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Stochastic n-person Prisoner's Dilemma: the Time-Consistency of Core and Shapley Value

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Abstract A cooperative finite-stage dynamic n-person prisoner's dilemma is considered. The time-consistent subset of the core is proposed. The the Shapley value for the stochastic model of the n-person prisoner's dilemma is calculated in explicit form.

Keywords: n-person prisoner's dilemma, coalition, dynamic game, core, Shapley value, time consistency.

1. Introduction

In recent times, great attention is paid to research processes of human interaction through Game theory models. Mathematical game theory is now booming. Dynamic games are examined. Dynamic games can play an important role in addressing the issue of politics, economics of monopolies and the distribution of market power, and some others. One of the fundamental models of the Game theory is the "prisoner's dilemma". Hamburger (Hamburger, 1973) has considered multi-agent behaviour effects throught implementation "n-person prisoner's dilemma".

A large number of players makes this survey more entertaining since even the characteristic function looks less trivial than in the two-agent model.

The existing literature on repeated and dynamic models of the "n-person prisoner's dilemma" is extensive and focuses particularly on theoretical analysis and searching empirical results.

In this paper we consider a new characteristic function introduced by Petrosyan's characteristic function (Petrosyan, 2019). Using this characteristic function the time-consistent subset of the core for dynamic n-person prisoner's dilemma is constructed.

2. The "n-person prisoner's dilemma" model description

A game $\Gamma = \langle N,\ X_1,\ \ldots,\ X_n,\ H_1(x_1,\ \ldots,\ x_n),\ \ldots,\ H_n(x_1,\ \ldots,\ x_n)\rangle$ is a static "n-person prisoner's dilemma" game, where N is a set of the players, $|N| = n$. We denote by $x_i \in \{C,\ D\} = X_i$ the pure strategies for each players $\forall i \in N$, where C means "to cooperate" strategy, but D means "to defect". The payoff function $H_i(x_1,\ \ldots,\ x_i,\ \ldots,\ x_n),\ \forall i \in N$ linearly depends on the number of players (x) who have chosen the "to cooperate" strategy:

$$H_i(x_1,\ldots,x_i,\ldots,x_n) = \begin{cases} C_i(x) = a_1 x + b_1,\ \forall\ x \in (0,n]\,, \text{if } x_i = C. \\ D_i(x) = a_2 x + b_2,\ \forall\ x \in [0,n)\,, \text{if } x_i = D. \end{cases}$$

This function meets the following requirements:

1. $D_i(x-1) > C_i(x), \forall x \in [1,n]$, i. e. the strategy "to defect" strictly dominates the strategy "to cooperate";
2. $C_i(n) > D_i(0)$, so the strategy profile $(C, \ \dots, \ C)$ is Pareto effective in contrast to $(D, \ \dots, \ D)$.
3. $D_i(x) \geq D_i(0), \forall x \in [0, n-1]$ and $C_i(x) \geq C_i(1), \forall x \in [1,n]$, therefore, payoffs of the players in case of the x cooperating players is at least not less than in case of the absence of cooperating players.
4. $C_i(x) = C_j(x)$ and $D_i(x) = D_j(x)$, this means that the players are symmetric.

Straffin (Straffin, 1993) introduce some of this principles in his book.

Considerable amount of literature use the table form of writing the payoff function. However, our investigation allows us to simplify the calculations of characteristic function and then describe the construction of the core and the computing of Shapley value. Moreover, such form of payoff function is useful for future research.

3. The core of the "n-person prisoner's dilemma"

Consider a dynamic game Γ_f which is played during K_f steps. This game consists of the set of f static games $(\gamma_1, \ \dots, \ \gamma_f)$, which can be described as the model of "n-person prisoner's dilemma". The games are realised with probabilities $p_1, \dots, p_f$ on each stage of the game Γ_f, $\sum_{j=1}^{f} p_j = 1$.

The payoff functions for each possible static game are:

$$H_i^{\gamma_j}(x_1, \dots, x_i, \dots, x_n) = \begin{cases} C_i^{\gamma_j}(x) = a_1^{\gamma_j} x^{\gamma_j} + b_1^{\gamma_j}, \ \forall \ x^{\gamma_j} \in (0, n], \text{if } x_i^{\gamma_j} = C \text{ and} \\ x^{\gamma_j} \text{ is a number of players, who plays the } C \text{ strategy,} \\ D_i^{\gamma_j}(x^{\gamma_j}) = a_2^{\gamma_j} x + b_2^{\gamma_j}, \ \forall \ x^{\gamma_j} \in [0, n), \text{if } x_i^{\gamma_j} = D \text{ and} \\ x^{\gamma_j} \text{ is a number of players, who plays the } C \text{ strategy.} \end{cases}$$

Let

$$V^{\gamma_j}(N) = \max_{x_1, \dots, x_i, \dots, x_n} \sum_{i \in N} H_i^{\gamma_j}(x_1, \dots, x_i, \dots, x_n)$$

is equal for all $\gamma_j : \ j \in [1, \ f]$.

Definition 1. A core of the game Γ_f is a set of possible allocations $(\alpha_1, \ \dots, \ \alpha_n)$ which doesn't contradict to the following statements:

1. individual rationality: $\alpha_i \geq V^{\Gamma_f}(i), \ \forall i \in N$;
2. coalitional rationality: $\sum_{i \in S} \alpha_i \geq V^{\Gamma_f}(S), \ \forall S \subset N$;
3. efficiency: $\sum_{i \in N} \alpha_i = V^{\Gamma_f}(N)$.

The value of characteristic function for each individual player in each stage of Γ_f equals to

$$V^{\gamma_j}(i) = D_i^{\gamma_j}(0) = b_2^{\gamma_j}.$$

Suppose, that S is a coalition: $S \subset N, \ |S| = s$. It can guarantee the payoff

$$V^{\gamma_j}(S) = \max_{r \in [0,s]} \left(r \left(a_1^{\gamma_j} r + b_1^{\gamma_j} \right) + (s - r) \left(a_2^{\gamma_j} r + b_2^{\gamma_j} \right) \right), \forall S \subset N.$$

$V^{\gamma_j}(N)$ is the same in all games $(\gamma_1, \ \ldots, \ \gamma_f)$ and it is equal to

$$V^{\gamma_j}(N) = \max_{s\in[0,n]} \left(s\left(a_1^{\gamma_j} s \ + \ b_1^{\gamma_j}\right) \ + \ (n \ - \ s)\left(a_2^{\gamma_j} s \ + \ b_2^{\gamma_j}\right)\right)$$

So, the core of the game Γ_f is the set of allocations which meets with following conditions:

$$\begin{cases} \sum_{i=1}^{N} \alpha_i = K_f \sum_{j=1}^{f} p_j \max_{s\in[0,n]} \left(a_1^{\gamma_j} s^2 + b_1^{\gamma_j} s + a_2^{\gamma_j} ns - a_2^{\gamma_j} s^2 + b_2^{\gamma_j} n - b_2^{\gamma_j} s\right); \\ \sum_{i=1}^{S} \alpha_i \geq K_f \sum_{j=1}^{f} p_j \max_{r\in[0, \ s]} \left(a_1^{\gamma_j} r^2 + b_1^{\gamma_j} r + a_2^{\gamma_j} sr - a_2^{\gamma_j} r^2 + b_2^{\gamma_j} s - b_2^{\gamma_j} r\right). \end{cases}$$

Definition 2. Define $W(S)$, $S \subset N$, as follows: $W(S) = \max_j V(\gamma_j, \ S)$. Denote by $D(\overline{\gamma_j})$ the set of imputations $\alpha^{\overline{\gamma}_j} = \left(\alpha_1^{\overline{\gamma}_j}, \ \ldots, \ \alpha_n^{\overline{\gamma}_j}\right)$ in Γ_f, satisfying the condition

$$\sum_{i\in S} \alpha_i^{\overline{\gamma}_j} \geq W(S), \ S \subset N, \ S \neq N,$$

$$\sum_{i\in N} \alpha_i^{\overline{\gamma}_j} = V(\gamma_j, \ N),$$

here $V(\gamma_j, \ N)$ is the maximum sum of players payoffs in the game Γ_f. (Petrosyan and Pankratova, 2018)

Definition 3. A set $\overline{D}(\overline{\gamma}_1)$ is called to be strongly time consistent in Γ_f if

1. $\overline{D}(\gamma_{l+1}) \neq \varnothing, j \in [1, f]$;
2. there exists imputation distribution procedure (See Petrosyan, 1993) $\beta = \left(\beta_1, \ldots, \beta_j, \ldots, \beta_{K_f}\right)$: $\overline{D}(\overline{\gamma}_1) \supset \sum_{j=1}^{l} \beta_j \oplus \overline{D}(\gamma_{l+1})$ for all allocations $\alpha^{\overline{\gamma}_1} \in \overline{D}(\overline{\gamma}_1)$.

Here the sign $\oplus$ means

$$\beta_j \oplus \overline{D}(\gamma_{l+1}) = \left\{\beta_j \oplus \overline{d}(\gamma_{l+1}) : \overline{d}(\gamma_{l+1}) \in \overline{D}(\gamma_{l+1})\right\} (\ PetrosyanandGrauer, 2004).$$

Theorem 1 (Petrosyan-Pankratova's subset of the core). *3-person game Γ_f has time-consistent subset of the core D which can be described by the following inequalities*

$$\alpha_i^D \geq K_f \max_{j\in[1,f]} b_2^{\gamma_j},$$

$$\alpha_i^D + \alpha_j^D \geq$$
$$\geq K_f \max\left(\max_{j\in[1,f]} \left(4a_1^{\gamma_j} + 2b_1^{\gamma_j}\right); \max_{j\in[1,f]} \left(a_1^{\gamma_j} + b_1^{\gamma_j} + a_2^{\gamma_j} + b_2^{\gamma_j}\right); \max_{j\in[1,f]} \left(2b_2^{\gamma_j}\right)\right),$$

$$\alpha_i^D + \alpha_j^D + \alpha_k^D = K_f \max_{j\in[1,f]} \left(9a_1^{\gamma_j} + 3b_1^{\gamma_j}\right), \ \forall \ i, \ j, \ k \in N.$$

Proof (of theorem). Since the dominant strategy for each players is D ("to defect"), the value of characteristic function for singletone in the game $\gamma_j,\ \forall j \in [1,\ f]$ is

$$V^{\gamma_j}(i) = H_i^{\gamma_j}(D, D, D) = b_2^{\gamma_j},\ \forall\ i \in N.$$

All of the players are symmetric, so the value of characteristic function for two-person coalition is the maximum of three sums $\sum_{i \in S} H_i^{\gamma_j}(D,\ D,\ D)$, $\sum_{i \in S} H_i^{\gamma_j}(C,\ D,\ D)$ or $\sum_{i \in S} H_i^{\gamma_j}(C,\ C,\ D)$, where $S = \{1, 2\}$. Therefore,

$$V^{\gamma_j}(1,2) = V^{\gamma_j}(1,3) = V^{\gamma_j}(2,3) = max\left(4a_1^{\gamma_j} + 2b_1^{\gamma_j}; a_1^{\gamma_j} + b_1^{\gamma_j} + a_2^{\gamma_j} + b_2^{\gamma_j}; 2b_2^{\gamma_j}\right)$$

Since as the profile of strategies $(C,\ C,\ C)$ is more effective than $(D,\ D,\ D)$,

$$V^{\gamma_j}(N) = \max\left\{9a_1^{\gamma_j} + 3b_1^{\gamma_j};\ 4a_1^{\gamma_j} + 2b_1^{\gamma_j} + 2a_2^{\gamma_j} + b_2^{\gamma_j};\ a_1^{\gamma_j} + b_1^{\gamma_j} + 2a_2^{\gamma_j} + 2b_2^{\gamma_j}\right\}.$$

Assume that

$$W(S) = \max_{j \in f} V(\gamma_j, S), S \subset N$$

Define $\overline{\gamma}_j$ as a subgame of Γ_f with the starting point j, where $j \in [1, K_f]$. Thus, the subgame $\overline{\gamma}_1$ coinsides with Γ_f.

Next define a new characteristic function $\overline{W}(\Gamma_f, S)$:

$$\overline{W}\left(\overline{\gamma}_j, S\right) = (K_f - j + 1)\, W(S),$$

where $j \in [1, K_f]$.

Since $V^{\gamma_j}(N) = \max_{x_1,\dots,x_i,\dots,x_n} \sum_{i \in N} H_i^{\gamma_j}(x_1, \dots, x_i, \dots, x_n)$ is equal for all $\gamma_j, j \in [1, f]$,

$$\alpha_i^D \geq K_f \max_{j \in [1,f]} b_2^{\gamma_j}.$$

$$\alpha_i^D + \alpha_j^D \geq \overline{W}(S),\ \text{where } |S| = 2 \text{ and}$$

$$\overline{W}(S) = \\ = K_f \max\left(\max_{j \in [1,f]}\left(4a_1^{\gamma_j} + 2b_1^{\gamma_j}\right); \max_{j \in [1,f]}\left(a_1^{\gamma_j} + b_1^{\gamma_j} + a_2^{\gamma_j} + b_2^{\gamma_j}\right); \max_{j \in [1,f]}\left(2b_2^{\gamma_j}\right)\right).$$

$$\sum_{i \in N} \alpha_i^D = \\ = K_f \max_{j \in [1,f]}\left\{9a_1^{\gamma_j} + 3b_1^{\gamma_j};\ 4a_1^{\gamma_j} + 2b_1^{\gamma_j} + 2a_2^{\gamma_j} + b_2^{\gamma_j};\ a_1^{\gamma_j} + b_1^{\gamma_j} + 2a_2^{\gamma_j} + 2b_2^{\gamma_j}\right\},$$

$$\forall j \in [1, f].$$

This inequalities proves that D is a subset of the core.

But as far as $W(N)$ is equal for all γ_j in Γ_f, $\overline{W}(N) = K_f W(N)$.

Therefore, we can define imputation distribution procedure, as

$$\beta_{i1} = \frac{\alpha_i^{\overline{\gamma}_1}}{K_f}.$$

Then, for all subgames of the Γ_f

$$\beta_{ij} = \frac{\alpha_i^{\overline{\gamma}_j}}{K_f - j + 1},$$

where $j \in [1, K_f]$.

The allocation α can be written using IDP (See Petrosyan, 1993) which gives us

$$\sum_{j\in[1,K_f]} \sum_{i\in S} \frac{\alpha_i^{\overline{\gamma}_j}}{K_f - j + 1} = \sum_{j=1}^{l} \sum_{i\in S} \frac{\alpha_i^{\overline{\gamma}_j}}{K_f - j + 1} + \sum_{j=l+1}^{K_f} \sum_{i\in S} \frac{\alpha_i^{\overline{\gamma}_j}}{K_f - j + 1} \geq$$

$$\geq lW(S) + (K_f - l)W(S) = \overline{W}(S).$$

Hence, we construct the strongly time-consistent Petrosyan-Pankratova's D-subset of the core.

□

Example 1 (D-subset of the core). Consider a dynamic 3-person prisoner's dilemma Γ_f, where $V^{\gamma_j}(N)$ are equal for all $j \in [1,\ f]$. Let us define $K_f = 5,\ f = 3$.

Table 1. Game γ_1
(the 1^{st} is row-player, the 2^{nd} is column player and the 3^{rd} is page-player)

C	C	D
C	(100, 100, 100)	(90, 115, 90)
D	(115, 90, 90)	(100, 100, 90)

D	C	D
C	(90, 90, 115)	(80, 100, 100)
D	(100, 80, 100)	(85, 85, 85)

Table 2. Game γ_2
(the 1^{st} is row-player, the 2^{nd} is column player and the 3^{rd} is page-player)

C	C	D
C	(100, 100, 100)	(50, 105, 50)
D	(105, 50, 50)	(90, 90, 0)

D	C	D
C	(50, 50, 105)	(0, 90, 90)
D	(90, 0, 90)	(75, 75, 75)

Table 3. Game γ_3
(the 1^{st} is row-player, the 2^{nd} is column player and the 3^{rd} is page-player)

C	C	D
C	(100, 100, 100)	(90, 110, 90)
D	(110, 90, 90)	(98, 98, 80)

D	C	D
C	(90, 90, 110)	(80, 98, 98)
D	(98, 80, 98)	(86, 86, 86)

The values of characteristic functions of the games $(\gamma_1,\ \gamma_2,\ \gamma_3)$ for each coalitions are:

Then we construct D-subset of the core for the game Γ_f

Table 4. The values of the characteristic functions of the games γ_1–γ_3

S	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
V^{γ_1}	85	85	85	**180**	**180**	**180**	**300**
V^{γ_2}	75	75	75	150	150	150	**300**
V^{γ_3}	**86**	**86**	**86**	**180**	**180**	**180**	**300**

$$\begin{cases} \alpha_i \geq 430, & \forall\, i \in N; \\ \alpha_i + \alpha_j \geq 900, & \forall\, i,\ j \in N,\ i \neq j; \\ \alpha_i + \alpha_j + \alpha_k = 1500, & \forall\, i,\ j,\ k \in N,\ i \neq j \neq k. \end{cases}$$

Consequently, D-subset of the core for this dynamic 3-person prisoner's dilemma contains imputations like (430, 470, 600), (500, 500, 500) (430, 535, 535), etc.

4. The Shapley value of stochastic "n-person prisoner's dilemma"

Definition 4. The Shapley value for the Γ_f is called an imputation $\left(Sh_1^{\Gamma_f}, \ldots, Sh_n^{\Gamma_f}\right)$ of the payoff $V^{\Gamma_f}(N)$ such that

$$Sh_i^{\Gamma_f} = \sum_{S \subseteq N} \frac{(s-1)!\,(n-s)!}{n!} \left[V^{\Gamma_f}(S) - V^{\Gamma_f}(S \setminus \{i\})\right] \ (Shapley, 1953).$$

It is well-known that the Shapley value satisfies:

1. efficiensy:

$$\sum_{i \in N} Sh_i^{\Gamma_f}\left(V^{\Gamma_f}\right) = V^{\Gamma_f}(N);$$

2. symmetry: if players i and j are symmetric in accordance with V^{Γ_f}

$$Sh_i^{\Gamma_f}\left(V^{\Gamma_f}\right) = Sh_j^{\Gamma_f}\left(V^{\Gamma_f}\right);$$

3. additivity: for two games V^{Γ_f} and W^{Γ_f}

$$Sh_S^{\Gamma_f}\left(V^{\Gamma_f}\right) + Sh_S^{\Gamma_f}\left(W^{\Gamma_f}\right) = Sh_S^{\Gamma_f}\left(V^{\Gamma_f} + W^{\Gamma_f}\right);$$

4. null-player: if $V^{\Gamma_f}(S \cup \{i\}) - V^{\Gamma_f}(S) = 0,\ \forall S \subseteq N \setminus \{i\}$

$$Sh_i^{\Gamma_f}\left(V^{\Gamma_f}\right) = 0$$

Consider now the generalisation of the game Γ_f, where $V^{\gamma_j}(N)$ can be not equal in different stages. Define the values of the characteristic functions of each games $\gamma_j,\ j \in [1, f]$ for the coalition N:

$$V^{\gamma_j}(N) = \max_{x_1, \ldots, x_i, \ldots, x_n} \sum_{i \in N} H_i^{\gamma_j}(x_1, \ldots, x_i, \ldots, x_n).$$

Assume that S_j is a coalition $|S_j| = s_j$, which select the strategy D. This coalition we call deviating coalition. Then $|N \setminus S_j| = n - s_j$.

The maximum of the characteristic function for the game γ_j is achieved when the benefit from the deviation of each additional player

$$D_i^{\gamma_j}\left(x=n-(s+1)\right)-C_i^{\gamma_j}\left(x=n-s\right)= \\ =\left(a_2^{\gamma_j}\left(n-s-1\right)+b^{\gamma_j}2\right)-\left(a_1^{\gamma_j}\left(n-s\right)+b_1^{\gamma_j}\right)$$

is less than the amount of losses of all other players, including those who have already deviated

$$\sum_{i=1}^{n-s_j-1}\left(C_i^{\gamma_j}\left(x^*\right)-C_i^{\gamma_j}\left(x^{**}\right)\right)+\sum_{i=1}^{s_j}\left(D_i^{\gamma_j}\left(x^*\right)-D_i^{\gamma_j}\left(x^{**}\right)\right),$$

where $x^*=n-s_j$, and $x^{**}=n-s-1$.

We shall find the number of players in the deviated coalition s_j, which gives the maximum values of the characteristic functions for each of the f possible realizations of γ_j in Γ_f:

$$a_1^{\gamma_j}n-a_1^{\gamma_j}s_j-a_1^{\gamma_j}+a_2^{\gamma_j}s_j>a_2^{\gamma_j}n-a_2^{\gamma_j}s_j-a_2^{\gamma_j}+b_2^{\gamma_j}-a_1^{\gamma_j}n-a_1^{\gamma_j}s_j-b_1^{\gamma_j}.$$

Then, the number of diviating players is

$$s_j=\left[\frac{\left(2a_1^{\gamma_j}-a_2^{\gamma_j}\right)n+\left(a_2^{\gamma_j}-b_2^{\gamma_j}+b_1^{\gamma_j}-a_1^{\gamma_j}\right)}{\left(2a_1^{\gamma_j}-2a_2^{\gamma_j}\right)}\right],\forall\gamma_j\in\Gamma_f.$$

Therefore, the values of the characteristic functions on each stage of the game Γ_f we can just share as

$$V^{\gamma_j}\left(N\right)=\left(a_1^{\gamma_j}\left(n-s_j\right)+b_1^{\gamma_j}\right)\left(n-s_j\right)+\left(a_2^{\gamma_j}\left(n-s_j\right)+b_2^{\gamma_j}\right)s_j,$$

where $s_j=\left[\frac{\left(2a_1^{\gamma_j}-a_2^{\gamma_j}\right)n+\left(a_2^{\gamma_j}-b_2^{\gamma_j}+b_1^{\gamma_j}-a_1^{\gamma_j}\right)}{\left(2a_1^{\gamma_j}-2a_2^{\gamma_j}\right)}\right],\ j\in[1,f]$.

Due to the efficiency and the symmetry axioms we can just shared equally the expected value of the characteristic function of Γ_f for the grand coalition.

$$Sh_i\left(V^{\Gamma_f}\right)= \\ =K_f\sum_{j=1}^{f}\frac{\left(\left(a_1^{\gamma_j}-a_2^{\gamma_j}\right)s_j^2+\left(a_2^{\gamma_j}n+b_2^{\gamma_j}-a_1^{\gamma_j}n-b_1^{\gamma_j}\right)s_j+a_1^{\gamma_j}n^2+b_1^{\gamma_j}n\right)p_j}{n},$$

for $\forall i\in N$, $s_j=\left[\frac{\left(2a_1^{\gamma_j}-a_2^{\gamma_j}\right)n+\left(a_2^{\gamma_j}-b_2^{\gamma_j}+b_1^{\gamma_j}-a_1^{\gamma_j}\right)}{\left(2a_1^{\gamma_j}-2a_2^{\gamma_j}\right)}\right]$.

And furthermore, on the each stage of Γ_f, the Shapley value is equal to

$$Sh_i\left(V^{\gamma_j}\right)=\sum_{j=1}^{f}\frac{\left(\left(a_1^{\gamma_j}-a_2^{\gamma_j}\right)s_j^2+\left(a_2^{\gamma_j}n+b_2^{\gamma_j}-a_1^{\gamma_j}n-b_1^{\gamma_j}\right)s_j+a_1^{\gamma_j}n^2+b_1^{\gamma_j}n\right)p_j}{n},$$

for $\forall i \in N$, $s_j = \left[\frac{\left(2a_1^{\gamma_j} - a_2^{\gamma_j}\right)n + \left(a_2^{\gamma_j} - b_2^{\gamma_j} + b_1^{\gamma_j} - a_1^{\gamma_j}\right)}{\left(2a_1^{\gamma_j} - 2a_2^{\gamma_j}\right)}\right]$. It does not change during the transition from one stage of the game to the next, given that the probabilities of each possible games $(\gamma_1, \ldots, \gamma_f)$ remain at all stages. Accordingly, the Shapley value of this game is time-consistent and belongs to the D subset of the core.

5. Conclusion

The payoff function is constructed for the arbitrary number of players in the n-person prisoner's dilemma. It would be correct to say that this payoff function can be used for multystage dynamic game where n players participate as in the the standard "prisoners dilemma".

The purpose of this study is to define the time-consistent subset of the core. It is found the Petrosjan's characteristic function, which gives the possibility to find Petrosjan-Pankratova's subset of the core.

Moreover, the Shapley value of the stochastic version of the n'person prisoner's dilemma was constructed in accordance with obtained payoff function.

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