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Shapley Value in Cooperative Working Capital Cost Game for Distributive Supply Network*

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Abstract Working capital management (WCM) is increasingly recognized as important means of liquidity and profitability improvement (Talonpoika et al., 2016), specifically in terms of globalization and growing competition between supply chains. At the same time, rising financial risk in supply chains (SCs) stimulated management to recognize that the financial side of supply chain management (SCM) is a promising area for improvements. Nevertheless, companies still focus on their individual SC issues and take their own interests into account rather than understanding the whole SC and cooperating with their partners (Wuttke et al., 2016). We address this gap by developing cooperative game of working capital management aimed at minimizing total financial costs associated with each SC stage. The model is verified on the grounds of the combination of game-theoretical modeling and case study of Russian collaborative SC. The suggested model analyses working capital management process for 3-stage supply network. The focal network is a distributive supply network consisting of N suppliers, one distributor and M retailers connected through material, information and financial flows. The members of the network can form coalitions with the distributor. Each member's working capital position is constrained by liquidity and profitability requirements. As such, they face the need to control and manage financial costs associated with each stage. We construct cooperative working capital cost game. For this cooperative game we investigate Shapley value as an optimal imputation. Theoretical results are illustrated with the numeric example of a real-life supply network from ICT industry. The investigated model provides financial illustration for the motivation of SC partners to cooperate in order to simultaneously achieve target levels of working capital investments and improve individual financial performance through collaborative actions.

Keywords: Working Capital Management, Supply Chain Finance, Cooperative Game, Cost Imputation, Nondominant Cost Imputation.

1. Introduction

Supply chains comprise a wide range of activities among various organizations, what induce challenges for effective collaboration among the participants. From scientific perspective, collaboration is a meta-concept, which might be interpreted differently. Overall all approaches to supply chain collaboration might be divided into two main groups: one focuses on process and another focuses on relationships. The former is based on efforts to coordinate supply chain activities in order to achieve required

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goals, while the latter implies to coordinate intangibles such as trust, responsibility and cooperation. For the purposes of our study we exploit the first approach.

If customers and suppliers, being significant cooperation agents, aim to achieve high levels of performance, they have to comprehend ways of co-creating value and sharing benefits among partners. It means they have to find satisfactory levels of effectiveness and efficiency of the relationships with their partners (Selnes and Sallis, 2003). Effectiveness implies development of new products and enhancing quality of the existing ones thereby intensifying competitiveness. Efficiency concerns optimization of costs, in-time deliveries and shortening lead times. However, these criteria are achievable only on the grounds of consistent improvement of relationships with each partner, or, simply put, collaborating parties are striving to provide more valuable product than it is possible individually.

Besides, "for an effective supply chain system, the management of upstream flow of money is as important as the management of downstream flow of goods"(Gupta and Dutta, 2011). From this perspective, working capital management (WCM) as an essential element of financial supply chain management (FSCM) has gained a lot of attention (Deloof, 2003; García-Teruel and Martínez-Solano, 2007; Johnson and Templar, 2011; Viskari et al., 2011; Viskari and Karri, 2012a; Matyac, 2015) due to the fact that it is a way to accelerate the cycle time of working capital (WC) and increase the profitability of the company in response to financial volatility in the business environment, e.g. the enacted Basel II, restraining external financing from banks.

Consequently, the demand for capital from within the SC, e.g. from companies directly involved in supply chain finance (SCF) schemes or acting as financial service providers (FSPs) has increased (Gelsomino et al., 2016; Hofmann and Kotzab, 2010; Kouvelis and Zhao, 2017; Song et al., 2018; Protopappa-Sieke and Seifert, 2017; Talonpoika et al., 2016; Xu et al., 2018). For this reason, the importance of effective WCM has raised dramatically, especially for SCs from emerging markets, which faced difficulties with access to capital, limited financial infrastructure and legal, regulatory and accounting uncertainties in the first place.

The coordinating mechanisms of WCM and SCF in SCs have received little attention because the role of financial coordinators (FSPs, banks, FinTech companies and other financial intermediaries) as core participants in facilitating and enabling SCF has only recently been identified in academic literature (Silvestro and Lustrato, 2014; Songet al., 2018; Martin and Hofmann, 2017; Protopappa-Sieke and Seifert, 2017).

We address these gaps and aim to develop a methodology for SC participants to cooperate with each other and unite into coalitions, what would lead to cost optimization of joint working capital and fair redistribution of optimized costs among the participants. As a result, we construct a cooperative game model providing optimal levels of cash conversion cycle to every business partner. This is only achieved by means of collaborative actions of capital reallocation along the SC under constraints of profitability-liquidity tradeoff. The model is verified on Russian collaborative SC data. The paper begins with a review of SC collaboration, WCM and SCF literature leading to the research question:

RQ: What are the cooperative solutions to the working capital cost game?

The selected methodology aiming at responding to the research question represents the upgrading of the approach proposed by (Hofmann and Kotzab, 2010). In response to the RQ, a model is developed; this is followed by numerical analysis and discussion of the findings. The paper closes with a conclusion, identifying further research directions.

2. Financial cooperation in supply chains

From the strategic management point of view, one of the most challenging collaboration perspectives is to extend the concept from collaboration within an organization to the level between organizations, since they do not exist in isolation (Gadde and Snehota, 2000; Håkansson and Snehota, 2006; Simatupang and Sridharan, 2002). Any organization, whether a large corporation, public body, or a small business, aims to meet the needs of its various customers and stakeholders, will need resources to do this, and will acquire many of its materials, equipment, facilities and supplies from other organizations. The performance of an organization is thus influenced by the actions of the organizations that make up the supply chain (Frohlich and Westbrook, 2001; Barratt, 2004; Kim, 2009, Kirca et al., 2005). Therefore, focus has moved from competition between firms at the same level in the production process to competition between supply chains, from raw materials to end customers (Beamon, 1998; Håkansson and Ford, 2002). A company's ability to create trust-based and long-term business relationships with customers, suppliers, and other strategic partners becomes a crucial competitive parameter. Though it is accepted that external relationships in SCM are strategically important, still many questions concerning operations integration with suppliers and customers in SC remain unanswered (Blome et al., 2014; Chen and Paulraj, 2004; Fairchild, 2005; Frochlich and Westbrook, 2001; Wuttke et al., 2013). SC collaboration is especially important to manage external relationships with suppliers and customers (Fawcett and Magnan, 2002). The empirical results indicate that SC collaboration considerably improves the collaborative advantage (Cao and Zhang, 2011), which in turn, has a significant positive effect on firms' financial performance (in particular, the mediator role of collaborative advantage is stronger for small firms than medium and large firms (Shi and Yu, 2013). Furthermore, a lack of collaboration may result in poor performance of the whole SC (Gunasekaran et al., 2004), such as: inaccurate forecasts, low capacity utilization, excessive inventory, inadequate customer service, inventory turns, inventory costs, time to market, order fulfillment response, quality, customer focus and customer satisfaction (Hendricks and Singhal, 2003; Ramdas and Spekman, 2000; Coyle et al., 2013), not to mention the perspective representing the "dark side" of inter-firm collaboration, which characterizes many buyer-supplier relationships (Rokkan et al., 2003; Noordhoff et al., 2011; Seggie et al., 2013).

It has been well documented by operations management scholars and practitioners, that communication between business partners is the essence of organizational life (Rokkan et al., 2003; Galaskiewicz, 2011). However, in empirical studies, researchers have typically considered inter-organizational communication as a part of a broader construct or have examined the extent to which the use of selected communication strategies by buyer firms enhances supplier firm operational performance. Furthermore, the majority of research focuses on the economic value for buyers or for suppliers; few studies investigate how strategic orientations of buyers and suppliers affect the relative relationship performance for the individual dyad members (Flynn et al., 2010; Paulraj et al., 2008). This being said, traditional perspectives that suppliers and buyers act as independent economic agents are being replaced with the understanding that these exchange partners are co-producers of value, and thus their performances are interlinked (Blackman et al., 2013; Flynn et al., 2016; Malshe and Agarwal, 2015; Silvestro and Lustrato, 2014; Stevens and Johnson, 2016; Yousefi and Pishvaee, 2018). Cachon and Lariviere (2005) published a paper analyzing the role of revenue sharing contracts in coordinating a supply chain. The idea is straightforward: organizations are self-serving entities maximizing individual profits, but sometimes this might result in a sub-optimal overall performance. However, a focal company can contractually coordinate the actions of other players in the supply chain in order to achieve optimal profit.

With this in mind, in the next paragraph we will mainly leave out of consideration a large body of working capital and cash management literature providing, solutions aimed at improving working capital position for a single company and thus neglecting the inter-organizational perspective of the issue (e.g. Deloof et al., 2003; Fedorova and Timofeev, 2015; García-Teruel and Martínez-Solano, 2007; Enqvist et al., 2014; Vázquez et al., 2016; Chauhan and Banerjee, 2017). Instead, we will focus on the recent papers outlining approaches to working capital management in the context of collaboration of business partners in a supply chain.

2.1. Working capital management in supply chains

Finance literature captures financial flow management as working capital management aimed to figure out a sufficient level of working capital, which will permit the company to achieve its strategic and financial goals. From this point of view, efficient business management comprises ability to leverage the working capital position in a way of maintaining sustainable balance between growth, profitability and liquidity.

Adequate working capital management is a paramount necessity for each company as inconsistent processes and operations within the supply chain, excessive inventories, inadequate terms of loans and credits lead to higher levels of working capital and lower levels of liquidity. If the first two factors are directly concerned with operational management of the supply chain, the last two are related to financial management. Therefore, the goals of a working capital management are (1) to evaluate the required level of inventory and receivables for the stable operation of the company; (2) to unlock additional liquidity; (3) to minimize capital blocked in current assets.

There are two main perspectives of working capital. The first one defines it as the ability of the company to cover its short-term debt with current assets. Jones (2006) defines the concept of this working capital perspective and describes it with the equation:

$$Working\,capital = Current\,assets - Current\,liabilities. \quad (1)$$

According to Jones (2006), current assets consist of cash, total inventory, accounts receivable, securities and cash equivalents. On the other side, current liabilities refer to accounts payable, accruals, notes payable and short-term debt. A positive result of working capital means that the amount of cash the company will receive in the next 12 months is bigger than what company needs to cover its liabilities. A negative result of working capital means that the company will not be able to cover its short-term debt (1).

Another perspective of working capital is widely used in most of the studies dedicated to operating working capital and consists of the total level of inventory, accounts receivable (A/R) and accounts payable (A/P). According to Pirttilä (2014) the equation is following:

$$Working\ capital = Inventories + Accounts\ receivable - Accounts\ payable. \quad (2)$$

The study by Talonpoika et al. (2014) included accrued expenses (A/E) in (2) as a separate component into the working capital cycle (usually is a part of A/P). Pirttilä (2014) states that the working capital cycle describes the main parts of the company's performance associated with financial flows.

The operational approach to evaluate working capital is a time-based measure of cash conversion cycle (CCC) introduced by (Richards and Laughlin, 1980) for measuring and controlling the effectiveness of working capital management on the basis of relative ratios (Figure 1).

The CCC has gained a strong position as a proxy of operational working capital management in the academic literature (Shin and Soenen, 1998; Deloof, 2003; Hutchison et al., 2007). It ignores the financial components of net working capital, such as cash, marketable securities, and short-term loans, and concentrate of the operational components. The CCC (3) presents the length (in days) of time a firm

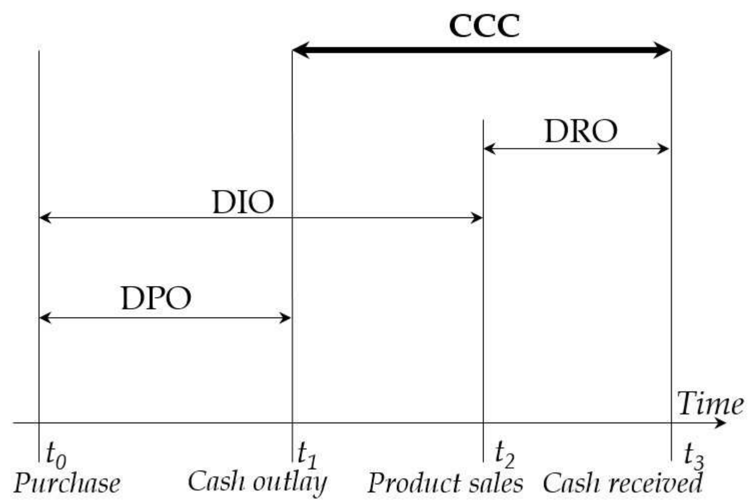

Fig. 1. Cash conversion cycle

has funds tied up in working capital, starting from the payment of purchases to the supplier and ending when remittance of sales is received from the customers. In other words, the CCC is a merge of three sub-cycles: the cycle times of inventories (DIO) as well as financial flows of accounts receivable (DRO) and accounts payable (DPO).

$$CCC = DIO + DRO - DPO. \quad (3)$$

CCC as working capital measure can be either negative or positive. Negative CCC means that the company has a low amount of inventory and the company receives money from its customers before it has to pay its A/R. In other words, in a negative CCC scenario, a company receives its A/R before it should pay A/P.

A large number of researchers believe that the lower CCC is the better a company can manage it cycles efficiently, although a too low CCC can cause problems with each component of the CCC (Cherkasova and Chadin, 2015; Garanina and Petrova, 2015; Volkov and Nikulin, 2012).

As such, DIO shows that the relationship between the level of inventory within a firm and the firm financial results is nor trivial. Inventory is a temporary physical asset, which a firm must possess to maintain its ongoing service of the customers. Therefore, managers have to leverage this metric not harming customer experience. Nonetheless, excessive levels of inventory keep the invested money tied up and might result in increased costs, for instance, warehousing or servicing the goods, but on the other hand appropriate reductions in inventory lead to loose cash and re-invest it in other aspects. Further, alteration of levels of inventory might have a bullwhip consequence effecting the upstream supply chain participants. leveraging inventory may either improve financial result or harass the overall performance of a company and a chain. Nonetheless there is considerable number of empirical test, which show that in general a shorter DIO correlates with a higher liquidity and superior financial results (Al-Shubiri and Aburumman, 2013). Besides, there are empirical results presenting negative correlation of high levels of inventory with firmвЂ™s operational and strategical output. Vendor Managed Inventory (VMI), replenishment systems, Lean/Just-In-Time management programs are examples of the methods, which allow to lower inventory levels avoiding the risk of out of stock situations. These techniques and frameworks via increasing productivity of information channels in the chain help to reduce excessive inventory (Chen and Paulraj, 2004).

DRO implies that cash received from the customers in a known period might enhance liquidity. This cash inflow might be re-invested in activities, which in their term might increase the sales volume. Thereafter, the less DRO, the higher chances that firm will re-invest the money. Moreover, there are considerable empirical evidences, which show the situation when a certain company spreads DRO via crediting sales, leads to a higher risk of not collecting the payments. According to these studies, it is supposed that a lower DRO positively correlates with a better financial results for a firm (Randall and Farris, 2009). Often decreased DRO is perceived as an unfavorable action for the customer, however, companies can smooth it via, for instance, discounts for paying in advance, thereby achieving lower DRO without straining the relationships with the customers.

DPO has as well contradictory relation with financial results. Delaying the payment to suppliers will obviously allow to keep the cash for longer period of time and thereby improve the liquidity. Nonetheless, when a company experiments with delaying the payments it might directly damage the relationships with its suppliers, moreover the whole supply chain in the long term might be damaged because suppliers lacking cash. Another negative collateral effect of such experiments are deteriorated level of service from suppliers due to the need of cash.

Considering the problem of identifying the CCC optimal value, there arises the issue of achieving target rates of return and, at the same time, maintaining the necessary level of liquidity (Garanina and Belova, 2015; Talonpoika et al., 2016; Yazdanfar and Öhman, 2014). In recent years the number of studies devoted to this issue has boomed, though the results are controversial and incomparable due to a number of reasons with research method selection among them (case studies (Farris and Hutchison, 2002; Randall and Farris, 2009); regression analysis of annual financial statements (Deloof, 2003; Garanina and Petrova, 2015; Garcia-Teruel and Martinez-Solano, 2007; Kroes and Manikas, 2014); optimization modeling (Hofmann, Kotzab, 2010; Gelsomino et al., 2018; Margolis et al., 2018; Yan et al., 2018).

As far as it goes, there are mixed evidences on the inverse relation between CCC and its components and profitability (Deloof, 2003; Garcia-Teruel and Martinez-Solano, 2007; Lazaridis and Tryfonidis, 2006; Randall and Farris, 2009; Shin and Soenen, 1998) as well as direct relation between CCC and its components and liquidity (Filbeck and Krueger, 2005). However, the conviction is the following: an increase of CCC will reduce risk and profitability on the one hand and will improve liquidity on the other.

Clearly, each company pursuing its target levels of liquidity and profitability implements a set of working capital policies (Kroes and Manikas, 2014) usually referred to as conservative, moderate or aggressive. The aggressive working capital policy implies estimation of current assets at the lowest possible level resulting in lower working capital requirements and higher risks. Conservative policy, on the contrary, is aimed at avoiding the maximum possible risks and guarantees smooth operations of the company, though the higher level of current assets leads to lower profitability. Moderate policy is assumed to be a trade-off between the aggressive and conservative policies providing reasonable accordance in profitability and liquidity.

In line with this classification, the contribution by (Talonpoika et al., 2016) suggests the theoretical typology of various financial working capital management strategies focusing on maximization or minimization of CCC components aiming to improve the financial working capital. Authors claim these strategies are to be pursued during the economic downturn, which make them possible to apply for companies from emerging markets, as they faced difficulties with access to capital, limited financial infrastructure and legal, regulatory and accounting uncertainties well before spreading volatility in the business environment as well as the enacted Basel II restrained getting financing from banks and in turn increased demand for capital from within the SC (Hofmann and Kotzab, 2010; Song et al., 2018; Protopappa-Sieke and Seifert, 2017; Talonpoika et al., 2016; Volkov and Nikulin, 2012). For this reasons, the practitioners' interest to effective WCM on inter-organizational level has increased dramatically, which resulted in a wave of publications (Marttonen et al., 2013; Protopappa-Sieke and Seifert, 2010; Protopappa-Sieke and Seifert, 2017; Pirttilä et al., 2014; Talonpoika at al., 2014; Talonpoika et al., 2016; Viskari et al., 2011; Viskari and Kärri, 2012a; Viskari et al., 2012b; Viskari et al., 2012c; Ylä-Kujala et al., 2016).

Motivation behind these research, besides the mentioned post-crisis challenges is the idea, that finance research on WCM has been focusing on company profitability instead of supply chain contribution, consequently, companies seek to optimize their individual performance; however, none of its elements can be truly managed by a company individually, but only in collaboration with business partners. It is important to note that individual financial performance optimization is to be considered in terms of a more holistic approach taking into account each participant's interests as well as the abilities to collaborate, or, in other words, supply chain orientation of a company.

With this consideration in mind, an initial assumption for optimization is, following Cachon and Lariviere (2005), collaboration of supply chain partners already motivated to maximize total profit of the chain. Alternatively, this motivation can be reformulated in terms of total financial costs minimization, and specifically financial costs associated with WCM.

2.2. Collaborative working capital management in supply chains

For the purposes of our study we consider a collaborative distributive three-stage supply network comprising three sets - K_1 suppliers, K_2 distributor and K_3 retailers at first, second and third stages respectively (see Figure 2). Initially collaborative

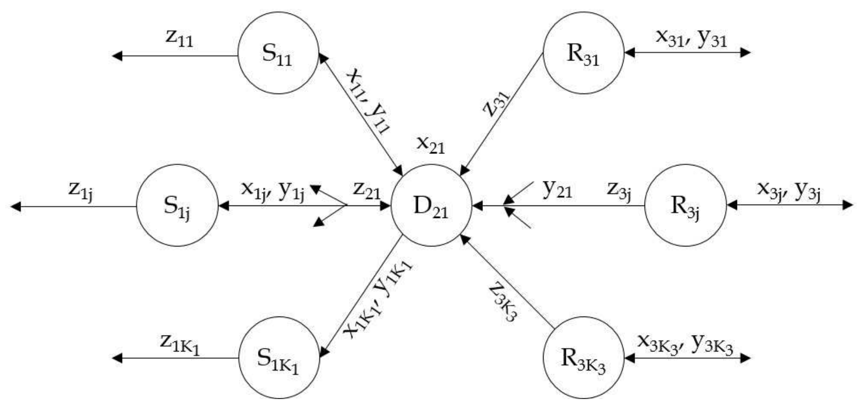

Fig. 2. Collaborative supply chain.

cash conversion cycle (CCCC) was considered as the concise consequence of an attempt to reduce CCC by solely one company (Figure 3), however leveraging CCC has an impact on all participants of the supply chain [Hofmann and Kotzab, 2010]. The research was conducted from the perspective that improving cash conversion cycle only within organization not considering other stakeholders might lead to the conflict of interests. In addition, the authors included in the research the aspects of joint risks and redistribution of costs along the chain among partners incurred by delay payments and excessive inventories.

Collaborative cash conversion cycle possesses the same benefits as CCC, however considers the whole supply chain thereby providing more precise estimation of working capital at part step of the chain. There are several limitations of the CCCC framework: operating with distinct suppliers and customers might cause internal competition and unwillingness to share information. In order to overcome this boundaries, it is advised to start the evaluation of CCCC from the pivotal firm and further extend it to the suppliers and customers.

There are two main purposes of CCCC: the first is to cut down overall costs of joint working capital and decrease collaborative cash conversion cycle. At the same time, the purpose is to minimize joint costs without violating constraints for each participant of the collaboration and global constraints for the whole chain. Further this paper considers costs minimization of a collaborative supply chain with the outlined on Figure 2 structure.

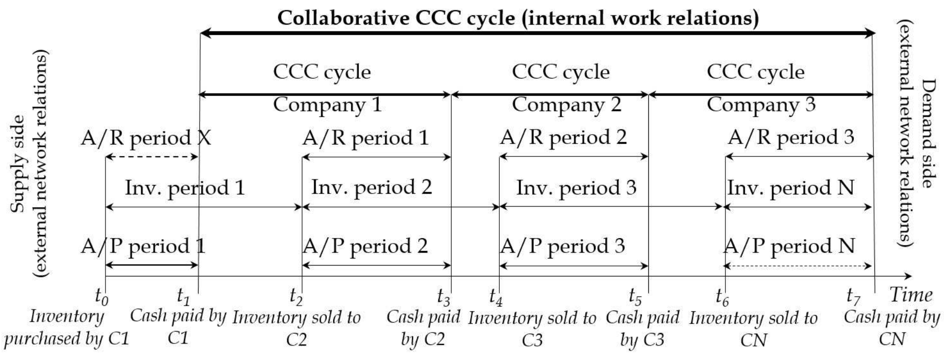

Fig. 3. Collaborative cash conversion cycle.

We denote $K_1 = \{(1,1),...,(1,k_1)\}$, $K_2 = \{(2,1)\}$ and $K_3 = \{(3,1),...,(3,k_3)\}$ as sets of suppliers, distributor and retailers in the chain respectively (Figure 2). For simplicity we introduce pair $(i,j) \in K$, where $K = K_1 \cup K_2 \cup K_3$ as a set of players, the former index presents the stage of the chain a participant belongs to: the suppliers ($i = 1$), the distributor ($i = 2$) or the retailers ($i = 3$). The latter index specifies the exact player belonging to the stage in question. As such, a pair $(i,j) \in K$ implies the participant (i,j) of the chain, for instance, the pair $(1,1)$ implies the first supplier (S_{11}).

Further we denote $DIO_{ij} = x_{ij}$, $DRO_{ij} = y_{ij}$ and $DPO_{ij} = z_{ij}$ and consequently $INV_{ij} = a_{ij}x_{ij}$, $AR_{ij} = b_{ij}y_{ij}$ and $AP_{ij} = c_{ij}z_{ij}$. Therefore the estimation of working capital financial costs for (i,j) participant of the chain (Viskari and Kärri, 2013) will take the following form:

$$FC_{ij}(x_{ij}, y_{ij}, z_{ij}) = a_{ij}x_{ij}\left[(1+r_{ij})^{\frac{x_{ij}}{365}} - 1\right] + b_{ij}y_{ij}\left[(1+r_{ij})^{\frac{y_{ij}}{365}} - 1\right] - \\ - c_{ij}z_{ij}\left[(1+r_{ij})^{\frac{z_{ij}}{365}} - 1\right]. \quad (4)$$

First of all, each participant has individual cash conversion cycle boundaries:

$$\underline{CCC}_{ij} \le x_{ij} + y_{ij} - z_{ij} \le \overline{CCC}_{ij}. \quad (5)$$

Moreover, several constrains arise from the outlined structure of the network (Figure 2) and the definition of collaborative conversion cycle. The suppliers are not able to leverage the days payable outstanding and the retailers are not able to leverage their days receivables outstanding:

$$z_{1i} = z^0_{1i}, i = 1, ...k_1; \\ y_{3j} = y^0_{3j}, j = 1, ...k_3. \quad (6)$$

The next constraint refer to days of accounts receivable of the distributor as the sum of days of accounts payable of the retailers:

$$y_{21} = \sum_{j=1}^{k_3} z_{3j}. \quad (7)$$

The same approach is applied to days of accounts payable of the distributor: we set it as the sum of days of accounts receivable of the suppliers:

$$z_{21} = \sum_{j=1}^{k_1} y_{1j}. \tag{8}$$

Further, there is an important recommendation on non-negativity and continuity of the CCCC elements (Figueira et al., 2005):

$$x_{ij} \geq 0, y_{ij} \geq 0, z_{ij} \geq 0,\ (i,j) \in K. \tag{9}$$

We define the joint financial costs of the supply chain as a sum of financial costs of all supply chain participants:

$$FC(...,\ x_{ij},\ y_{ij},\ z_{ij},\ ...) = \\ = \sum_{i=1}^{k_1} FC_{1i}(x_{1i},\ y_{1i},\ z_{1i}) + FC_{21}(x_{21},\ y_{21},\ z_{21}) + \sum_{j=1}^{k_3} FC_{3j}(x_{3j},\ y_{3j},\ z_{3j}). \tag{10}$$

As the result we aim to solve a minimization problem with the objective function (10) and the set of the constrains (4) – (9). The outlined problem comprises the CCCC configuration for the case when players form a maximum coalition K.

Previously it was inferred that the participants of the supply chain are motivated to cooperate with each other and collaborate in order to reduce the collaborative cash conversion cycle and the cost of the joint working capital of the whole supply chain. However, having achieved positive result of decreasing total financial costs and optimizing the length of CCCC, the participants of the coalition face the next issue. Since the solution of the cost minimization issue is a vector comprising of new individual CCC components $(...,\ x^*_{ij},\ y^*_{ij}\ z^*_{ij},\ ...)$ it is not fixed that the next condition is fulfilled:

$$FC(...,\ x^*_{ij}, y^*_{ij}, z^*_{ij},\ ...) \leq FC(...,\ x^0_{ij}, y^0_{ij}, z^0_{i},\ ...), (i,j) \in K \tag{11}$$

where $x^0_{ij},\ y^0_{ij},\ z^0_{ij}$ – parameters of the participant (i,j) before optimization, and $x^*_{ij},\ y^*_{ij},\ z^*_{ij}$ – parameters of the participant (i,j) after optimization.

In other words, there might be a situation, when working capital costs of a certain participant have increased after optimization. Therefore, it is not beneficial for him to participate in such a coalition. If there are no further actions in the coalition regarding this issue, this participant being individually rational will leave the coalition thereby affecting all the participants of the chain. This issue of cost distribution policy is still to be solved.

3. Cooperative working capital cost game

3.1. Characteristic cost function

In our study the characteristic function of a game with a multitude of players N is the real function defined on all possible coalitions $S \subseteq K$, and for any pair of non-overlapping coalitions T, S $(T \subset K,\ S \subset K)$ the sub-additivity condition is satisfied [Kunter, 2012; Leng and Parlar, 2009]:

$$v(T) + v(S) \geq v(T \cup S),\ v(\emptyset) = 0. \tag{12}$$

The inequality (12) implies that the opportunities of the joint coalition are not worse compared to two non-overlapping coalitions acting independently of each other. Therefore, the participant of the game is motivated to unite into the maximum coalition K.

From the perspective of this paper and problem stated characteristic cost function $\upsilon(S)$, $S \subset K$ has the next form:

$$\upsilon(S) = \min_{x_{ij}, y_{ij}, z_{ij},\,(i,j)\in S} \;\max_{x_{ij}, y_{ij}, z_{ij},\,(i,j)\notin S} FC_S(...,\, x_{ij}, y_{ij}, z_{ij},\, ...), \qquad (13)$$

where $S \subset K$, $FC_S(...,\, x_{ij}, y_{ij}, z_{ij},\, ...) = \sum_{(i,j)\in S} FC_{ij}(x_{ij}, y_{ij}, z_{ij})$.

If we define $\upsilon(S)$ as in (13), the sub-addivity condition (12) is satisfied meaning that the participant of two different and not overlapping coalitions has opportunities to reduce their costs further via uniting in a larger coalition.

The vector $\alpha = (...,\, \alpha_{ij},\, ...)$ satisfying the following conditions (Petrosyan and Zenkevich, 2016):

$$\alpha_{ij} \le \upsilon(i,j),\; (i,j) \in K, \qquad (14)$$

$$\sum_{(i,j)\in K} \alpha_{ij} = \upsilon(K), \qquad (15)$$

where $\upsilon(i,j)$ – is the value of the characteristic function for one element coalition $S = \{(i,j)\}$ and α_{ij} is an imputation. The multitude of all imputations in cooperative game $G = \langle K,\, \upsilon\rangle$ is further marked as $I(G)$.

The condition (14) is individual rationality condition implying that each participant of the coalition obtains at least the same value playing individually and not joining the coalition and not having support from any other players. The condition (15) is collective rationality condition implying that there is no other imputation vector, according to which a player will obtain more value or the players are dividing not existing gain and such imputation is not feasible.

Further the imputation $\varphi[\upsilon] = (..., \varphi_{ij}[\upsilon], ...)$ is assigned as a cooperative solution of the cooperative game $G = \langle K,\, \upsilon\rangle$, the components of which will be interpreted as winnings received by players as a result of an agreement or decision of an arbiter.

Further in order to solve the issues of optimization and costs redistribution for the many-one-many supply chain structure it is necessary to list each possible coalition and build the characteristic function of each one. In the given structure (Figure 2) there are eight possible substructures of united participants: coalition of the distributor, coalition of a set of suppliers, coalition of a set of retailers, coalition of a set of suppliers and the distributor, coalition of a set of retailers and the distributor, coalition of a set of suppliers and a set of retailers, coalition of a set of suppliers, the distributor and a set of retailers, coalition of all participant in the chain. The next sections are dedicated to the process of constructing the characteristic function for each situation.

Value of characteristic function for the distributor coalition. Let us examine the coalition consisting only of the distributor, the rest of the players (the suppliers and the retailers) are playing against him trying to maximize the cost of the working capital of the distributor. Therefore the value $\upsilon(2,1)$ of the characteristic function will have the following form:

$$\upsilon(2,1) = \min_{x_{21}} \max_{y_{21}, z_{21}} FC_{21}(x_{21}, y_{21}, z_{21}). \quad (16)$$

The financial cost function has the form:

$$FC_{21}(x_{21},\ y_{21},\ z_{21}) = a_{21}x_{21}\left[(1+r_{21})^{\frac{x_{21}}{365}} - 1\right] + b_{21}y_{21}\left[(1+r_{21})^{\frac{y_{21}}{365}} - 1\right] - \\ - c_{21}z_{21}\left[(1+r_{21})^{\frac{z_{21}}{365}} - 1\right]. \quad (17)$$

The suppliers and retailers have an ability to influence both DRO_{21} and DPO_{21} of the distributor according to the equations (7) and (8). In order to maximize the characteristic function the counterparts have to minimize z_{21} and maximize y_{21}, moreover the distributor has the cash conversion cycle constraints:

$$z_{21} \to \min;$$

$$y_{21} \to \max;$$

$$\underline{CCC}_{21} \le x_{21} + y_{21} + z_{21} \le \overline{CCC}_{21}.$$

The minimum of z_{21} is 0 according to condition (9), while the maximizing value of y_{21} is:

$$y_{21} = \overline{CCC}_{21} - x_{21}. \quad (18)$$

Therefore, in order to build the characteristic function the next step is to minimize the cost in the next form:

$$FC_{21}(x_{21},\ y_{21},\ z_{21}) = a_{21}x_{21}\left[(1+r_{21})^{\frac{x_{21}}{365}} - 1\right] + b_{21}(\overline{CCC}_{21} - \\ - x_2)\left[(1+r_2)^{\frac{\overline{CCC}_{21} - x_{21}}{365}} - 1\right] - -c_{21}z_{21}\left[(1+r_{21})^{\frac{z_{21}}{365}} - 1\right] \to \min_{x_{21}}. \quad (19)$$

Value of characteristic function for a set of suppliers coalition. As a further step we consider the suppliers coalition $S \subset K_1$ with the rest of the players acting against the the coalition trying to maximize the cost of the working capital of the coalition in question. In this case $\upsilon(S)$ will have the following form:

$$\upsilon(S) = \min_{x_{1j}, y_{1j}} \max_{y_{1j}} \sum_{(1,j)\in S} FC_{1j}(x_{1j}, y_{1j}, z_{1j}). \quad (20)$$

Where the financial cost function FC_S has the form:

$$FC_S = \sum_{(1,j)\in S} FC_{1j}(x_{1j},\ y_{1j},\ z_{1j}) = \sum_{(1,j)\in S} a_{1j}x_{1j}\left[(1+r_{1j})^{\frac{x_{1j}}{365}} - 1\right] + \\ \sum_{(1,j)\in S} b_{1j}y_{1j}\left[(1+r_{1j})^{\frac{y_{1j}}{365}} - 1\right] - \sum_{(1,j)\in S} c_{1j}z^0_{1j}\left[(1+r_{1j})^{\frac{z^0_{1j}}{365}} - 1\right]. \quad (21)$$

The players exterior to the coalition have an ability to influence days receivable outstanding ($\sum_{(1,j)\in S} y_{1j}$) of the coalition group via delaying payments according to the equation (8). In order to maximize the characteristic function the counterparts have to maximize $\sum_{(1,j)\in S} y_{1j}$), moreover the coalition has the cash conversion cycle constraints and constraints for each participant.

$$\underline{CCC}_{1j} \leq x_{1j} + y_{1j} - z^0_{1j} \leq \overline{CCC}_{1j},\ (1,j) \in K_1$$

Therefore the maximum values the counterparts can achieve follow the next rule:

$$\sum_{(1,j)\in S} y_{1j} = \sum_{(1,j)\in S} \overline{CCC}_{1j} - \sum_{(1,j)\in S} x_{1j} + \sum_{(1,j)\in S} z^0_{1j}. \tag{22}$$

Therefore the coalition can minimize its cost function (21) via leveraging its y_{1j} along the constrain (22) and managing its set of x_{1j} as well.

Value of characteristic function for a set of retailers coalition. The further coalition represents a group of retailers $S \subset K_3$ with the rest of the players performing against it trying to maximize the cost of the working capital of the coalition. Therefore the characteristic function will have the following form:

$$\upsilon(S) = \min_{x_{3l}, z_{3l}} \max_{z_{3l}} \sum_{(3,l)\in S} FC_{3l}(x_{3l},\ y_{3l},\ z_{3l}). \tag{23}$$

Where the financial cost function has the form:

$$FC_S = \sum_{(3,l)\in S} FC_{3l}(x_{3l},\ y_{3l},\ z_{3l}) = \sum_{(3,l)\in S} a_{3l}x_{3l}\left[(1+r_{3l})^{\frac{x_{3l}}{365}} - 1\right] + \sum_{(3,l)\in S} b_{3l}y^0_{3l}\left[(1+r_{3l})^{\frac{y^0_{3l}}{365}} - 1\right] - \sum_{(3,l)\in S} c_{3l}z_{3l}\left[(1+r_{3l})^{\frac{z_{3l}}{365}} - 1\right]. \tag{24}$$

The counterparts of the coalition have an ability to influence days payable outstanding $\sum_{(3,l)\in S} z_{3l}$ of the coalition group via shortening payments period according to the equation (8). In order to maximize the characteristic function the counterparts have to minimize $\sum_{(3,l)\in S} z_{3l}$, moreover the coalition has the cash conversion cycle constraints and constrains on each participant:

$$\underline{CCC}_{3l} \leq x_{3l} + y^0_{3l} - z_{3l} \leq \overline{CCC}_{3l},\ (3,l) \in S$$

Therefore the maximum values the counterparts can achieve comply with the next rule:

$$\sum_{(3,l)\in S} z_{3l} = 0. \tag{25}$$

Further the coalition can minimize its cost function (24) via leveraging its z_{3l} along the constraint (25) and managing its set of x_{3l} keeping in mind the limitations on cash conversion cycle.

Value of characteristic function for a set of suppliers and distributor coalition. We consider the coalition comprising a group of suppliers ($U \subset K_1$) and the distributor while the rest $K_1 \setminus U$ suppliers and all the retailers K_3 are playing against it trying to maximize the cost of the working capital of the coalition. Therefore the value of characteristic function for the coalition $S = U \cup K_2$ will have the following form:

$$v(S) = v(U \cup K_2) = \\ = \min_{x_{1j}, y_{1j}, z_{21}} \max_{y_{21}} \Bigg(\sum_{(1,j)\in U} FC_{1j}(x_{1j},\, y_{1j},\, z_{1j}) + FC_{21}(x_{21},\, y_{21},\, z_{21}) \Bigg). \quad (26)$$

Where the financial cost function has the form:

$$\sum_{(1,j)\in U} FC_{1j}(x_{1j},\, y_{1j},\, z_{1j}) + FC_{21}(x_{21},\, y_{21},\, z_{21}) = \\ = \sum_{(1,j)\in U} a_{1j}x_{1j}\left[(1+r_{1j})^{\frac{x_{1j}}{365}} - 1\right] + \sum_{(1,j)\in U} b_{1j}y_{1j}\left[(1+r_{1j})^{\frac{y_{1j}}{365}} - 1\right] - \\ - \sum_{(1,j)\in U} c_{1j}z_{1j}\left[(1+r_{1j})^{\frac{z_{1j}}{365}} - 1\right] + a_{21}x_{21}\left[(1+r_{21})^{\frac{x_{21}}{365}} - 1\right] + \\ + b_{21}y_{21}\left[(1+r_{21})^{\frac{y_{21}}{365}} - 1\right] - c_{21}z_{21}\left[(1+r_{21})^{\frac{z_{21}}{365}} - 1\right]. \quad (27)$$

The coalition opponents have power to influence days payable outstanding and days receivable outstanding of the coalition group via shortening payments period of the distributor and delaying payments to the distributor. In order to maximize the characteristic function the opponents have to maximize y_{21} and minimize z_{21}, in addition the coalition has the coalition has the cash conversion cycle constraints and constrains on each participant.

$$\underline{CCC}_{1j} \le x_{1j} + y_{1j} - z^0_{1j} \le \overline{CCC}_{1j},\ (1,j) \in U, \quad (28)$$

$$\underline{CCC}_{21} \le x_{21} + y^0_{21} - z_{21} \le \overline{CCC}_{21},\ (2,1) = K_2. \quad (29)$$

The minimum of z_{21} according to the condition (9) and the structure of the coalition is:

$$z_{21} = \sum_{(1,j)\in U} y_{1j}.$$

While the maximizing value of y_{21} is:

$$y_{21} = \overline{CCC}_{21} - x_{21} + \sum_{(1,j)\in U} y_{1j}.$$

Further the coalition can minimize its cost function (27) via leveraging its $\sum_{(1,j)\in U} y_{1j}$ and operating its set of x_{21}, x_{1j}, $(1,j) \in U$ acknowledging the cash conversion cycle boundaries.

Value of characteristic function for a set of retailers and the distributor coalition. Next we consider the coalition V consisting of a group of retailers ($V \subset K_3$) and the distributor, while the rest $K_3 \setminus V$ retailers and all the suppliers are playing against it trying to maximize the cost of the working capital of the coalition. Therefore the value of characteristic function for the coalition $S = V \cup K_2$ will have the following form:

$$v(S) = v(V \cup K_2) = \\ = \min_{x_{ij}, z_{21}, y_{3l}} \max_{y_{21}} \Big(FC_{21}(x_{21},\, y_{21},\, z_{21}) + \sum_{(3,l)\in V} FC_{3l}(x_{3l},\, y_{3l},\, z_{3l}) \Big). \quad (30)$$

Where the financial cost function has the form:

$$FC_S = FC_{21}(x_{21},\, y_{21},\, z_{21}) + \sum_{(3,l)\in V} FC_{3l}(x_{3l},\, y_{3l},\, z_{3l}) = \\ = a_{21}x_{21}\Big[(1+r_{21})^{\frac{x_{21}}{365}} - 1\Big] + b_{21}y_{21}\Big[(1+r_{21})^{\frac{y_{21}}{365}} - 1\Big] - \\ - c_{21}z_{21}\Big[(1+r_{21})^{\frac{z_{21}}{365}} - 1\Big] + \sum_{(3,l)\in V} a_{3l}x_{3l}\Big[(1+r_{3l})^{\frac{x_{3l}}{365}} - 1\Big] + \\ + \sum_{(3,l)\in V} b_{3l}y^0_{3l}\Big[(1+r_{3l})^{\frac{y^0_{3l}}{365}} - 1\Big] - \sum_{(3,l)\in V} c_{3l}z_{3l}\Big[(1+r_{3l})^{\frac{z_{3l}}{365}} - 1\Big]. \quad (31)$$

The opponents attempting to maximize (31) have an impact on days payable outstanding and days receivable outstanding of the coalition group via shortening payments period of the distributor and delaying payments to the distributor. In order to maximize the characteristic function the opponents have to maximize y_{21} and minimize z_{21}, in addition the coalition has the cash conversion cycle constraints and constrains on each participant:

$$\underline{CCC}_{3l} \le x_{3l} + y^0_{3l} - z_{3l} \le \overline{CCC}_{3l},\ (3,l) \in V \quad (32)$$

$$\underline{CCC}_{21} \le x_{21} + y_{21} - z_{21} \le \overline{CCC}_{21}. \quad (33)$$

Therefore the maximum values the counterparts can achieve comply with the rule:

$$z_{21} = \sum_{(1,j)\in K_1} y_{1j} = 0. \quad (34)$$

While the maximizing value of y_{21} is:

$$y_{21} = \overline{CCC}_{21} - x_{21}.$$

Further the coalition can minimize its cost function (31) via leveraging its $\sum_{(3,l)\in V} z_{3l}$ and manipulating its set of $x_{21},\ x_{3l},\ (3,l) \in V$ taking into consideration the cash conversion cycle boundaries.

Value of characteristic function for a set of suppliers and a set of retailers coalition. The further coalition represents a group of suppliers U ($U \subset K_1$) and a group of retailers V ($V \subset K_3$). The rest of the $K_1 \setminus U$ suppliers, the distributor and the $K_3 \setminus V$ retailers are playing against it trying to maximize the cost of the working capital of the coalition. Therefore the value of the characteristic function for the coalition $S = U \cup V$ will have the following form:

$$v(S) = v(U \cup V) = \\ = \min_{x_{1j}, y_{1j}, x_{3l}, z_{3l}} \max_{y_{1j}, z_{3l}} \Big(\sum_{(1,j)\in U} FC_{1j}(x_{1j},\, y_{1j},\, z_{1j}) + \sum_{(3,l)\in V} FC_{3l}(x_{3l},\, y_{3l},\, z_{3l}) \Big). \tag{35}$$

Where the financial cost function has the form:

$$FC_S = \sum_{(1,j)\in U} FC_{1j}(x_{1j},\, y_{1j},\, z_{1j}) + \sum_{(3,l)\in V} FC_{3l}(x_{3l},\, y_{3l},\, z_{3l}) = \\ = \sum_{(1,j)\in U} a_{1j}x_{1j}\left[(1+r_{1j})^{\frac{x_{1j}}{365}} - 1\right] + \sum_{(1,j)\in U} b_{1j}y_{1j}\left[(1+r_{1j})^{\frac{y_{1j}}{365}} - 1\right] - \\ - \sum_{(1,j)\in U} c_{1j}z^0_{1j}\left[(1+r_{1j})^{\frac{z^0_{1j}}{365}} - 1\right] + \sum_{(3,l)\in V} a_{3l}x_{3l}\left[(1+r_{3l})^{\frac{x_{3l}}{365}} - 1\right] + \\ + \sum_{(3,l)\in V} b_{3l}y_{3l}\left[(1+r_{3l})^{\frac{y_{3l}^0}{365}} - 1\right] - \sum_{(3,l)\in V} c_{3l}z^0_{3l}\left[(1+r_{3l})^{\frac{z_{3l}}{365}} - 1\right]. \tag{36}$$

The opponents have power to influence days payable outstanding and days receivable outstanding of the coalition group via shortening payments period of the retailers and delaying payments to the suppliers. In order to maximize the characteristic function the opponents have to maximize y_{21} and minimize z_{21}, in addition the coalition has the cash conversion cycle constraints and constrains on each participant:

$$\underline{CCC}_{1j} \le x_{1j} + y_{1j} - z^0_{1j} \le \overline{CCC}_{1j},\ (1,j) \in U, \tag{37}$$

$$\underline{CCC}_{3l} \le x_{3l} + y^0_{3l} - z_{3l} \le \overline{CCC}_{3l},\ (3,l) \in V. \tag{38}$$

Therefore the maximum values the counterparts can achieve follow the next rules:

$$\sum_{(1,j)\in U} y_{1j} = \sum_{(1,j)\in U} \overline{CCC}_{1j} - \sum_{(1,j)\in U} x_{1j} + \sum_{(1,j)\in U} z^0_{1j}; \tag{39}$$

$$\sum_{(3,l)\in V} z_{3l} = 0. \tag{40}$$

Further the coalition can minimize its cost function (35) via leveraging its $\sum_{(1,j)\in U} y_{1j}$ and operating its set of x_{1j}, x_{3l}, $(1,j) \in U$ and $(3,l) \in V$ keeping in mind the cash conversion cycle boundaries.

In general this coalition structure represents a combined form of two previous structures: a set of retails and suppliers, therefore the processes of building characteristic function are just combines as well.

Value of characteristic function for a set of suppliers, the distributor and a set of retailers coalition. This coalition is the most complicated and comprises a group of suppliers U, the distributor K_2 and a group of retailers V, while the rest $K_1 \setminus U$ suppliers and $K_3 \setminus V$ retailers are playing against it trying to maximize the cost of the working capital of the coalition. Therefore the characteristic function of coalition $S = U \cup K_2 \cup V$ will have the following form:

$$v(S) = v(U \cup K_2 \cup V) = \min_{x_{ij}, y_{ij}, z_{ij}} \max_{x_{ij}, y_{ij}, z_{ij}} (\sum_{(1,j) \in U} FC_{1j}(x_{1j},\, y_{1j},\, z_{1j}) + + FC_{21}(x_{21},\, y_{21},\, z_{21}) + \sum_{(3,l) \in V} FC_{3l}(x_{3l},\, y_{3l},\, z_{3l})). \quad (41)$$

Where the financial cost function has the form:

$$\sum_{(1,j)\in U} FC_{1j}(x_{1j},\, y_{1j},\, z_{1j}) + FC_{21}(x_{21},\, y_{21},\, z_{21}) + \sum_{(3,l)\in V} FC_{3l}(x_{3l},\, y_{3l},\, z_{3l}) =$$
$$= \sum_{(1,j)\in U} a_{1j}x_{1j}\left[(1+r_{1j})^{\frac{x_{1j}}{365}} - 1\right] + \sum_{(1,j)\in U} b_{1j}y_{1j}\left[(1+r_{1j})^{\frac{y_{1j}}{365}} - 1\right] -$$
$$- \sum_{(1,j)\in U} c_{1j}z_{1j}\left[(1+r_{1j})^{\frac{z_{1j}}{365}} - 1\right] +$$
$$+ a_{21}x_2\left[(1+r_{21})^{\frac{x_{21}}{365}} - 1\right] + b_{21}y_{21}\left[(1+r_{21})^{\frac{y_{21}}{365}} - 1\right] - c_{21}z_{21}\left[(1+r_{21})^{\frac{z_{21}}{365}} - 1\right] +$$
$$+ \sum_{(3,l)\in V} a_{3l}x_{3l}\left[(1+r_{3l})^{\frac{x_{3l}}{365}} - 1\right] + \sum_{(3,l)\in V} b_{3l}y_{3l}\left[(1+r_{3l})^{\frac{y_{3l}}{365}} - 1\right] -$$
$$- \sum_{(3,l)\in V} c_{3l}z_{3l}\left[(1+r_{3l})^{\frac{z_{3l}}{365}} - 1\right]. \quad (42)$$

The coalition opponents have power to influence days payable outstanding and days receivable outstanding of the coalition group via shortening payments period of the distributor and delaying payments to the distributor. In order to maximize the characteristic function the opponents have to maximize y_{21} and minimize z_{21}, in addition the coalition has the coalition has the cash conversion cycle constraints and constrains on each participant:

$$\underline{CCC}_{1j} \leq x_{1j} + y_{1j} - z^0_{1j} \leq \overline{CCC}_{1j},\ (1,j) \in U, \quad (43)$$

$$\underline{CCC}_{21} \leq x_{21} + y_{21} - z_{21} \leq \overline{CCC}_{21}; \quad (44)$$

$$\underline{CCC}_{3l} \leq x_{3l} + y^0_{3l} - z_{3l} \leq \overline{CCC}_{3l},\ (3,l) \in V. \quad (45)$$

Therefore the maximum values the counterparts can achieve comply with the following rule:

$$z_{21} = \sum_{(1,j)\in U} y_{1j};$$

$$y_{21} = \overline{CCC}_{21} - x_{21} + \sum_{(1,j)\in U} y_{1j}.$$

Further the coalition can minimize its cost function (41) via leveraging its $\sum_{(1,j)\in U} y_{1j}$, $\sum_{(3,l)\in V} z_{3l}$ and manipulating its set of $x_{1j} \in U$, x_{21} and $x_{3l} \in V$ keeping in mind the cash conversion cycle boundaries.

Value of characteristic function for grand coalition. The last coalition K consists of all participants in the supply chain. Since all partners have a common goal, the only step to build the characteristic function is to minimize the cost function of joint working capital.

$$v(K) = v(K_1, K_2, K_3) = \min_{x_{ij},y_{ij},z_{ij}} (\sum_{(1,i)\in K_1} FC_{1i} + FC_{21} + \sum_{(3,j)\in K_3} FC_{3j}). \quad (46)$$

After this stage having exploited the possible structures of the chain, the goal is to build the Shapley value of the game and check whether it belongs to C-core. It will be the solution of the costs redistribution problem.

3.2. Shapley value and C-core

Let $\varphi : \{\langle (N, v) \rangle\} \to R^n$ – function complying to each game $G = \langle N, v\rangle$ the imputation $\varphi[v] = (\varphi_1[v], \ldots, \varphi_n[v])$, which satisfies the Shapley's axioms . This vector $\varphi[v] = (\varphi_1[v], \ldots, \varphi_n[v])$ is named Shapley value of the game $G = \langle N, v\rangle$ [Shapley, 1953].

In arbitrary game $G = \langle N, v\rangle$ exists unique Shapley value. The components of Shapley value are calculated according to the following formula [Shapley, 1953]:

$$\varphi_i[v] = \sum_{\{S|i\in S\subset N\}} \frac{(s-1)!(n-s)!}{n!}[v(S) - v(S \setminus i)],\ for\, \forall i \in N, \quad (47)$$

where s is the number of players in coalition S.

Shapley value has the next implications. it is assumed that the players have coordinated to meet up in a certain place in order to conduct the negotiations of redistribution the gain from the maximum possible coalition. Naturally due to some random delay each of them arrives at different time. it is assumed hat each sequence of arriving players has the same probability and if the player i arriving find the others in the coalition $S \setminus i$, then the player i receives the gain equivalent $v(S) - v(S \setminus i)$. In other words the gain of the player i is the value added by this player to the maximum guaranteed gain of the coalition. Shapley value provides a mathematical solution of the cost distribution problem. The values of the vector correspond to the cost each player should bear after the optimization.

The theory and concepts above allow to introduce a methodology which purpose to provide a solid solution of the costs redistribution after the optimization. The algorithm includes the further steps:

1. Define the participant of the supply chain;
2. Define all the possible coalitions within the supply chain;
3. Introduce the working capital cost function;
4. Build the characteristic function for each coalition;
   (a) Implement the maximizing constraints on the coalition;
   (b) Minimize the cost function of the coalition;
5. Build the Shapley value of the game;
6. Test Shapley value for belonging to C-core.

4. Numerical example.

The case study represents the numerical optimization in information and communications technology industry (ICT). This choice is justified by several reasons. First of all this industry possesses deeply integrated structure, rapidly implements new technologies (Pirttilä et al., 2014), Moreover, being service oriented, the industry has wide range of the customers. In addition, there is an obvious absence of thorough attention to the ICT supply chain in the scientific literature (Lind et al., 2012).

Figure 4. depicts a financial supply chain of Russian provider of telecommunication services. According to the chain structure considered in the paper, the company plays a role of the distributor D_{21}. The focal company provides a wide range of products and services: long-distance and mobile telephony services, data transmission, television. The strategy of the company is to achieve a shift towards being a provider of completely integrated services via enhancing technological aspects. According to this goal the company invests substantial amounts of money into modernization of operational software, for instance, one of the direction is procurement optimization programs. Therefore the firm is highly motivated to cooperate with the other participants avoiding any discrimination of both either small or medium participants.

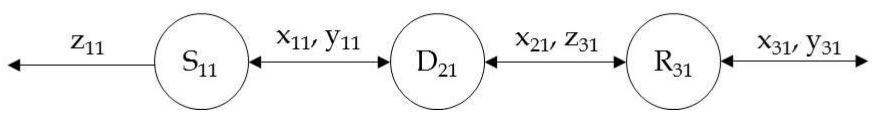

Fig. 4. Financial flows of the ICT chain.

The system integrator S_{11} (Figure 4.) is a large player on the domestic market as well operating in Europe. The business of the integrator is primarily concentrated on development of ICT infrastructure: energy appliance, information hubs, engineering solutions for industries. The firm is the major supplier of the telecommunication services provider highly involved in its procurement.

Mobile phone company R_{31} (Figure 4.) being deeply integrated with the provider of telecommunication services sells services such as mobile internet, mobile telecommunications across Russia. It business has the model of providing superior products at affordable price. This approach along with high demand on the products allows the firm to perform better in terms of growing the number of the subscribers. Nonetheless, the firms financial accomplishment lack behind due to the construction of a new network, which required significant leverage.

The data was gathered from the annual financial reports of the considered companies and represented in Table 1 and Table 2.

Table 1. Initial data before optimization.

	System integrator	Telecommunication services provider	Mobile phone company
	1	**2**	**3**
DIO	77.2	184	64.2
DRO	67.6	5.7	6.6
DPO	51.8	67.6	5.6
INV	1 342	11 593	972
AR	1 374	458	119
AP	901	4 256	85
CCC	**93**	**122.1**	**65.2**
FC	**32.5**	**237.7**	**2.7**

Table 2. Parameters of the SC players.

	System integrator	Telecommunication services provider	Mobile phone company
	1	**2**	**3**
r	8.2%	4.7%	3.4%
Revenue	7 419	29 792	6 588
COGS	6 345	22 981	5 528

The model requires optimization of CCC part in certain possible interval. This paper uses the interval between -17 and 61.50 for cash conversion cycle of information and communications technology industry defined by (Garanina and Petrova, 2015).

Further, according to the methodology in order to optimize costs along the supply chain and obtain the cost distribution strategy it is necessary to go through 6 steps.

Table 3. Data after optimization in grand coalition.

	System integrator	Telecommunication services provider	Mobile phone company
	1	**2**	**3**
DIO	0	52.65	18.35
DRO	112	42.35	6.6
DPO	51.8	112	42.45
INV	0	3 314	278
AR	2 276	3 456	126
AP	1 946	2 667	106
CCC	**60**	**-17**	**-17**
FC	**45.4**	**-102**	**-1.7**

Table 3 and Table 4 represent the rezults of optimization and comparative change in controllable variables and the best possible value of the joint cost function. Interpreting the results of the optimization it is possible to see that in general for the supply chain it is beneficial to reduce the amount of the inventories. For instance, the model implies that the supplier should decrease its inventories as much as possible, ideally to 0.

Table 4. Comparative analysis

	System integrator	Telecommunication services provider	Mobile phone company
	1	**2**	**3**
DIO	0%	29%	29%
DRO	166%	423%	106%
DPO	100 %	243%	755%
INV	0%	29%	29%
AR	166%	423%	106%
AP	100 %	243%	755%
CCC	**65%**	**-717%**	**-382%**
FC	**140%**	**-43%**	**-67%**

Another significant point is that according to the model the coalition should prolong the payments due for the mobile company by the telecommunication services provider. In addition the system integrator should prolong the payments due to the telecommunication services provider as well.

Considering the changes it is obvious that the cash conversion cycle of each participant took boundary values of the range of stability. Furthermore while the telecommunication services provider and the mobile phone company the CCC values took the left boundary, the CCC of the system integrator received the value of the right boundary. Since the optimization was conducted along the cost of working capital function, therefore the main cost contributor was the telecommunication services provider and the minimization of its contribute to the cost brought more value for the coalition.

Further analyzing the reduce in the cost of the joint capital two issues arise. The first is that the final value of the function is negative. The second issue is that according to the model the system integrator should bear higher costs on its working capital after the optimization. Considering the second issue on the first sight being rational the system integrator should not accept these terms of the agreement and leave the coalition thereby saving its own financial resources bot negatively affecting the rest player. Nonetheless, the results of the optimization are positive in terms of cost reduction of the joint working capital. Therefore, the participant being interested in this cooperation should develop a fair distribution strategy.

At this point we construct the cooperative game according to the methodology described, calculate the value of characteristic function for the existing coalitions (Table 5) and evaluation of Shapley value of the game (Table 6).

Table 5. Characteristic function.

S	1	2	3	12	13	23	123
$v(S)$	15.29	16.26.	0.08	10.69	15.37	0.07	-59.02

The Shapley value implies the way how the final cost should be optimally re-distributed. In other words, the supplier should get additional decrease of the costs by -12.98, which is less compared to 15.29 - costs when not participating in the coalition. The distributor should also have a significant decrease in working capital costs – -20.14, which is considerably higher in comparison with 16,26 associated

Table 6. Shapley value.

$\varphi(1)$	$\varphi(2)$	$\varphi(3)$
-12.98	-20.14	-25.90

with not participating in coalition. Finally considering the optimization for the retailer the final costs should be -25.90 compared to 0,08 of not being a participant of the maximum coalition.

Besides we can say that the obtained Shapley value belongs to the C-core:

$$\varphi(1) + \varphi(2) = -33.12 < 10.69 = \upsilon(1,2),$$

$$\varphi(1) + \varphi(3) = -38.88 < 15.36 = \upsilon(1,3),$$

$$\varphi(2) + \varphi(3) = -46.04 < 0.07 = \upsilon(2,3),$$

$$\varphi(1) + \varphi(2) + \varphi(3) = -59.02 = \upsilon(1,2,3).$$

It proves that this cost imputation is strictly nondominant and there are no coalitions that can be opportunistic in such a game.

Nonetheless this case reveals several constrains which are still to be discovered further. The first one is the base for the assumption that each participant of the chain is able to obtain its cash conversion cycle on the boundary values. In the reality there might be conditions and situations, for instance delays in the supplies due to infrastructure breakdowns. The second assumption is that possessing a optimal negative cost function of the joint working capital a supply chain participant might face not willingness to be financed by outer participants of the chain, for instance the final customer might not be ready to pay in advance for the product and decide to switch to another retailer or the earlier step supplier might require earlier payments.

5. Discussion and conclusions

The main purpose of the study was to introduce a methodology, which would allow the participants of a supply chain willing to cooperate to optimize the costs on joint working capital and develop cost redistribution policy.

Critical literature analysis showed that competent management of financial supply chains has direct positive impact on the liquidity of a company. Moreover it was discover that the most prominent approach in this field is management of the working capital. Cash conversion cycle is a widely admitted metrics to measure the effectiveness of working capital management. This metrics contains three important parts: days inventory outstanding, days accounts receivable and days accounts payable, which describe financial flows of the supply chain. Reduce of the cash conversion cycle generally leads to increase in the liquidity. However, CCC reduction on the level of a certain form is highly limited, therefore the optimization should be done along several consequent parts of the chain. Therefore collaborative cash conversion cycle should be considered.

Further even if the participants of a certain chain decided to cooperate with each other in order to optimize the collaborative cash conversion cycle and the costs of the joint working capital they might face the problem of redistribution of costs, because there may be cases in which participants obtain higher costs on their working capital. However the described case shows that Shapley value lies in the

C-core and is strictly nondominant. It implies that all supply chain participants have no motives to oppose cooperation and behave opportunistically.

The methodology provides a mathematical overview of the cost redistribution problem allowing the participant to bear costs according the value they add participating in the coalition. The example was considered a supply chain with one-one-one structure in information telecommunication industry of three participants: a system integrator, a telecommunication service provider and a mobile phone company. In this example the costs of the joint working capital were reduced to -59 mln dollars a year, with final cost redistribution to each participant: -12.98 to the supplier, -20.14 to the distributor and -25.98 to the retailer. Moreover the CCC of each participants took the next values: 60 for the supplier, -17 for the distributor and -17 for the retailer.

Further research should be done is two directions: mathematical and managerial. Regarding the former, a limitation which was obtained empirically is that each additional participant in a supply chain compounds calculation process of characteristic function for coalitions and Shapley value. Therefore it is needed to consider quantitative method of optimization or analytical approach, which would permit to aggregate some types of coalitions.

Further fruitful area of research in mathematical terms is dynamic cooperative working capital cost game allowing to redistribute the costs on a regular day-to-day basis

Considering the second field, it is necessary to develop a framework or technique which would on the one hand figure out the possibility of maintaining boundary levels of the conversion cash cycle of the coalition participants in a particular industry, and on the other hand will discover the possibility for the chain to be financed from the outer participants.

References

Akkermans, H., Van Wassenhove, L. N. (2018). *Supply Chain Tsunamis: Research on Low-Probability, High-Impact Disruptions.* Journal of Supply Chain Management, **54(1)**, 64–76.

Barratt, M. (2004). *Understanding the meaning of collaboration in the supply chain.* Supply Chain Management: An International Journal, **9(1)**, 30–42.

Beamon, B. M. (1998). *Supply chain design and analysis: Models and methods.* International Journal of Production Economics, **55(3)**, 281–294.

Blackman, I. D., Holland, C., Westcott, T. (2013). *Motorola's global financial supply chain strategy.* Supply Chain Management: An International Journal, **18(2)**, 132–147.

Blome, C., Paulraj, A., Schuetz, K. (2014). *Supply chain collaboration and sustainability: a profile deviation analysis.* International Journal of Operations and Production Management, **34(5)**, 639–663.

Cachon, G. P., Lariviere, M. A. (2005). *Supply chain coordination with revenue-sharing contracts: strengths and limitations.* Management science, **51(1)**, 30–44.

Cao, M., Zhang, Q. (2011). *Supply chain collaboration: Impact on collaborative advantage and firm performance.* Journal of Operations Management, **29(3)**, 163–180.

Chauhan, G. S., Banerjee, P. (2018). *Financial constraints and optimal working capital-evidence from an emerging market.* International Journal of Managerial Finance, **14(1)**, 37–53.

Chen, I. J., Paulraj, A. (2004). *Towards a theory of supply chain management: the constructs and measurements.* Journal of Operations Management, **22(2)**, 119–150.

Cherkasova, V., Chadin, V. (2015). *Working capital determinants in the concept of life cycle of the organization.* Journal of Corporate Finance Research, **9(2)**, 79–99.

Coyle, J. J., Langley, J. C., Novack, R. A., Gibson, B. J. (2013). Supply Chain Management: A Logistics Perspective. Ninth Edition, South-Western, Cengage Learning: Mason, USA.

Deloof, M. (2003). *Does working capital management affect profitability of Belgian firms?* Journal of Business Finance and Accounting, **30(3-4)**, 573–87.

Enqvist, J., Graham, M., Nikkinen, J. (2014). *The impact of working capital management on firm profitability in different business cycles: Evidence from Finland.* Research in International Business and Finance, **32**, 36–49.

Fairchild, A. (2005). *Intelligent matching: integrating efficiencies in the financial supply chain.* Supply Chain Management: An International Journal, **10(4)**, 244–248.

Fawcett, S. E., Magnan, G. M. (2002). *The rhetoric and reality of supply chain integration.* International Journal of Physical Distribution and Logistics Management, **32(5)**, 339.

Fedorova, E., Timofeev, Y. (2015). *Standards of financial stability of Russian companies: industry-specific features.* Journal of Corporate Finance Research, **9(1)**, 38–47.

Filbeck, G., Krueger, T. M. (2005). *An analysis of working capital management results across industries.* American Journal of Business, **20(2)**, 11–20.

Flynn, B. B., Huo, B., Zhao, X. (2010). *The impact of supply chain integration on performance: A contingency and configuration approach.* Journal of Operations Management, **28(1)**, 58–71.

Flynn, B. B., Koufteros, X., Lu, G. (2016). *On theory in supply chain uncertainty and its implications for supply chain integration.* Journal of Supply Chain Management, **52(3)**, 3–27.

Frohlich, M. T., Westbrook, R. (2001). *Arcs of integration: An international study of supply chain strategies.* Journal of Operations Management, **19(2)**, 185–200.

Gadde, L.-E., Snehota, I. (2000). *Making the Most of Supplier Relationships.* Industrial Marketing Management, **29(4)**, 305–316.

Galaskiewicz, J. (2011). *Studying supply chains from a social network perspective.* Journal of Supply Chain Management, **47(1)**, 4–8.

Garanina, T. A., Belova, O. A. (2015). *Liquidity, cash conversion cycle and financial performance: case of Russian companies.* Investment Management and Financial Innovations, **12(1)**, 90–100.

Garanina, T., Petrova, O. (2015). *Relationship between liquidity, cash conversion cycle and returns of Russian companies.* Journal of Corporate Finance Research, **9(1)**, 5–21.

García-Teruel, P. J., Martínez-Solano, P. (2007). *Effects of Working Capital Management on SME Profitability.* International Journal of Managerial Finance, **3(2)**, 164–177.

Gelsomino, L. M., de Boer, R., Steeman, M., Perego, A. (2018). *An optimisation strategy for concurrent Supply Chain Finance schemes.* Journal of Purchasing and Supply Management. DOI: 10.1016/j.pursup.2018.07.004

Gelsomino, L. M., Mangiaracina, R., Perego, A., Tumino, A. (2016). *Supply chain finance: a literature review.* International Journal of Physical Distribution & Logistics Management, **46(4)**, 348–366.

Gunasekaran, A., Patel, C., McGaughey, R. E. (2004). *A framework for supply chain performance measurement.* International Journal of Production Economics, **87(3)**, 333–347.

Gupta, S., Dutta, K. (2011). *Modeling of financial supply chain.* European Journal of Operational Research, **211(1)**, 47–56.

Håkansson, H., Ford, D. (2002). *How should companies interact in business networks?* Journal of Business Research, **55**, 133–139.

Håkansson, H., Snehota, I. (2006). *No business is an island: The network concept of business strategy.* Scandinavian Journal of Management, **22**, 256–270

Hendricks, K. B., Singhal, V. R. (2003). *The effect of supply chain glitches on shareholder wealth.* Journal of Operations Management, **21(5)**, 501–522.

Hofmann, E., Kotzab, H. (2010). *A Supply Chain-Oriented Approach of Working Capital Management.* Journal of Business Logistics, **31(2)**, 305–330.

Hutchison, P. D., Farris II, M. T., Anders, S. B. (2007). *Cash-to-cash analysis and management.* The CPA Journal, **77(8)**, 42.

Johnson, M., Templar, S. (2011). *The relationships between supply chain and firm performance: The development and testing of a unified proxy.* International Journal of Physical Distribution and Logistics Management, **41(2)**, 88–103.

Jones, M. (2006). Financial accounting. Wiley.

Kim, S. W. (2009). *An investigation on the direct and indirect effect of supply chain integration on firm performance.* International Journal of Production Economics, **119(2)**, 328–346.

Kirca, A. H., Jayachandran, S., Bearden, W. O. (2005). *Market Orientation: A Meta-Analytic Review and Assessment of Its Antecedents and Impact on Performance.* Journal of Marketing, **69(2)**, 24–41.

Kouvelis, P., Chambers, C., Wang, H. (2006). *Supply Chain Management Research and Review, Trends, and Opportunities.* Production and Operations Management, *15(3)*, 449–69.

Kouvelis, P., Zhao, W. (2017). *Who should finance the supply chain? Impact of credit ratings on supply chain decisions.* Manufacturing & Service Operations Management, **20(1)**, 19–35.

Kroes, J. R., Manikas, A. S. (2014). *Cash flow management and manufacturing firm financial performance: A longitudinal perspective.* International Journal of Production Economics, **148**, 37–50.

Kunter, M. (2012). *Coordination via cost and revenue sharing in manufacturer-retailer channels.* European Journal of Operational Research, **216(2)**, 477–486.

Lazaridis, I., Tryfonidis, D. (2006). *Relationship between working capital management and profitability of listed companies in the Athens stock exchange.* Journal of Financial Management and Analysis, **19(1)**.

Leng, M., Parlar, M. (2009). *Allocation of cost savings in a three-level supply chain with demand information sharing: A cooperative-game approach.* Operations Research, **57(1)**, 200–213.

Lind, L., Pirttilä, M., Viskari, S., Schupp, F., Kärri, T. (2012). *Working capital management in the automotive industry: Financial value chain analysis.* Journal of purchasing and supply management, **18(2)**, 92–100.

Malshe, A., Agarwal, M. K. (2015). *From Finance to Marketing: The Impact of Financial Leverage on Customer Satisfaction.* Journal of Marketing, **79**(September), 21–38.

Margolis, J. T., Sullivan, K. M., Mason, S. J., Magagnotti, M. (2018). *A multi-objective optimization model for designing resilient supply chain networks.* International Journal of Production Economics, **204**, 174–185.

Martin, J., Hofmann, E. (2017). *Involving financial service providers in supply chain finance practices: Company needs and service requirements.* Journal of Applied Accounting Research, **18(1)**, 42–62.

Marttonen, S., Monto, S., Kärri, T. (2013). *Profitable working capital management in industrial maintenance companies.* Journal of Quality in Maintenance Engineering, **19(4)**, 429–446.

Matyac, E. (2015). *Financial supply chain management.* Strategic Finance, **96(9)**, 62.

Noordhoff, C. S., Kyriakopoulos, K., Moorman, C., Pauwels, P., Dellaert, B. G. (2011). *The bright side and dark side of embedded ties in business-to-business innovation.* Journal of Marketing, **75(5)**, 34–52.

Paulraj, A., Lado, A. A., Chen, I. J. (2008). *Inter-organizational communication as a relational competency: Antecedents and performance outcomes in collaborative buyer-supplier relationships.* Journal of Operations Management, **26(1)**, 45–64.

Petrosyan, L. A., Zenkevich, N. A. (2016). Game Theory. World Scientific Publishing – 564.

Pirttilä, M., Viskari, S., Lind, L., Kärri, T. (2014). *Benchmarking working capital management in the inter-organisational context.* International Journal of Business Innovation and Research, **8(2)**, 119–136.

Protopappa-Sieke, M., Seifert, R. W. (2017). *Benefits of working capital sharing in supply chains.* Journal of the Operational Research Society, **68(5)**, 521–532.

Protopappa-Sieke, M., Seifert, R. W. (2010). *Interrelating operational and financial performance measurements in inventory control.* European Journal of Operational Research, **204(3)**, 439–448.

Ramdas, K., Spekman, R. E. (2000). *Chain or shackles: understanding what drives supply-chain performance.* Interfaces, **30(4)**, 3–21.

Randall, W. S., Theodore Farris, M. (2009). *Supply chain financing: using cash-to-cash variables to strengthen the supply chain.* International Journal of Physical Distribution & Logistics Management, **39(8)**, 669–689.

Richards, V. D., Laughlin, E. J. (1980). *A cash conversion cycle approach to liquidity analysis.* Financial management, **9(1)**, 32–38.

Rokkan, A. I., Heide, J. B., Wathne, K. H. (2003). *Specific Investments in Marketing Relationships: Expropriation and Bonding Effects.* Journal of Marketing Research, **40**(May), 210–224.

Seggie, S. H., Griffith, D. A., Jap, S. D. (2013). *Passive and active opportunism in interorganizational exchange.* Journal of Marketing, **77(6)**, 73–90.

Shapley, L. S. (1953). *A value for n-person games.* Contributions to the Theory of Games, **2(28)**, 307–317.

Shin, H. H., Soenen, L. (1998). *Efficiency of working capital management and corporate profitability.* Financial practice and education, **8**, 37–45.

Silvestro, R., Lustrato, P. (2014). *Integrating financial and physical supply chains: the role of banks in enabling supply chain integration.* International Journal of Operations and Production Management, **34(3)**, 298–324.

Simatupang, T. M., Sridharan, R. (2002). *The Collaborative Supply Chain.* The International Journal of Logistics Management, **13(1)**, 15–30.

Song, H., Yu, K., Lu, Q. (2018). *Financial service providers and banks' role in helping SMEs to access finance.* International Journal of Physical Distribution and Logistics Management, **48(1)**, 69–92.

Stevens, G., Johnson, M. (2016). *Integrating the Supply Chain... 25 Years On.* International Journal of Physical Distribution and Logistics Management, **46(1)**, 1–30.

Talonpoika, A. M., Kärri, T., Pirttilä, M., Monto, S. (2016). *Defined strategies for financial working capital management.* International Journal of Managerial Finance, **12(3)**, 277–294.

Talonpoika, A. M., Monto, S., Pirttilä, M., Kärri, T. (2014). *Modifying the cash conversion cycle: revealing concealed advance payments.* International Journal of Productivity and Performance Management, **63(3)**, 341–353.

Vázquez, X. H., Sartal, A., Lozano-Lozano, L. M. (2016). *Watch the working capital of tier-two suppliers: a financial perspective of supply chain collaboration in the automotive industry.* Supply Chain Management: An International Journal, **21(3)**, 321–333.

Viskari, S., Kärri, T. (2013). *A cycle time model for analysing the efficiency of working capital management in a value chain.* International Journal of Business Performance and Supply Chain Modelling, **5(3)**, 221–238.

Viskari, S., Kärri, T. (2012). *A model for working capital management in the inter-organisational context.* International Journal of Integrated Supply Management, **7(1-3)**, 61–79.

Viskari, S., Lind, L., Kärri, T., Schupp, F. (2012). *Using working capital management to improve profitability in the value chain of automotive industry.* International Journal of Services and Operations Management, **13(1)**, 42–64.

Viskari, S., Pirttilä, M., Kärri, T. (2011). *Improving profitability by managing working capital in the value chain of pulp and paper industry.* International Journal of Managerial and Financial Accounting, **3(4)**, 348–366.

Viskari, S., Ruokola, A., Pirttilä, M., Kärri, T. (2012). *Advanced model for working capital management: bridging theory and practice.* International Journal of Applied Management Science, **4(1)**, 1–17.

Volkov, D., Nikulin, E. (2012). *Current State and Prospects for Research on Firm's Working Capital Management.* Journal of Corporate Finance Research, **6(3)**, 61–69.

Wuttke, D. A., Blome, C., Henke, M. (2013). *Focusing the financial flow of supply chains: An empirical investigation of financial supply chain management.* International Journal of Production Economics, **145(2)**, 773–789.

Xu, X., Chen, X., Jia, F., Brown, S., Gong, Y., Xu, Y. (2018). *Supply chain finance: A systematic literature review and bibliometric analysis.* International Journal of Production Economics, **204**, 160–173.

Yan, N., He, X., Liu, Y. (2018). *Financing the capital-constrained supply chain with loss aversion: supplier finance vs. supplier investment.* Omega. DOI: 10.1016/j.omega.2018.08.003

Yazdanfar, D., Öhman, P. (2014). *The impact of cash conversion cycle on firm profitability: An empirical study based on Swedish data.* International Journal of Managerial Finance, **10(4)**, 442–452.

Ylä-Kujala, A., Marttonen, S., Kärri, T., Sinkkonen, T., Baglee, D. (2016). *Inter-organisational asset management: linking an operational and a strategic view.* International Journal of Process Management and Benchmarking, **6(3)**, 366–385.

Yousefi, A., Pishvaee, M. S. (2018). *A Fuzzy Optimization Approach to Integration of Physical and Financial Flows in a Global Supply Chain Under Exchange Rate Uncertainty.* International Journal of Fuzzy Systems, **20(8)**, 2415–2439.