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Stochastic Game of Data Transmission with Three Asymmetric Players*

Fengyan Sun[1,2] **and Elena Parilina**[1]

[1] *St. Petersburg State University,*
7/9 Universitetskaya nab., Saint Petersburg 199034, Russia;
[2] *School of Mathematics and Statistics, Qingdao University,*
Qingdao 266071, PR China;
E-mail: sunfengyan1212@outlook.com;
e.parilina@spbu.ru

Abstract In the paper, we consider a stochastic game model of data transmission with three asymmetric players (i.e. network nodes), in which the network is given and does not change over time. The players aim to transmit as many packages as possible to the corresponding terminal nodes through the common node whose capacity is two. We assume that each player has a finite capacity buffer for storing data packages. The dynamic process of data transmission is modeled as a stochastic game with finite set of states. Existence of the Nash equilibrium and a cooperative solution is proved. We find the cooperative strategy profile and Nash equilibrium in pure strategies. The estimation of the price of anarchy is calculated for a numerical example.

Keywords: ALOHA-like scheme, stochastic game, data transmission, price of anarchy.

1. Introduction

Theory of stochastic games is an important part of dynamic game theory. In conflict-controlled systems with probabilistic transition from state to state, the process can be modeled by a stochastic game with the finite or infinite set of states. For example, in (Bure and Parilina, 2019), a data transmission model with finite set of states is presented. A game-theoretical model of data transmission in a network is introduced in (Afghah et al., 2013). The data transmission models with ALOHA-like scheme are considered by Altman et al. (2004); Marban et al. (2013) and Sagduyu and Ephremides (2006). The model of two players (i.e. network nodes) is introduced, in which players or nodes aim to transmit packages to the terminal nodes independently or under cooperation. But the transmission of packages must go through the common node which have the given capacity. In the model, the appearance of packages at different node has different probabilities and we assume that the probabilities do not vary on time (see Bure and Parilina, 2019).

Other data transmission models for the networks with different structures are also introduced in (Bure and Parilina, 2017a, 2017b). Bure and Parilina (2017b) consider the data transmission model with imperfect information on the presence of packages in the other player's queue. They find the Nash equlibrium and the profile of cooperative strategies. In order to show the comparison between these two equlibria, the price of anarchy is calculated. Fink (1964), Raghavan and Filar (1991) consider the non-cooperative stochastic games. In (Raghavan and Filar, 1991) the

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algorithms of finding the Nash equilibria are considered. The existence of the Nash equilibrium in n-person stochastic game is proved in Fink (1964). Moreover, the construction of a model of a cooperative game of data transmission is represented in (Bure and Parilina, 2019).

In the paper, we consider a game theoretic model of data transmission with three players based on the model proposed in (Bure and Parilina, 2019), in which they consider a two-person stochastic game. We make an assumption that each node has a buffer of finite capacity for storing packages. The next extension of the model in (Bure and Parilina, 2019) is that the capacity of the common node is two. We consider non-cooperative and cooperative scenarios of the game and define the pure strategy Nash equilibrium and the profile of cooperative strategies.

The structure of the paper is the following. In Section 2, the model of data transmission process is introduced. In Section 3, stochastic game is constructed based on assumptions given in Section 2. In Section 4, we make a simulation study. Section 5 concludes. The table containing transition probabilities used in three-person stochastic game model is represented in Appendix.

2. Model

We consider a network represented in Fig. 1 which is a data transmission ALOHA-like scheme. There are three players who are nodes 1, 2 and 3 aiming to send as many data packages as possible to the nodes r_1, r_2, r_3, respectively. From the data transmission scheme, the packages should go through a common node whose capacity is two. This node can be used by any of three players without any restrictions. Player $i \in \{1, 2, 3\}$ has a buffer of capacity k_i which means that it can store from 0 to k_i data packages of unit capacities at each time period. If at the beginning of each period, Player i possesses less than k_i packages, then he may receive a data package of a unit capacity with probability $\upsilon_i \in (0, 1)$, respectively.

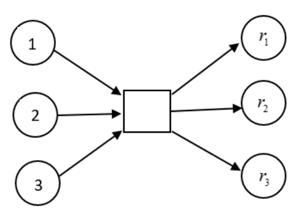

Fig. 1. Data transmission scheme.

We assume that player can transmit zero, one and two packages to the destination node in each time period. Each player can transmit two packages simultaneously and they will be successfully delivered if other players do not transmit. If three players simultaneously transit the packages, the packages are back to the Nodes. We use some parameters to model the data transmission. If the package is successfully delivered to the destination node, the payoff of this player is 1 minus the costs of a package transmission equal to $c \in (0, 1)$. And the costs of a player for one time period delay per each unit package is $d \in [0, 1)$, $d \ll 1$.

We define the state of the system in time period t as $(\omega_1(t), \omega_2(t), \omega_3(t))$, where $\omega_i(t) \in \{0, 1, \ldots, k_i\}$ is the number of data packages at Player i's buffer, $i = 1, 2, 3$. The set of system states at any time period t is denoted by Ω, $|\Omega| = (k_1 + 1)(k_2 + 1)(k_3 + 1)$.

Under the given assumptions, we define a stochastic game with a finite set of states and finite set of actions.

3. Stochastic Game

We assume that time is discrete. The set of states at each time period is $\Omega = \{\omega = (\omega_1, \omega_2, \omega_3) : \omega_i \in [0, k_i], i = 1, 2, 3\}$. Suppose that any player does not know the number of packages at other players' buffers. At state ω the set of player i's actions is A_i^ω, that is

$$A_i^\omega = \begin{cases} \{t, w\}, & \text{if } \omega_i > 0, \\ \{w\}, & \text{if } \omega_i = 0, \end{cases} \tag{1}$$

where action t means "to transmit" a package, w means "to wait".

We consider all possible states and define the players' payoff functions:

1. If $\omega = (\omega_1, \omega_2, \omega_3)$, where $\omega_1 = \omega_2 = \omega_3 = 0$, the payoff function of player i is $u_i^\omega(a_1^\omega, a_2^\omega, a_3^\omega) = 0$ for any $i = 1, 2, 3$ and $a_1^\omega = a_2^\omega = a_3^\omega = w$.
2. If $\omega = (\omega_i, \omega_j, \omega_l)$ such that $\omega_i > 0$, $\omega_j = 0$, $\omega_l = 0$, then $a_i^\omega \in \{t, w\}$ $a_j^\omega = a_l^\omega = w$. And $u_i^\omega(a_i^\omega, w, w) = 1 - c - d(\omega_i - 1)$ when $a_i^\omega = t$ and $u_i^\omega(a_i^\omega, w, w) = -d\omega_i$ when $a_i^\omega = w$. The payoff functions of player j and l are $u_j^\omega(a_i^\omega, w, w) = u_l^\omega(a_i^\omega, w, w) = 0$ for any a_i^ω.
3. If $\omega = (\omega_i, \omega_j, \omega_l)$ such that $\omega_i > 0, \omega_j > 0$ and $\omega_l = 0$, then the payoff matrices are defined as follows

$$\text{Player } i\text{:} \quad \begin{pmatrix} 1 - c - d(\omega_i - 1) & 1 - c - d(\omega_i - 1) \\ -d\omega_i & -d\omega_i \end{pmatrix},$$

$$\text{Player } j\text{:} \quad \begin{pmatrix} 1 - c - d(\omega_j - 1) & -d\omega_j \\ 1 - c - d(\omega_j - 1) & -d\omega_j \end{pmatrix},$$

$$\text{Player } l\text{:} \quad \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}.$$

   where the Player i chooses a row (row 1 corresponds to action t, row 2 corresponds to action w) and Player j chooses a column (column 1 corresponds to action t, column 2 corresponds to action w). Player l has a unique action w.
4. If the state is $\omega = (\omega_i, \omega_j, \omega_l)$ such that $\omega_i > 0, \omega_j > 0, \omega_l > 0$, where any Player i, j and l has two actions t and w. Define the payoff matrices in the following way. If Player l chooses action t, then the payoff matrices of Players i, j and l correspondingly take the forms:

$$\text{Player } i\text{:} \quad \begin{pmatrix} -c - d\omega_i & 1 - c - d(\omega_i - 1) \\ -d\omega_i & -d\omega_i \end{pmatrix},$$

$$\text{Player } j\text{:} \quad \begin{pmatrix} -c - d\omega_j & -d\omega_j \\ 1 - c - d(\omega_j - 1) & -d\omega_j \end{pmatrix},$$

$$\text{Player } l\text{:} \quad \begin{pmatrix} -c - d\omega_l & 1 - c - d(\omega_l - 1) \\ 1 - c - d(\omega_l - 1) & 1 - c - d(\omega_l - 1) \end{pmatrix}.$$

If Player l chooses the action w, then the payoff matrix takes the form

$$\text{Player } i\text{:} \quad \begin{pmatrix} 1-c-d(\omega_i-1) & 1-c-d(\omega_i-1) \\ -d\omega_i & -d\omega_i \end{pmatrix},$$

$$\text{Player } j\text{:} \quad \begin{pmatrix} 1-c-d(\omega_j-1) & -d\omega_j \\ 1-c-d(\omega_j-1) & -d\omega_j \end{pmatrix},$$

$$\text{Player } l\text{:} \quad \begin{pmatrix} -d\omega_l & -d\omega_l \\ -d\omega_l & -d\omega_l \end{pmatrix}.$$

In the last six matrices Player i chooses a row (row 1 corresponds to action t, row 2 corresponds to action w) and Player j chooses a column (column 1 corresponds to action t, column 2 corresponds to action w).

We assume that the players' strategies in the whole game are stationary. The stationary strategy depends on the state and does not depend on time and the history of the stage. We assume that Player i uses the same strategy in any state $(\omega_i, \omega_j, \omega_l)$, where $\omega_i \in \{1, \ldots, k_i - 1\}$, and he may use another strategy in state $(\omega_i, \omega_j, \omega_l)$, where $\omega_i = k_i$. Therefore, Player i's mixed stationary strategy η_i is (p_i^f, p_i^{nf}), $i = 1, 2, 3$, where $p_i^f \in [0, 1]$ is a probability of choosing action t in any state $(\omega_i, \omega_j, \omega_l)$ when $\omega_i = k_i$, $\omega_j \in [0, k_j]$, $\omega_l \in [0, k_l]$, or p_i^f is the probability of transmitting a package when the buffer of player i is full. Let $p_i^{nf} \in [0, 1]$ is a probability of choosing action t in any state $(\omega_i, \omega_j, \omega_l)$, when $\omega_i \in [1, k_i - 1]$, $\omega_j \in [0, k_j]$, $\omega_l \in [0, k_l]$, or p_i^{nf} is probability of transmitting a package when the buffer of player i is not full. The stationary strategy profile is $(\eta_1, \eta_2, \eta_3) = ((p_1^f, p_1^{nf}), (p_2^f, p_2^{nf}), (p_3^f, p_3^{nf}))$. Denote by Ξ_i the set of stationary strategies of Player i. The set of pure stationary strategies of Player $i = 1, 2, 3$ is $\{(0, 0), (0, 1), (1, 0), (1, 1)\}$.

Next we need to define the transition probabilities $\pi(\omega''/\omega', \eta)$ which are calculated for any states $\omega', \omega'' \in \Omega$ and any strategy profile η. The transition probabilities $\pi(\omega''/\omega', \eta)$ are represented in Table 1 (see Appendix), where $\omega' = (\omega_i', \omega_j', \omega_l')$, $\omega' = (\omega_i'', \omega_j'', \omega_l'')$, $i, j, l = 1, 2, 3$, $i \neq j \neq l$ and

$$\eta = (\eta_i, \eta_j, \eta_l) = ((p_i^f, p_i^{nf}), (p_j^f, p_j^{nf}), (p_l^f, p_l^{nf})).$$

Therefore, we define three-person stochastic game G by a tuple

$$\langle \Omega, \{A_i^\omega\}_{i=1,2,3;\omega\in\Omega}, \{\Xi_i\}_{i=1,2,3}, \{\pi(\omega''/\omega', \eta)\}_{\omega'\in\Omega,\omega''\in\Omega,\eta\in\Xi_1\times\Xi_2\times\Xi_3}, \delta\rangle,$$

where $\delta \in (0, 1)$ is a common discount rate.

We calculate the discounted expected payoff in stochastic game G given by

$$E_i(\eta) = \pi_0(\mathbb{I} - \delta\Pi(\eta))^{-1}u_i(\eta), \tag{2}$$

where strategy profile η is given and π_0 is an initial probability distribution over the set of states Ω, $\mathbb{I}$ is an identity matrix of size m, $\Pi(\eta)$ is a $m \times m$ matrix of transition probabilities $\pi(\cdot/\cdot, \eta)$ whose $(l, n)th$ entry is a probability of transition from state $\omega^{(l)}$ to state $\omega^{(n)}$ under the realization of profile η. The vector $u_i(\eta)$ is $(u_i^\omega(\eta(\omega)) : \omega \in \Omega)$ which the collection of payoffs in states under realization of the action profiles corresponding to strategy profile η. Here m is the cardinality of the set of states, i. e., $m = |\Omega|$.

We consider two approaches (cooperative and non-cooperative) to find a solution in game G. We consider the Nash equilibrium as an optimality principle within a non-cooperative approach. Following the cooperative approach, we find the profile of cooperative stationary strategies maximizing the total players' expected payoff in game G.

Theorem 1 (Fink, 1964). *In game G, there exist the Nash equilibrium and the profile of cooperative stationary strategies.*

To estimate the selfishness in the network we calculate the price of anarchy (Koutsoupias and Papadimitriou, 1999) given by

$$PoA(G) = \frac{\sum_{i=1}^{3} E_i(\eta^*)}{\min_{\eta \in NE(G)} \sum_{i=1}^{3} E_i(\eta)}, \tag{3}$$

where $NE(G)$ is the set of the Nash equilibrium in game G, η^* is the profile of cooperative strategies. One can notice the PoA is not defined if the sum in the denominator in (3) is null.

4. Simulation study

We consider game G with the following parameters: $c = 0.2$, $d = 0.03$. Let Player 1 have a buffer of the smallest capacity $k_1 = 1$, and Players 2 and 3 have buffers of capacities $k_2 = 2$ and $k_3 = 3$, respectively. Any player has a discount factor $\delta = 0.99$. The probabilities that the package occurs at Player 1, 2 and 3 are $v_1 = 0.6$, $v_2 = 0.6$ and $v_3 = 0.3$, respectively. There are 24 states in the game. We assume that it starts from the state $(0,0,0)$, when there are no packages at the nodes. The set of pure stationary strategy for Player 1 is $\{(0,0),(1,0)\}$. The sets of pure stationary strategies of Player 2 and 3 are the same, which is $\{(0,0),(0,1),(1,0),(1,1)\}$. Player 1's mixed strategy is defined by a probability p_1^f. Player 2 and 3's mixed strategies are defined by a vector (p_i^f, p_i^{nf}), $i = 2,3$. And the size of transition matrix is 24×24.

In order to find the profile of cooperative strategies, we need to calculate the total players' payoff in the whole game for each pure stationary strategy profile. There are 32 profiles in pure stationary strategies in the game with given parameters. The maximal expected total payoff is obtained under the profile of pure stationary strategies. In non-cooperative setting, we focus on the Nash equilibria in pure stationary strategies to avoid computational difficulties. In order to find the Nash equilibria, we calculate the matrices of Players' expected payoffs by given above formulae as follows. In each payoff matrix, Player 1 chooses rows, Player 2 chooses columns and the Player 3 chooses matrices.

When Player 3 chooses the pure stationary strategy $(0,0)$, the payoff matrix for Player 1 is A_1, for Player 2 is B_1, and for Player 3 is C_1:

$$A_1 = \begin{pmatrix} -2.9503 & -2.9503 & -2.9503 & -2.9503 \\ 47.5200 & 47.5200 & 47.5200 & 47.5200 \end{pmatrix}$$

$$B_1 = \begin{pmatrix} -5.8518 & 47.5200 & 43.7829 & 47.5200 \\ -5.8518 & 47.5200 & 43.7829 & 47.5200 \end{pmatrix}$$

$$C_1 = \begin{pmatrix} -8.4263 & -8.4263 & -8.4263 & -8.4263 \\ -8.4263 & -8.4263 & -8.4263 & -8.4263 \end{pmatrix}$$

When Player 3 chooses the pure stationary strategy $(0,1)$, the payoff matrix for Player 1 is A_2, for Player 2 is B_2, and for Player 3 is C_2:

$$A_2 = \begin{pmatrix} -2.9503 & -2.9503 & -2.9503 & -2.9503 \\ 47.5200 & 46.4694 & 43.4095 & 43.3403 \end{pmatrix}$$

$$B_2 = \begin{pmatrix} -5.8518 & 47.5200 & 43.7829 & 47.5200 \\ -5.8518 & -1.0626 & 39.6724 & 43.7959 \end{pmatrix}$$

$$C_2 = \begin{pmatrix} 23.7600 & 23.7600 & 23.7600 & 23.7600 \\ 23.7600 & 21.2132 & -7.3135 & -7.8366 \end{pmatrix}$$

When Player 3 chooses the pure stationary strategy $(1,0)$, the payoff matrix for Player 1 is A_3, for Player 2 is B_3, and for Player 3 is C_3:

$$A_3 = \begin{pmatrix} -2.9503 & -2.9503 & -2.9503 & -2.9503 \\ 47.5200 & 47.9689 & -13.4913 & -13.5204 \end{pmatrix}$$

$$B_3 = \begin{pmatrix} -5.8518 & 47.5200 & 43.7829 & 47.5200 \\ -5.8518 & 2.2479 & -17.2284 & -16.0455 \end{pmatrix}$$

$$C_3 = \begin{pmatrix} 16.5273 & 16.5273 & 16.5273 & 16.5273 \\ 16.5273 & 17.3129 & -24.1583 & -24.1777 \end{pmatrix}$$

When Player 3 chooses the pure stationary strategy $(1,1)$, the payoff matrix for Player 1 is A_4, for Player 2 is B_4, and for Player 3 is C_4:

$$A_4 = \begin{pmatrix} -2.9503 & -2.9503 & -2.9503 & -2.9503 \\ 47.5200 & 46.4402 & -16.6203 & -17.7001 \end{pmatrix}$$

$$B_4 = \begin{pmatrix} -5.8518 & 47.5200 & 43.7829 & 47.5200 \\ -5.8518 & -2.4115 & -20.3574 & -20.3981 \end{pmatrix}$$

$$C_4 = \begin{pmatrix} 23.7600 & 23.7600 & 23.7600 & 23.7600 \\ 23.7600 & 23.3522 & -24.1474 & -24.9539 \end{pmatrix}$$

There are three Nash equilibria in pure strategies in the game defined by payoff matrices (A_1,B_1,C_1), (A_2,B_2,C_2), (A_3,B_3,C_3), (A_4,B_4,C_4). The first one is $\xi_1=(0,1)$, $\xi_2=(0,0,0,1)$, $\xi_3=(0,1,0,0)$ and the payoff of Player 1, 2 and 3 is 43.3403, 43.7959, -7.8366, respectively. The second one is $\xi_1=(1,0)$, $\xi_2=(0,0,0,1)$, $\xi_3=(0,0,0,1)$ and the payoff of Player 1, 2 and 3 is -2.9503, 47.5200, 23.7600, respectively. The third one is $\xi_1=(0,1)$, $\xi_2=(0,1,0,0)$, $\xi_3=(0,0,0,1)$ and the payoff of Player 1, 2 and 3 is 46.4402, -2.4115, 23.3522, respectively. Among all of these payoffs, the "worst" Nash equilibrium with the smallest total payoff is the third one $\xi_1=(0,1)$, $\xi_2=(0,1,0,0)$, $\xi_3=(0,0,0,1)$. This profile of strategies in three-person normal-form game corresponds to the profile in pure stationary strategies $p_1^f=1$ (to transit a package with probability 1 when the buffer is full), $p_2^f=0$, $p_2^{nf}=1$ (to transit a package with probability 0 when the buffer is full, and to transit a package with probability 1 when the buffer is not full), $p_3^f=1$, $p_3^{nf}=1$ (to transit a package with probability 1 when the buffer is full, and to transit a package with probability 1 when the buffer is not full). This total payoff in this pure strategy Nash equilibria is 67.3809.

There are two profile of cooperative strategies (η_1, η_2, η_3), where $\eta_1 = (0,1)$, $\eta_2 = (0,1,0,0)$, $\eta_3 = (1,0,0,0)$ and the other one is $\eta_1 = (0,1)$, $\eta_2 = (0,0,0,1)$, $\eta_3 = (1,0,0,0)$, with the total payoff 86.6137. In cooperation, Player 3 never transmits packages whatever the other two players choose. Then the price of anarchy in game G is not less than 1.2854. We calculate the lower bound of the price of anarchy because we focus on the pure Nash equilibria. If we calculate all the equilibria including mixed-strategies Nash equilibria, the price of anarchy may increase.

5. Conclusion

We have considered a model of data transmission with a given network topology as a three-person stochastic game with finite set of states. The players have the buffers of finite capacities for storing the data packages. We have represented a simulation study of the model. The profile of cooperative strategies and the pure strategy Nash equilibria have been calculated, and lower bound of the price of anarchy has been calculated.

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Appendix

In Table 1 the probabilities are grouped by any three rows. In any group, the first row contains the transition probabilities $\pi(\omega''/\omega', \eta)$ from state ω' defined in the second row to state ω'' defined in the third row. In the cases which are not described in Table 1, transition probabilities equal zero.

Table 1. Transition probabilities.

$\pi(\omega''/\omega',\eta)$	$v_iv_jv_z$	$v_i(1-v_j)(1-v_z)$	$v_iv_j(1-v_z)$
$\omega'=(\omega'_i,\omega'_j,\omega'_z)$	$\omega'_i=\omega'_j=\omega'_z=0$	$\omega'_i=\omega'_j=\omega'_z=0$	$\omega'_i=\omega'_j=\omega'_z=0$
$\omega''=(\omega''_i,\omega''_j,\omega''_z)$	$\omega''_i=\omega''_j=\omega''_z=1$	$\omega''_i=1,$ $\omega''_j=\omega''_z=0$	$\omega''_i=\omega''_j=1,$ $\omega''_z=0$
$\pi(\omega''/\omega',\eta)$	$(1-v_i)(1-v_j)\cdot$ $\cdot(1-v_z)$	$(1-v_i)(1-v_j)\cdot$ $\cdot p_z^{nf}(1-v_z)$	$(1-v_i)(1-v_j)\cdot$ $\cdot[p_z^{nf}v_z+(1-v_z)\cdot$ $\cdot(1-p_z^{nf})]$
$\omega'=(\omega'_i,\omega'_j,\omega'_z)$	$\omega'_i=\omega'_j=\omega'_z=0$	$\omega'_i=\omega'_j=0,$ $\omega'_z\in[1,k_z-1]$	$\omega'_i=\omega'_j=0,$ $\omega'_z\in[1,k_z-1]$
$\omega''=(\omega''_i,\omega''_j,\omega''_z)$	$\omega''_i=\omega''_j=\omega''_z=0$	$\omega''_i=\omega''_j=0,$ $\omega''_z=\omega'_z-1$	$\omega''_i=\omega''_j=0,$ $\omega''_z=\omega'_z$
$\pi(\omega''/\omega',\eta)$	$(1-v_i)(1-v_j)\cdot$ $\cdot(1-p_z^{nf})v_z$	$(1-v_i)v_j\cdot$ $\cdot p_z^{nf}(1-v_z)$	$(1-v_i)v_j\cdot$ $\cdot[(1-v_z)(1-p_z^{nf})]+$ $+p_z^{nf}v_z$
$\omega'=(\omega'_i,\omega'_j,\omega'_z)$	$\omega'_i=\omega'_j=0,$ $\omega'_z\in[1,k_z-1]$	$\omega'_i=\omega'_j=0,$ $\omega'_z\in[1,k_z-1]$	$\omega'_i=\omega'_j=0,$ $\omega'_z\in[1,k_z-1]$
$\omega''=(\omega''_i,\omega''_j,\omega''_z)$	$\omega''_i=\omega''_j=0,$ $\omega''_z=\omega'_z+1$	$\omega''_i=0,\omega''_j=1,$ $\omega''_z=\omega'_z-1$	$\omega''_i=0,\omega''_j=1,$ $\omega''_z=\omega'_z$
$\pi(\omega''/\omega',\eta)$	$(1-v_i)v_j\cdot$ $\cdot(1-p_z^{nf})v_z$	$v_iv_j\cdot$ $\cdot p_z^{nf}(1-v_z)$	$v_iv_j[p_z^{nf}v_z+$ $+(1-v_z)(1-p_z^{nf})]$
$\omega'=(\omega'_i,\omega'_j,\omega'_z)$	$\omega'_i=\omega'_j=0,$ $\omega'_z\in[1,k_z-1]$	$\omega'_i=\omega'_j=0,$ $\omega'_z\in[1,k_z-1]$	$\omega'_i=\omega'_j=0,$ $\omega'_z\in[1,k_z-1]$
$\omega''=(\omega''_i,\omega''_j,\omega''_z)$	$\omega''_i=0,\omega''_j=1,$ $\omega''_z=\omega'_z+1$	$\omega''_i=\omega''_j=1,$ $\omega''_z=\omega'_z-1$	$\omega''_i=\omega''_j=1,$ $\omega''_z=\omega'_z$
$\pi(\omega''/\omega',\eta)$	$v_iv_j\cdot$ $\cdot(1-p_z^{nf})v_z$	$(1-v_i)p_j^{nf}(1-v_j)\cdot$ $\cdot p_z^{nf}(1-v_z)$	$(1-v_i)p_j^{nf}(1-v_j)\cdot$ $\cdot[p_z^{nf}v_z+(1-v_z)\cdot$ $\cdot(1-p_z^{nf})]$
$\omega'=(\omega'_i,\omega'_j,\omega'_z)$	$\omega'_i=\omega'_j=0,$ $\omega'_z\in[1,k_z-1]$	$\omega'_i=0,$ $\omega'_j\in[1,k_j-1],$ $\omega'_z\in[1,k_z-1]$	$\omega'_i=0,$ $\omega'_j\in[1,k_j-1],$ $\omega'_z\in[1,k_z-1]$
$\omega''=(\omega''_i,\omega''_j,\omega''_z)$	$\omega''_i=\omega''_j=1,$ $\omega''_z=\omega'_z+1$	$\omega''_i=0,$ $\omega''_j=\omega'_j-1,$ $\omega''_z=\omega'_z-1$	$\omega''_i=0,$ $\omega''_j=\omega'_j-1,$ $\omega''_z=\omega'_z$
$\pi(\omega''/\omega',\eta)$	$(1-v_i)\cdot$ $\cdot p_j^{nf}(1-v_j)\cdot$ $\cdot(1-p_z^{nf})v_z$	$(1-v_i)[p_j^{nf}v_j+$ $(1-p_j^{nf})(1-v_j)]\cdot$ $\cdot[p_z^{nf}v_z+(1-p_z^{nf})\cdot$ $\cdot(1-v_z)]$	$(1-v_i)(1-p_z^{nf})\cdot$ $\cdot v_z[p_j^{nf}v_j+$ $+(1-v_j)(1-p_j^{nf})]$
$\omega'=(\omega'_i,\omega'_j,\omega'_z)$	$\omega'_i=0,$ $\omega'_j\in[1,k_j-1],$ $\omega'_z\in[1,k_z-1]$	$\omega'_i=0,$ $\omega'_j\in[1,k_j-1],$ $\omega'_z\in[1,k_z-1]$	$\omega'_i=0,$ $\omega'_j\in[1,k_j-1],$ $\omega'_z\in[1,k_z-1]$
$\omega''=(\omega''_i,\omega''_j,\omega''_z)$	$\omega''_i=0,$ $\omega''_j=\omega'_j-1,$ $\omega''_z=\omega'_z+1$	$\omega''_i=0,$ $\omega''_j=\omega'_j,$ $\omega''_z=\omega'_z$	$\omega''_i=0,$ $\omega''_j=\omega'_j,$ $\omega''_z=\omega'_z+1$

$\pi(\omega''/\omega',\eta)$	$(1-v_i)\cdot$ $\cdot(1-p_j^{nf})v_j\cdot$ $\cdot(1-p_z^{nf})v_z$	$v_ip_j^{nf}(1-v_j)\cdot$ $\cdot p_z^{nf}(1-v_z)$	$v_ip_j^{nf}(1-v_j)\cdot$ $\cdot[v_zp_z^{nf}+(1-v_z)\cdot$ $\cdot(1-p_z^{nf})]$
$\omega'=(\omega'_i,\omega'_j,\omega'_z)$	$\omega'_i=0,$ $\omega'_j\in[1,k_j-1],$ $\omega'_z\in[1,k_z-1]$	$\omega'_i=0,$ $\omega'_j\in[1,k_j-1],$ $\omega'_z\in[1,k_z-1]$	$\omega'_i=0,$ $\omega'_j\in[1,k_j-1],$ $\omega'_z\in[1,k_z-1]$
$\omega''=(\omega''_i,\omega''_j,\omega''_z)$	$\omega''_i=0,$ $\omega''_j=\omega'_j+1,$ $\omega''_z=\omega'_z+1$	$\omega''_i=1,$ $\omega''_j=\omega'_j-1,$ $\omega''_z=\omega'_z-1$	$\omega''_i=1,$ $\omega''_j=\omega'_j-1,$ $\omega''_z=\omega'_z$
$\pi(\omega''/\omega',\eta)$	$v_ip_j^{nf}(1-v_j)\cdot$ $\cdot(1-p_z^{nf})v_z$	$v_i[p_j^{nf}v_j+$ $(1-p_j^{nf})(1-v_j)]\cdot$ $\cdot[p_z^{nf}v_z+(1-p_z^{nf})\cdot$ $\cdot(1-v_z)]$	$v_i(1-p_z^{nf})v_z\cdot$ $\cdot[v_jp_j^{nf}+(1-v_j)\cdot$ $\cdot(1-p_j^{nf})]$
$\omega'=(\omega'_i,\omega'_j,\omega'_z)$	$\omega'_i=0,$ $\omega'_j\in[1,k_j-1],$ $\omega'_z\in[1,k_z-1]$	$\omega'_i=0,$ $\omega'_j\in[1,k_j-1],$ $\omega'_z\in[1,k_z-1]$	$\omega'_i=0,$ $\omega'_j\in[1,k_j-1],$ $\omega'_z\in[1,k_z-1]$
$\omega''=(\omega''_i,\omega''_j,\omega''_z)$	$\omega''_i=1,$ $\omega''_j=\omega'_j-1,$ $\omega''_z=\omega'_z+1$	$\omega''_i=1,$ $\omega''_j=\omega'_j,$ $\omega''_z=\omega'_z$	$\omega''_i=1,$ $\omega''_j=\omega'_j,$ $\omega''_z=\omega'_z+1$
$\pi(\omega''/\omega',\eta)$	$v_i(1-p_j^{nf})v_j\cdot$ $\cdot(1-p_z^{nf})v_z$	$(1-v_i)p_i^{nf}\cdot$ $\cdot p_j^{nf}(1-v_j)\cdot$ $\cdot(1-p_z^{nf})(1-v_z)$	$(1-v_i)p_i^{nf}\cdot$ $\cdot p_j^{nf}(1-v_j)\cdot$ $\cdot(1-p_z^{nf})v_z$
$\omega'=(\omega'_i,\omega'_j,\omega'_z)$	$\omega'_i=0,$ $\omega'_j\in[1,k_j-1],$ $\omega'_z\in[1,k_z-1]$	$\omega'_i\in[1,k_i-1],$ $\omega'_j\in[1,k_j-1],$ $\omega'_z\in[1,k_z-1]$	$\omega'_i\in[1,k_i-1],$ $\omega'_j\in[1,k_j-1],$ $\omega'_z\in[1,k_z-1]$
$\omega''=(\omega''_i,\omega''_j,\omega''_z)$	$\omega''_i=1,$ $\omega''_j=\omega'_j+1,$ $\omega''_z=\omega'_z+1$	$\omega''_i=\omega'_i-1,$ $\omega''_j=\omega'_j-1,$ $\omega''_z=\omega'_z$	$\omega''_i=\omega'_i-1,$ $\omega''_j=\omega'_j-1,$ $\omega''_z=\omega'_z+1$
$\pi(\omega''/\omega',\eta)$	$(1-v_i)p_i^{nf}\{v_j\cdot$ $\cdot p_j^{nf}(1-p_z^{nf})(1-v_z)+$ $+(1-p_j^{nf})(1-v_j)\cdot$ $\cdot[p_z^{nf}v_z+(1-p_z^{nf})\cdot$ $\cdot(1-v_z)]\}$	$(1-v_i)p_i^{nf}v_z\cdot$ $\cdot(1-p_z^{nf})[(1-p_j^{nf})\cdot$ $\cdot(1-v_j)+p_j^{nf}v_{j]}$	$(1-v_i)p_i^{nf}\cdot$ $\cdot(1-p_j^{nf})v_j\cdot$ $\cdot(1-p_z^{nf})v_z$
$\omega'=(\omega'_i,\omega'_j,\omega'_z)$	$\omega'_i\in[1,k_i-1],$ $\omega'_j\in[1,k_j-1],$ $\omega'_z\in[1,k_z-1]$	$\omega'_i\in[1,k_i-1],$ $\omega'_j\in[1,k_j-1],$ $\omega'_z\in[1,k_z-1]$	$\omega'_i\in[1,k_i-1],$ $\omega'_j\in[1,k_j-1],$ $\omega'_z\in[1,k_z-1]$
$\omega''=(\omega''_i,\omega''_j,\omega''_z)$	$\omega''_i=\omega'_i-1,$ $\omega''_j=\omega'_j,$ $\omega''_z=\omega'_z$	$\omega''_i=\omega'_i-1,$ $\omega''_j=\omega'_j,$ $\omega''_z=\omega'_z+1$	$\omega''_i=\omega'_i-1,$ $\omega''_j=\omega'_j+1,$ $\omega''_z=\omega'_z+1$

$\pi(\omega''/\omega',\eta)$	$p_i^{nf}p_j^{nf}p_z^{nf}(1-v_i)\cdot$ $\cdot(1-v_j)v_z+p_i^{nf}p_j^{nf}\cdot$ $\cdot(1-p_z^{nf})v_iv_jv_z+$ $+p_i^{nf}(1-p_j^{nf})(1-p_z^{nf})\cdot$ $\cdot v_i(1-v_j)v_z+(1-p_i^{nf})\cdot$ $\cdot p_j^{nf}(1-p_z^{nf})(1-v_i)v_jv_z+$ $+(1-p_i^{nf})(1-p_j^{nf})(1-p_z^{nf})\cdot$ $\cdot(1-v_i)(1-v_j)v_z$	$p_i^{nf}p_j^{nf}p_z^{nf}(1-v_i)\cdot$ $\cdot(1-v_j)(1-v_z)+p_i^{nf}p_j^{nf}\cdot$ $\cdot(1-p_z^{nf})v_iv_j(1-v_z)+$ $+p_i^{nf}(1-p_j^{nf})p_z^{nf}\cdot$ $\cdot v_i(1-v_j)v_z+p_j^{nf}\cdot$ $\cdot p_z^{nf}(1-p_i^{nf})(1-v_i)v_jv_z+$ $+p_i^{nf}(1-p_j^{nf})(1-p_z^{nf})v_i\cdot$ $\cdot(1-v_j)(1-v_z)+$ $+(1-p_i^{nf})p_j^{nf}(1-p_z^{nf})\cdot$ $\cdot(1-v_i)v_j(1-v_z)+$ $+(1-p_i^{nf})(1-p_j^{nf})p_z^{nf}\cdot$ $\cdot(1-v_i)(1-v_j)v_z+$ $+(1-p_i^{nf})(1-p_j^{nf})(1-p_z^{nf})\cdot$ $\cdot(1-v_i)(1-v_j)(1-v_z)$
$\omega'=(\omega'_i,\omega'_j,\omega'_z)$	$\omega'_i\in[1,k_i-1]$, $\omega'_j\in[1,k_j-1]$, $\omega'_z\in[1,k_z-1]$	$\omega'_i\in[1,k_i-1]$, $\omega'_j\in[1,k_j-1]$, $\omega'_z\in[1,k_z-1]$
$\omega''=(\omega''_i,\omega''_j,\omega''_z)$	$\omega''_i=\omega'_i$, $\omega''_j=\omega'_j$, $\omega''_z=\omega'_z+1$	$\omega''_i=\omega'_i$, $\omega''_j=\omega'_j$, $\omega''_z=\omega'_z$
$\pi(\omega''/\omega',\eta)$	$p_i^{nf}p_j^{nf}p_z^{nf}(1-v_i)v_jv_z+$ $+(1-p_j^{nf})v_j(1-p_z^{nf})v_z[p_i^{nf}v_i\cdot$ $\cdot(1-p_i^{nf})(1-v_i)]$	$p_i^{nf}p_j^{nf}p_z^{nf}v_iv_jv_z+(1-p_i^{nf})\cdot$ $\cdot(1-p_j^{nf})(1-p_z^{nf})v_iv_jv_z$
$\omega'=(\omega'_i,\omega'_j,\omega'_z)$	$\omega'_i\in[1,k_i-1]$, $\omega'_j\in[1,k_j-1]$, $\omega'_z\in[1,k_z-1]$	$\omega'_i\in[1,k_i-1]$, $\omega'_j\in[1,k_j-1]$, $\omega'_z\in[1,k_z-1]$
$\omega''=(\omega''_i,\omega''_j,\omega''_z)$	$\omega''_i=\omega'_i$, $\omega''_j=\omega'_j+1$, $\omega''_z=\omega'_z+1$	$\omega''_i=\omega'_i+1$, $\omega''_j=\omega'_j+1$, $\omega''_z=\omega'_z+1$
$\pi(\omega''/\omega',\eta)$	$(1-v_i)(1-v_j)p_z^f(1-v_z)$	$(1-v_i)(1-v_j)[1-p_z^f(1-v_z)]$
$\omega'=(\omega'_i,\omega'_j,\omega'_z)$	$\omega'_i=\omega'_j=0$, $\omega'_z=k_z$	$\omega'_i=\omega'_j=0$, $\omega'_z=k_z$
$\omega''=(\omega''_i,\omega''_j,\omega''_z)$	$\omega''_i=\omega''_j=0$, $\omega''_z=\omega'_z-1$	$\omega''_i=\omega''_j=0$, $\omega''_z=\omega'_z$
$\pi(\omega''/\omega',\eta)$	$(1-v_i)v_jp_z^f(1-v_z)$	$(1-v_i)v_j[1-p_z^f(1-v_z)]$
$\omega'=(\omega'_i,\omega'_j,\omega'_z)$	$\omega'_i=\omega'_j=0$, $\omega'_z=k_z$	$\omega'_i=\omega'_j=0$, $\omega'_z=k_z$
$\omega''=(\omega''_i,\omega''_j,\omega''_z)$	$\omega''_i=0,\omega''_j=1$, $\omega''_z=\omega'_z-1$	$\omega''_i=0,\omega''_j=1$, $\omega''_z=\omega'_z$
$\pi(\omega''/\omega',\eta)$	$v_iv_jp_z^f(1-v_z)$	$v_iv_j[1-p_z^f(1-v_z)]$
$\omega'=(\omega'_i,\omega'_j,\omega'_z)$	$\omega'_i=\omega'_j=0$, $\omega'_z=k_z$	$\omega'_i=\omega'_j=0$, $\omega'_z=k_z$
$\omega''=(\omega''_i,\omega''_j,\omega''_z)$	$\omega''_i=\omega''_j=1$, $\omega''_z=\omega'_z-1$	$\omega''_i=\omega''_j=1$, $\omega''_z=\omega'_z$

$\pi(\omega''/\omega',\eta)$	$(1-v_i)p_j^f(1-v_j)\cdot$ $\cdot p_z^f(1-v_z)$	$(1-v_i)p_j^f(1-v_j)\cdot$ $\cdot[1-p_z^f(1-v_z)]$	$(1-v_i)[1-p_j^f(1-v_j)]\cdot$ $\cdot[1-p_z^f(1-v_z)]$
$\omega'=(\omega'_i,\omega'_j,\omega'_z)$	$\omega'_i=0,\omega'_j=k_j,$ $\omega'_z=k_z$	$\omega'_i=0,\omega'_j=k_j,$ $\omega'_z=k_z$	$\omega'_i=0,\omega'_j=k_j,$ $\omega'_z=k_z$
$\omega''=(\omega''_i,\omega''_j,\omega''_z)$	$\omega''_i=0,$ $\omega''_j=\omega'_j-1,$ $\omega''_z=\omega'_z-1$	$\omega''_i=0,$ $\omega''_j=\omega'_j-1,$ $\omega''_z=\omega'_z$	$\omega''_i=0,$ $\omega''_j=\omega'_j,$ $\omega''_z=\omega'_z$
$\pi(\omega''/\omega',\eta)$	$v_ip_j^f(1-v_j)\cdot$ $\cdot p_z^f(1-v_z)$	$v_ip_j^f(1-v_j)\cdot$ $\cdot[1-p_z^f(1-v_z)]$	$v_i[1-p_j^f(1-v_j)]\cdot$ $\cdot[1-p_z^f(1-v_z)]$
$\omega'=(\omega'_i,\omega'_j,\omega'_z)$	$\omega'_i=0,\omega'_j=k_j,$ $\omega'_z=k_z$	$\omega'_i=0,\omega'_j=k_j,$ $\omega'_z=k_z$	$\omega'_i=0,\omega'_j=k_j,$ $\omega'_z=k_z$
$\omega''=(\omega''_i,\omega''_j,\omega''_z)$	$\omega''_i=1,$ $\omega''_j=\omega'_j-1,$ $\omega''_z=\omega'_z-1$	$\omega''_i=1,$ $\omega''_j=\omega'_j-1,$ $\omega''_z=\omega'_z$	$\omega''_i=1,$ $\omega''_j=\omega'_j,$ $\omega''_z=\omega'_z$
$\pi(\omega''/\omega',\eta)$	$p_i^f(1-v_i)p_j^f\cdot$ $\cdot(1-v_j)(1-p_z^f)$	$p_i^f(1-v_i)[p_j^f\cdot$ $\cdot v_j(1-p_z^f)+$ $+(1-p_j^f)\cdot$ $\cdot[1-p_z^f(1-v_z)]$	$p_i^fp_j^fp_z^f+$ $+p_i^fp_j^f(1-p_z^f)v_iv_j+$ $+p_i^f(1-p_j^f)p_z^fv_iv_z+$ $+(1-p_i^f)p_j^fp_z^fv_jv_z+$ $+p_i^f(1-p_j^f)(1-p_z^f)v_i+$ $+(1-p_i^f)p_j^f(1-p_z^f)v_j+$ $+(1-p_i^f)(1-p_j^f)p_z^fv_z+$ $+(1-p_i^f)(1-p_j^f)\cdot$ $\cdot(1-p_z^f)$
$\omega'=(\omega'_i,\omega'_j,\omega'_z)$	$\omega'_i=k_i,\omega'_j=k_j,$ $\omega'_z=k_z$	$\omega'_i=k_i,\omega'_j=k_j,$ $\omega'_z=k_z$	$\omega'_i=k_i,\omega'_j=k_j,$ $\omega'_z=k_z$
$\omega''=(\omega''_i,\omega''_j,\omega''_z)$	$\omega''_i=\omega'_i-1,$ $\omega''_j=\omega'_j-1,$ $\omega''_z=\omega'_z$	$\omega''_i=\omega'_i-1,$ $\omega''_j=\omega'_j,$ $\omega''_z=\omega'_z$	$\omega''_i=\omega'_i,$ $\omega''_j=\omega'_j,$ $\omega''_z=\omega'_z$
$\pi(\omega''/\omega',\eta)$	$(1-v_i)p_i^{nf}(1-v_j)\cdot$ $\cdot p_j^f(1-p_z^f)$	$(1-v_i)p_i^{nf}\cdot$ $\cdot\{p_j^fv_j(1-p_z^f)+$ $+(1-p_j^f)\cdot$ $\cdot[1-p_z^f(1-v_z)]\}$	$(1-v_i)(1-p_i^{nf})p_j^f\cdot$ $\cdot(1-v_j)p_z^f(1-v_z)$
$\omega'=(\omega'_i,\omega'_j,\omega'_z)$	$\omega'_i\in[1,k_i-1],$ $\omega'_j=k_j,$ $\omega'_z=k_z$	$\omega'_i\in[1,k_i-1],$ $\omega'_j=k_j,$ $\omega'_z=k_z$	$\omega'_i\in[1,k_i-1],$ $\omega'_j=k_j,$ $\omega'_z=k_z$
$\omega''=(\omega''_i,\omega''_j,\omega''_z)$	$\omega''_i=\omega'_i-1,$ $\omega''_j=\omega'_j-1,$ $\omega''_z=\omega'_z$	$\omega''_i=\omega'_i-1,$ $\omega''_j=\omega'_j,$ $\omega''_z=\omega'_z$	$\omega''_i=\omega'_i,$ $\omega''_j=\omega'_j-1,$ $\omega''_z=\omega'_z-1$
$\pi(\omega''/\omega',\eta)$	$(1-v_j)p_j^f\cdot$ $\cdot\{p_i^{nf}v_i(1-p_z^f)+$ $+(1-p_i^{nf})(1-v_i)\cdot$ $\cdot[1-p_z^f(1-v_z)]\}$	$v_i(1-p_i^{nf})p_j^f\cdot$ $\cdot(1-v_j)p_z^f(1-v_z)$	$v_i(1-p_i^{nf})\cdot$ $\cdot p_j^f(1-v_j)\cdot$ $\cdot[1-p_z^f(1-v_z)]$
$\omega'=(\omega'_i,\omega'_j,\omega'_z)$	$\omega'_i\in[1,k_i-1],$ $\omega'_j=k_j,$ $\omega'_z=k_z$	$\omega'_i\in[1,k_i-1],$ $\omega'_j=k_j,$ $\omega'_z=k_z$	$\omega'_i\in[1,k_i-1],$ $\omega'_j=k_j,$ $\omega'_z=k_z$
$\omega''=(\omega''_i,\omega''_j,\omega''_z)$	$\omega''_i=\omega'_i,$ $\omega''_j=\omega'_j-1,$ $\omega''_z=\omega'_z$	$\omega''_i=\omega'_i+1,$ $\omega''_j=\omega'_j-1,$ $\omega''_z=\omega'_z-1$	$\omega''_i=\omega'_i+1,$ $\omega''_j=\omega'_j-1,$ $\omega''_z=\omega'_z$

$\pi(\omega''/\omega',\eta)$	$(1-p_i^{nf})v_i\cdot$ $\cdot[1-p_j^f(1-v_i)]\cdot$ $\cdot[1-p_z^f(1-v_z)]$	$(1-v_i)p_i^{nf}\cdot$ $\cdot(1-v_j)p_j^{nf}\cdot$ $\cdot(1-p_z^{f)}$	$(1-v_i)p_i^{nf}\cdot$ $\cdot(1-v_z)p_z^f\cdot$ $\cdot(1-p_j^{nf})(1-v_j)$
$\omega'=(\omega'_i,\omega'_j,\omega'_z)$	$\omega'_i\in[1,k_i-1],$ $\omega'_j=k_j,$ $\omega'_z=k_z$	$\omega'_i\in[1,k_i-1],$ $\omega'_j\in[1,k_j-1],$ $\omega'_z=k_z$	$\omega'_i\in[1,k_i-1],$ $\omega'_j\in[1,k_j-1],$ $\omega'_z=k_z$
$\omega''=(\omega''_i,\omega''_j,\omega''_z)$	$\omega''_i=\omega'_i+1,$ $\omega''_j=\omega'_j,$ $\omega''_z=\omega'_z$	$\omega''_i=\omega'_i-1,$ $\omega''_j=\omega'_j-1,$ $\omega''_z=\omega'_z$	$\omega''_i=\omega'_i-1,$ $\omega''_j=\omega'_j,$ $\omega''_z=\omega'_z-1$
$\pi(\omega''/\omega',\eta)$	$p_i^{nf}(1-v_i)\cdot$ $\cdot\{p_j^{nf}v_j(1-p_z^f)+$ $+(1-v_j)(1-p_j^{nf})\cdot$ $\cdot[1-p_z^f(1-v_z)]\}$	$(1-v_i)p_i^{nf}\cdot$ $\cdot v_j(1-p_j^{nf})\cdot$ $\cdot p_z^f(1-v_z)$	$(1-v_i)p_i^{nf}\cdot$ $\cdot v_j(1-p_j^{nf})\cdot$ $\cdot[1-p_z^f(1-v_z)]$
$\omega'=(\omega'_i,\omega'_j,\omega'_z)$	$\omega'_i\in[1,k_i-1],$ $\omega'_j\in[1,k_j-1],$ $\omega'_z=k_z$	$\omega'_i\in[1,k_i-1],$ $\omega'_j\in[1,k_j-1],$ $\omega'_z=k_z$	$\omega'_i\in[1,k_i-1],$ $\omega'_j\in[1,k_j-1],$ $\omega'_z=k_z$
$\omega''=(\omega''_i,\omega''_j,\omega''_z)$	$\omega''_i=\omega'_i-1,$ $\omega''_j=\omega'_j,$ $\omega''_z=\omega'_z$	$\omega''_i=\omega'_i-1,$ $\omega''_j=\omega'_j+1,$ $\omega''_z=\omega'_z-1$	$\omega''_i=\omega'_i-1,$ $\omega''_j=\omega'_j+1,$ $\omega''_z=\omega'_z$
$\pi(\omega''/\omega',\eta)$	$p_z^f(1-v_z)\cdot$ $\cdot\{p_i^{nf}v_i(1-p_j^{nf})\cdot$ $\cdot(1-v_j)+(1-p_i^{nf})\cdot$ $\cdot(1-v_i)[p_j^{nf}v_j+$ $+(1-p_j^{nf})(1-v_j)]\}$	$(1-v_z)p_z^f\cdot$ $\cdot v_j(1-p_j^{nf})\cdot$ $\cdot[p_i^{nf}v_i+$ $+(1-p_i^{nf})(1-v_i)]$	$(1-p_j^{nf})v_j\cdot$ $\cdot[p_i^{nf}v_ip_z^fv_z+$ $+p_i^{nf}v_i(1-p_z^f)+p_z^f\cdot$ $\cdot v_z(1-p_i^{nf})(1-v_i)+$ $+(1-p_i^{nf})(1-v_i)\cdot$ $\cdot(1-p_z^f)]+p_i^{nf}p_j^{nf}p_z^f\cdot$ $\cdot(1-v_i)v_j$
$\omega'=(\omega'_i,\omega'_j,\omega'_z)$	$\omega'_i\in[1,k_i-1],$ $\omega'_j\in[1,k_j-1],$ $\omega'_z=k_z$	$\omega'_i\in[1,k_i-1],$ $\omega'_j\in[1,k_j-1],$ $\omega'_z=k_z$	$\omega'_i\in[1,k_i-1],$ $\omega'_j\in[1,k_j-1],$ $\omega'_z=k_z$
$\omega''=(\omega''_i,\omega''_j,\omega''_z)$	$\omega''_i=\omega'_i,$ $\omega''_j=\omega'_j,$ $\omega''_z=\omega'_z-1$	$\omega''_i=\omega'_i,$ $\omega''_j=\omega'_j+1,$ $\omega''_z=\omega'_z-1$	$\omega''_i=\omega'_i,$ $\omega''_j=\omega'_j+1,$ $\omega''_z=\omega'_z$
$\pi(\omega''/\omega',\eta)$	$(1-p_i^{nf})v_i\cdot$ $\cdot(1-p_j^{nf})v_j\cdot$ $\cdot p_z^f(1-v_z)$	$(1-p_i^{nf})v_i\cdot$ $\cdot(1-p_j^{nf})v_j\cdot$ $\cdot[1-p_z^f(1-v_z)]+$ $+p_i^{nf}p_j^{nf}p_z^fv_iv_j$	$(1-v_i)p_i^{nf}\cdot$ $\cdot(1-v_j)p_z^f(1-v_z)$
$\omega'=(\omega'_i,\omega'_j,\omega'_z)$	$\omega'_i\in[1,k_i-1],$ $\omega'_j\in[1,k_j-1],$ $\omega'_z=k_z$	$\omega'_i\in[1,k_i-1],$ $\omega'_j\in[1,k_j-1],$ $\omega'_z=k_z$	$\omega'_i=0,$ $\omega'_j\in[1,k_j-1],$ $\omega'_z=k_z$
$\omega''=(\omega''_i,\omega''_j,\omega''_z)$	$\omega''_i=\omega'_i+1,$ $\omega''_j=\omega'_j+1,$ $\omega''_z=\omega'_z-1$	$\omega''_i=\omega'_i+1,$ $\omega''_j=\omega'_j+1,$ $\omega''_z=\omega'_z$	$\omega''_i=0,$ $\omega''_j=\omega'_j-1,$ $\omega''_z=\omega'_z-1$
$\pi(\omega''/\omega',\eta)$	$(1-v_i)p_j^{nf}\cdot$ $\cdot(1-v_j)[p_z^fv_z+$ $+(1-p_z^f)]$	$(1-v_i)[p_j^{nf}v_j+$ $+(1-v_j)(1-p_j^{nf})]\cdot$ $\cdot p_z^f(1-v_z)$	$(1-v_i)[p_j^{nf}v_j+$ $+(1-v_j)(1-p_j^{nf})]\cdot$ $\cdot[p_z^fv_z+(1-p_z^f)]$
$\omega'=(\omega'_i,\omega'_j,\omega'_z)$	$\omega'_i=0,$ $\omega'_j\in[1,k_j-1],$ $\omega'_z=k_z$	$\omega'_i=0,$ $\omega'_j\in[1,k_j-1],$ $\omega'_z=k_z$	$\omega'_i=0,$ $\omega'_j\in[1,k_j-1],$ $\omega'_z=k_z$
$\omega''=(\omega''_i,\omega''_j,\omega''_z)$	$\omega''_i=0,$ $\omega''_j=\omega'_j-1,$ $\omega''_z=\omega'_z$	$\omega''_i=0,$ $\omega''_j=\omega'_j,$ $\omega''_z=\omega'_z-1$	$\omega''_i=0,$ $\omega''_j=\omega'_j,$ $\omega''_z=\omega'_z$

$\pi(\omega''/\omega',\eta)$	$(1-v_i)(1-p_j^{nf})v_j\cdot$ $\cdot p_z^f(1-v_z)$	$(1-v_i)(1-p_j^{nf})v_j\cdot$ $\cdot[p_z^f v_z+(1-p_z^f)]$
$\omega'=(\omega'_i,\omega'_j,\omega'_z)$	$\omega'_i=0,$ $\omega'_j\in[1,k_j-1],$ $\omega'_z=k_z$	$\omega'_i=0,$ $\omega'_j\in[1,k_j-1],$ $\omega'_z=k_z$
$\omega''=(\omega''_i,\omega''_j,\omega''_z)$	$\omega''_i=0,$ $\omega''_j=\omega'_j+1,$ $\omega''_z=\omega'_z-1$	$\omega''_i=0,$ $\omega''_j=\omega'_j+1,$ $\omega''_z=\omega'_z$
$\pi(\omega''/\omega',\eta)$	$p_i^{nf}p_j^f p_z^f(1-v_i)+$ $+p_i^{nf}p_j^f(1-p_z^f)v_iv_j+$ $+p_i^{nf}(1-p_j^f)p_z^f v_iv_z+$ $+(1-p_i^{nf})p_j^f p_z^f(1-v_i)v_jv_z+$ $+p_i^{nf}(1-p_j^f)(1-p_z^f)v_i+$ $+(1-p_i^{nf})p_j^f(1-p_z^f)(1-v_i)v_j+$ $+(1-p_i^{nf})(1-p_j^f)p_z^f(1-v_i)v_z+$ $+(1-p_i^{nf})(1-p_j^f)(1-p_z^f)(1-v_i)$	$p_i^{nf}p_j^{nf}p_z^f(1-v_i)(1-v_j)+$ $+p_i^{nf}p_j^{nf}(1-p_z^f)v_iv_j+$ $+p_i^{nf}(1-p_j^{nf})p_z^f v_i(1-v_j)v_z+$ $+(1-p_i^{nf})p_j^{nf}p_z^f(1-v_i)v_jv_z+$ $+p_i^{nf}(1-p_j^{nf})(1-p_z^f)\cdot$ $\cdot v_i(1-v_j)+(1-p_i^{nf})p_j^{nf}\cdot$ $\cdot(1-p_z^{nf})(1-v_i)v_j+$ $+(1-p_i^{nf})(1-p_j^{nf})p_z^f\cdot$ $\cdot(1-v_i)(1-v_j)v_z+$ $+(1-p_i^{nf})(1-p_j^{nf})\cdot$ $\cdot(1-p_z^f)(1-v_i)(1-v_j)$
$\omega'=(\omega'_i,\omega'_j,\omega'_z)$	$\omega'_i\in[1,k_i-1],$ $\omega'_j=k_j,$ $\omega'_z=k_z$	$\omega'_i\in[1,k_i-1],$ $\omega'_j\in[1,k_j-1],$ $\omega'_z=k_z$
$\omega''=(\omega''_i,\omega''_j,\omega''_z)$	$\omega''_i=\omega'_i,$ $\omega''_j=\omega'_j,$ $\omega''_z=\omega'_z$	$\omega''_i=\omega'_i,$ $\omega''_j=\omega'_j,$ $\omega''_z=\omega'_z$