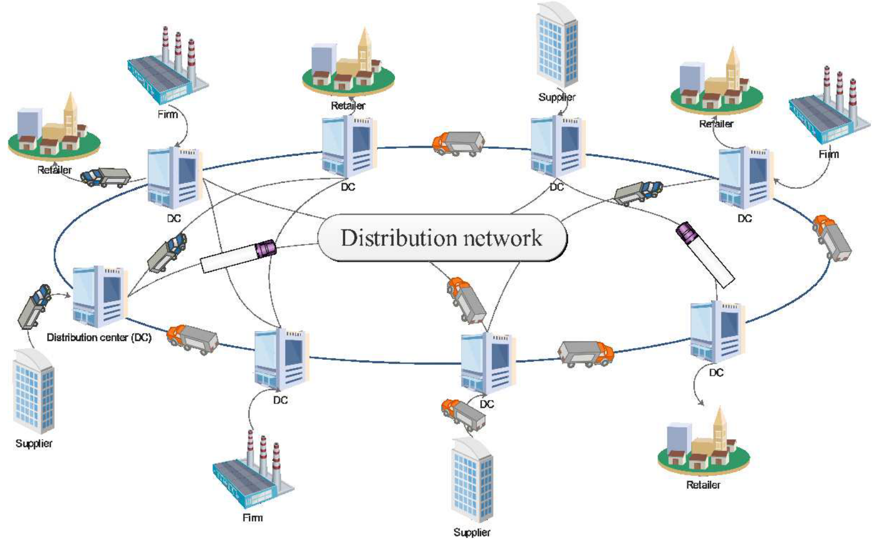

Fig. 1. Distribution network

of (domestic and global) supplier relationships and strategic alliances enabling distribution network to cover availability of new products frequently and rapidly, access different sources and sell globally, minimize inventory and reduce costs across the supply network; and offer personalized one-to-one solutions to customers (Dehning et al., 2007).

Here an online store consists of 31 distribution centers in Iran is under investigation as an empirical example to test the applicability of our proposed model as shown in Fig. 2.

3.2. Model notations and assumptions

In this section we provide a list of notations and assumptions for the formulation of key leader' and key player' position in the mentioned distribution network game, and proposing the model in the basis of game theory.

To illustrate the main ideas of proposed model in this paper, we hereby present some important notations in the table 1.

We consider a distribution network consists of 31 independent distribution centers (players) each of them located in one province of Iran. They coordinate together through an online platform to facilitate the flow of material and goods among autonomous agents including retailers, manufacturers, and suppliers. The numbers and positions of agents in the network are not stable as well as types of products. Consequently, the flow of material and goods is uncertain in each province. All DCs are connected through roads and the delivering of material and goods conducts through road transportation with different vehicles that each DC has the authority to decide about it. Here for the establishment of coordination among all players, there is a contract under supervision and control of the online platform. The performance of each node considered based on the total profit. So the following assumptions will be considered in this framework:

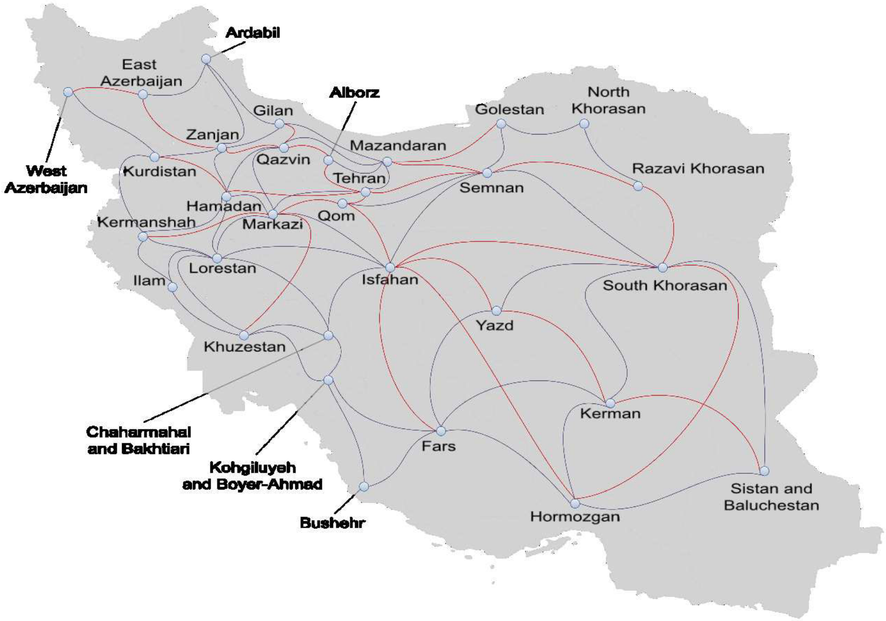

Fig. 2. Logistics network of an online store in Iran

Table 1. Notations

symbol	Definition
i, j	Players.
(i, j)	The link between player i and player j.
π_i	Total profit of player i per unit of investment.
P_i	Profit per unit of investment for player i along the link (i, j).
DP_i	Direct profit.
IP_{ij}	Indirect profit along link (i, j).
TC_i	Total cost per unit of investment.
A'	The transpose of matrix A.
$\mathbf{0}$	The zero matrix
A^D	A matrix with diagonal entries $A^D_{ij} = a_{ij}, i = 1, \cdots, N$, and off-diagonal entries $A^D_{ij} = 0, \ \ i \neq \ j$.
$\boldsymbol{I}$	Identity or unit matrix.
$\mathbf{1}$	$n \times n$ matrix with all elements equal to 1.
$\langle x, \ y \rangle$	The inner product of two column vectors x, y.

1. All players are connected to each other in a symmetric way and through different routes to meet the demands of all players.
2. Each player should allocate special infrastructure to each route to decrease extremely delivery time because of a large number of orders per unit of time (i.e. it is not possible to use the same vehicle in two different routes).
3. The system of delivering is DC to DC and all agents will receive their orders from the DC, but their relations are not within the borders of this research.
4. Each player just should deliver merchandises to its direct neighbors and this work will continue to meet the demands and the online platform manage all these transportations.
5. Total profit of each player along the link is considered as the sum of total direct profit for each player per unit of investment and total profit per unit of investment for each player along the link minus total direct costs per unit of investment.
6. The profit per unit of investment for player i along the link ij can be measured based on the market data in each province dependent on flows and routs. But for simplicity, we assume that the rate of orders and demands (flows) in each DC are similar and consequently we consider the same profit along the link for all DCs. Also, having a property through investment by each DC in buying a vehicle is associated with an extra profit for DC that we assume it is similar among all players.
7. The Total costs per unit of investment will be considered as a quadratic form which is strictly concave in each unit of investment to capture diminishing marginal cost.
8. The amount of profit along the link is considered as strategic complementarity from the perspective of each player.
9. Cost of investment can be estimated based on the interest rate and other environmental factors and governmental supports which is different in each province but for simplicity, we will consider it similar among all players.
10. The profit along the link is defined based on the conditions of each route, cost of transportation, and other criteria between mutual players which are defined by platform as a weighted graph.

3.3. Model formulation

In this sub-section based on the proposed assumptions and notations the following payoff function for each player as total profit (π_i) of player i along the link (i,j) per unit of investment (E).

$$P_{ij} = \theta_i + \beta_{ij}E_j, \tag{1}$$

$$IP_{ij} = (\theta_i + \beta_{ij}E_j)E_i = \theta_i E_i + \beta_{ij}E_j E_i, \tag{2}$$

$$DP_i = \mu_i E_i, \tag{3}$$

$$TC_i = \gamma_i E_i^2, \tag{4}$$

$$\pi_{ij} = DP_i + IP_{ij} - TC_i = (\theta_i + \mu_i)\,E_i + \beta_{ij}E_j E_i - \gamma_i E_i^2. \tag{5}$$

Then we can extend it to the following form to encompass total network relations.

$$\pi_i = (\theta_i + \mu_i)\, E_i + \sum_{j=1}^{m} \beta_{ij} E_i E_j - \gamma_i E_i^2, \quad i \neq j. \tag{6}$$

I. The equation (1), as the profit per unit of investment for player i along the link (i, j) (P_{ij}) was estimated based on the previous data of investments in current distribution centers as a linear form. It illustrates that if player i invests in distribution infrastructure (e.g. vehicles) so it can access the next player with which it has a link (direct neighbor). Then direct neighbor also will do the same to deliver the goods to the player which ordered them. Hence, each player with its investment can gain profit along the link from both its capability to deliver goods to the direct neighbors (θ_i) and from investments of other players with direct link to deliver the orders to clients $(\beta_{ij} E_j)$. Therefore, based on the online platform there are a fix amount of profit for each rout (β_{ij}) between DC i and j based on the length of the rout and related expenses that can be defined according to a symmetric weighted graph $(0 < \beta_{ij} \ll 1)$.

II. In the expression (2), the indirect profit of player i per unit of investment along the link (i, j) (IP_{ij}) is calculated through multiplication of profit per unit of investment for player i along the link (i, j) by amount of its investment.

III. In the equation (3), the direct profit of player i (DP_i) is defined as multiplication of the profit per unit of investment through having a property (μ_i) by amount of its investment. The value of μ_i is estimated as inflation rate.

IV. In the expression (4), the total cost for each player (TC_i) is defined as a quadratic form which is strictly concave in amount of investment and can capture the diminishing marginal cost per unit of investment. The value of γ_i is considered as interest rate and impact of environmental factors that can provide cost of investment for each player.

V. In the equation (5), the profit (π_{ij}) as aggregate payoff along link (i, j) is calculated just by substitution in the formula and summarization to finally reach the total profit (π_{i}) (6).

We show that a denser and larger network of local interactions increases the aggregate equilibrium outcome due to the relation between equilibrium strategic decision and network structure. This is almost because both the number and the weight of network paths increase with the network connections and the length of paths.

In our model, the network designer is able to make the leader informed about the profit of investment via experiencing the market and situation. When the leader knows the situation, the followers should guess the necessary information by observing the leader' investment. This is in the same condition of previous works such as Zhou and Chen (2015), Ballester et al. (2006), and Zhou and Chen (2016). We will follow their procedures to recalculation leader index and key player index proposed in Zhou and Chen (2015) and Zhou and Chen (2016) considering our new assumptions and payoff function.

3.4. Simultaneous-move in distribution network game with complete information

According to the proposed payoff function, all players make a decision about their investment level, simultaneously. Therefore, on the fundament of strategic complementarity situation and direct profits per unit of investment by each DC, following the research of Zhou and Chen (2015) and (2016) the best response function (BR) for each player will be measured as follow:

$$BR_i\left(E_{-i}\right) = (\theta_i + \mu_i) + \sum_{j \neq i} \beta_{ij} E_j, \tag{7}$$

$$E_i^N = BR_i\left(E_{-i}^N\right) = (\theta_i + \mu_i) + \sum_{j \neq i} \beta_{ij} E_j. \tag{8}$$

Then, with rewriting it in the matrix notation form we will have:

$$E^N = (\theta + \mu) + BE^N \Leftrightarrow E^N = (\theta + \mu)[I - B]^{-1} \equiv b\left(B, \theta + \mu\right). \tag{9}$$

Where $\theta + \mu = \left((\theta_1 + \mu_1), \ldots, (\theta_N + \mu_N)\right)'$ and the variable $b(B, \theta + \mu)$ is called the Katz-Bonacich centrality of matrix B and weight vector $(\boldsymbol{\theta} + \mu)$. This measure is in connection to the Nash equilibrium of the simultaneous-move game (Ballester et al., 2006; Zhou and Chen, 2015; 2016). Let we assume $M = [I - B]^{-1}$, then the expansion formula will be as follow:

$$M = \sum_{k=0}^{+\infty} B^k = I + B + B^2 + B^3 + \cdots = I + B. \tag{10}$$

Therefore, based on the assumptions of $\mathbf{0} < B \ll \mathbf{1}$, we will have:

$$m_{ij} = \sum_{k=0}^{+\infty} \beta_{ij}^{[k]} = I + \beta_{ij} + \beta_{ij}^{[2]} + \cdots = ((I + B)^{-1})_{ij}. \tag{11}$$

Where, $\beta_{ij}^{[k]}$ is the ij entry of β^k. Note that $\beta_{ij}^{[k]}$ counts the number of routs from i to j with length k and $B^0 = I$. Using m_{ij} notation, we can rewrite it as (Zhou and Chen, 2015; 2016):

$$E_i^N = b_i\left(B, \theta + \mu\right) = \sum_{j=1}^{n} m_{ij} (\theta + \mu)_j. \tag{12}$$

3.5. Sequential-move in distribution network game with complete information

In this sub-section based on the research of Zhou and Chen (2015) and (2016) we will consider the game as two-stage game with two groups. Let us rewrite the model considering leader group L and follower group F in a block matrix form as follow:

$$B = \begin{pmatrix} B_{LL} \; B_{LF} \\ B_{FL} \; B_{FF} \end{pmatrix}. \tag{13}$$

We also define vectors E_L and E_F as the amount of investments chosen by the DCs in L and F, respectively.

Proposition 1. *In the unique sub-game perfect Nash equilibrium of the two-stage game, players' contributions are given by (Zhou and Chen, 2015; 2016):*

$$\begin{pmatrix} E_L^* \\ E_F^* \end{pmatrix} = S \begin{pmatrix} (\theta + \mu)_L \\ (\theta + \mu)_F \end{pmatrix}, \tag{14}$$

$$S = \begin{pmatrix} U & UT_{LF}Q \\ QT_{FL}U & Q + QT_{FL}UT_{LF}Q \end{pmatrix}, \tag{15}$$

where, $T = B_{LL} + B_{LF}QB_{FL}$*,* $Q = [I - B_{FF}]^{-1}$*, and* $U = [I - (T + T^D)]^{-1}$*. Moreover,* $\begin{pmatrix} E_L^* \\ E_F^* \end{pmatrix} \geq \begin{pmatrix} E_L^N \\ E_F^N \end{pmatrix}$.

According to the proposition1, an increase in the value of total direct profit of investment $(\theta + \mu)$ for each player has direct positive effects on all the DCs' investments. Therefore, leader (in set L) intend to contribute more to gain more profit and consequently because of strategic complementarity, the followers have an incentive to invest more.

3.6. Single-leader problem

Following the research of Zhou and Chen (2015) and (2016) we must choose a single player as the first mover (i.e., $|L| = 1$) which led to maximum aggregate investment by all DCs based on the equilibrium investments in Proposition 1:

$$\max_{L \subset N: |L|=1} \{E_L + E_F\}. \tag{16}$$

To identify the key leader we use equation (15) and the case of $L = \{i\}$. Thus, we can characterize the key leader as player i which moves first to calculate the L-index for all players and investigate leader with high L-index in the network. For this purpose, we rewrite matrix B as follows:

$$B = \begin{pmatrix} 0 & \beta_{\cdot i} \\ \beta_{i\cdot} & B_{-l} \end{pmatrix}. \tag{17}$$

Where $\beta_{\cdot i} \equiv (\beta_{1i}, \ \ldots, \beta_{(i-1)i}, \beta_{(i+1)i}, \ \ldots, \ \beta_{Ni})'$ and $\beta_{i\cdot} \equiv \big(\beta_{i1}, \ \ldots, \beta_{i(i-1)}, \beta_{i(i+1)}, \ldots, \beta_{iN}\big) = \beta'_{\cdot i}$. Here, player i chooses its level of investment in the first stage, then in the second stage the rest of players $-i$ will determine their level of investments based on the first stage in the network game. Then, with anticipating these responses from the followers, player i in the first stage will choose E_i to maximize:

$$\pi\left(E_i, E^*_{-i}(E_i)\right) = (\theta + \mu)_i E_i - \gamma_i E_i^2 + E_i \left\langle \beta_{\cdot i}, E^*_{-i}(E_i)\right\rangle. \tag{18}$$

Where the value of total direct profit of investment for player $-i$ is modified from $(\theta + \mu)_j$ to $(\theta + \mu)_j + \beta_{ji}E_i$ and the equilibrium investments of $-i$ players will be:

$$E^*_{-i}(E_i) = b\left(B_{-i}, (\theta + \mu)_{-i} + E_i\beta_{\cdot i}\right) = (I - B_{-i})^{-1}\left((\theta + \mu)_{-i} + E_i\beta_{\cdot i}\right). \tag{19}$$

Therefore, the equilibrium investment for leader in the sequential-move (E_i^L) can be simplified as:

$$E_i^L = \frac{(\theta + \mu)_i + \left\langle \beta_{\cdot i}, (I - B_{-i})^{-1}(\theta + \mu)_{-i}\right\rangle}{1 - 2\left\langle \beta_{\cdot i}, (I - B_{-i})^{-1}\beta_{\cdot i}\right\rangle}. \tag{20}$$

Also, the equilibrium investment for leader in the simultaneous-move (E_i^N) based on the definition of Nash equilibrium can be proposed as:

$$E_i^N = \frac{(\theta + \mu)_i + \left\langle \beta_{\cdot i}, (I - B_{-i})^{-1}(\theta + \mu)_{-i}\right\rangle}{1 - \left\langle \beta_{\cdot i}, (I - B_{-i})^{-1}\beta_{\cdot i}\right\rangle}. \tag{21}$$

Moreover, comparing (12) and (21) we will have (Zhou and Chen, 2015; 2016):

$$m_{ii} = \frac{1}{1 - \langle \beta_{\cdot i}, (I - B_{-i})^{-1} \beta_{\cdot i} \rangle} \ , \tag{22}$$

$\frac{m_{ij}}{m_{ii}}$ =j-th entry of $(I - B_{-i})^{-1}\beta_{\cdot i}$ for$j \neq i$.

We can then derive the equilibrium investments of other players. Afterward, we compare across scenarios with different leaders to determine the key leader. If the leader chooses E_i^N in the first stage, the followers' best responses are to choose E_{-i}^N. Thus, the difference of aggregate investment between the sequential-move and simultaneous-move can be interpreted as leader impact. In other word, if i is selected as the leader, the change of the aggregate investment is:

$$\left(1 + \sum_{j \neq i} \frac{m_{ij}}{m_{ii}}\right) \left(E_i^L - E_i^N\right) = \left(1 + \sum_{j \neq i} \frac{m_{ij}}{m_{ii}}\right) \left(\frac{b_i\left(B, \theta + \mu\right)}{2 - m_{ii}} - b_i\left(B, \theta + \mu\right)\right) =$$

$$= \frac{(m_{ii} - 1)}{(2 - m_{ii})} \frac{b_i\left(B, 1\right)}{m_{ii}} b_i\left(B, \theta + \mu\right). \tag{23}$$

This leads to the L-index specified in the second proposition.

Proposition 2. *The key leader (i^*) is player which maximizes the leading index (L-index) (Zhou and Chen, 2015; 2016):*

$$L_i = \frac{(m_{ii} - 1)}{(2 - m_{ii})} \frac{b_i\left(B, 1\right)}{m_{ii}} b_i\left(B, \theta + \mu\right). \tag{24}$$

This approach can be considered as an alternative derivation of the intercentrality measure defined in Ballester et al. (2006). In other words, in that study, the purpose was to remove player i from the network and afterward, the rest of the players will determine their level of investment in the network. Thus, we can calculate the impact of this removal on the aggregate investment by all players. Therefore, based on the change in the aggregate contribution we can measure key players using C-index for each player. It goes without saying that the key player can be different than key leaders in the network and we will present Proposition 3 as follow:

Proposition 3. *The key player is the player (i^*) which maximizes the C-index as intercentrality measure of player i (Zhou and Chen, 2015; 2016):*

$$c_i = \frac{b_i\left(B, 1\right)}{m_{ii}} b_i\left(B, \theta + \mu\right). \tag{25}$$

Therefore, using the two Prepositions (2) and (3) on the fundament of equations (24) and (25) we can find key leader and key player according to their decision on the level of investment in the network.

4. Data collection and case study results

In this section based on the proposed Propositions (2 and 3), payoff function (equation 6), and assumptions in the previous section each player can determine its investment level in the distribution network game. In the following, we investigate the scenario based on the autonomous rational players with symmetric information and provided game structure. For simplicity, we assume that the parameters of μ and θ for all players are the same and equal to 0.1 and 0.2, respectively. The weighted graph (β_{ij}) is given according to Fig. 2 as the bellow matrix in the Table 3 (all data have expressed in 0.0001 scale US dollar).

Table 2. List of distribution centers in provinces

Province	DC	Province	DC	Province	DC
Alborz	1	East Azarbaijan	12	Qazvin	23
Markazi	2	Tehran	13	Qom	24
Ardabil	3	Lorestan	14	Kerman	25
West Azarbaijan	4	Gilan	15	Kermanshah	26
Esfahan	5	Sistan and Baluchistan	16	Golestan	27
Khuzestan	6	Zanjan	17	Razavi Khorasan	28
Ilam	7	Mazandaran	18	Hamadan	29
North Khorasan	8	Semnan	19	Kohkiluyeh and Buyer-Ahmad	30
Bushehr	9	Kurdistan	20	Yazd	31
Hormozgan	10	Chaharmahaal and Bakhtiari	21		
South Khorasan	11	Fars	22		

Then, based on the equations (24) and (25) we will calculate L_i and C_i indexes, respectively. Where m_{ii} is the diagonal element of matrix $M(B,\ (\theta+\mu))$ (defined in equation (22)). Table 4 gives the Katz-Bonacich centrality and intercentrality measures for all players.

According to measurements, the analysis of results shows that player (16) has the highest profit of direct and indirect links through her weighted graph. As a result, the player (16) has the highest Katz-Bonacich centrality and thus contributes to the highest level of investments. According to the weighted graph and the assigned profit to each route, the key player can be varied. Now, by considering the same values for μ and θ for all players, indirect effects matter more, and player (16) again has the highest joint direct and indirect effects on aggregate outcomes.

5. Conclusion and discussions

In the supply network context, investigation of the key player and key leader is critical tasks for managers to decide about investment and network design tasks. Their decision affect the performance of the network and with a wrong decision, the failure of the network is not far from our thought. To prevent this event, managers should formulate the position of leaders and key players in their network by considering existent situations. This research with addressing this issue tried to formulate and measure the key leader and key player in a real case study in the distribution network game consisting of 31 distribution centers belong to a popular online store in Iran

We used the theory of power which is very popular in the literature to investigate leaders and key players in the networks of players. According to the literature, this theory can be measured on the fundament of centrality concept.

We used Katz-Bonacich centrality method to measure key leader which counts the number of all paths that originate from a specific player, weighted by a factor. This method can be applicable in DN context because the number of high-weighted paths originates from a player, the more amount of player's profit in the network.

Table 3. Weighted graph (β_{ij}) matrix

B	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	0	0.007	0.0135	0.0179	0.0109	0.0203	0.0168	0.0185	0.0262	0.0329	0.0296	0.0144	0.0012	0.0121	0.007
2	0.007	0	0.0211	0.0197	0.0072	0.0145	0.0129	0.0252	0.0217	0.0311	0.0402	0.0196	0.0073	0.0052	0.0144
3	0.0135	0.0211	0	0.0132	0.0258	0.0326	0.0244	0.027	0.0403	0.0481	0.0454	0.0055	0.0148	0.0233	0.0067
4	0.0179	0.0197	0.0132	0	0.0269	0.0266	0.0192	0.0405	0.0387	0.0507	0.0555	0.0077	0.0227	0.0218	0.0198
5	0.0109	0.0072	0.0258	0.0269	0	0.0186	0.017	0.0288	0.0145	0.0244	0.0293	0.026	0.011	0.0093	0.0191
6	0.0203	0.0145	0.0326	0.0266	0.0186	0	0.0112	0.0397	0.0115	0.032	0.048	0.0269	0.0219	0.0094	0.026
7	0.0168	0.0129	0.0244	0.0192	0.017	0.0112	0	0.0356	0.0233	0.0432	0.0447	0.0193	0.0178	0.0077	0.0194
8	0.0185	0.0252	0.027	0.0405	0.0288	0.0397	0.0356	0	0.0485	0.0407	0.0184	0.0328	0.0178	0.0303	0.0204
9	0.0262	0.0217	0.0403	0.0387	0.0145	0.0115	0.0233	0.0485	0	0.0232	0.04	0.039	0.0307	0.0215	0.0381
10	0.0329	0.0311	0.0481	0.0507	0.0244	0.032	0.0432	0.0407	0.0232	0	0.0303	0.0483	0.0334	0.0332	0.0415
11	0.0296	0.0402	0.0454	0.0555	0.0293	0.048	0.0447	0.0184	0.04	0.0303	0	0.0478	0.0328	0.0386	0.0387
12	0.0144	0.0196	0.0055	0.0077	0.026	0.0269	0.0193	0.0328	0.039	0.0483	0.0478	0	0.015	0.022	0.0121
13	0.0012	0.0073	0.0148	0.0227	0.011	0.0219	0.0178	0.0178	0.0307	0.0334	0.0328	0.015	0	0.0125	0.0081
14	0.0121	0.0052	0.0233	0.0218	0.0093	0.0094	0.0077	0.0303	0.0215	0.0332	0.0386	0.022	0.0125	0	0.0166
15	0.007	0.0144	0.0067	0.0198	0.0191	0.026	0.0194	0.0204	0.0381	0.0415	0.0387	0.0121	0.0081	0.0166	0
16	0.0379	0.032	0.054	0.0566	0.0298	0.044	0.0467	0.0301	0.0351	0.0186	0.0118	0.0542	0.0392	0.039	0.0473
17	0.0072	0.0126	0.0094	0.0147	0.0189	0.0242	0.015	0.0258	0.0335	0.0413	0.0406	0.007	0.008	0.0148	0.0087
18	0.0075	0.014	0.0159	0.0294	0.0177	0.0285	0.0244	0.0112	0.0374	0.04	0.0295	0.0217	0.0067	0.0192	0.0092
19	0.0067	0.0132	0.0207	0.0286	0.0169	0.0278	0.0237	0.0136	0.0366	0.0393	0.0285	0.0209	0.0059	0.0184	0.014
20	0.0122	0.0085	0.0164	0.0112	0.0157	0.0156	0.008	0.0304	0.0277	0.0396	0.0454	0.0113	0.0125	0.0107	0.0141
21	0.0134	0.0098	0.0284	0.0295	0.0026	0.0212	0.018	0.0314	0.0171	0.0265	0.0319	0.0286	0.0136	0.0119	0.0217
22	0.0228	0.0193	0.0379	0.039	0.0121	0.0165	0.0275	0.0409	0.0076	0.0155	0.0331	0.0381	0.0231	0.0214	0.0312
23	0.0026	0.0076	0.0113	0.0191	0.012	0.0221	0.0154	0.0216	0.0265	0.0364	0.0366	0.0114	0.0038	0.0127	0.0046
24	0.0046	0.0034	0.0181	0.026	0.007	0.0179	0.0162	0.0211	0.0219	0.0286	0.0361	0.0183	0.0033	0.0085	0.0114
25	0.0255	0.0237	0.0407	0.0434	0.0165	0.0308	0.0335	0.0286	0.0219	0.0121	0.025	0.0409	0.026	0.0258	0.0341
26	0.0125	0.0091	0.0198	0.0146	0.0163	0.0122	0.0046	0.031	0.0243	0.0442	0.045	0.0147	0.0132	0.008	0.0148
27	0.0108	0.0173	0.0191	0.0326	0.0209	0.0318	0.0277	0.0079	0.0406	0.0433	0.0263	0.0249	0.0099	0.0224	0.0125
28	0.0236	0.0297	0.0333	0.045	0.0306	0.0442	0.0401	0.0063	0.0412	0.0344	0.012	0.0373	0.0224	0.0348	0.0267
29	0.008	0.0044	0.0167	0.0153	0.0116	0.016	0.0093	0.0263	0.0261	0.0355	0.0409	0.0152	0.0084	0.0066	0.01
30	0.019	0.0147	0.0332	0.0343	0.0075	0.0108	0.0244	0.0363	0.007	0.0198	0.0351	0.0334	0.0185	0.0175	0.2016
31	0.0165	0.0147	0.0317	0.0344	0.0075	0.027	0.0245	0.0348	0.0182	0.0169	0.0218	0.0319	0.0169	0.0168	0.0251

B	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
1	0.0379	0.0072	0.0075	0.0067	0.0122	0.0134	0.0228	0.0026	0.0046	0.0255	0.0125	0.0108	0.0236	0.008	0.019	0.0165
2	0.032	0.0126	0.014	0.0132	0.0085	0.0098	0.0193	0.0076	0.0034	0.0237	0.0091	0.0173	0.0297	0.0044	0.0147	0.0147
3	0.054	0.0094	0.0159	0.0207	0.0164	0.0284	0.0379	0.0113	0.0181	0.0407	0.0198	0.0191	0.0333	0.0167	0.0332	0.0317
4	0.0566	0.0147	0.0294	0.0286	0.0112	0.0295	0.039	0.0191	0.026	0.0434	0.0146	0.0326	0.045	0.0153	0.0343	0.0344
5	0.0298	0.0189	0.0177	0.0169	0.0157	0.0026	0.0121	0.012	0.007	0.0165	0.0163	0.0209	0.0306	0.0116	0.0075	0.0075
6	0.044	0.0242	0.0285	0.0278	0.0156	0.0212	0.0165	0.0221	0.0179	0.0308	0.0122	0.0318	0.0442	0.016	0.0108	0.027
7	0.0467	0.015	0.0244	0.0237	0.008	0.018	0.0275	0.0154	0.0162	0.0335	0.0046	0.0277	0.0401	0.0093	0.0244	0.0245
8	0.0301	0.0258	0.0112	0.0136	0.0304	0.0314	0.0409	0.0216	0.0211	0.0286	0.031	0.0079	0.0063	0.0263	0.0363	0.0348
9	0.0351	0.0335	0.0374	0.0366	0.0277	0.0171	0.0076	0.0265	0.0219	0.0219	0.0243	0.0406	0.0412	0.0261	0.007	0.0182
10	0.0186	0.0413	0.04	0.0393	0.0396	0.0265	0.0155	0.0364	0.0286	0.0121	0.0442	0.0433	0.0344	0.0355	0.0198	0.0169
11	0.0118	0.0406	0.0295	0.0285	0.0454	0.0319	0.0331	0.0366	0.0361	0.025	0.045	0.0263	0.012	0.0409	0.0351	0.0218
12	0.0542	0.007	0.0217	0.0209	0.0113	0.0286	0.0381	0.0114	0.0183	0.0409	0.0147	0.0249	0.0373	0.0152	0.0334	0.0319
13	0.0392	0.008	0.0067	0.0059	0.0125	0.0136	0.0231	0.0038	0.0033	0.026	0.0132	0.0099	0.0224	0.0084	0.0185	0.0169
14	0.039	0.0148	0.0192	0.0184	0.0107	0.0119	0.0214	0.0127	0.0085	0.0258	0.008	0.0224	0.0348	0.0066	0.0175	0.0168
15	0.0473	0.0087	0.0092	0.014	0.0141	0.0217	0.0312	0.0046	0.0114	0.0341	0.0148	0.0125	0.0267	0.01	0.2016	0.0251
16	0	0.0472	0.0413	0.0402	0.0455	0.0324	0.0275	0.0429	0.0344	0.0132	0.0454	0.038	0.0238	0.0414	0.0319	0.0223
17	0.0472	0	0.0147	0.0139	0.007	0.0216	0.0311	0.0044	0.0113	0.0339	0.0104	0.0179	0.0303	0.0082	0.0264	0.0249
18	0.0413	0.0147	0	0.0051	0.0192	0.0203	0.0298	0.0104	0.01	0.0326	0.0198	0.0033	0.0175	0.0151	0.0251	0.0236
19	0.0402	0.0139	0.0051	0	0.0184	0.0195	0.029	0.0097	0.0092	0.0319	0.0191	0.0094	0.0165	0.0143	0.0244	0.0228
20	0.0455	0.007	0.0192	0.0184	0	0.0183	0.0278	0.0113	0.0119	0.0322	0.0034	0.0225	0.0349	0.0041	0.0232	0.0232
21	0.0324	0.0216	0.0203	0.0195	0.0183	0	0.0147	0.0146	0.0092	0.0191	0.0183	0.0235	0.0332	0.0142	0.0057	0.0101
22	0.0275	0.0311	0.0298	0.029	0.0278	0.0147	0	0.0241	0.0191	0.0143	0.0278	0.033	0.0344	0.0237	0.0044	0.0106
23	0.0429	0.0044	0.0104	0.0097	0.0113	0.0146	0.0241	0	0.0071	0.0293	0.0108	0.0137	0.0261	0.0061	0.0195	0.0195
24	0.0344	0.0113	0.01	0.0092	0.0119	0.0092	0.0191	0.0071	0	0.0212	0.0125	0.0132	0.0257	0.0072	0.0149	0.0121
25	0.0132	0.0339	0.0326	0.0319	0.0322	0.0191	0.0143	0.0293	0.0212	0	0.0322	0.0359	0.0222	0.0281	0.0186	0.009
26	0.0454	0.0104	0.0198	0.0191	0.0034	0.0183	0.0278	0.0108	0.0125	0.0322	0	0.0231	0.0355	0.0047	0.0238	0.0238
27	0.038	0.0179	0.0033	0.0094	0.0225	0.0235	0.033	0.0137	0.0132	0.0359	0.0231	0	0.0142	0.0184	0.0284	0.0269
28	0.0238	0.0303	0.0175	0.0165	0.0349	0.0332	0.0344	0.0261	0.0257	0.0222	0.0355	0.0142	0	0.0308	0.0364	0.0231
29	0.0414	0.0082	0.0151	0.0143	0.0041	0.0142	0.0237	0.0061	0.0072	0.0281	0.0047	0.0184	0.0308	0	0.0191	0.0184
30	0.0319	0.0264	0.0251	0.0244	0.0232	0.0057	0.0044	0.0195	0.0149	0.0186	0.0238	0.0284	0.0364	0.0191	0	0.0133
31	0.0223	0.0249	0.0236	0.0228	0.0232	0.0101	0.0106	0.0195	0.0121	0.009	0.0238	0.0269	0.0231	0.0184	0.0133	0

Table 4. Katz-Bonacich centrality (L-index) and intercentrality (C-index) measures for all players

Number of player	Provinces	$b_i\,(B,\theta+\mu)$	$b_i\,(B,1)$	L_i	C_i
1	Alborz	0.8241	2.7471	0.069	2.197
2	Markazi	0.8497	2.8325	0.08	2.33
3	Ardabil	1.1255	3.7515	0.314	3.932
4	West Azarbaijan	1.2389	4.1296	0.489	4.673
5	Esfahan	0.8703	2.9011	0.089	2.439
6	Khuzestan	1.0980	3.6599	0.276	3.762
7	Ilam	1.0493	3.4976	0.224	3.46
8	North Khorasan	1.1755	3.9184	0.378	4.259
9	Bushehr	1.1964	3.9879	0.411	4.395
10	Hormozgan	1.3444	4.4814	0.701	5.404
11	South Khorasan	1.3732	4.5773	0.774	5.606
12	East Azarbaijan	1.1259	3.7531	0.315	3.934
13	Tehran	0.8474	2.8246	0.079	2.317
14	Lorestan	0.9226	3.0753	0.12	2.722
15	Gilan	1.1859	3.9529	0.542	4.207
16	Sistan and Baluchistan	1.4345	4.7816	**0.948** *	**6.04** *
17	Zanjan	0.9608	3.2025	0.149	2.935
18	Mazandaran	0.9681	3.2270	0.152	2.979
19	Semnan	0.9673	3.2242	0.151	2.975
20	Kurdistan	0.9579	3.1931	0.146	2.919
21	Chaharmahaal and Bakhtiari	0.9418	3.1393	0.131	2.831
22	Fars	1.0850	3.6168	0.259	3.682
23	Qazvin	0.8644	2.8813	0.088	2.405
24	Qom	0.8282	2.7606	0.07	2.218
25	Kerman	1.1391	3.7971	0.324	4.026
26	Kermanshah	0.9727	3.2424	0.158	3.004
27	Golestan	1.0348	3.4495	0.208	3.373
28	Razavi Khorasan	1.2091	4.0303	0.429	4.481
29	Hamadan	0.8774	2.9248	0.095	2.475
30	Kohgiluyeh and Buyer-Ahmad	1.2142	4.0473	0.589	4.395
31	Yazd	0.9691	3.2303	0.149	2.989

Also, we used intercentrality concept to measure key players because the intercentrality measure internalizes all the network payoff externalities which players exert on each other whereas, Katz-Bonacich centrality fails to internalize them all.

We considered these two methods through using a weighted graph as an increasing form with a length of the path to show Butterfly effect in the network, which was underinvestigated in previous studies. On the fundament of these concepts, we formulated two indexes to measure the key leader and key player in the network. For this result, several assumptions relevant to the situation of real empirical study were specified, and a payoff function for calculating the aggregate profit of each player based on its level of investment was defined.

Finally, with applying the proposed model and using game theory method which considers the interaction of a group of players in a complex situation we could identify the key leader and key player in the provided DN problem.

The results show that the DC located in the Sistan and Baluchistan province is both the key leader and key player based on its relevant indexes. Also, the DC located in the Alborz province is more likely to be removed in the network with its less key leader and key player indexes.

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