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Monotone Properties of Information Control in a Game with Uncertainty

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Abstract This study sets out to investigate the impact of information control. We used our previous reflexive analysis of a game to find the sensitivity of strategies and utility functions to increasing beliefs about thresholds. The game itself is constructed by using a normal form game and making suggestions on the agents's believes and knowledge weaker. We found domains of parameters where monotonicity of the impact holds too. Together, these results provide important insights into the impact of reflexive analysis on the properties of information control.

1. Introduction

Let's say there are a set of agents $N = \{1, \ldots, n\}$, a set real, non-negative strategies $X = \{X_1, \ldots, X_n\}$, and a set of utility functions $F(A) = \{f_1(A), ..., f_n(A)\}$ with a parameter A. One can consider a game

$$G =< N, X, F(A) > .$$

We have investigated a case when agents don't have consensus on the value of A. We used a formal grammar

$$\varphi = \bot \mid p \mid \neg\varphi \mid (\varphi \to \psi) \mid K_i\varphi \mid B_i\varphi \mid C_K\varphi \mid C_B\varphi,$$

where $\bot$ is False. Elementary propositions are elements of a set $P \in \{(A = x)|x \in R\}$. $\{K_i\varphi\}$ is a set of knowledge operators of agents that describes their knowledge about a value φ. $\{B_i\varphi\}$ is a set of belief operators of agents that describes their beliefs about a value ϕ. $C_K\varphi$ means that φ is common knowledge among agents in N. $C_B\varphi$ means that φ is a common belief among agents in N. We will write $G_I =< N, X, F(A), I >$ for a game $G =< N, X, F(A) >$ with a given logic assumptions or axioms of informational structure I. The ordinary case is $G =< N, X, F(A), C_K(A = A_0) >$, where A_0 is an actual value of a parameter A. It is just a game $G =< N, X, F(A_0) >$ in a normal form. Note that Nash equilibria for $G_{C_K} =< N, X, F(A), C_K(A = A_0) >$ and $G_{C_B} =< N, X, F(A), C_B(A = A_0) >$ coincides though resulting values of utility functions could differ since $C_K(A = A_0) \to (A = A0)$ but there is no such theorem for $C_B(A = A_0)$. A belief could be false even if it is a common belief.

There is a well-known way to investigate this game using Nash equilibria. Each Nash equilibrium is a vector $y = (y_1, ..., y_n)$ such that $\forall x_i \in X_i$

$$f_i(y_1, ..., y_{i-1}, y_i, y_{i+1}, ..., y_n) \geq f_i(y_1, ..., y_{i-1}, x_i, y_{i+1}, ..., y_n)$$

We denote $G_\phi =< N, X, F(A), \varphi >$ e.g. $G_{\forall i B_i(A=A_i)} =< N, X, F(A), \forall i B_i(A = A_i) >$

We knew the equilibria for some games (Fedyanin, 2019). There were functional dependencies strategies and utility on beliefs about the parameter A. So we found the intervals of monotonicity using derivatives of the functional dependencies. Some expressions were very obvious or easy to find but others were very difficult for analysis. We expressed our results in several theorems.

2. An example of game

We will continue investigations of a game of collective actions (Fedyanin and Chkhartishvili, 2011). There are a set of agents $N = \{1, \ldots, n\}$, a set of real not negative strategies and a set of utility functions

$$f_i(x_1, ..., x_n, r_1, ..., r_n, A_1, ...A_n) = x_i \left(\sum_{j \in N} x_j - A \right) - \frac{x_i^2}{r_i}, \forall i \in N$$

where $0 < r_i < 1$.

The corresponding practical interpretation lies in that the agents apply the strategies and it appears successful (provides a positive contribution to the utility functions of the agents) when the total effort exceeds a specific threshold; the latter is set equal to 1. With the strategy being successful, the agent's gain(the first term in utility function) increases with the increasing effort of the agent. On the other hand, the agent's effort itself results in a negative contribution to the utility function (see the second term) which depends on the type r_i. The larger the type of variable, the "easier" the agent applies the strategy (for instance, in a psychological sense, it could be explained by the agent's greater loyalty or liking for the joint action) (Fedyanin and Chkhartishvili, 2011).

The Cournot oligopoly model (Cournot, 1960) looks similar but it is not the same because of different utility functions

$$f_i = x_i \left(A - \sum_{j \in N} x_j \right) - \frac{x_i^2}{r_i}.$$

The corresponding practical interpretation of the Cournot oligopoly is the following: strategies are the amounts of sold products, utility functions are the amounts of products multiplied by a price that decreases when the total amount of sold products increases minus costs.

There are some important differences that make the game of collective actions look like a combination of the Cournot oligopoly and the game theoretical modification of Granovetter (Granovetter, 1978) and not just the Cournot oligopoly. The Breer Threshold model (Breer et al., 2017) is the one where utility functions are

$$f_i = x_i \left(A - \sum_{j \in N} x_j \right)$$

and a set of strategies is restricted to binary values - strategy is equal either 0 or 1. Anyway we can apply all ideas below for the Cournot oligopoly as well but we haven't applied them yet.

In this paper we propose to consider A as an uncertain parameter for agents and they have to make some suggestion about it.

3. Results

3.1. Players with common knowledge

We can model it by a game

$$G_{C_K} =< N, X, F(A), C_K(A = A_0) >$$

There is a well-known way to find the Nash equilibrium. It is to compose and solve a system of equations where the strategy of each player equals his or her best response

$$x_i = BR_i(x - i) = \frac{2r_i}{1 - ri}\left(\sum_{j \neq i} x_j - A\right) + \epsilon_i, \forall i \in N$$

where

$$\epsilon_i = \begin{cases} 0, & \frac{2r_i}{1-ri}\left(\sum_{j \neq i} x_j - A\right) \geq 0 \\ -\frac{2r_i}{1-ri}\left(\sum_{j \neq i} x_j - A\right), & \frac{2r_i}{1-ri}\left(\sum_{j \neq i} x_j - A\right) < 0 \end{cases}$$

Zero Nash Equilibrium for G_{C_K}

Theorem 1. *An existance of a zero Nash equilibrium doesn't depend on the value of the threshold in the game G_{C_K}.*

Proof. There is always a solution in the game G_{C_K}. Actions of agents.

$$x_i = 0, \forall i \in N$$

Values of agents' utilities.

$$f_i = 0, \forall i \in N$$

□

Nonzero Nash Equilibrium for G_{C_K} If

$$\sum_{j \in N} \frac{r_j}{2 - r_j} > 1$$

then there is one more solution.

Theorem 2. *Let*

$$\sum_{j \in N} \frac{r_j}{2 - r_j} > 1.$$

The larger threshold the larger strategy of an agent in the game G_{C_K}.

Proof. Strategies of agents.

$$x_i = \frac{A\dfrac{r_i}{2 - r_i}}{\displaystyle\sum_{j \in N} \frac{r_j}{2 - r_j} - 1}, \forall i \in N,$$

Derivative.

$$\frac{\partial x_i}{\partial A} = \frac{\dfrac{r_i}{2 - r_i}}{\displaystyle\sum_{j \in N} \frac{r_j}{2 - r_j} - 1} > 0, \forall i \in N,$$

□

If

$$\sum_{j \in N} \frac{r_j}{2 - r_j} > 1$$

then there is one more solution

Theorem 3. *Let*

$$\sum_{j \in N} \frac{r_j}{2 - r_j} > 1.$$

The larger threshold the larger value of utility functions of agent in the game G_{C_K}.

Proof. Values of agents' utilities.

$$f_i = \frac{A^2 (1 - r_i) r_i}{\left(\sum_{j \in N} \frac{r_j}{2 - r_j} - 1 \right)^2 (2 - r_i)^2}, \forall i \in N$$

Derivative.

$$\frac{\partial f_i}{\partial A} = \frac{2A(1 - r_i) r_i}{\left(\sum_{j \in N} \frac{r_j}{2 - r_j} - 1 \right)^2 (2 - r_i)^2} \geq 0, \forall i \in N$$

□

Thus if one wants to increase the utility of agents, they should increase the plans that these agents should try to exceed. This conclusion looks very reasonable, regardless of the model.

3.2. Players with communication and consensus

We can model this case by a game

$$G_{C_B} =< N, X, F(A), C_B(A = A_0) >$$

There could be a communication between agents and they can communicate according the de Groot model (DeGroot, 1974). There is no difference if an existence of such communication to the common knowledge among all agents or it is not. Let their influences be w_j then one should compose and solve the system

$$x_i = BR_i(x_{-i}) = \frac{2r_i}{1 - r_i} \left(\sum_{i \neq j} x_j - \sum_j w_j A_i \right)$$

for each i.

Zero Nash Equilibrium for G_{C_B} There is always a zero solution in the game.

Theorem 4. *An existance of a zero Nash equilibrium doesn't depend on value of the threshold in the game* G_{C_B}.

Proof. Strategies of agents.

$$x_i = 0, \forall i \in N$$

Values of agents' utilities.

$$f_i = 0, \forall i \in N$$

□

Nonzero Nash Equilibrium for G_{C_B} If

$$\sum_{j \in N} \frac{r_j}{2 - r_j} > 1$$

then there is one more solution

Theorem 5. *Let*

$$\sum_{j \in N} \frac{r_j}{2 - r_j} > 1.$$

If an agent in the game G_{C_B} has a nonzero influence on consensus opinion, then the larger his belief about the threshold, the larger the strategies of all agents. The true value of the threshold doesn't affect the strategies in the G_{C_B}.

Proof. Strategies of agents.

$$x_i = \frac{\sum_{j \in N} w_j A_j \frac{r_i}{2 - r_i}}{\sum_{j \in N} \frac{r_j}{1 - r_j} - 1}, \forall i \in N$$

Derivative.

$$\frac{\partial x_i}{\partial A_j} = \frac{w_j \frac{r_i}{2 - r_i}}{\sum_{j \in N} \frac{r_j}{1 - r_j} - 1} \geq 0, \forall i \in N$$

$$\frac{\partial x_i}{\partial A_i} = \frac{w_i \frac{r_i}{2 - r_i}}{\sum_{j \in N} \frac{r_j}{1 - r_j} - 1} \geq 0, \forall i \in N$$

$$\frac{\partial x_i}{\partial A} = 0, \forall i \in N$$

□

If

$$\sum_{j \in N} \frac{r_j}{2 - r_j} > 1$$

then there is one more solution

Theorem 6. *Let*

$$\sum_{j \in N} \frac{r_j}{2 - r_j} > 1.$$

If an agent has a nonzero influence on consensus opinion, then the larger his belief about the threshold, the larger the utility for each agent. The true value of the threshold doesn't affect the utilities for agents.

Proof. Values of agents' utilities.

$$f_i = \frac{\left(\sum_{j\in N} w_j A_j\right)^2 (1-r_i)r_i}{\left(\sum_{j\in N} \frac{r_j}{2-r_j} - 1\right)^2 (2-r_i)^2}, \forall i \in N$$

Derivative.

$$\frac{\partial f_i}{\partial A_j} = \frac{w_j \sum_{j\in N} w_j A_j (1-r_i)r_i}{\left(\sum_{j\in N} \frac{r_j}{2-r_j} - 1\right)^2 (2-r_i)^2} \geq 0, \forall i \in N, j \neq i$$

$$\frac{\partial f_i}{\partial A_i} = \frac{w_i \sum_{j\in N} w_j A_j (1-r_i)r_i}{\left(\sum_{j\in N} \frac{r_j}{2-r_j} - 1\right)^2 (2-r_i)^2} \geq 0, \forall i \in N$$

$$\frac{\partial f_i}{\partial A} = 0, \forall i \in N$$

□

3.3. Players without communication

We can model this case by games

$$G_{\forall i B_i C_B(A=A_i)} =< N, X, F(A), \forall i B_i C_B(A = A_i) >$$

$$G_{\forall i B_i C_K(A=A_i)} =< N, X, F(A), \forall i B_i C_K(A = A_i) >$$

We formulated axioms to make an informational system complete.

$$G_{\forall i B_i C_B(A=A_i)\wedge B_i(A=A_i)} =< N, X, F(A), \forall i (B_i C_B(A = A_i) \wedge B_i(A = A_i)) >$$

$$G_{\forall i B_i C_K(A=A_i)\wedge B_i(A=A_i)} =< N, X, F(A), \forall i (B_i C_K(A = A_i) \wedge B_i(A = A_i)) >$$

Player i could believe that all utility functions are

$$f_i = x_i(\sum_{j\in N} x_j - A_i) - x_i^2/r_i.$$

It coincides with the Nash equilibrium with a certain value of parameter A, if there is $A = A_i$ for any i common knowledge that $A = A_i$.

The strategy of each player which equals to their best response that are

$$x_i = BR_i(x_{-i}) = \frac{2r_i}{1-r_i}(\sum_{j\neq i} x_j - A_i).$$

Agent i makes a best response for all other agents according to their beliefs. Thus, from the i-th player's point of view it looks like they should compose and solve the system for the following best responses

$$x_j = BR_i(x_{-i}) = \frac{2r_j}{1-r_j}(\sum_{k\neq j} x_k - A_i)$$

for each j.

Zero Nash Equilibrium for $G_{\forall i B_i C_B(A=A_i)\wedge B_i(A=A_i)}$ There is always a solution.

Theorem 7. *An existance of a zero Nash equilibrium doesn't depend on value of the threshold in the game* $G_{\forall i B_i C_K(A=A_i)\wedge B_i(A=A_i)}$.

Proof. Strategies of agents.

$$x_i = 0, \forall i \in N$$

Values of agents' utilities.

$$f_i = 0, \forall i \in N$$

□

Nonzero Nash Equilibrium for $G_{\forall i B_i C_B(A=A_i)\wedge B_i(A=A_i)}$ If

$$\sum_{j\in N} \frac{r_j}{2-r_j} > 1$$

then there is one more solution.

Theorem 8. *Let*

$$\sum_{j\in N} \frac{r_j}{2-r_j} > 1.$$

One can change a strategy of an agent in the game $G_{\forall i B_i C_K(A=A_i)\wedge B_i(A=A_i)} =< N, X, F(A), \forall i(B_i C_K(A = A_i) \wedge B_i(A = A_i))$ *if and only if she change his belief about the threshold. In this case the larger belief about threshold will lead to the larger strategy.*

Proof. Strategies of agents.

$$x_i = \frac{A_i \frac{r_i}{2-r_i}}{\sum_{j\in N} \frac{r_j}{2-r_j} - 1}, \forall i \in N$$

Derivative.

$$\frac{\partial x_i}{\partial A_i} = \frac{\frac{r_i}{2-r_i}}{\sum_{j\in N} \frac{r_j}{2-r_j} - 1} > 0, \forall i \in N,$$

$$\frac{\partial x_i}{\partial A_j} = 0, \forall i \in N, j \neq i.$$

$$\frac{\partial x_i}{\partial A} = 0, \forall i \in N$$

□

Thus one cannot change a strategy of an agent if she doesn't change his own belief about A.

Theorem 9. *Let*

$$\sum_{j\in N}\frac{r_j}{2-r_j}>1.$$

The utility of an agent in the game $G_{\forall i B_i C_B(A=A_i)\wedge B_i(A=A_i)}$ *will increase when his belief* A_i *will increase if and only if this statement holds*

$$2A_i\frac{1-r_i}{2-r_i}<\sum_{j\neq i}A_j\frac{r_j}{2-r_j}-A\left(\sum_{j\in N}\frac{r_j}{2-r_j}-1\right).$$

Proof. Values of agents' utilities.

$$f_i=\frac{\frac{r_i}{2-r_i}}{\left(\sum\limits_{j\in N}\frac{r_i}{2-r_i}-1\right)^2}\left(A_i\sum_{j\in N}A_j\frac{r_j}{2-r_j}-A_iA\left(\sum_{j\in N}\frac{r_i}{2-r_i}-1\right)-A_i^2\frac{1}{2-r_i}\right),$$

$$\forall i\in N$$

Derivative

$$\frac{\partial f_i}{\partial A}=\frac{A_i\frac{r_i}{2-r_i}}{\left(\sum\limits_{j\in N}\frac{r_j}{2-r_j}-1\right)}>0,\forall i\in N\,j\neq i$$

$$\frac{\partial f_i}{\partial A_i}=A_iA_j\frac{r_ir_j}{(2-r_i)(2-r_j)}\left(\sum_{j\in N}\frac{r_j}{2-r_j}-1\right)^{-2}>0,\forall i\in N\,j\neq i$$

$$\frac{\partial f_i}{\partial A_i}=\frac{\frac{r_i}{2-r_i}}{\left(\sum\limits_{j\in N}\frac{r_j}{2-r_j}-1\right)^2}\times$$

$$\times\left(A_i\frac{r_i}{2-r_i}+\sum_{j\in N}A_j\frac{r_j}{2-r_j}-A\left(\sum_{j\in N}\frac{r_j}{2-r_j}-1\right)-2A_i\frac{1}{2-r_i}\right)=$$

$$=\frac{\frac{r_i}{2-r_i}}{\left(\sum\limits_{j\in N}\frac{r_j}{2-r_j}-1\right)^2}\left(\sum_{j\in N}A_j\frac{r_j}{2-r_j}-A\left(\sum_{j\in N}\frac{r_j}{2-r_j}-1\right)-A_i\right).$$

□

It means that sometimes we may need a combined information control - no separately chosen beliefs about thresholds.

One can get some details from this theorem.

Theorem 10. *Let*

$$\sum_{j\in N}\frac{r_j}{2-r_j}>1.$$

If this statement

$$\sum_{j\in N}\frac{r_j}{2-r_j}(A_j-A)>0,\forall i\in N$$

doesn't hold then the utility of all agents in the game $G_{\forall iB_iC_B(A=A_i)\wedge B_i(A=A_i)}$ won't increase at the same time when all their beliefs A_i increase.

Proof. Let's list all inequalities

$$\sum_{j\in N}A_j\frac{r_j}{2-r_j}-A\left(\sum_{j\in N}\frac{r_j}{2-r_j}-1\right)-A_i>0,\forall i\in N$$

$$\frac{r_i}{2-r_i}\sum_{j\in N}A_j\frac{r_j}{2-r_j}-A\frac{r_i}{2-r_i}\left(\sum_{j\in N}\frac{r_j}{2-r_j}-1\right)-A_i\frac{r_i}{2-r_i}>0,\forall i\in N$$

$$\sum_{j\in N}\frac{r_j}{2-r_j}\sum_{m\in N}A_m\frac{r_m}{2-r_m}-A\sum_{j\in N}\frac{r_j}{2-r_j}\left(\sum_{j\in N}\frac{r_j}{2-r_j}-1\right)-\sum_{j\in N}A_j\frac{r_j}{2-r_j}>0,$$
$$\forall i\in N$$

$$\left(\sum_{j\in N}\frac{r_j}{2-r_j}-1\right)\left(\sum_{j\in N}A_j\frac{r_j}{2-r_j}-A\sum_{j\in N}\frac{r_j}{2-r_j}\right)>0,\forall i\in N$$

$$\left(\sum_{j\in N}\frac{r_j}{2-r_j}-1\right)\sum_{j\in N}\frac{r_j}{2-r_j}(A_j-A)>0,\forall i\in N$$

$$\sum_{j\in N}\frac{r_j}{2-r_j}(A_j-A)>0,\forall i\in N$$

□

3.4. Stubborn players with communication without consensus

We can model this case by games

$$G_{B_iC_B(A=A_i)}=<N,X,F(A),\forall iB_iC_B(A=A_i)>$$

$$G_{B_iC_K(A=A_i)}=<N,X,F(A),\forall iB_iC_K(A=A_i)>$$

We formulated axioms to make an informational system complete.

$$G_{B_iC_B(A=A_i)\wedge B_i(A=A_i))}=<N,X,F(A),\forall i(B_iC_B(A=A_i)\wedge B_i(A=A_i))>$$

$$G_{B_iC_K(A=A_i)\wedge B_i(A=A_i)}=<N,X,F(A),\forall i(B_iC_K(A=A_i)\wedge B_i(A=A_i))>$$

If there is a communication with no trust at all then all agents 'become stubborn" and other opinions donвЂ™t change their opinions. There is no difference if an existence of such communication is a common knowledge among all agents or it is not. The important information is that A_i is a common knowledge and that all agents are stubborn in our sense. Thus, from the i player's point of view, they should compose and solve the system for the following best responses

$$x_j = BR_i(x_{-i}) = \frac{2r_i}{1-r_i}\left(\sum_{j\neq i} x_j - A_i\right)$$

for each i.

Zero Nash Equilibrium for $G_{\forall i(B_iC_K(A=A_i)\wedge B_i(A=A_i))}$ There is always a solution for $G_{B_iC_K(A=A_i)\wedge B_i(A=A_i)}$ and $G_{B_iC_B(A=A_i)\wedge B_i(A=A_i))}$.

Theorem 11. *An existance of a zero Nash equilibrium doesn't depend on value of the threshold in the games $G_{B_iC_K(A=A_i)\wedge B_i(A=A_i)}$ and $G_{B_iC_B(A=A_i)\wedge B_i(A=A_i))}$.*

Proof. Strategies of agents.

$$x_i = 0, \forall i \in N$$

Values of agents' utilities.

$$f_i = 0, \forall i \in N$$

□

Nonzero Nash Equilibrium for $G_{\forall i(B_iC_K(A=A_i)\wedge B_i(A=A_i))}$ If

$$\sum_{j\in N}\frac{r_j}{2-r_j} > 1$$

then there is one more solution.

Theorem 12. *Let*

$$\sum_{j\in N}\frac{r_j}{2-r_j} > 1.$$

Then there is

$$\frac{\partial x_i}{\partial A_j} > 0; \forall i \in N j \neq i,$$

$$\frac{\partial x_i}{\partial A_i} > 0$$

in the game $G_{\forall i(B_iC_K(A=A_i)\wedge B_i(A=A_i))}$ if and only if

$$A_i < 1, \forall i \in N.$$

Proof. Strategies of agents.

$$x_i = \frac{\frac{r_i}{2-r_i}}{\sum_{j\in N}\frac{r_j}{2-r_j} - 1}\left(A_i + \sum_{j\in N}\frac{r_j}{2-r_j}\left(A_j - A_i\right)\right), \forall i \in N$$

Derivatives.

$$\frac{\partial x_i}{\partial A_j} = \frac{\frac{r_i}{2-r_i}\sum_{j\neq i}\frac{r_j}{2-r_j}}{\sum_{j\in N}\frac{r_j}{2-r_j}-1} > 0, \forall i \in N, \forall j \neq i$$

$$\frac{\partial x_i}{\partial A_i} = \frac{\frac{r_i}{2-r_i}}{\sum_{j\in N}\frac{r_j}{2-r_j}-1}\left(1 - A_i\sum_{j\in N}\frac{r_j}{2-r_j}\right), \forall i \in N$$

$$\frac{1}{A_i} > \sum_{j\in N}\frac{r_j}{2-r_j} > 1,$$

$$\frac{\partial x_i}{\partial A} = 0, \forall i \in N$$

□

Theorem 13. *Let*

$$\sum_{j\in N}\frac{r_j}{2-r_j} > 1.$$

The larger the threshold A the smaller the value of the utility function of each agent in the game $G_{\forall i(B_i C_K(A=A_i)\wedge B_i(A=A_i))}$

Proof. Let's find an universal expression for derivatives of utility functions.

If

$$\frac{\partial A}{\partial y} = 1$$

then

$$\frac{\partial f_i}{\partial A} = -x_i - \frac{\partial x_i}{\partial A}A.$$

Thus

$$\frac{\partial f_i}{\partial A} = -x_i = -\frac{\frac{r_i}{2-r_i}}{\sum_{j\in N}\frac{r_j}{2-r_j}-1}\left(A_i + \sum_{j\in N}\frac{r_j}{2-r_j}(A_j - A_i)\right) \leq 0, \forall i \in N$$

□

Values of agents' utilities

$$f_i(x_1, ..., x_n, r_1, ..., r_n, A_1, ...A_n) =$$

$$= \frac{\frac{r_i}{2-r_i}}{\left(\sum_{j\in N}\frac{r_j}{2-r_j}-1\right)^2}\left(A_i + \sum_{j\in N}\frac{r_j}{2-r_j}(A_j - A_i)\right)\times$$

$$\left(\sum_{j\in N}\frac{A_j r_j}{2-r_j}\left(\sum_{m\in N}\frac{r_m}{2-r_m}-1\right)+n\sum_{j\in N}\frac{A_j r_j}{2-r_j}-A\right)\times$$

$$\times\frac{\frac{r_i^2}{(2-r_i)^2}}{\left(\sum_{j\in N}\frac{r_j}{2-r_j}-1\right)^2 r_i}\left(A_i+\sum_{j\in N}\frac{r_j}{2-r_j}(A_j-A_i)\right)^2=$$

$$=\frac{\frac{r_i}{2-r_i}}{\left(\sum_{j\in N}\frac{r_j}{2-r_j}-1\right)^2}\left(A_i+\sum_{j\in N}\frac{r_j}{2-r_j}(A_j-A_i)\right)\times$$

$$\left(\sum_{j\in N}\frac{A_j r_j}{2-r_j}\left(\sum_{j\in N}\frac{r_j}{2-r_j}+n-1\right)-A\left(\sum_{j\in N}\frac{r_j}{2-r_j}-1\right)^2-\right.$$

$$\left.-\frac{1}{2-r_i}\left(A_i+\sum_{j\in N}\frac{r_j}{2-r_j}(A_j-A_i)\right)^2\right),\forall i\in N$$

Theorem 14. *Let*

$$\sum_{j\in N}\frac{r_j}{2-r_j}>1.$$

If there are

$$\frac{\partial x_i}{\partial y}>0$$

and

$$x_i<r_i\frac{2-A}{2}$$

in the game $G_{\forall i(B_iC_K(A=A_i)\wedge B_i(A=A_i))}$ *then*

$$\frac{\partial f_i}{\partial y}>0.$$

Proof. Since $x_i\geq 0$ then

$$\frac{\partial f_i}{\partial A}\leq 0.$$

If

$$\frac{\partial A}{\partial y}=0$$

then

$$\frac{\partial f_i}{\partial y}=\frac{\partial x_i}{\partial y}+\sum_{j\in N}\frac{\partial x_j}{\partial y}-A\frac{\partial x_i}{\partial y}-2\frac{\partial x_i}{\partial y}*\frac{x_i}{r_i},$$

$$\frac{\partial f_i}{\partial y} = 2\frac{\partial x_i}{\partial y}\left(1 - \frac{A}{2} - \frac{x_i}{r_i}\right) + \sum_{j\neq i}\frac{\partial x_j}{\partial y},$$

□

Theorem 15. *Let*

$$\sum_{j\in N}\frac{r_j}{2-r_j} > 1.$$

Then

$$\frac{\partial f_i}{\partial A_j} =$$

$$2\frac{\frac{r_i}{2-r_i}\sum_{j\neq i}\frac{r_j}{2-r_j}}{\sum_{j\in N}\frac{r_j}{2-r_j} - 1}\left(1 - \frac{A}{2} - \frac{1}{r_i}\frac{\frac{r_i}{2-r_i}}{\sum_{j\in N}\frac{r_j}{2-r_j} - 1}\left(A_i + \sum_{j\in N}\frac{r_j}{2-r_j}(A_j - A_i)\right)\right) +$$

$$+\sum_{k\neq i,j}\frac{\frac{r_k}{2-r_k}\sum_{m\neq k}\frac{r_m}{2-r_m}}{\sum_{m\in N}\frac{r_m}{2-r_m} - 1} + \frac{\frac{r_j}{2-r_j}}{\sum_{m\in N}\frac{r_m}{2-r_m} - 1}\left(1 - A_k\sum_{m\in N}\frac{r_m}{2-r_m}\right),$$

in the game $G_{\forall i(B_iC_K(A=A_i)\wedge B_i(A=A_i))}$.

Theorem 16. *Let*

$$\sum_{j\in N}\frac{r_j}{2-r_j} > 1.$$

Then there is

$$\frac{\partial f_i}{\partial A_i} = 2\frac{\frac{r_i}{2-r_i}}{\sum_{j\in N}\frac{r_j}{2-r_j} - 1}\left(1 - A_i\sum_{j\in N}\frac{r_j}{2-r_j}\right)\times$$

$$\times\left(1 - \frac{A}{2} - \frac{1}{r_i}\frac{\frac{r_i}{2-r_i}}{\sum_{j\in N}\frac{r_j}{2-r_j} - 1}\left(A_i + \sum_{j\in N}\frac{r_j}{2-r_j}(A_j - A_i)\right)\right) +$$

$$+\sum_{j\neq i}\frac{\frac{r_j}{2-r_j}\sum_{m\neq j}\frac{r_m}{2-r_m}}{\sum_{m\in N}\frac{r_m}{2-r_m} - 1}, \quad \forall i \in N, \forall j \neq i$$

in the game $G_{\forall i(B_iC_K(A=A_i)\wedge B_i(A=A_i))}$.

4. Conclusion

In this paper we considered a game with a parameter A and suggested that this is an uncertain parameter for agents and they have to make some suggestion about it. We made a table where listed behavior of derivative of strategies and utilities of agents. Having this table one can predict the reaction of a system and thus choose appropriate informational control.

Table 1. Please write your table caption here

Control	Game	Strategy x_i	Utility f_i
$\partial/\partial A$	Players with common knowledge G_{C_K}	≥ 0	≥ 0
	Players with communication and consensus G_{C_B}	$= 0$	$= 0$
	Players without communication $G_{\forall i B_i C_B(A=A_i)\wedge B_i(A=A_i)}$	$= 0$	≥ 0
	Stubborn players with communication $G_{\forall i(B_i C_K(A=A_i)\wedge B_i(A=A_i))}$	$= 0$	≤ 0
$\partial/\partial A_j$	Players with common knowledge G_{C_K}	NA	NA
	Players with communication and consensus G_{C_B}	≥ 0	≥ 0
	Players without communication $G_{\forall i B_i C_B(A=A_i)\wedge B_i(A=A_i)}$	≥ 0	?
	Stubborn players with communication $G_{\forall i(B_i C_K(A=A_i)\wedge B_i(A=A_i))}$	?	??
$\partial/\partial A_i$	Players with common knowledge G_{C_K}	NA	NA
	Players with communication and consensus G_{C_B}	≥ 0	≥ 0
	Players without communication $G_{\forall i B_i C_B(A=A_i)\wedge B_i(A=A_i)}$	≥ 0	≥ 0
	Stubborn players with communication $G_{\forall i(B_i C_K(A=A_i)\wedge B_i(A=A_i))}$	≥ 0	??

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